Package ‘tensorMQR1’

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Author Xu Liu [aut,cre],
Jian Huang [aut],
Heng Lian [aut],
Xinyuan Song [aut],
Yuling Jiao [aut]

Maintainer Xu Liu <liu.xu@sufe.edu.cn>

Description Symmetric tensor estimation for multiresponse quadratic regression. The number of predictors can be diverged as sample size increases, in which the penalty LASSO, MCP or SCAD can be used.

License GPL (>= 2)

Imports Rcpp (>= 0.11.15), RcppEigen (>= 0.3.2.3.0)

LinkingTo Rcpp, RcppEigen

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R topics documented:

`tensorMQR1-package` *Symmetric Tensor Estimation for Quadratic Regression.*

Description

For a high-dimensional multiresponse quadratic regression (MQR) with or without aparsity assumptions, treating the coefficients as a third-order tensor and borrowing Tucker decomposition to reduce the number of parameters. The multivariate sparse group lasso (mcp or scad) and the steepest gradient descent algorithm are used to estimate tensor for sparsity situation.

Details

This section should provide a more detailed overview of how to use the package, including the most important functions.

Author(s)

Xu Liu

Maintainer: Xu Liu <liu.xu@sufe.edu.cn>

References

Symmetric Tensor Estimation for Quadratic Regression.

`generateData` *Generate data from MQR model.*

Description

Generate data for a high-dimensional multiresponse quadratic regression, with or without aparsity assumptions.

Usage

```
generateData(n, q, s, p, D3,SigmaX=diag(p-1),sigma2=0.2,seed_id, t=0.0, rho=0.0)
```

Arguments

`n`	Sample size.
`q`	The number of responses, $q \geq 1$.
`s`	The true covariates associating to response, $s \geq 1$.
`p`	The number of covariates, $p \geq 1$.
`D3`	The mode of unfolding $D_{(3)}$.
`SigmaX`	Covariance of X. Default is identity matrix.
`sigma2`	err variance. Default is 0.1.
`seed_id`	Seed of generator.
`rho`	The correlation of ϵ_j and ϵ_k, where $j, k \in \{1, \cdots, q\}$.

Details

This function gives $qp(p+1)/2$ coefficients' estimators of MQR. The core tensor is a $r_1 \times r_2 \times r_3$-tensor. We choose r_1, r_2 and r_3 by `BIC` or `CV`.

Value

`Y`	Response, a $n \times q$-matrix.
`X`	Design matrix, a $n \times p$-matrix.

References

Symmetric Tensor Estimation for Quadratic Regression.

See Also

mam_sparse

Examples

```
# Example 1

D3 <- matrix(runif(72, 0.7, 1), 2, 36)
mydata <- generateData(200, 2, 6, 6, D3)

Y <- mydata$Y
X <- mydata$X


# Example 2
n <- 500
p <- 10
q <- 10
s <- 7
s0 <- s
r10=r20=r30=2
S3 <- matrix(runif(r10*r20*r30,3,7),nrow = r30)
T1 <- matrix(rnorm(s0*r10),nrow = s0)
U1 <- qr.Q(qr(T1))
T1 <- matrix(rnorm(q*r30),nrow = q)
U3 <- qr.Q(qr(T1))
D3 <- U3%*%S3%*%t(kronecker(U1,U1))
mydata <- generateData(n,q,s0,p,D3)
```

`mqr` *Fit MQR without sparsity assumption and with fixed ranks.*

Description

Fit a low-dimensional multiresponse quadratic regression without aparsity assumptions and with given r_1, r_2, r_3. The steepest gradient descent algorithm is used to estimate tensor.

Usage

```
mqr(Y, X, r1 = NULL, r3 = NULL, SUV = NULL, isSym = TRUE,
                          eps = 1e-6, max_step = 20, max_step1=20)
```

Arguments

Y	A $n \times q$ numeric matrix of responses.
X	A $n \times p$ numeric design matrix for the model.
r1	The first dimension of single value matrix of the tensor. Default is 2.
r3	The third dimension of single value matrix of the tensor. Default is 2.
SUV	A user-specified list of initial coefficient matrix of S, U, V. By default, initial matrices are provided randomly.
isSym	A logical value indicating whether restrict tensor to be symmetric. If isSym is `TRUE` (the default), we decompose the tensor to be $D_{(3)} = VS_{(3)}(U \otimes U)^T$, where the core tensor S is symmetric, and both U and V belong to Stiefel manifold. If isSym is `FALSE`, we decompose the tensor to S, A, B, C, that is $D_{(3)} = CS_{(3)}(B \otimes A)^T$, where the tensor is treated as being asymmetric.
eps	Convergence threshhold. The algorithm iterates until the loss function change in any coefficient is less than `eps`. Default is `1e-6`.
maxstep	The maximum iterates number of the steepest gradient descent method. Default is `20`.
max_step1	Maximum number of outer iterations. Default is 20.

Details

This function gives $qp(p+1)/2$ coefficients' estimators of MQR. The core tensor is a $r_1 \times r_2 \times r_3$-tensor. We fixed r_1, r_2 and r_3 in the function `mqr`, but one can choose r_1, r_2 and r_3 by `BIC` or `CV`. See details in function `mqr_bic` or `mqr_cv`.

Value

Dnew	Estimator of $D_{(3)}$.
rss	Residual sum of squares (RSS).
Y	Response Y.
X	Design matrix X.

References

Symmetric Tensor Estimation for Quadratic Regression.

See Also

mqr_sparse, mqr_bic, mqr_cv

Examples

```
D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2
mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr(mydata$Y, mydata$X, r1=4, r3= 2)
D3hat <- fit$Dnew
```

---

mqr_dr *Fit MQR without sparsity assumption, and with ranks selected by* BIC *or* CV.

Description

Fit a low-dimensional multiresponse quadratic regression without aparsity assumptions and with ranks r_1, r_3 selected by BIC (the default), AIC, EBIC, CV, or GCV..

Usage

```
mqr_dr(Y, X, r1_index = NULL, r3_index = NULL, method = "BIC", ncv = 10,
              SUV = NULL, isSym = TRUE, eps = 1e-6, max_step = 20, max_step1 = 20)
```

Arguments

Y	A $n \times q$ numeric matrix of responses.
X	A $n \times p$ numeric design matrix for the model.
r1_index	A user-specified sequence of r_1 values, where r_1 is the first dimension of single value matrix of the tensor. Default is r1_index= $1, \cdots, \min(\log(n)\rceil, p)$.
r3_index	A user-specified sequence of r_3 values, where r_3 is the third dimension of single value matrix of the tensor. Default is r3_index= $1, \cdots, \min(\log(n)\rceil, q)$.
method	The method to be applied to select parameters. Either BIC (the default), AIC, EBIC, CV, or GCV.
ncv	The number of cross-validation folds. Default is 10. If method is not "CV", ncv is useless.
SUV	A user-specified list of initial coefficient matrix of S, U, V, which is a list with values S, U, V. By default, initial matrices are provided randomly.
isSym	A logical value indicating whether restrict tensor to be symmetric. If isSym is TRUE (the default), we decompose the tensor to be $D_{(3)} = VS_{(3)}(U \otimes U)^T$, where the core tensor S is symmetric, and both U and V belong to Stiefel manifold. If isSym is FALSE, we decompose the tensor to S, A, B, C, that is $D_{(3)} = CS_{(3)}(B \otimes A)^T$, where the tensor is treated as being asymmetric.
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps. Default is 1e-6.
maxstep	The maximum iterates number of the steepest gradient descent method. Default is 20.
max_step1	Maximum number of outer iterations. Default is 20.

Details

This function gives $qp(p+1)/2$ coefficients' estimators of MQR. The core tensor is a $r_1 \times r_2 \times r_3$-tensor. We choose r_1, r_2 and r_3 by BIC or CV.

Value

Dnew	Estimator of $D_{(3)}$.
rss	Residual sum of squares (RSS).
rk_opt	The optimal parametres that slected by BIC or CV. It is a vector with length 4, which are selected r_1 and r_3.
selected	Which λ is selection.
Y	Response Y.
X	Design matrix X.

References

Symmetric Tensor Estimation for Quadratic Regression.

See Also

mqr, mqr_sparse_dr

Examples

```
D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2
mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr_dr(mydata$Y, mydata$X)
D3hat <- fit$Dnew
opt <- fit$rk_opt
```

---

mqr_sparse *Fit MQR with sparsity assumption and fixed ranks.*

Description

Fit a high-dimensional multiresponse quadratic regression with or without aparsity assumptions, and given ranks given ranks r_1, r_2, r_3. The multivariate sparse group lasso (mcp or scad) and the steepest gradient descent algorithm are used to estimate tensor for sparsity situation. The tuning parameter is selected by BIC (the default), AIC, EBIC, CV, or GCV.

Usage

```
mqr_sparse(Y, X, r1 = NULL, r3 = NULL, method = "BIC", ncv = 10, isPenU = 0, isPenColumn = 1,
          penalty = "LASSO", lambda = NULL, SUV = NULL,  nlam = 50,isSym=TRUE,
          initMethod="LASSO", lam_min = 1e-4, ftol = 1e-6, maxstep = 20,  maxstep1 = 20,
          eps = 1e-4,  thresh=1e-4, gamma_pen = 2,  dfmax = NULL, alpha = 1)
```

Arguments

`Y`	A $n \times q$ numeric matrix of responses.
`X`	A $n \times q$ numeric design matrix for the model.
`r1`	The first dimension of single value matrix of the tensor. Default is 2.
`r3`	The third dimension of single value matrix of the tensor. Default is 2.
`method`	The method to be applied to select parameters. Either `BIC` (the default), `AIC`, `EBIC`, `CV`, or `GCV`.
`ncv`	The number of cross-validation folds. Default is 10. If `method` is not `CV`, `ncv` is useless.
`isPenU`	A logical value indicating whether the rows of U is penalized. Default is `FALSE`. If `isPenU` is `FALSE`, the coefficients associating with X_j is penalized for each $j \in \{1, \cdots, p\}$.
`isPenColumn`	A logical value indicating whether the coefficients associating with X_j that affects whole response y is penalized. Default is `TRUE`. If `isPenU` is `TRUE`, the coefficients associating with X_j that affects whole response y is penalized for each $j \in \{1, \cdots, p\}$. If `isPenU` is `FALSE`, the coefficients associating with X_j that affects single response y_l is penalized for each $j \in \{1, \cdots, p\}$, where $l \in \{1, \cdots, q\}$.
`penalty`	The penalty to be applied to the model. Either "LASSO" (the default), "SCAD", or "MCP".
`lambda`	A user-specified sequence of lambda values. By default, a sequence of values of length `nlam` is computed, equally spaced on the log scale.
`SUV`	A user-specified list of initial coefficient matrix of S, U, V. By default, initial matrices are provided randomly.
`isSym`	A logical value indicating whether restrict tensor to be symmetric. If isSym is `TRUE` (the default), we decompose the tensor to be $D_{(3)} = VS_{(3)}(U \otimes U)^T$, where the core tensor S is symmetric, and both U and V belong to Stiefel manifold. If isSym is `FALSE`, we decompose the tensor to S, A, B, C, that is $D_{(3)} = CS_{(3)}(B \otimes A)^T$, where the tensor is treated as being asymmetric.
`initMethod`	One can estimate the initial tensor $D_{(3)}$ as a metrix by choosing a penalty to penalize group-column wise. `initMethod` can be `LASSO`, `MCP` or `SCAD`. The default is `LASSO`.
`nlam`	The number of lambda values. Default is 20.
`lam_min`	The smallest value for lambda, as a fraction of lambda.max. Default is 1e-3.
`ftol`	Convergence threshhold for the Curvilinear search. The algorithm iterates until the relative change in any coefficient is less than `eps1`. Default is `1e-6`.
`maxstep`	The maximum iterates number of the steepest gradient descent method. Default is `20`.
`max_step1`	Maximum number of outer iterations. Default is 20.
`thresh`	The threshold to numerically determine which coefficients are zeros. Since the steepest projected gradient descent method with the approximated penalty can not shrink the estimated row of true zero row of U to exactly zero, we need to determine a numerical threshold. Default is `1e-6`.
`eps`	Convergence threshhold for the outer loop. The algorithm iterates until the relative change in any coefficient is less than `eps1`. Default is `1e-4`.
`gamma_pen`	The tuning parameter of the MCP/SCAD penalty (see details).

dfmax	Upper bound for the number of nonzero coefficients. Default is no upper bound. However, for large data sets, computational burden may be heavy for models with a large number of nonzero coefficients.
alpha	Tuning parameter for the Mnet estimator which controls the relative contributions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha may be arbitrarily small, but not exactly 0.

Details

This function gives $qp(p+1)/2$ coefficients' estimators of MQR. The core tensor is a $r_1 \times r_1 \times r_3$-tensor. r_1 and r_3 are fixed.

Value

betapath	Solution path of β.
rss	Residual sum of squares (RSS).
df	Degrees of freedom.
lambda	The sequence of regularization parameter values in the path.
lambda_opt	The value of lambda with the minimum BIC value.
selectedID	The index of lambda corresponding to lambda_opt.
activeF	The active set of U. If isPenColumn is TRUE, activeF is same as activeX
activeX	The active set of coefficients associtating with X. If isPenColumn is TRUE, activeX is same as activeF
Snew	Estimator of S_3.
Unew	Estimator of U.
Vnew	Estimator of V.
Y	Response Y.
X	Design matrix X.

References

Symmetric Tensor Estimation for Quadratic Regression.

See Also

mqr, mqr_sparse_dr

Examples

```
D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2
mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr_sparse(mydata$Y, mydata$X)
activeX <- fit$activeX
```

mqr_sparse_dr *Fit MQR with sparsity assumption and ranks selected by* BIC *or* CV*.*

Description

Fit a high-dimensional multiresponse quadratic regression with or with aparsity assumptions. The multivariate sparse group lasso (mcp or scad) and the steepest gradient descent algorithm are used to estimate tensor for sparsity situation. The tuning parameter and ranks are selected by BIC (the default), AIC, EBIC, CV, or GCV, which matchs the method of rank selection.

Usage

```
mqr_sparse_dr(Y, X, r1_index = NULL, r3_index = NULL, method = "BIC", ncv = 10, penalty = "LASSO",
              isPenU = 0, isPenColumn = 1, lambda = NULL, SUV = NULL, nlam = 50,isSym=TRUE,
              initMethod="LASSO", lam_min = 0.001, ftol = 1e-6, max_step = 20, max_step1 = 20,
              eps = 1e-4, thresh = 1e-4, gamma_pen = 2, dfmax = NULL, alpha = 1)
```

Arguments

- Y: A $n \times q$ numeric matrix of responses.
- X: A $n \times q$ numeric design matrix for the model.
- r1_index: A user-specified sequence of r_1 values, where r_1 is the first dimension of single value matrix of the tensor. Default is r1_index$= 1, \cdots, \min(\lceil\log(n)\rceil, p)$.
- r3_index: A user-specified sequence of r_3 values, where r_3 is the third dimension of single value matrix of the tensor. Default is r3_index$= 1, \cdots, \min(\lceil\log(n)\rceil, q)$.
- method: The method to be applied to select parameters. Either BIC (the default), AIC, EBIC, CV, or GCV.
- ncv: The number of cross-validation folds. Default is 10. If method is not CV, ncv is useless.
- penalty: The penalty to be applied to the model. Either "LASSO" (the default), "SCAD", or "MCP".
- isPenU: A logical value indicating whether the rows of U is penalized. Default is FALSE. If isPenU is FALSE, the coefficients associating with X_j is penalized for each $j \in \{1, \cdots, p\}$.
- isPenColumn: A logical value indicating whether the coefficients associating with X_j that affects whole response y is penalized. Default is TRUE. If isPenU is TRUE, the coefficients associating with X_j that affects whole response y is penalized for each $j \in \{1, \cdots, p\}$. If isPenU is FALSE, the coefficients associating with X_j that affects single response y_l is penalized for each $j \in \{1, \cdots, p\}$, where $l \in \{1, \cdots, q\}$.
- lambda: A user-specified sequence of lambda values. By default, a sequence of values of length nlam is computed, equally spaced on the log scale.
- SUV: A user-specified list of initial coefficient matrix of S, U, V. By default, initial matrices are provided randomly.
- isSym: A logical value indicating whether restrict tensor to be symmetric. If isSym is TRUE (the default), we decompose the tensor to be $D_{(3)} = VS_{(3)}(U \otimes U)^T$, where the core tensor S is symmetric, and both U and V belong to Stiefel manifold. If isSym is FALSE, we decompose the tensor to S, A, B, C, that is $D_{(3)} = CS_{(3)}(B \otimes A)^T$, where the tensor is treated as being asymmetric.

initMethod	One can estimate the initial tensor $D_{(3)}$ as a metrix by choosing a penalty to penalize group-column wise. initMethod can be LASSO, MCP or SCAD. The default is LASSO.
nlam	The number of lambda values. Default is 50.
lam_min	The smallest value for lambda, as a fraction of lambda.max. Default is 1e-2.
ftol	Convergence threshhold for the Curvilinear search. The algorithm iterates until the relative change in any coefficient is less than eps1. Default is 1e-6.
maxstep	The maximum iterates number of the steepest gradient descent method. Default is 20.
max_step1	Maximum number of outer iterations. Default is 20.
eps	Convergence threshhold for the outer loop. The algorithm iterates until the relative change in any coefficient is less than eps1. Default is 1e-4.
thresh	The threshold to numerically determine which coefficients are zeros. Since the steepest projected gradient descent method with the approximated penalty can not shrink the estimated row of true zero row of U to exactly zero, we need to determine a numerical threshold. Default is 1e-6.
gamma_pen	The tuning parameter of the MCP/SCAD penalty (see details).
dfmax	Upper bound for the number of nonzero coefficients. Default is no upper bound. However, for large data sets, computational burden may be heavy for models with a large number of nonzero coefficients.
alpha	Tuning parameter for the Mnet estimator which controls the relative contributions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha may be arbitrarily small, but not exactly 0.

Details

This function gives $qp(p+1)/2$ coefficients' estimators of MAM. The core tensor is a $r_1 \times r_1 \times r_3$-tensor. We choose r_1 and r_3 by BIC or CV.

Value

rss	Residual sum of squares (RSS).
df	Degrees of freedom.
activeF	The active set of U. If isPenColumn is TRUE, activeF is same as activeX
activeX	The active set of coefficients associtating with X. If isPenColumn is TRUE, activeX is same as activeF
Snew	Estimator of S_3.
Unew	Estimator of U.
Vnew	Estimator of V.
lambda	The sequence of regularization parameter values in the path.
selectedID	The index of lambda corresponding to lambda_opt.
lambda_opt	The value of lambda with the minimum BIC or CV value.
RSS	The values of BIC or CV, which is a vector.
rk_opt	The optimal parametres that slected by BIC or CV. It is a vector with length 2, which are selected r_1 and r_3.
Y	Response Y.
X	Design matrix X.

References

Symmetric Tensor Estimation for Quadratic Regression.

See Also

mqr_dr, mqr_sparse

Examples

```
#Example 1

D3 <- matrix(runif(72, 0.7, 1), 2, 36) # tensor with size 6*6*2
mydata <- generateData(200, 2, 6, 6, D3)

fit <- mqr_sparse_dr(mydata$Y, mydata$X)
S3hat <- fit$Snew
opt <- fit$rk_opt
```

`mvrblockwise` *Estimate coefficients of high-dimensional multivariate regression for the grouped-column-wise*

Description

This function provides the coefficient matrix estimator of high-dimensional multivariate regression (MVR) with penalty `LASSO`, `MCP` or `SCAD`). The tuning parameter is selected by `BIC` (the default), `AIC`, `EBIC`, `CV`, or `GCV`.

Usage

```
mvrblockwise(Y, X ,Z=NULL, method="BIC", ncv=10, penalty="LASSO", isPenColumn=TRUE,
             group=NULL, lambda=NULL, nlam=50, intercept=TRUE, lam_min=1e-4,
             eps=1e-6, max_step=50, gamma_pen=2, dfmax=NULL, alpha=1)
```

Arguments

`Y`	The response, a vector of size n or a matrix of size $n \times q$.
`X`	The covariates to be penalized, a matrix with dimension $n \times p$.
`Z`	The covariates without penalization, a matrix with dimension $n \times d$. The default is `NULL`.
`method`	The method to be applied to select parameters. Either `BIC` (the default), `AIC`, `EBIC`, `CV`, or `GCV`.
`ncv`	The number of cross-validation folds. Default is 10. If `method` is not `CV`, `ncv` is useless.
`penalty`	The penalty to be applied to the model. Either `LASSO` (the default), `MCP` or `SCAD`.
`isPenColumn`	A logical value indicating whether the coefficients associating with X_j that affects whole response Y is penalized. Default is `TRUE`. If `isPenColumn` is `TRUE`, the coefficients associating with X_j that affects simultaneously whole response y is penalized for each $j \in \{1, \cdots, p\}$. If `isPenColumn` is `FALSE`, the coefficients associating with X_j that affects single response Y_l is penalized for each $j \in \{1, \cdots, p\}$, where $l \in \{1, \cdots, q\}$.

`group`	A integer vector describing the grouping of the coefficients. For example, we can preset `group = rep(1:G,each=K)`. If no grouping, `group = rep(1:ncol(X))`. The default is `group = rep(1:ncol(X))`.
`lambda`	A user-specified sequence of lambda values. By default, a sequence of values of length `nlam` is computed, equally spaced on the log scale.
`nlam`	The number of lambda values. Default is 50.
`intercept`	Should intercept(s) be fitted (default=`TRUE`) or set to zero (`FALSE`)?
`lam_min`	The smallest value for lambda, as a fraction of lambda.max. Default is 1e-3.
`eps`	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than `eps1`. Default is `1e-4`.
`max_step`	Maximum number of iterations. Default is `50`.
`gamma_pen`	The tuning parameter of the MCP/SCAD penalty (see details).
`dfmax`	Upper bound for the number of nonzero coefficients. Default is no upper bound. However, for large data sets, computational burden may be heavy for models with a large number of nonzero coefficients.
`alpha`	Tuning parameter for the Mnet estimator which controls the relative contributions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. `alpha=1` is equivalent to LASSO, MCP/SCAD penalty, while `alpha=0` would be equivalent to ridge regression. However, `alpha=0` is not supported; `alpha` may be arbitrarily small, but not exactly 0.

Value

`Bhat`	Estimator of coefficients of X.
`rss`	Residual sum of squares (RSS).
`activeX`	The active set of X. It is a p dimensional vector.
`lambda`	The sequence of regularization parameter values in the path.
`selectedID`	The index of `lambda` corresponding to `lambda_opt`.
`lambda_opt`	The value of `lambda` with the minimum `BIC` value.
`bic`	BIC value used to select variables.
`muhat`	Estimator of intercept μ. It is `NULL` if `intercept` is `FALSE`.
`Chat`	Estimator of coefficients of Z. `Chat` is `NULL` if `Z` is `NULL`.
`group`	The input group.
`Y`	Response Y.
`X`	Design matrix X.

References

Symmetric Tensor Estimation for Quadratic Regression. Manuscript.

Examples

```
library(tensorMQR1)

#example 1
n <- 200
q <- 5
```

```
s <- 3
p <- 100
B <- matrix(runif(q*s, 2,3), s)
X <- matrix(rnorm(n*p),n,p)
Y <- X[,1:s]%*%B + matrix(rnorm(n*q),n)
fit <- mvrblockwise(Y,X)
fit$activeX
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$muhat

#example 2
K = 5
n <- 200
q <- 5
s <- 4
p <- 100
B1 <- matrix(runif(q*K, 2,3), K)
B2 <- matrix(0,2*K,q)
B3 <- matrix(runif(q*(s-1)*K, 2,3), (s-1)*K)
B <- rbind(B1,B2,B3)
X <- matrix(rnorm(n*p*K),n)
Y <- X[,1:((s+2)*K)]%*%B + matrix(rnorm(n*q),n)
group <- rep(1:p,each=K)
fit <- mvrblockwise(Y,X,group=group,isPenColumn=TRUE)
which(fit$activeX==1)
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$muhat

#example 3
K = 5
n <- 200
q <- 5
s <- 4
d <- 3
p <- 100
B1 <- matrix(runif(q*K, 2,3), K)
B2 <- matrix(0,2*K,q)
B3 <- matrix(runif(q*(s-1)*K, 2,3), (s-1)*K)
B <- rbind(B1,B2,B3)
C <- matrix(runif(q*d, 1,2), d)
X <- matrix(rnorm(n*p*K),n)
Z <- matrix(rnorm(n*d),n)
Y <- X[,1:((s+2)*K)]%*%B + Z%*%C + matrix(rnorm(n*q),n)
group <- rep(1:p,each=K)
fit <- mvrblockwise(Y,X,Z,group=group,isPenColumn=TRUE)
which(fit$activeX==1)
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$Chat
fit$muhat
```

`mvrcolwise` *Estimate coefficients of high-dimensional multivariate regression for the column-wise*

Description

This function provides the coefficient matrix estimator of high-dimensional multivariate regression (MVR) with penalty LASSO, MCP or SCAD). The tuning parameter is selected by BIC (the default), AIC, EBIC, CV, or GCV.

Usage

```
mvrcolwise(Y, X ,Z=NULL, method="BIC", ncv=10, penalty="LASSO",
           isPenColumn=TRUE,lambda=NULL, nlam=50, intercept=TRUE,
           lam_min=1e-4,eps=1e-6, max_step=50, gamma_pen=2, dfmax=NULL, alpha=1)
```

Arguments

Y	The response, a vector of size n or a matrix of size $n \times q$.
X	The covariates to be penalized, a matrix with dimension $n \times p$.
Z	The covariates without penalization, a matrix with dimension $n \times d$. The default is NULL.
method	The method to be applied to select parameters. Either BIC (the default), AIC, EBIC, CV, or GCV.
ncv	The number of cross-validation folds. Default is 10. If method is not CV, ncv is useless.
penalty	The penalty to be applied to the model. Either LASSO (the default), MCP or SCAD.
isPenColumn	A logical value indicating whether the coefficients associating with X_j that affects whole response Y is penalized. Default is TRUE. If isPenColumn is TRUE, the coefficients associating with X_j that affects simultaneously whole response y is penalized for each $j \in \{1, \cdots, p\}$. If isPenColumn is FALSE, the coefficients associating with X_j that affects single response Y_l is penalized for each $j \in \{1, \cdots, p\}$, where $l \in \{1, \cdots, q\}$.
lambda	A user-specified sequence of lambda values. By default, a sequence of values of length nlam is computed, equally spaced on the log scale.
nlam	The number of lambda values. Default is 50.
intercept	Should intercept(s) be fitted (default=TRUE) or set to zero (FALSE)?
lam_min	The smallest value for lambda, as a fraction of lambda.max. Default is 1e-3.
eps	Convergence threshhold. The algorithm iterates until the relative change in any coefficient is less than eps1. Default is 1e-4.
max_step	Maximum number of iterations. Default is 50.
gamma_pen	The tuning parameter of the MCP/SCAD penalty (see details).
dfmax	Upper bound for the number of nonzero coefficients. Default is no upper bound. However, for large data sets, computational burden may be heavy for models with a large number of nonzero coefficients.
alpha	Tuning parameter for the Mnet estimator which controls the relative contributions from the LASSO, MCP/SCAD penalty and the ridge, or L2 penalty. alpha=1 is equivalent to LASSO, MCP/SCAD penalty, while alpha=0 would be equivalent to ridge regression. However, alpha=0 is not supported; alpha may be arbitrarily small, but not exactly 0.

Value

Bhat	Estimator of coefficients of X.
rss	Residual sum of squares (RSS).
activeX	The active set of X. It is a p dimensional vector.
lambda	The sequence of regularization parameter values in the path.
selectedID	The index of lambda corresponding to lambda_opt.
lambda_opt	The value of lambda with the minimum BIC value.
bic	BIC value used to select variables.
muhat	Estimator of intercept μ. It is NULL if intercept is FALSE.
Chat	Estimator of coefficients of Z. Chat is NULL if Z is NULL.
Y	Response Y.
X	Design matrix X.

References

Symmetric Tensor Estimation for Quadratic Regression. Manuscript.

Examples

```
library(tensorMQR1)

#example 1
n <- 200
q <- 5
s <- 3
p <- 100
B <- matrix(runif(q*s, 2,3), s)
X <- matrix(rnorm(n*p),n,p)
Y <- X[,1:s]%*%B + matrix(rnorm(n*q),n)
fit <- mvrcolwise(Y,X)
fit$activeX
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$muhat

#example 2
n <- 200
q <- 5
s <- 3
d <- 3
p <- 100
B <- matrix(runif(q*s, 2,3), s)
C <- matrix(runif(q*d, 1,2), d)
X <- matrix(rnorm(n*p),n,p)
Z <- matrix(rnorm(n*d),n)
Y <- X[,1:s]%*%B + Z%*%C + matrix(rnorm(n*q),n)
fit <- mvrcolwise(Y,X,Z)
fit$activeX
fit$Bhat
which(rowSums(fit$Bhat^2)>0)
fit$Chat
fit$muhat
```

`TransferModalUnfoldings` *Transfer a tensor's modal unfoldings to another.*

Description

Transfer a tensor's modal unfoldings to another.

Usage

```
TransferModalUnfoldings(S, d1, d2 , r1, r2, r3)
```

Arguments

`S`	A mode-d1-unfolding of a tensor with size $r_1 \times r_2 \times r_3$, input unfolding
`d1`	An integer, the mode of unfolding $S_{(d_1)}$
`d2`	An integer, the mode of output unfolding $S_{(d_2)}$
`r1`	The fist dimension of tensor
`r2`	The second dimension of tensor
`r3`	The third dimension of tensor

Details

This function transfers an input mode-d1-unfolding $S_{(d_1)}$ to mode-d2-unfolding $S_{(d_2)}$

Value

`D`	The output mode-d2-unfolding, $S_{(d_2)}$

References

Symmetric Tensor Estimation for Quadratic Regression.

Examples

```
D1 <- matrix(1:24,nrow = 4) # A tensor unfolding with size 4*6
D2 <- TransferModalUnfoldings(D1,1,2,4,3,2)
```

`TransferP2T` *Transfer coefficients of the multivariate quadritic model to a tensor's modal unfoldings.*

Description

Transfer coefficients of the multivariate quadritic model to a tensor's modal unfoldings.

Usage

```
TransferP2T(coef, d , p, q)
```

Arguments

`coef`	The coefficients of the multivariate quadritic model, which is a vector with length $qp(p+1)/2$
`d`	An integer, the mode of unfolding $S_{(d)}$
`p`	The fist dimension of tensor
`q`	The third dimension of tensor

Details

This function transfers coefficients of the multivariate quadritic model `coef` to a mode-d-unfolding $D_{(d)}$ of a tensor.

Value

`Dd`	A mode-d-unfolding of a tensor with size $p \times pq$, input unfolding.

References

Symmetric Tensor Estimation for Quadratic Regression.

Examples

```
p <- 4
q <- 3
D1 <- NULL
for(j in 1:q){
  D0 <- matrix(runif(p^2,1,3),p)
  D1 <- cbind(D1,(D0+t(D0))/2)
}
coef <- TransferT2P(D1, 1 , p, q)
D1 <- TransferP2T(coef, 1 , p, q)
D2 <- TransferP2T(coef, 2 , p, q)
coef2 <- TransferT2P(D2, 2 , p, q)
```

TransferT2P *Transfer a tensor's modal unfoldings to coefficients of the multivariate quadritic model.*

Description

Transfer a tensor's modal unfoldings to coefficients of the multivariate quadritic model.

Usage

```
TransferT2P(S, d , p, q)
```

Arguments

S	A mode-d-unfolding of a tensor with size $p \times pq$, input unfolding
d	An integer, the mode of unfolding $S_{(d)}$
p	The fist dimension of tensor
q	The third dimension of tensor

Details

This function transfers an input mode-d-unfolding $S_{(d)}$ to coefficients of the multivariate quadritic model `coef`.

Value

coef	The coefficients of the multivariate quadritic model. `coef` is a vector with length $qp(p+1)/2$.

References

Symmetric Tensor Estimation for Quadratic Regression.

Examples

```
p <- 4
q <- 3
D1 <- NULL
for(j in 1:q){
  D0 <- matrix(runif(p^2,1,3),p)
  D1 <- cbind(D1,(D0+t(D0))/2)
}
coef <- TransferT2P(D1, 1 , p, q)
```

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