ECON 35101 International Macroeconomics and Trade: Comprehension Check 2

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1 Setup

Preferences: Representative consumers have CES utility

$$C_j = \left(\sum_{i=1}^{n} \psi_{ij}^{\frac{1-\sigma}{\sigma}} C_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Consumers maximize utility given budget constraint $\sum_{i=1}^{n} P_{ij} C_{ij} \leq Y_j$. The first order condition is given by

$$\begin{aligned}
&\left(\sum_{i=1}^{n} \psi_{ij}^{\frac{1-\sigma}{\sigma}} C_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}-1} \psi_{ij}^{\frac{1-\sigma}{\sigma}} C_{ij}^{-\frac{1}{\sigma}} = \lambda P_{ij} \\
\implies & \frac{\psi_{ij}^{\frac{1-\sigma}{\sigma}}}{\psi_{kj}^{\frac{1-\sigma}{\sigma}}} \frac{C_{ij}^{-\frac{1}{\sigma}}}{C_{kj}^{-\frac{1}{\sigma}}} = \frac{P_{ij}}{P_{kj}} \\
\implies & \frac{P_{kj} C_{kj}}{P_{ij} C_{ij}} = \frac{\psi_{kj}^{1-\sigma}}{\psi_{ij}^{1-\sigma}} \left(\frac{P_{kj}}{P_{ij}} \right)^{1-\sigma} \\
\implies & \frac{Y_j}{P_{ij} C_{ij}} = P_j^{1-\sigma} \psi_{ij}^{\sigma-1} P_{ij}^{\sigma-1}
\end{aligned}$$

where the associated price index is

$$P_j = \left(\sum_{i=1}^{n} \psi_{ij}^{1-\sigma} P_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

This gives rise to the demand function

$$C_{ij} = \frac{\psi_{ij}^{1-\sigma} P_{ij}^{-\sigma}}{P_j^{1-\sigma}} Y_j$$

Trade Costs: Iceberg trade cost implies that

$$P_{ij} = \tau_{ij} P_{ii} = \tau_{ij} \frac{Y_i}{Q_i}$$

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Trade Flows: The *value* of imports from i to j is given by

$$X_{ij} = P_{ij}C_{ij} = \frac{\psi_{ij}^{1-\sigma}P_{ij}^{1-\sigma}}{P_j^{1-\sigma}}E_j$$

where E_j denotes total expenditure of country j. Plug bilateral trade price into the equation above, and we get

$$\begin{aligned} X_{ij} &= \frac{\psi_{ij}^{1-\sigma}\left(\tau_{ij}\frac{Y_i}{Q_i}\right)^{1-\sigma}}{\sum_{k=1}^n \psi_{kj}^{1-\sigma}\left(\tau_{kj}\frac{Y_k}{Q_k}\right)^{1-\sigma}}E_j \\ &= \frac{(\tau_{ij}Y_i)^{1-\sigma}\left(\frac{Q_i}{\psi_{ij}}\right)^{\sigma-1}}{\sum_{k=1}^n (\tau_{kj}Y_k)^{1-\sigma}\left(\frac{Q_k}{\psi_{kj}}\right)^{\sigma-1}}E_j \end{aligned}$$

Competitive Equilibrium: In a competitive equilibrium, total income equal total expenditure $Y_i = E_i$. Goods market clears $Y_i = \sum_{j=1}^n X_{ij}$. Combining two conditions, we get

$$Y_i = \sum_{j=1}^n \frac{(\tau_{ij}Y_i)^{1-\sigma}\left(\frac{Q_i}{\psi_{ij}}\right)^{\sigma-1}}{\sum_{k=1}^n (\tau_{kj}Y_k)^{1-\sigma}\left(\frac{Q_k}{\psi_{kj}}\right)^{\sigma-1}}Y_j$$

Assume that $\psi_{ij} = 1$ for all i, j, which simplifies the equation above.

$$Y_i = \sum_{j=1}^n \frac{(\tau_{ij}Y_i)^{1-\sigma}Q_i^{\sigma-1}}{\sum_{k=1}^n (\tau_{kj}Y_k)^{1-\sigma}Q_k^{\sigma-1}}Y_j \tag{1}$$

2 Free-Trade Equilibriums

Under free trade $\tau_{ij} = 1$, (1) becomes

$$Y_i = \sum_{j=1}^n \frac{Y_i^{1-\sigma}Q_i^{\sigma-1}}{\sum_{k=1}^n Y_k^{1-\sigma}Q_k^{\sigma-1}}Y_j$$

Re-arrange terms and we get

$$\sum_{k=1}^n \left(\frac{Y_k}{Y_i}\right)^{1-\sigma}\left(\frac{Q_k}{Q_i}\right)^{\sigma-1} = \sum_{j=1}^n \frac{Y_j}{Y_i}$$

Guess the solution takes the form $Y_i = Q_i^{\frac{\sigma-1}{\sigma}}$. We have

$$LHS = \sum_{k=1}^n \left(\frac{Q_k}{Q_i}\right)^{\frac{\sigma-1}{\sigma}(1-\sigma)}\left(\frac{Q_k}{Q_i}\right)^{\sigma-1} = \sum_{k=1}^n \left(\frac{Q_k}{Q_i}\right)^{\frac{\sigma-1}{\sigma}} = \sum_{j=1}^n \left(\frac{Q_j}{Q_i}\right)^{\frac{\sigma-1}{\sigma}} = RHS$$

Here if we let $\varepsilon = \sigma - 1$, we have $Y_i = Q_i^{\frac{\varepsilon}{\varepsilon+1}}$.

3 Welfare in Free-Trade Equilibrium

Real income is given by

$$\begin{aligned}
\frac{Y_j}{P_j} &= \frac{Q_j^{\frac{\varepsilon}{\varepsilon+1}}}{\left(\sum_{i=1}^n P_{ij}^{-\varepsilon}\right)^{\frac{1}{-\varepsilon}}} \\
&= Q_j^{\frac{\varepsilon}{\varepsilon+1}} \left(\sum_{i=1}^n \left(\frac{Y_i}{Q_i}\right)^{-\varepsilon}\right)^{\frac{1}{\varepsilon}} \\
&= Q_j^{\frac{\varepsilon}{\varepsilon+1}} \left(\sum_{i=1}^n Q_i^{\frac{\varepsilon}{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}} \qquad (2)
\end{aligned}$$

4 Identification

Take equation (1) and write

$$Y_i = \sum_{j=1}^n \frac{\left(\frac{\tau_{ij}}{Q_i}\right)^{1-\sigma} Y_i^{1-\sigma}}{\sum_{k=1}^n \left(\frac{\tau_{kj}}{Q_k}\right)^{1-\sigma} Y_k^{1-\sigma}} Y_j$$

We see that $\{Y_i\}$ is only determined by the ratio $\{\frac{\tau_{ij}}{Q_i}\}$ besides σ. In the two economies, these ratios are the same $\frac{\tau_{ij}}{Q_i} = \frac{\tau'_{ij}}{Q'_i}$ for all i, j. Therefore, country 1's share of world income in the prime economy is the same as that in the original economy.

5 Immiserizing growth

From (2), calculate the partical derivative w.r.t. Q_j

$$\begin{aligned}
\frac{\partial \frac{Y_j}{P_j}}{\partial Q_j} &= \frac{\varepsilon}{\varepsilon+1} Q_j^{\frac{-1}{\varepsilon+1}} \left(\sum_{i=1}^n Q_i^{\frac{\varepsilon}{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}} + Q_j^{\frac{\varepsilon}{\varepsilon+1}} \frac{1}{\varepsilon} \left(\sum_{i=1}^n Q_i^{\frac{\varepsilon}{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}-1} \frac{\varepsilon}{\varepsilon+1} Q_j^{\frac{-1}{\varepsilon+1}} \\
&= \underbrace{\frac{1}{\varepsilon+1} Q_j^{\frac{-1}{\varepsilon+1}} \left(\sum_{i=1}^n Q_i^{\frac{\varepsilon}{\varepsilon+1}}\right)^{\frac{1}{\varepsilon}}}_{(+)} \left(\varepsilon + \frac{Q_j^{\frac{\varepsilon}{\varepsilon+1}}}{\sum_{i=1}^n Q_i^{\frac{\varepsilon}{\varepsilon+1}}}\right)
\end{aligned}$$

Since $\sigma = \varepsilon + 1 > 0$, the sign of the derivative depends on the last term.

- If $\sigma > 1$ as in Costinot and Rodríguez-Clare (2014), $\varepsilon > 0$ and $\frac{\partial \frac{Y_j}{P_j}}{\partial Q_j} > 0$. Hence, immiserizing growth is not possible with Armington under free trade.
- If $\sigma < 1$, $\varepsilon + \frac{Q_j^{\frac{\varepsilon}{\varepsilon+1}}}{\sum_{i=1}^n Q_i^{\frac{\varepsilon}{\varepsilon+1}}} < 0$ when ε is small, and immiserizing growth is possible.