ECON 35101 International Macroeconomics and Trade: Assignment 1

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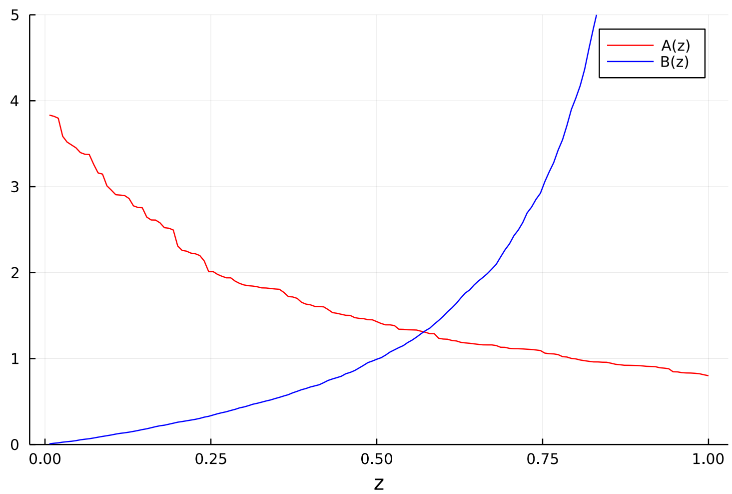
Replicating DFS (1977) Figure 1: See Figure 1.

Figure 1: Replicated figure 1 in DFS (1977)



Notes: Figure only shows the range of $A(z)$ and $B(z)$ between 0 and 5. Varieties (x-axis) are normalized to be in the range of $[0, 1]$.

Welfare and Gains from Trade: Note that log indirect utility is given by

$$v \equiv \ln V = \ln wL - \int_0^1 b(z) \ln p(z) dz$$

In the case of autarky, home utility, denoted as v^a, is

$$v^a = \ln L - \int_0^1 b(z) \ln a(z) dz \tag{1}$$

With trade, home utility, denoted as v, is

$$v = \ln L - \int_0^{\bar{z}} b(z) \ln a(z) dz - \int_{\bar{z}}^1 b(z) \ln \frac{a^*(z)}{\omega g} dz \tag{2}$$

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Hence, home gains from trade are

$$GFT = v - v^a = \int_{\bar{z}}^{1} b(z) \ln \frac{\omega g}{A(z)} dz \tag{3}$$

Note that $A(\bar{z}) = \frac{a^*(\bar{z})}{a(\bar{z})} = \omega g$, then $\frac{\omega g}{A(z)} = \frac{A(\bar{z})}{A(z)} \geq 1$ when $z \geq \bar{z}$ since $A(\cdot)$ is decreasing. This means that the integrand is nonnegative, which implies that home gains from trade are nonnegative.

Similarly, foreign utility in the case of autarky is given by

$$v^{*a} = \ln L^* - \int_0^1 b(z) \ln a^*(z) dz \tag{4}$$

and foreign utility with trade is given by

$$v^* = \ln \frac{L^*}{\omega} - \int_0^{\bar{z}^*} b(z) \ln \frac{a(z)}{g} dz - \int_{\bar{z}^*}^{1} b(z) \ln \frac{a^*(z)}{\omega} dz \tag{5}$$

Hence, foreign gains from trade are

$$GFT^* = v^* - v^{*a} = \int_0^{\bar{z}^*} b(z) \ln \frac{A(z) g}{\omega} dz \tag{6}$$

Note that $A(\bar{z}^*) = \frac{\omega}{g}$, then $\frac{A(z)g}{\omega} = \frac{A(z)}{A(\bar{z}^*)} \geq 1$ when $z \leq \bar{z}^*$ since $A(\cdot)$ is decreasing. This means that the integrand is nonnegative, which implies that foreign gains from trade are nonnegative as well.

Uniform Foreign Technology Progress: See Table 1.

Table 1: Welfare and gains from trade with uniform foreign technology progress

Foreign technological change, $\hat{a}^*(z)$	1.0		0.9		0.8	
	Home	Foreign	Home	Foreign	Home	Foreign
Autarky welfare	0.421	0.0	0.421	0.105	0.421	0.223
Trade welfare	0.527	0.26	0.537	0.354	0.554	0.452
Gains from trade	0.106	0.26	0.116	0.248	0.133	0.229

Volume of Trade: Here, volume of trade is defined as the value of home imports plus exports.

$$VOT = \int_{\bar{z}}^{1} b(z) L dz + \int_0^{\bar{z}^*} b(z) \frac{L^*}{\omega} dz \tag{7}$$

Comparing gains from trade (3) and (6) with volume of trade (7), we see that given $\{L, L^*, g\}$, gains from trade and volume of trade are pinned down by $A(z)$ and $b(z)$. One important difference, however, is that gains from trade depend on the curvature of $A(z)$ while volume of trade does not.

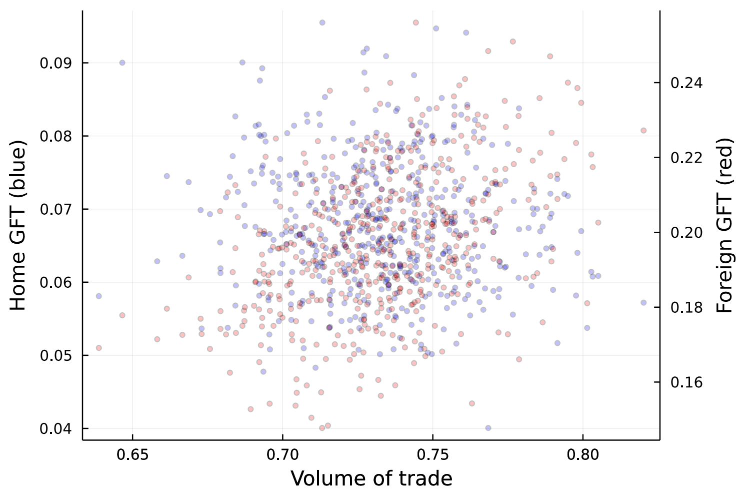
To check the existence of multiple equilibria, we adopt the following method. Fix $L = L^* = 1$ and $g = 0.9$, let total volume of trade be $VOT(A, b)$, and denote gains from trade be $GFT(A, b)$ and $GFT^*(A, b)$. We simulate (A, b) N times, and for each simulation, we compute $GFT(A, b)$, $GFT^*(A, b)$, and $VOT(A, b)$.

Table 2 shows an example of multiple equilibria. We see that for a given volume of trade, gains from trade can be very different. In addition, Figure 2 below shows the plot of GFT and VOT from $N = 500$ simulations. We conclude that multiple equilibria exist at different levels of VOT.

Table 2: Multiple equilibria when (A, b) are free

	$\bar{\omega}$	$\bar{z}$	$\bar{z}^*$	GFT	GFT^*	VOT
(1)	1.249	95.0	72.0	0.069	0.206	0.753
(2)	1.218	94.0	71.0	0.064	0.203	0.753
(3)	1.272	96.0	74.0	0.064	0.191	0.753
(4)	1.343	94.0	76.0	0.085	0.191	0.753

Figure 2: Simulated volume of trade and gains from trade with A free



If we hold $b(z)$ fixed and vary $A(z)$, multiple equilibria still exist. Table 3 provides an example, and Figure 3 confirms the existence of multiple equilibria. This is because, given $\{b, L, L^*, g\}$, the $B(z)$ schedule is fixed. However, there exist infinite $A(z)$ schedules crossing $B(z)$ which yield the same $\{\bar{\omega}, \bar{z}, \bar{z}^*\}$ (see Figure 4 for example). In these cases, volumes of trade are the same, but gains from trade are different since, as noted above, they depend on the curvature of $A(z)$. This implies that we cannot say much about the exact magnitudes of the gains from trade in this model if we observe the equilibrium volume of trade and do not observe autarky prices.

Table 3: Multiple equilibria when A is free

	$\bar{\omega}$	$\bar{z}$	$\bar{z}^*$	GFT	GFT^*	VOT
(1)	1.236	93.0	70.0	0.065	0.209	0.753
(2)	1.343	93.0	76.0	0.083	0.19	0.753
(3)	1.206	93.0	68.0	0.052	0.187	0.753
(4)	1.303	93.0	73.0	0.081	0.185	0.753

Figure 3: Simulated volume of trade and gains from trade with A free but b fixed

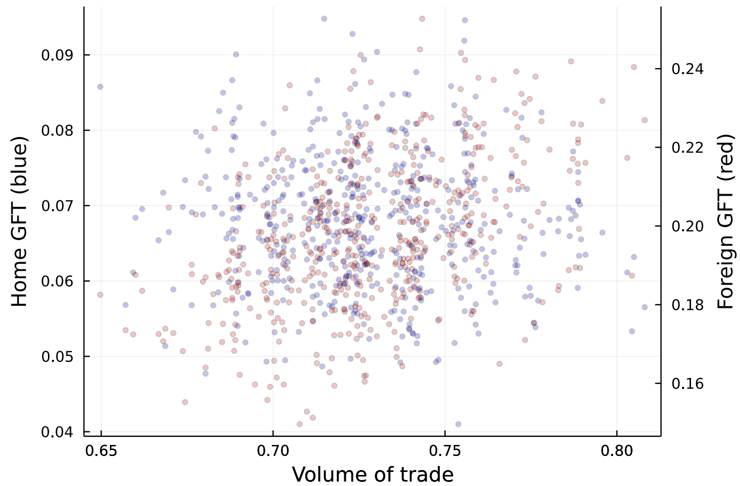


Figure 4: Multiple equilibria with different A