# Package 'tensorMQR1'

January 6, 2020

Type Package

Title tensorMQR1
Version 0.1.0
<b>Date</b> 2019-09-01
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<b>Description</b> Symmetric tensor estimation for multiresponse quadratic regression. The number of predictors can be diverged as sample size increases, in which the penalty LASSO, MCP or SCAD can be used.
License GPL (>= 2)
<b>Imports</b> Rcpp (>= 0.11.15), RcppEigen (>= 0.3.2.3.0)
LinkingTo Rcpp, RcppEigen
RoxygenNote 6.0.1
NeedsCompilation yes
Repository github
<pre>URL https://github.com/xliusufe/tensorMQR1</pre>
Encoding UTF-8
R topics documented:
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tensorMOR1-package	Symmetric Tensor Estimation for Quadratic Regression.
terisorrigiti package	Symmetric Tensor Estimation for Quadratic Regression.

## Description

For a high-dimensional multiresponse quadratic regression (MQR) with or without aparsity assumptions, treating the coefficients as a third-order tensor and borrowing Tucker decomposition to reduce the number of parameters. The multivariate sparse group lasso (mcp or scad) and the steepest gradient descent algorithm are used to estimate tensor for sparsity situation.

#### **Details**

This section should provide a more detailed overview of how to use the package, including the most important functions.

#### Author(s)

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#### References

Symmetric Tensor Estimation for Quadratic Regression.

#### **Description**

Generate data for a high-dimensional multiresponse quadratic regression, with or without aparsity assumptions.

#### Usage

```
generateData(n, q, s, p, D3,SigmaX=diag(p-1),sigma2=0.2,seed_id, t=0.0, rho=0.0)
```

#### **Arguments**

n	Sample size.
q	The number of responses, $q \ge 1$ .
S	The true covariates associating to response, $s \ge 1$ .
р	The number of covariates, $p \ge 1$ .
D3	The mode of unfolding $D_{(3)}$ .
SigmaX	Covariance of $X$ . Default is identity matrix.
sigma2	err variance. Default is 0.1.
seed_id	Seed of generator.
rho	The correlation of $\epsilon_i$ and $\epsilon_k$ , where $j, k \in \{1, \dots, q\}$ .

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#### **Details**

This function gives qp(p+1)/2 coefficients' estimators of MQR. The core tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC or CV.

#### Value

```
Y Response, a n \times q-matrix.

X Design matrix, a n \times p-matrix.
```

#### References

Symmetric Tensor Estimation for Quadratic Regression.

#### See Also

mam\_sparse

### **Examples**

```
# Example 1
D3 <- matrix(runif(72, 0.7, 1), 2, 36)
mydata <- generateData(200, 2, 6, 6, D3)</pre>
Y <- mydata$Y
X <- mydata$X
# Example 2
n <- 500
p <- 10
q <- 10
s <- 7
s0 <- s
r10=r20=r30=2
S3 \leftarrow matrix(runif(r10*r20*r30,3,7),nrow = r30)
T1 <- matrix(rnorm(s0*r10), nrow = s0)
U1 <- qr.Q(qr(T1))
T1 <- matrix(rnorm(q*r30), nrow = q)
U3 \leftarrow qr.Q(qr(T1))
D3 <- U3%*%S3%*%t(kronecker(U1,U1))
mydata <- generateData(n,q,s0,p,D3)</pre>
```

mqr

Fit MQR without sparsity assumption and with fixed ranks.

## Description

Fit a low-dimensional multiresponse quadratic regression without aparsity assumptions and with given  $r_1, r_2, r_3$ . The steepest gradient descent algorithm is used to estimate tensor.

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#### Usage

```
mqr(Y, X, r1 = NULL, r3 = NULL, SUV = NULL, isSym = TRUE,

eps = 1e-6, max\_step = 20, max\_step1=20)
```

## **Arguments**

Υ	A $n \times q$ numeric matrix of responses.
Χ	A $n \times p$ numeric design matrix for the model.
r1	The first dimension of single value matrix of the tensor. Default is 2.
r3	The third dimension of single value matrix of the tensor. Default is 2.
SUV	A user-specified list of initial coefficient matrix of $S,U,V.$ By default, initial matrices are provided randomly.
isSym	A logical value indicating whether restrict tensor to be symmetric. If isSym is TRUE (the default), the core tensor is symmetric, and both $U$ and $V$ belong to Stiefel manifold. If isSym is FALSE, we decompose the tensor to $S,A,B,C,$ where the tensor is treated to be asymmetric.
eps	Convergence threshhold. The algorithm iterates until the loss function change in any coefficient is less than eps. Default is 1e-6.
maxstep	The maximum iterates number of the steepest gradient descent method. Default is 20.
max_step1	Maximum number of outer iterations. Default is 20.

#### **Details**

This function gives qp(p+1)/2 coefficients' estimators of MQR. The core tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We fixed  $r_1$ ,  $r_2$  and  $r_3$  in the function mqr, but one can choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC or CV. See details in function mqr\_bic or mqr\_cv.

## Value

Dnew	Estimator of $D_{(3)}$ .
rss	Residual sum of squares (RSS).
Υ	Response $Y$ .
Χ	Design matrix $X$ .

#### References

Symmetric Tensor Estimation for Quadratic Regression.

## See Also

```
mqr_sparse, mqr_bic, mqr_cv
```

```
D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2 mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr(mydata$Y, mydata$X, r1=4, r3= 2)

D3hat <- fit$Dnew
```

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mqr_dr	Fit MQR without sparsity assumption, and with ranks selected by BIC or CV.

## Description

Fit a low-dimensional multiresponse quadratic regression without aparsity assumptions and with ranks  $r_1, r_3$  selected by BIC (the default), AIC, EBIC, CV, or GCV..

## Usage

## Arguments

Υ	A $n \times q$ numeric matrix of responses.
X	A $n \times p$ numeric design matrix for the model.
r1_index	A user-specified sequence of $r_1$ values, where $r_1$ is the first dimension of single value matrix of the tensor. Default is r1_index= $1, \cdots, \min(\log(n) \rceil, p)$ .
r3_index	A user-specified sequence of $r_3$ values, where $r_3$ is the third dimension of single value matrix of the tensor. Default is $r3\_index = 1, \cdots, \min(\log(n)\rceil, q)$ .
method	The method to be applied to select parameters. Either BIC (the default), AIC, EBIC, ${\sf CV}$ , or ${\sf GCV}$ .
ncv	The number of cross-validation folds. Default is $10$ . If method is not "CV", ncv is useless.
SUV	A user-specified list of initial coefficient matrix of $S,U,V,$ which is a list with values $S,U,V.$ By default, initial matrices are provided randomly.
isSym	A logical value indicating whether restrict tensor to be symmetric. If isSym is TRUE (the default), the core tensor is symmetric, and both $U$ and $V$ belong to Stiefel manifold. If isSym is FALSE, we decompose the tensor to $S$ , $A$ , $B$ , $C$ , where the tensor is treated to be asymmetric.
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-6.
maxstep	The maximum iterates number of the steepest gradient descent method. Default is 20.
max_step1	Maximum number of outer iterations. Default is 20.

## **Details**

This function gives qp(p+1)/2 coefficients' estimators of MQR. The core tensor is a  $r_1 \times r_2 \times r_3$ -tensor. We choose  $r_1$ ,  $r_2$  and  $r_3$  by BIC or CV.

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#### Value

Dnew	Estimator of $D_{(3)}$ .
rss	Residual sum of squares (RSS).
rk_opt	The optimal parametres that slected by BIC or CV. It is a vector with length 4, which are selected $r_1$ and $r_3$ .
selected	Which $\lambda$ is selection.
Υ	Response $Y$ .
Χ	Design matrix $X$ .

#### References

Symmetric Tensor Estimation for Quadratic Regression.

#### See Also

```
mqr, mqr_sparse_dr
```

#### **Examples**

```
D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2 mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr_dr(mydata$Y, mydata$X)
D3hat <- fit$Dnew
opt <- fit$rk_opt
```

mqr\_sparse

Fit MQR with sparsity assumption and fixed ranks.

## Description

Fit a high-dimensional multiresponse quadratic regression with or without aparsity assumptions, and given ranks given ranks  $r_1, r_2, r_3$ . The multivariate sparse group lasso (mcp or scad) and the steepest gradient descent algorithm are used to estimate tensor for sparsity situation. The tuning parameter is selected by BIC (the default), AIC, EBIC, CV, or GCV.

## Usage

```
mqr_sparse(Y, X, r1 = NULL, r3 = NULL, method = "BIC", ncv = 10, isPenU = 0,
    isPenColumn = 1, penalty = "LASSO", lambda = NULL, SUV = NULL,
    nlam = 50, lam_min = 1e-4, ftol = 1e-6, maxstep = 20, maxstep1 = 20,
    eps = 1e-4, thresh=1e-4, gamma_pen = 2, dfmax = NULL, alpha = 1)
```

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#### **Arguments**

Y A  $n \times q$  numeric matrix of responses.

X A  $n \times q$  numeric design matrix for the model.

r1 The first dimension of single value matrix of the tensor. Default is 2. r3 The third dimension of single value matrix of the tensor. Default is 2.

method The method to be applied to select parameters. Either BIC (the default), AIC,

EBIC, CV, or GCV.

ncv The number of cross-validation folds. Default is 10. If method is not CV, ncv is

useless.

is PenU A logical value indicating whether the rows of U is penalized. Default is FALSE.

If is PenU is FALSE, the coefficients associating with  $X_j$  is penalized for each

 $j \in \{1, \cdots, p\}.$ 

isPenColumn A logical value indicating whether the coefficients associating with  $X_i$  that af-

fects whole response y is penalized. Default is TRUE. If isPenU is TRUE, the coefficients associating with  $X_j$  that affects whole response y is penalized for each  $j \in \{1, \cdots, p\}$ . If isPenU is FALSE, the coefficients associating with  $X_j$  that affects single response  $y_l$  is penalized for each  $j \in \{1, \cdots, p\}$ , where

 $l \in \{1, \cdots, q\}.$ 

penalty The penalty to be applied to the model. Either "LASSO" (the default), "SCAD",

or "MCP".

lambda A user-specified sequence of lambda values. By default, a sequence of values of

length nlam is computed, equally spaced on the log scale.

SUV A user-specified list of initial coefficient matrix of S, U, V. By default, initial

matrices are provided randomly.

nlam The number of lambda values. Default is 20.

1am\_min The smallest value for lambda, as a fraction of lambda.max. Default is 1e-3.

ftol Convergence threshhold for the Curvilinear search. The algorithm iterates until

the relative change in any coefficient is less than eps1. Default is 1e-6.

maxstep The maximum iterates number of the steepest gradient descent method. Default

is 20.

max\_step1 Maximum number of outer iterations. Default is 20.

thresh The threshold to numerically determine which coefficients are zeros. Since the

steepest projected gradient descent method with the approximated penalty can not shrink the estimated row of true zero row of U to exactly zero, we need to

determine a numerical threshold. Default is 1e-6.

eps Convergence threshhold for the outer loop. The algorithm iterates until the rel-

ative change in any coefficient is less than eps1. Default is 1e-4.

gamma\_pen The tuning parameter of the MCP/SCAD penalty (see details).

dfmax Upper bound for the number of nonzero coefficients. Default is no upper bound.

However, for large data sets, computational burden may be heavy for models

with a large number of nonzero coefficients.

alpha Tuning parameter for the Mnet estimator which controls the relative contri-

butions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha

may be arbitrarily small, but not exactly 0.

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#### **Details**

This function gives qp(p+1)/2 coefficients' estimators of MQR. The core tensor is a  $r_1 \times r_1 \times r_3$ -tensor.  $r_1$  and  $r_3$  are fixed.

#### Value

betapath Solution path of  $\beta$ .

rss Residual sum of squares (RSS).

df Degrees of freedom.

lambda The sequence of regularization parameter values in the path.

lambda\_opt The value of lambda with the minimum BIC value.
selectedID The index of lambda corresponding to lambda\_opt.

activeF  $\qquad \qquad \text{The active set of $U$. If isPenColumn is TRUE, activeF is same as activeX}$ 

activeX The active set of coefficients associtating with X. If isPenColumn is TRUE,

activeX is same as activeF

 $\begin{array}{lll} \text{Snew} & \text{Estimator of } S_3. \\ \text{Unew} & \text{Estimator of } U. \\ \text{Vnew} & \text{Estimator of } V. \\ \text{Y} & \text{Response } Y. \\ \text{X} & \text{Design matrix } X. \end{array}$ 

#### References

Symmetric Tensor Estimation for Quadratic Regression.

#### See Also

```
mqr, mqr_sparse_dr
```

## **Examples**

```
D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2 mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr_sparse(mydata$Y, mydata$X)
activeX <- fit$activeX
```

mqr\_sparse\_dr

Fit MQR with sparsity assumption and ranks selected by BIC or CV.

#### **Description**

Fit a high-dimensional multiresponse quadratic regression with or with aparsity assumptions. The multivariate sparse group lasso (mcp or scad) and the steepest gradient descent algorithm are used to estimate tensor for sparsity situation. The tuning parameter and ranks are selected by BIC (the default), AIC, EBIC, CV, or GCV, which matchs the method of rank selection.

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#### **Usage**

#### **Arguments**

Y A  $n \times q$  numeric matrix of responses.

X A  $n \times q$  numeric design matrix for the model.

r1\_index A user-specified sequence of  $r_1$  values, where  $r_1$  is the first dimension of single

value matrix of the tensor. Default is r1\_index=  $1, \dots, \min(\lceil \log(n) \rceil, p)$ .

r3\_index A user-specified sequence of  $r_3$  values, where  $r_3$  is the third dimension of single

value matrix of the tensor. Default is r3\_index=  $1, \dots, \min(\lceil \log(n) \rceil, q)$ .

method The method to be applied to select parameters. Either BIC (the default), AIC,

EBIC, CV, or GCV.

ncv The number of cross-validation folds. Default is 10. If method is not CV, ncv is

useless.

penalty The penalty to be applied to the model. Either "LASSO" (the default), "SCAD",

or "MCP".

is PenU A logical value indicating whether the rows of U is penalized. Default is FALSE.

If isPenU is FALSE, the coefficients associating with  $X_i$  is penalized for each

 $j \in \{1, \cdots, p\}.$ 

isPenColumn A logical value indicating whether the coefficients associating with  $X_i$  that af-

fects whole response y is penalized. Default is TRUE. If isPenU is TRUE, the coefficients associating with  $X_j$  that affects whole response y is penalized for each  $j \in \{1, \cdots, p\}$ . If isPenU is FALSE, the coefficients associating with  $X_j$  that affects single response  $y_l$  is penalized for each  $j \in \{1, \cdots, p\}$ , where

 $l \in \{1, \cdots, q\}.$ 

lambda A user-specified sequence of lambda values. By default, a sequence of values of

length nlam is computed, equally spaced on the log scale.

SUV A user-specified list of initial coefficient matrix of  $S,\,U,\,V.$  By default, initial

matrices are provided randomly.

nlam The number of lambda values. Default is 50.

lam\_min The smallest value for lambda, as a fraction of lambda.max. Default is 1e-2.

ftol Convergence threshhold for the Curvilinear search. The algorithm iterates until

the relative change in any coefficient is less than eps1. Default is 1e-6.

maxstep The maximum iterates number of the steepest gradient descent method. Default

is 20.

max\_step1 Maximum number of outer iterations. Default is 20.

eps Convergence threshhold for the outer loop. The algorithm iterates until the rel-

ative change in any coefficient is less than eps1. Default is 1e-4.

thresh The threshold to numerically determine which coefficients are zeros. Since the

steepest projected gradient descent method with the approximated penalty can not shrink the estimated row of true zero row of U to exactly zero, we need to

determine a numerical threshold. Default is 1e-6.

gamma\_pen The tuning parameter of the MCP/SCAD penalty (see details).

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dfmax Upper bound for the number of nonzero coefficients. Default is no upper bound.

However, for large data sets, computational burden may be heavy for models

with a large number of nonzero coefficients.

alpha Tuning parameter for the Mnet estimator which controls the relative contri-

butions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha

may be arbitrarily small, but not exactly 0.

#### **Details**

This function gives qp(p+1)/2 coefficients' estimators of MAM. The core tensor is a  $r_1 \times r_1 \times r_3$ -tensor. We choose  $r_1$  and  $r_3$  by BIC or CV.

#### Value

rss Residual sum of squares (RSS).

df Degrees of freedom.

activeF The active set of U. If isPenColumn is TRUE, activeF is same as activeX activeX The active set of coefficients associtating with X. If isPenColumn is TRUE,

activeX is same as activeF

lambda The sequence of regularization parameter values in the path.

selectedID The index of lambda corresponding to lambda\_opt.

lambda\_opt The value of lambda with the minimum BIC or CV value.

RSS The values of BIC or CV, which is a vector.

rk\_opt The optimal parametres that slected by BIC or CV. It is a vector with length 2,

which are selected  $r_1$  and  $r_3$ .

Y Response Y.X Design matrix X.

#### References

Symmetric Tensor Estimation for Quadratic Regression.

#### See Also

```
mqr_dr, mqr_sparse
```

```
#Example 1
D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2
mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr_sparse_dr(mydata$Y, mydata$X)
S3hat <- fit$Snew
opt <- fit$rk_opt</pre>
```

## TransferModalUnfoldings

Transfer a tensor's modal unfoldings to another.

## **Description**

Transfer a tensor's modal unfoldings to another.

## Usage

```
TransferModalUnfoldings(S, d1, d2 , r1, r2, r3)
```

## **Arguments**

S	A mode-d1-unfolding of a tensor with size $r_1 \times r_2 \times r_3$ , input unfolding
d1	An integer, the mode of unfolding $S_{(d_1)}$
d2	An integer, the mode of output unfolding $S_{\left(d_{2}\right)}$
r1	The fist dimension of tensor
r2	The second dimension of tensor
r3	The third dimension of tensor

## **Details**

This function transfers an input mode-d1-unfolding  $S_{(d_1)}$  to mode-d2-unfolding  $S_{(d_2)}$ 

## Value

D The output mode-d2-unfolding,  $S_{(d_2)}$ 

## References

Symmetric Tensor Estimation for Quadratic Regression.

```
D1 <- matrix(1:24,nrow = 4) # A tensor unfolding with size 4*6
D2 <- TransferModalUnfoldings(D1,1,2,4,3,2)
```

TransferP2T

TransferP2T	Transfer coefficients of the multivariate quadritic model to a tensor's modal unfoldings.

## **Description**

Transfer coefficients of the multivariate quadritic model to a tensor's modal unfoldings.

#### Usage

```
TransferP2T(coef, d , p, q)
```

## **Arguments**

coef	The coefficients of the multivariate quadritic model, which is a vector with length $qp(p+1)/2$
d	An integer, the mode of unfolding $S_{(d)}$
p	The fist dimension of tensor
q	The third dimension of tensor

#### **Details**

This function transfers coefficients of the multivariate quadritic model coef to a mode-d-unfolding  $D_{(d)}$  of a tensor.

#### Value

Dd

A mode-d-unfolding of a tensor with size  $p \times pq$ , input unfolding.

#### References

Symmetric Tensor Estimation for Quadratic Regression.

```
p <- 4
q <- 3
D1 <- NULL
for(j in 1:q){
    D0 <- matrix(runif(p^2,1,3),p)
    D1 <- cbind(D1,(D0+t(D0))/2)
}
coef <- TransferT2P(D1, 1 , p, q)
D1 <- TransferP2T(coef, 1 , p, q)
D2 <- TransferP2T(coef, 2 , p, q)
coef2 <- TransferT2P(D2, 2 , p, q)</pre>
```

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TransferT2P	Transfer a tensor's modal unfoldings to coefficients of the multivariate quadritic model.
-------------	---

#### **Description**

Transfer a tensor's modal unfoldings to coefficients of the multivariate quadritic model.

## Usage

```
TransferT2P(S, d , p, q)
```

## Arguments

S	A mode- $d$ -unfolding of a tensor with size $p \times pq$ , input unfolding
d	An integer, the mode of unfolding $S_{(d)}$
p	The fist dimension of tensor
q	The third dimension of tensor

#### **Details**

This function transfers an input mode-d-unfolding  $S_{(d)}$  to coefficients of the multivariate quadritic model coef.

## Value

coef The coefficients of the multivariate quadritic model. coef is a vector with length qp(p+1)/2.

## References

Symmetric Tensor Estimation for Quadratic Regression.

```
p <- 4
q <- 3
D1 <- NULL
for(j in 1:q){
    D0 <- matrix(runif(p^2,1,3),p)
    D1 <- cbind(D1,(D0+t(D0))/2)
}
coef <- TransferT2P(D1, 1 , p, q)</pre>
```

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