

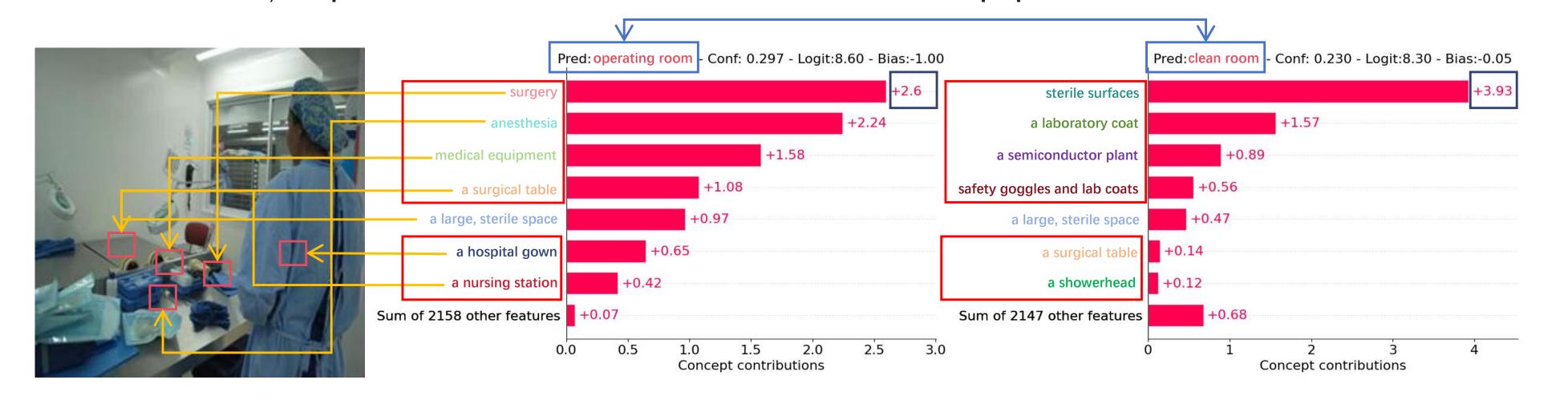
# FAITHFUL VISION-LANGUAGE INTERPRETATION VIA CONCEPT BOTTLENECK MODELS



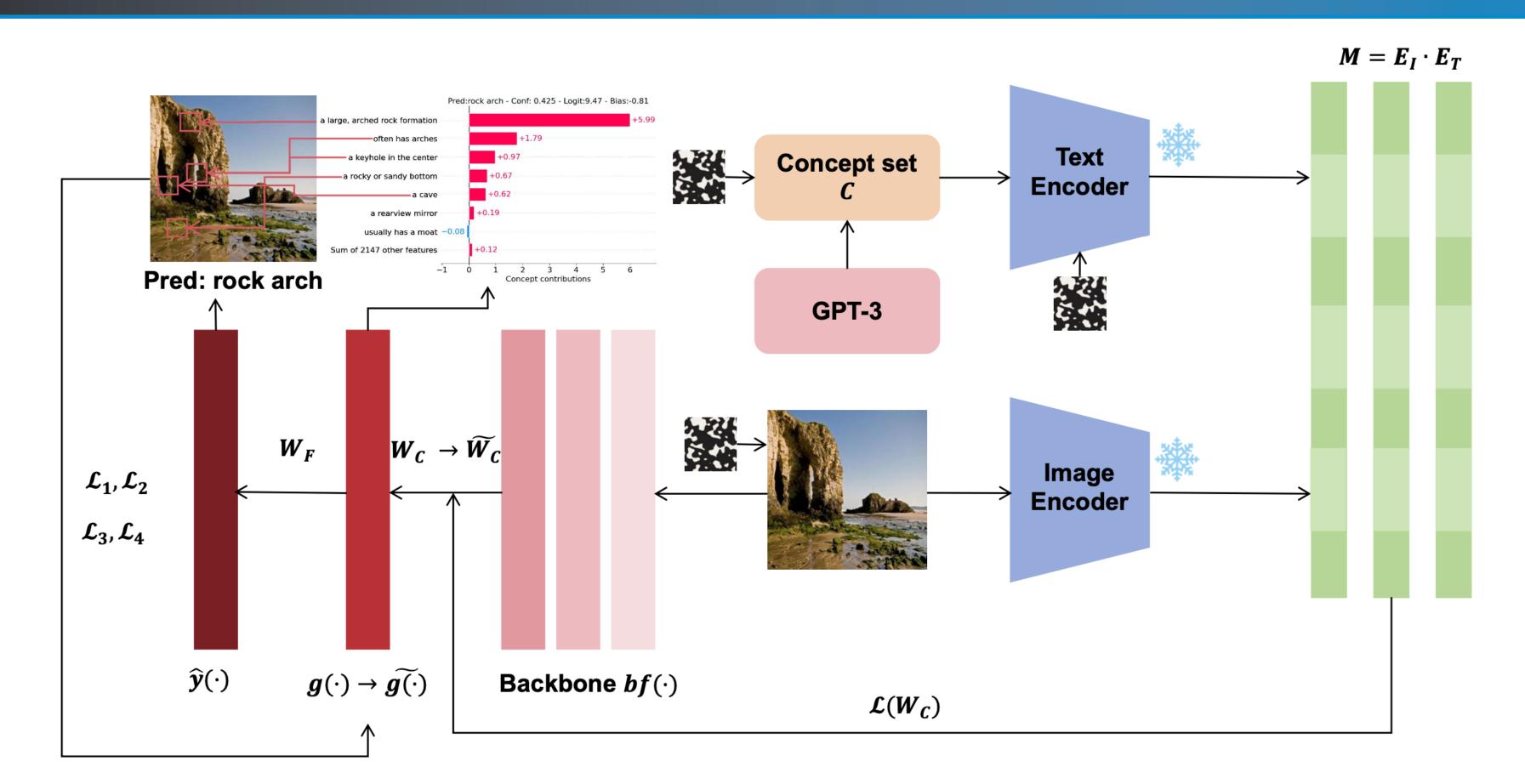
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#### INTRODUCTION

Traditional Concept Bottleneck Models (CBMs) require substantial manual annotation, which label-free CBM effectively addresses by leveraging factual information from pre-trained models. However, this convenience comes with inherent instability in pre-trained models. We addressed it in this paper.



#### DEFINITION AND FRAMEWORK

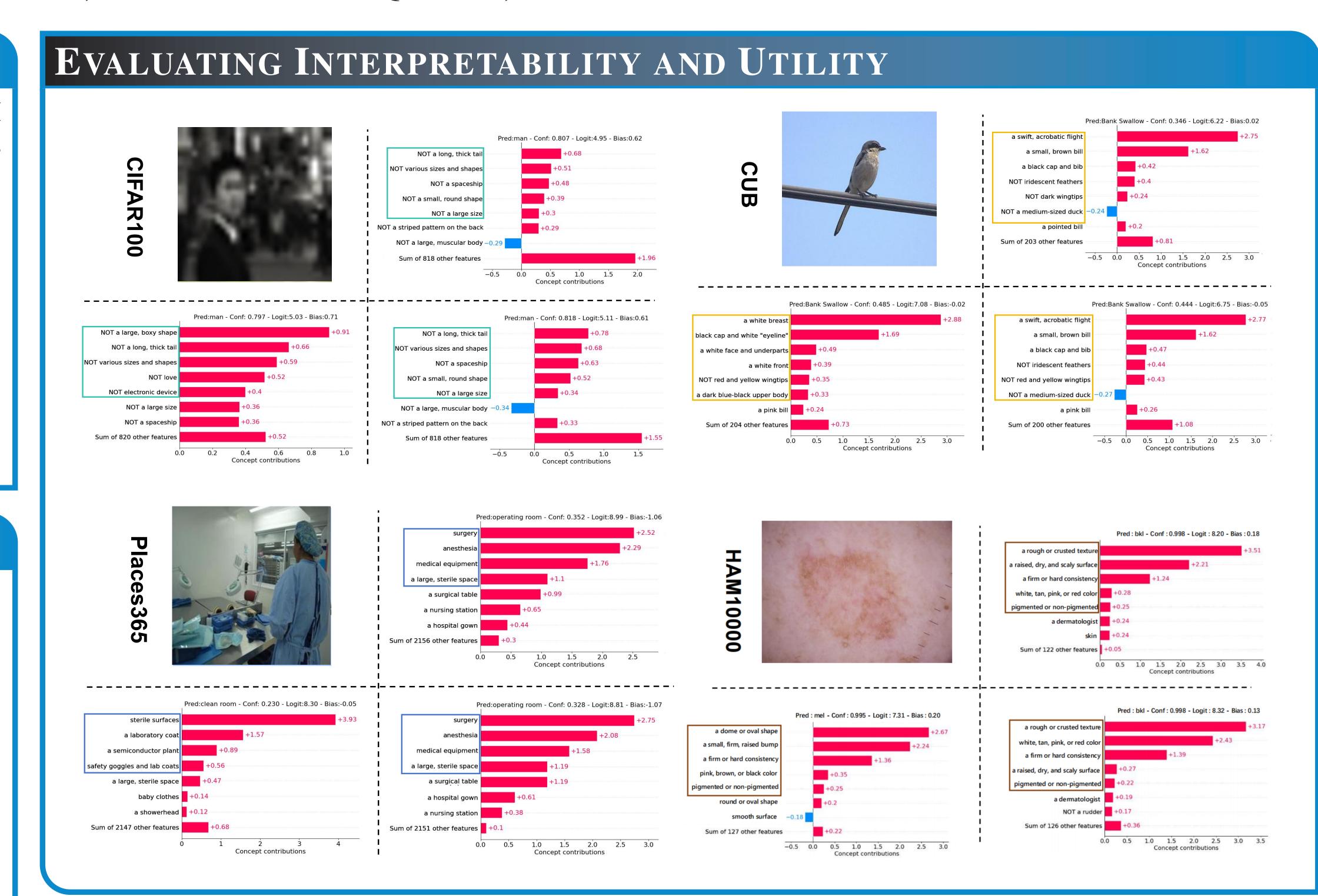


$$\min_{\tilde{W}_c} \mathbb{E}_x \left[ \lambda_1 \underbrace{D(y(x,\tilde{\boldsymbol{c}}),y(x,\boldsymbol{c}))}_{\mathcal{L}_1} + \lambda_2 \underbrace{\mathcal{L}_{k_1}(\tilde{\boldsymbol{g}}(x),\boldsymbol{g}(x))}_{\mathcal{L}_2} + \lambda_3 \underbrace{\max_{||\delta|| \leq R_2} D(y(x,\tilde{\boldsymbol{c}}),y(x,\tilde{\boldsymbol{c}}+\boldsymbol{\delta}))}_{\mathcal{L}_3} + \lambda_4 \underbrace{\max_{||\rho|| \leq R_1} \mathcal{L}_{k_2}(\tilde{\boldsymbol{g}}(x),\tilde{\boldsymbol{g}}(x)+\boldsymbol{\rho})]}_{\mathcal{L}_4} \right].$$

**Definition 1** (**Faithful Vision-Language Concept**). Under the same concept space, i.e., under the set of concepts generated by GPT3 at one time, we call a matrix  $\tilde{W}_c$  is a  $(D, R, \alpha, \beta, k_1, k_2)$ -Faithful Vision-Language Concept (FVLC) model for the vanilla concept if it satisfies for any input x:

- (Similarity of Explanation)  $V_{k_1}(\tilde{\boldsymbol{g}}(x), \boldsymbol{g}(x)) \geq \beta_1$  for some  $1 \geq \beta_1 \geq 0$ ;
- (Stability of Explanation)  $V_{k_2}(\tilde{\boldsymbol{g}}(x), \tilde{\boldsymbol{g}}(x) + \boldsymbol{\rho}) \geq \beta_2$  for some  $1 \geq \beta_2 \geq 0$  and all  $\|\boldsymbol{\rho}\| \leq R_1$ , where  $\|\cdot\|$  is a norm and  $R_1 \geq 0$ ;
- (Closeness of Prediction)  $D(y(x, \tilde{c}), y(x, c)) \leq \alpha_1$  for some  $\alpha_1 \geq 0$ , where D is some probability distance or divergence;
- (Stability of Prediction)  $D(y(x, \tilde{c}), y(x, \tilde{c} + \delta)) \le \alpha_2$  for all  $\|\delta\| \le R_2$ , where D is some probability distance or divergence,  $\|\cdot\|$  is a norm and  $R_2 \ge 0$ ,

where  $\tilde{\boldsymbol{g}}(x) = \tilde{W}_c b f(x)$ ,  $y(x, \boldsymbol{c}) = W_F g(x)$ , and  $y(x, \tilde{\boldsymbol{c}}) = W_F \tilde{\boldsymbol{g}}(x)$ ,  $y(x, \tilde{\boldsymbol{c}} + \boldsymbol{\delta}) = W_F (\tilde{\boldsymbol{g}}(x) + \boldsymbol{\delta})$ . For any given  $x, \tilde{c} = \tilde{\boldsymbol{g}}(x)$  is a  $(D, R, \alpha, \beta, k_1, k_2)$ -FVLC. Here,  $\alpha = \min\{\alpha_1, \alpha_2\}$ ,  $\beta = \max\{\beta_1, \beta_2\}$ , and  $R = \min\{R_1, R_2\}$ .



#### STABILITY EVALUATION

Method	CIFAR10	CIFAR100	CUB	Places365
Standard (No interpretability)	88.80%	70.10%	76.70%	48.56%
P-CBM (CLIP)	84.50%	56.00%	N/A	N/A
Label-free CBM	86.32%	65.42%	74.23%	43.63%
WP1(5%) - base	86.47%	65.13%	74.08%	43.57%
WP1(5%) - <b>FVLC</b>	86.34%	65.43%	73.96%	43.67%
WP1(10%) - base	86.25%	65.09%	73.97%	43.67%
WP1(10%) - <b>FVLC</b>	86.39%	64.90%	73.92%	43.62%
WP2 - base	86.41%	65.16%	73.96%	43.54%
WP2 - FVLC	86.22%	65.34%	74.44%	44.55%
IP - base	86.62%	65.36%	74.39%	43.64%
IP - FVLC	86.88%	65.29%	74.01%	43.71%
WP1(5%)+WP2 - base	86.49%	65.17%	73.90%	43.67%
WP1(5%)+WP2 - <b>FVLC</b>	86.43%	65.33%	73.92%	43.49%
WP1(10%)+WP2 - base	86.30%	64.87%	73.82%	43.61%
WP1(10%)+WP2 - <b>FVLC</b>	86.38%	65.06%	74.01%	43.44%
WP1(10%)+WP2+IP - base	85.96%	64.41%	73.74%	43.32%
WP1(10%)+WP2+IP - <b>FVLC</b>	86.70%	65.14%	74.36%	43.46%

Method	CIFA	AR10	CIFAR100		CUB		Places365	
Method	TCPC	TOPC	TCPC	TOPC	TCPC	TOPC	C TCPC TO	TOPC
WP1(5%) - base	1.55E-01	6.32E-02	1.01E-01	7.17E-02	1.26E-01	1.85E-01	1.59E-01	6.40E-0
WP1(5%) - <b>FVLC</b>	1.12E-03	8.55E-03	2.81E-03	4.51E-03	1.05E-02	1.50E-03	1.38E-03	1.30E-0
WP1(10%) - base	1.99E-01	8.36E-02	1.94E-01	1.31E-01	2.32E-01	3.41E-01	2.26E-01	1.14E-0
WP1(10%) - <b>FVLC</b>	1.19E-03	7.40E-03	3.67E-03	4.55E-03	1.19E-02	1.53E-03	1.39E-03	1.25E-0
WP2 - base	1.53E-01	4.99E-02	1.36E-01	6.67E-02	1.43E-01	1.73E-01	1.40E-01	6.37E-0
WP2 - FVLC	1.10E-02	8.72E-03	3.35E-03	4.55E-03	1.05E-02	1.53E-03	1.55E-03	1.29E-(
IP - base	1.68E-01	6.28E-02	1.38E-01	8.81E-02	1.71E-01	2.23E-01	1.73E-01	8.09E-0
IP - FVLC	8.02E-03	8.29E-03	3.24E-03	4.56E-03	1.04E-02	1.59E-03	1.50E-03	1.25E-(
WP1(5%)+WP2 - base	1.85E-01	3.50E-02	1.28E-01	6.65E-02	1.44E-01	1.79E-01	1.60E-01	6.32E-0
WP1(5%)+WP2 - <b>FVLC</b>	1.20E-02	7.46E-03	3.67E-03	4.55E-03	9.81E-02	1.51E-03	1.54E-03	1.28E-0
WP1(10%)+WP2 - base	1.17E-01	8.62E-02	1.93E-01	1.32E-01	1.76E-01	3.45E-01	2.52E-01	1.17E-0
WP1(10%)+WP2 - <b>FVLC</b>	1.18E-02	9.41E-03	2.06E-02	1.44E-02	1.87E-02	3.79E-02	2.74E-02	1.18E-0
WP1(10%)+WP2+IP - base	1.36E-01	1.05E-01	2.22E-01	1.55E-01	1.95E-01	3.54E-01	2.62E-01	1.44E-0
WP1(10%)+WP2+IP - <b>FVLC</b>	1.43E-02	1.11E-02	2.39E-02	1.77E-02	2.21E-02	4.54E-02	3.35E-02	1.34E-0

## ABLATION STUDY

Method	Setting		CIFAR10		CIFAR100		CUB		Places365		
	$\mathcal{L}_2$	$\mathcal{L}_3$	$\mathcal{L}_4$	TCPC	TOPC	TCPC	TOPC	TCPC	TOPC	TCPC	TOPC
WP1(10%) - FVLC				1.99E-01	8.36E-02	1.94E-01	1.31E-01	2.32E-01	3.41E-01	2.26E-01	1.14E-01
	$\checkmark$			2.09E-02	3.14E-02	2.81E-02	4.88E-02	4.08E-01	7.56E-02	4.69E-02	6.14E-02
		$\checkmark$		1.80E-02	1.79E-02	2.01E-02	2.85E-02	3.77E-01	4.50E-02	4.48E-02	3.68E-02
			$\checkmark$	4.78E-03	3.11E-02	1.67E-02	2.19E-02	5.69E-02	5.52E-03	6.04E-03	4.98E-03
	$\checkmark$	$\checkmark$		1.67E-02	1.11E-02	5.39E-02	6.85E-02	1.69E-01	2.12E-02	1.98E-02	1.81E-02
	$\checkmark$		$\checkmark$	1.65E-03	1.01E-02	5.08E-03	6.43E-03	1.61E-02	2.14E-03	1.95E-03	1.72E-03
		$\checkmark$	$\checkmark$	1.63E-03	1.02E-02	5.04E-03	6.27E-03	1.63E-02	2.10E-03	1.94E-03	1.71E-03
	$\checkmark$	$\checkmark$	$\checkmark$	1.19E-03	7.40E-03	3.67E-03	4.55E-03	1.19E-02	1.53E-03	1.39E-03	1.25E-0
WP2 - FVLC				1.53E-01	4.99E-02	1.36E-01	6.67E-02	1.43E-01	1.73E-01	1.40E-01	6.37E-0
	$\checkmark$			7.62E-02	2.29E-02	2.02E-02	2.08E-02	6.09E-02	5.22E-03	1.04E-02	5.31E-0
		$\checkmark$		5.37E-02	1.84E-02	1.46E-02	1.16E-02	6.80E-02	2.95E-03	1.06E-02	2.36E-0
			$\checkmark$	5.19E-02	1.46E-02	1.31E-02	9.34E-03	5.30E-02	3.24E-03	6.83E-03	2.12E-0
	$\checkmark$	$\checkmark$		4.57E-02	1.82E-02	1.45E-02	1.01E-02	5.01E-02	3.46E-03	6.30E-03	2.94E-0
	$\checkmark$		$\checkmark$	2.73E-02	6.96E-03	7.10E-03	4.60E-03	2.42E-02	1.91E-03	3.97E-03	2.36E-0
		$\checkmark$	$\checkmark$	2.47E-02	8.00E-03	5.29E-03	5.25E-03	2.40E-02	1.70E-03	3.82E-03	1.96E-0
	$\checkmark$	$\checkmark$	$\checkmark$	1.10E-02	8.72E-03	3.35E-03	4.55E-03	1.05E-02	1.53E-03	1.55E-03	1.29E-0
IP - FVLC				1.68E-01	6.28E-02	1.38E-01	8.81E-02	1.71E-01	2.23E-01	1.73E-01	8.09E-0
	$\checkmark$			6.39E-02	2.55E-02	2.56E-02	3.57E-02	8.04E-02	1.27E-02	1.16E-02	9.80E-0
		$\checkmark$		3.82E-02	3.91E-02	1.53E-02	2.16E-02	4.77E-02	7.66E-03	6.99E-03	5.88E-0
			$\checkmark$	1.63E-02	1.73E-02	6.35E-03	9.35E-03	2.14E-02	3.30E-03	3.01E-03	2.59E-0
	$\checkmark$	$\checkmark$		2.04E-02	2.10E-02	8.35E-03	1.15E-02	2.67E-02	4.04E-03	3.77E-03	3.18E-0
	$\checkmark$		$\checkmark$	1.23E-02	1.27E-02	5.09E-03	6.94E-03	1.65E-02	2.45E-03	2.35E-03	1.94E-0
		$\checkmark$	$\checkmark$	1.26E-02	1.09E-02	5.07E-03	7.00E-03	1.59E-02	2.53E-03	2.29E-03	1.95E-0
	$\checkmark$	$\checkmark$	$\checkmark$	8.02E-03	8.29E-03	3.24E-03	4.56E-03	1.04E-02	1.59E-03	1.50E-03	1.25E-0
WP1(10%)+WP2 - FVLC				1.17E-01	8.62E-02	1.93E-01	1.32E-01	1.76E-01	3.45E-01	2.52E-01	1.17E-0
	$\checkmark$			5.97E-02	6.28E-02	1.43E-01	9.94E-02	1.24E-01	2.58E-01	1.86E-01	8.24E-0
		$\checkmark$		3.76E-02	6.34E-02	1.48E-01	1.03E-01	1.21E-01	2.63E-01	1.90E-01	8.13E-02
			$\checkmark$	3.52E-02	2.54E-02	6.03E-02	4.13E-02	5.08E-02	1.07E-01	7.63E-02	3.31E-0
	$\checkmark$	$\checkmark$		9.21E-02	5.45E-02	1.28E-01	8.56E-02	1.04E-01	2.24E-01	1.60E-01	6.82E-02
	$\checkmark$		$\checkmark$	3.15E-02	2.90E-02	6.88E-02	4.71E-02	5.76E-02	1.21E-01	8.86E-02	3.72E-0
		$\checkmark$	$\checkmark$	1.53E-02	1.18E-02	2.78E-02	1.89E-02	2.37E-02	4.88E-02	3.62E-02	1.55E-02
	$\checkmark$	$\checkmark$	$\checkmark$	1.18E-02	9.41E-03	2.06E-02	1.44E-02	1.87E-02	3.79E-02	2.74E-02	1.18E-0
WP1(10%)+WP2+IP - FVLC				1.36E-01	1.05E-01	2.22E-01	1.55E-01	1.95E-01	3.54E-01	2.62E-01	1.44E-0
	$\checkmark$			7.37E-02	5.75E-02	1.29E-01	9.24E-02	1.21E-01	2.34E-01	1.74E-01	7.30E-0
		$\checkmark$		7.85E-02	5.85E-02	1.23E-01	9.17E-02	1.14E-01	2.43E-01	1.70E-01	6.80E-0
			$\checkmark$	4.59E-02	3.67E-02	7.62E-02	5.57E-02	6.74E-02	1.41E-01	1.08E-01	4.23E-0
	$\checkmark$	$\checkmark$		5.81E-02	4.52E-02	9.69E-02	7.11E-02	8.99E-02	1.83E-01	1.34E-01	5.49E-0
	$\checkmark$		$\checkmark$	5.84E-02	4.53E-02	9.75E-02	7.17E-02	8.94E-02	1.85E-01	1.36E-01	5.41E-0
		$\checkmark$	$\checkmark$	2.91E-02	2.31E-02	4.85E-02	3.56E-02	4.46E-02	9.33E-02	6.78E-02	2.81E-0
	$\checkmark$	$\checkmark$	$\checkmark$	1.43E-02	1.11E-02	2.39E-02	1.77E-02	2.21E-02	4.54E-02	3.35E-02	1.34E-02

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