Lecture 11:

Measuring Light: Radiometry and Cameras

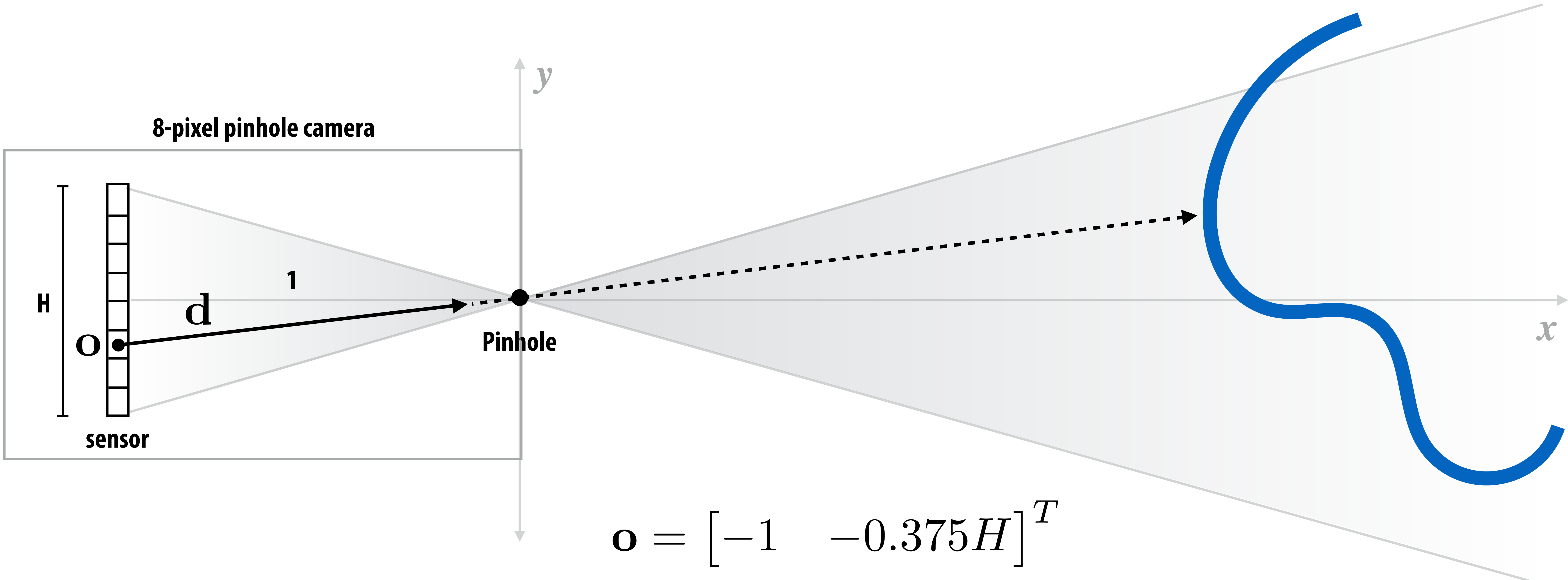
Computer Graphics
CMU 15-462/15-662, Fall 2015

Slides credit: a majority of these slides were created by Matt Pharr and Pat Hanrahan

Simulating a pinhole camera

Imprecise statements you've heard me say so far in this class:

- **What is the "color" of the surface visible at each sample point?**
- **How much light arrives at sensor point through pinhole?**



$$\mathbf{o} = \begin{bmatrix} -1 & -0.375H \end{bmatrix}^T$$

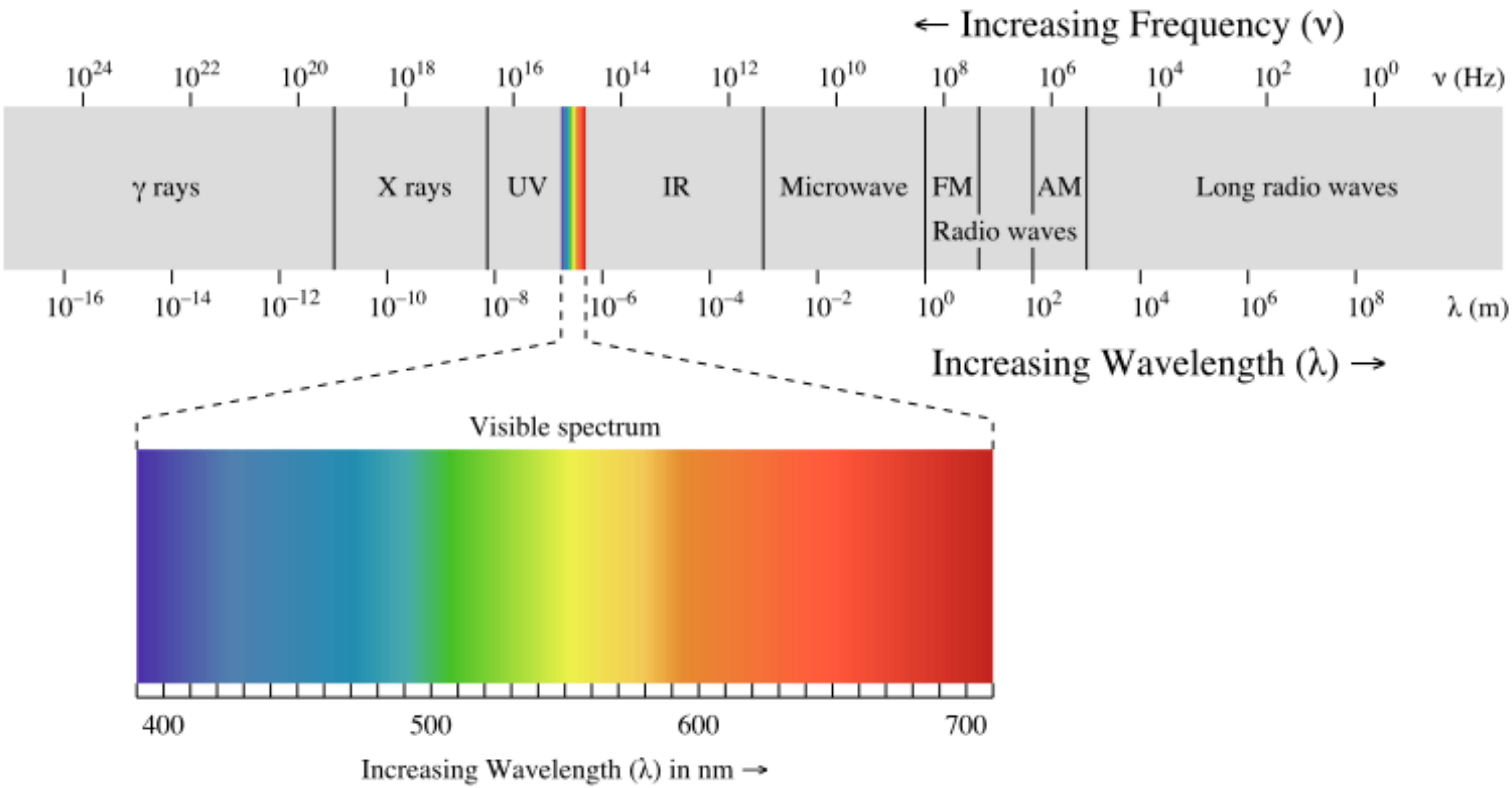
$$\mathbf{d} = \frac{\begin{bmatrix} 1 & 0.375H \end{bmatrix}^T}{\left\| \begin{bmatrix} 1 & 0.375H \end{bmatrix}^T \right\|}$$

Radiometry

- **System of units and measures for measuring illumination**

- **Geometric optics model of light**
  - **Photons travel in straight lines, represented by rays**
  - **Good approximation when wavelength << size of objects light interacts with**
    - **Doesn't model diffraction, interference, ...**

Light is electromagnetic radiation that is visible to eye



Lights: what do they do?

Cree 11 W LED light bulb
("60 Watt" incandescent replacement)

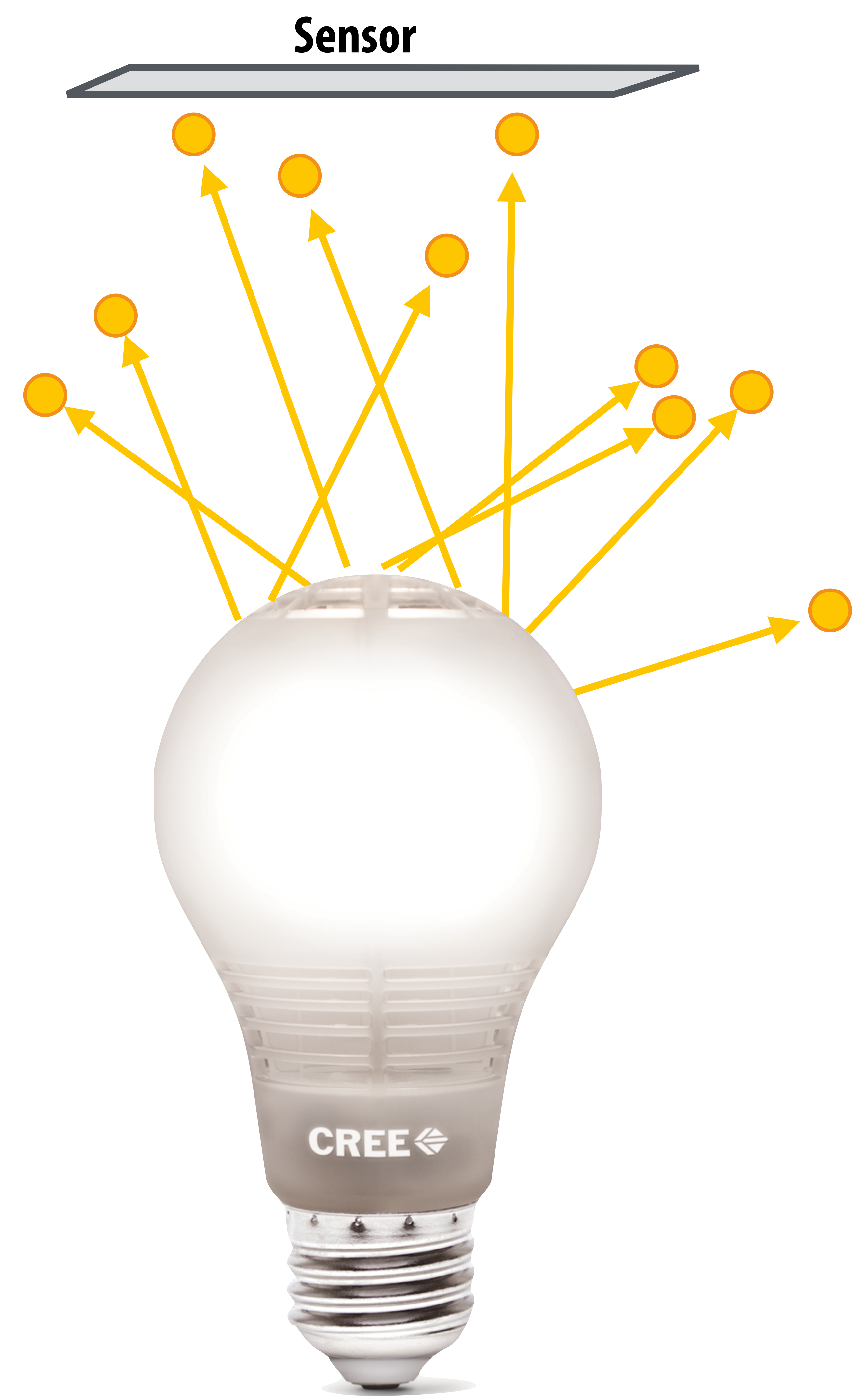
- Physical process converts input energy into photons
  - Each photon carries a small amount of energy
- Over some amount of time, light fixture consumes some amount of energy, Joules
  - Some input energy is turned into heat, some into photons
- Energy of photons hitting an object ~ exposure
  - Film, sensors, sunburn, solar panels, …
- Graphics: generally assume "*steady state*" process
  - Rate of energy consumption = power, Watts (Joules/second)

Measuring illumination: radiant flux (power)

- Given a sensor, we can count how many photons reach it
  - Over a period of time, gives the power received by the sensor
- Given a light, consider counting the number of photons emitted by it
  - Over a period of time, gives the power emitted by the light
- Energy carried by a photon:

$$Q = \frac{hc}{\lambda}$$

$$h \approx 6.626 \times 10^{-34}$$



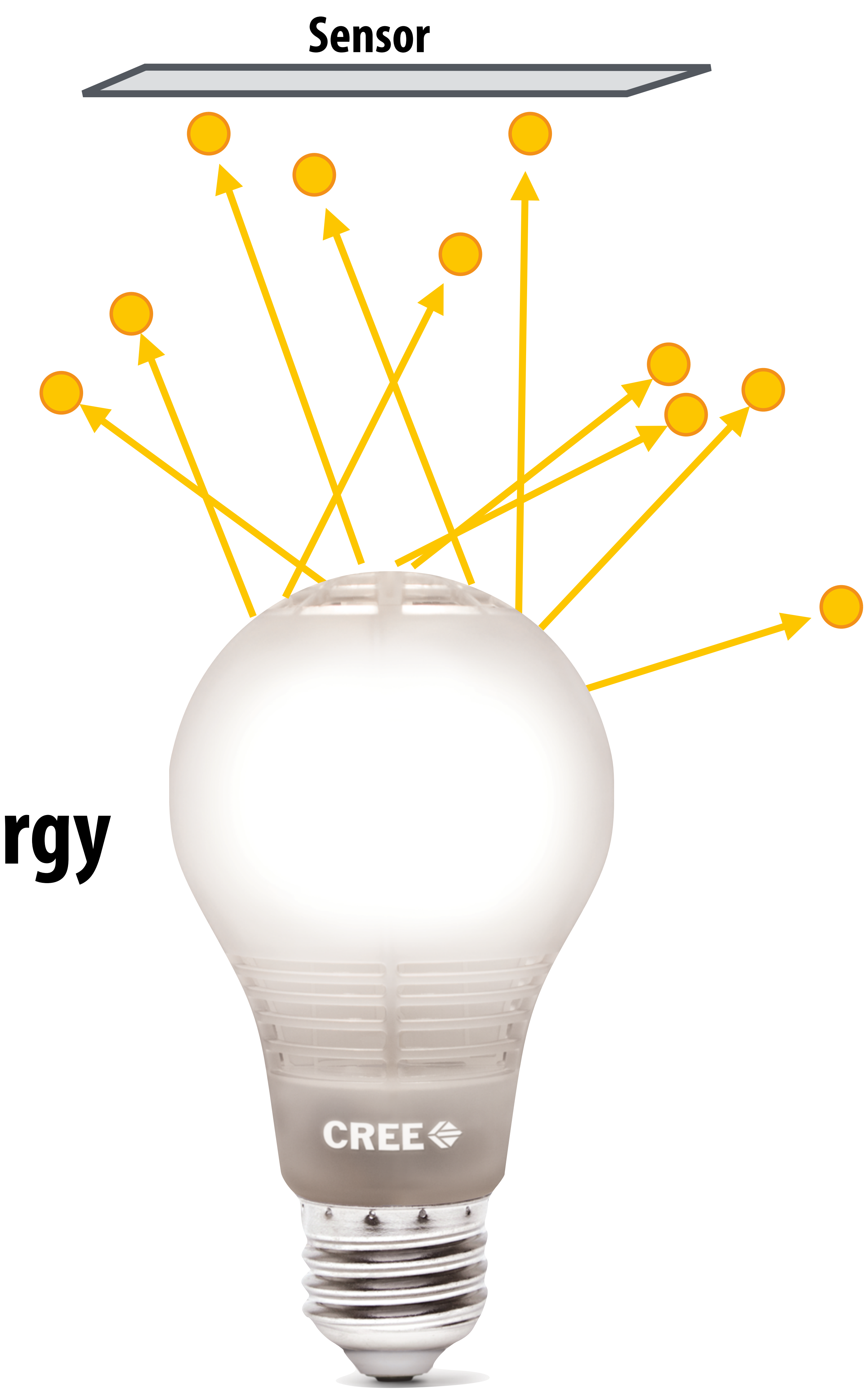
Measuring illumination: radiant flux (power)

- **Flux: energy per unit time (Watts) received by the sensor (or emitted by the light)**

$$\Phi = \lim_{\Delta \to 0} \frac{\Delta Q}{\Delta t} = \frac{\mathrm{d}Q}{\mathrm{d}t} \quad \left[\frac{\mathrm{J}}{\mathrm{s}}\right]$$

- **Time integral of flux is total radiant energy**

$$Q = \int_{t_0}^{t_1} \Phi(t)\,\mathrm{d}t$$



Spectral power distribution

- Describes distribution of energy by wavelength

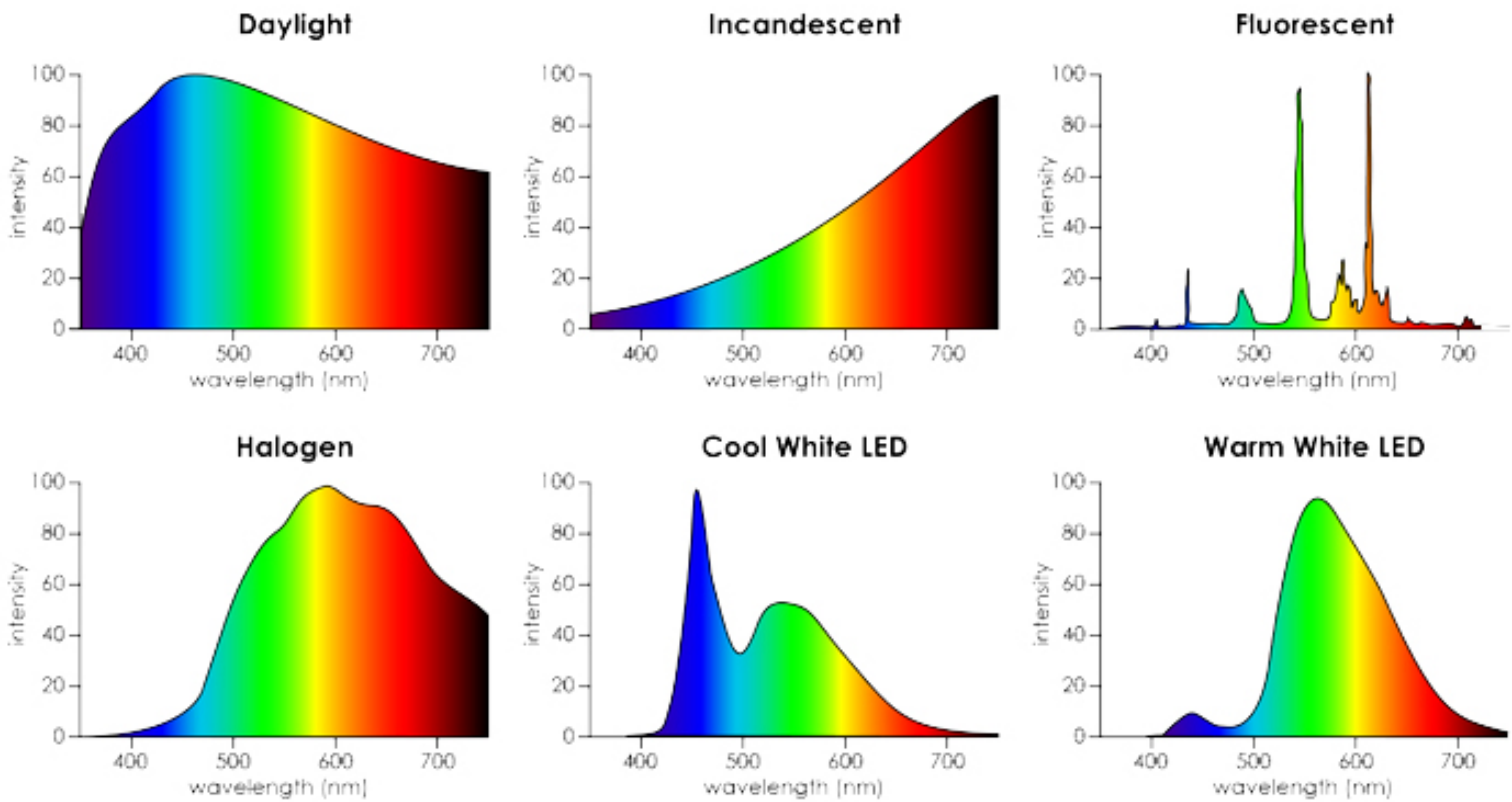
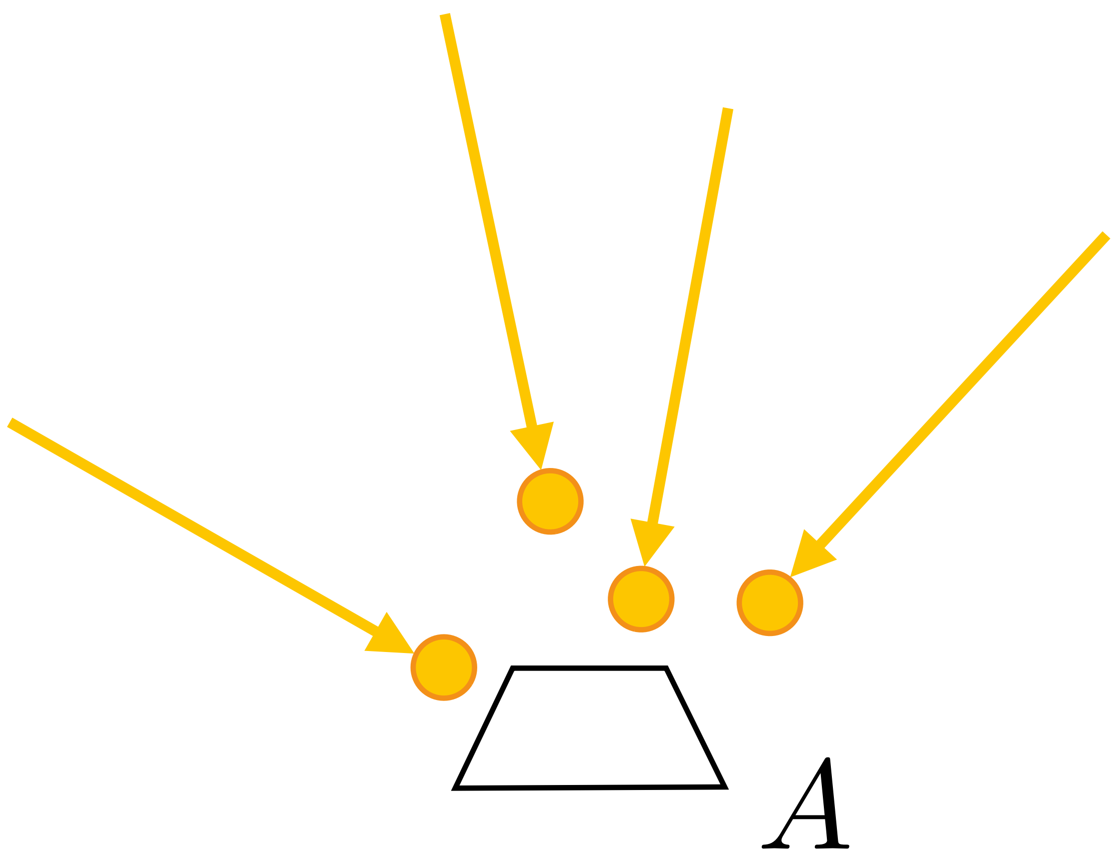


Figure credit:

admesy
ADVANCED MEASUREMENT SYSTEMS

Measuring illumination: irradiance

- **Flux: time density of energy**
- **Irradiance: area density of flux**



Given a sensor of with area *A*, we can consider the average flux over the entire sensor area:

$$\frac{\Phi}{A}$$

Irradiance (E) is given by taking the limit of area at a single point on the sensor:

$$E(\mathrm{p}) = \lim_{\Delta \to 0} \frac{\Delta\Phi(\mathrm{p})}{\Delta A} = \frac{\mathrm{d}\Phi(\mathrm{p})}{\mathrm{d}A} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2}\right]$$

Beam power in terms of irradiance

Consider beam with flux Φ incident on surface with area A

$$E = \frac{\Phi}{A}$$

$$\Phi = EA$$

Projected area

Consider beam with flux Φ incident on angled surface with area *A′*

$$A = A' \cos\theta$$

A = projected area of surface relative to direction of beam

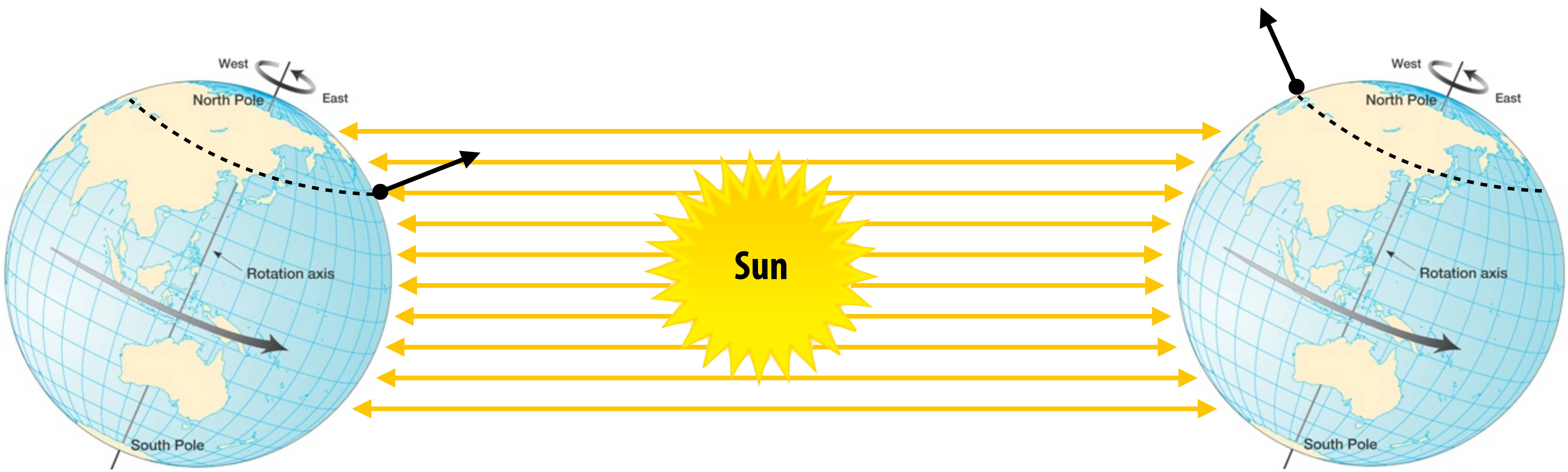
Lambert's Law

Irradiance at surface is proportional to cosine of angle between light direction and surface normal.

$$A = A' \cos\theta$$

$$E = \frac{\Phi}{A'} = \frac{\Phi \cos\theta}{A}$$

Why do we have seasons?



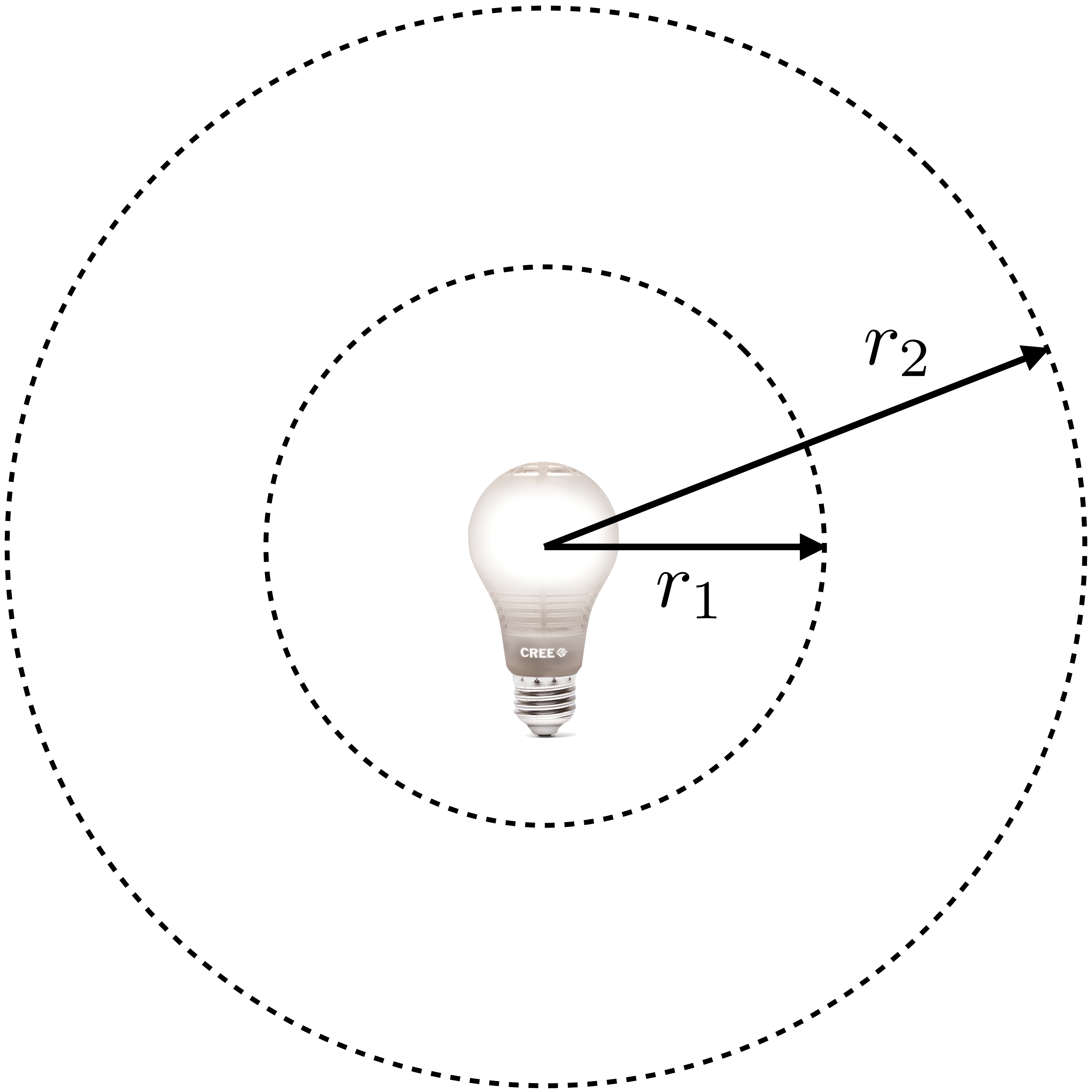
Summer
(Northern hemisphere)

Winter
(Northern hemisphere)

Earth's axis of rotation: ~23.5° off axis

[Image credit: Pearson Prentice Hall]

Irradiance falloff with distance



Assume light is emitting flux Φ in a uniform angular distribution

Compare irradiance at surface of two spheres:

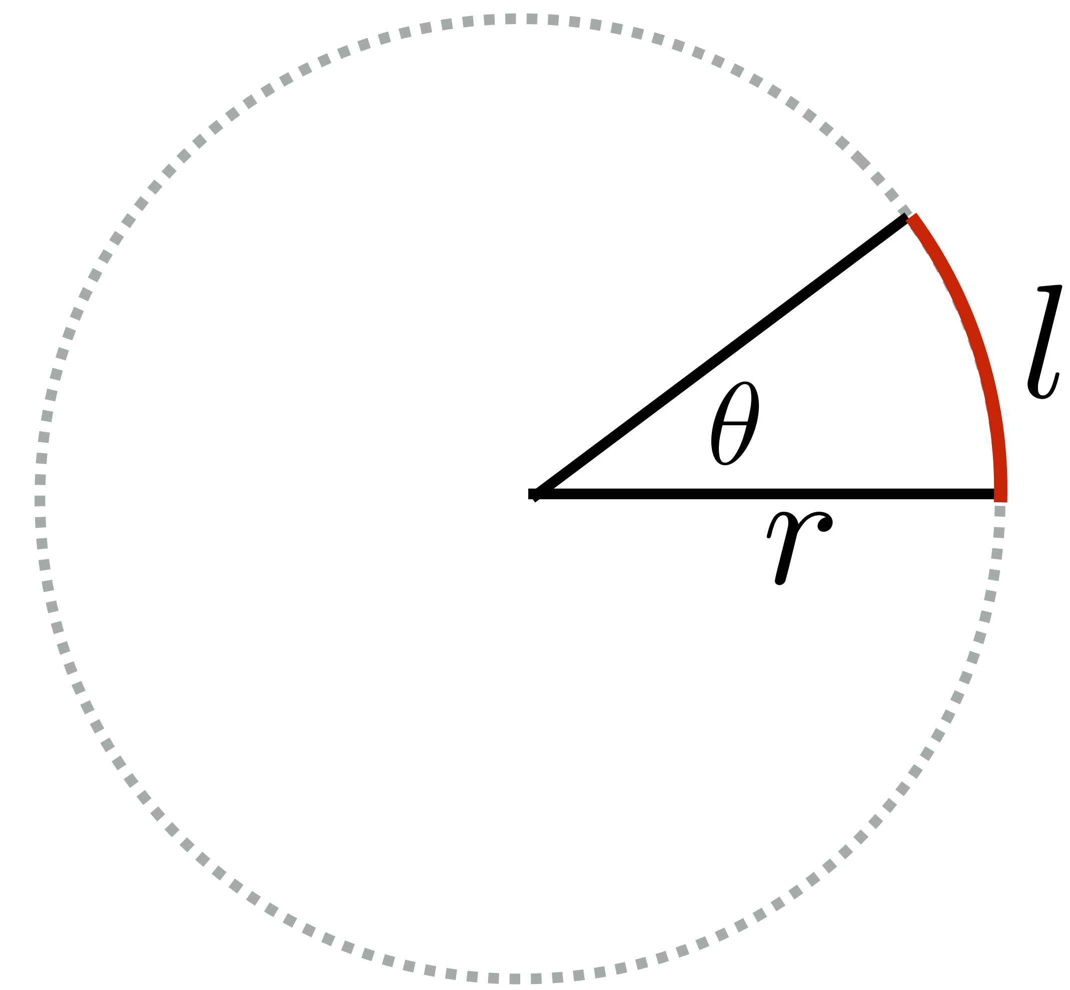
$$E_1 = \frac{\Phi}{4\pi r_1^2} \rightarrow \Phi = 4\pi r_1^2 E_1$$

$$E_2 = \frac{\Phi}{4\pi r_2^2} \rightarrow \Phi = 4\pi r_2^2 E_2$$

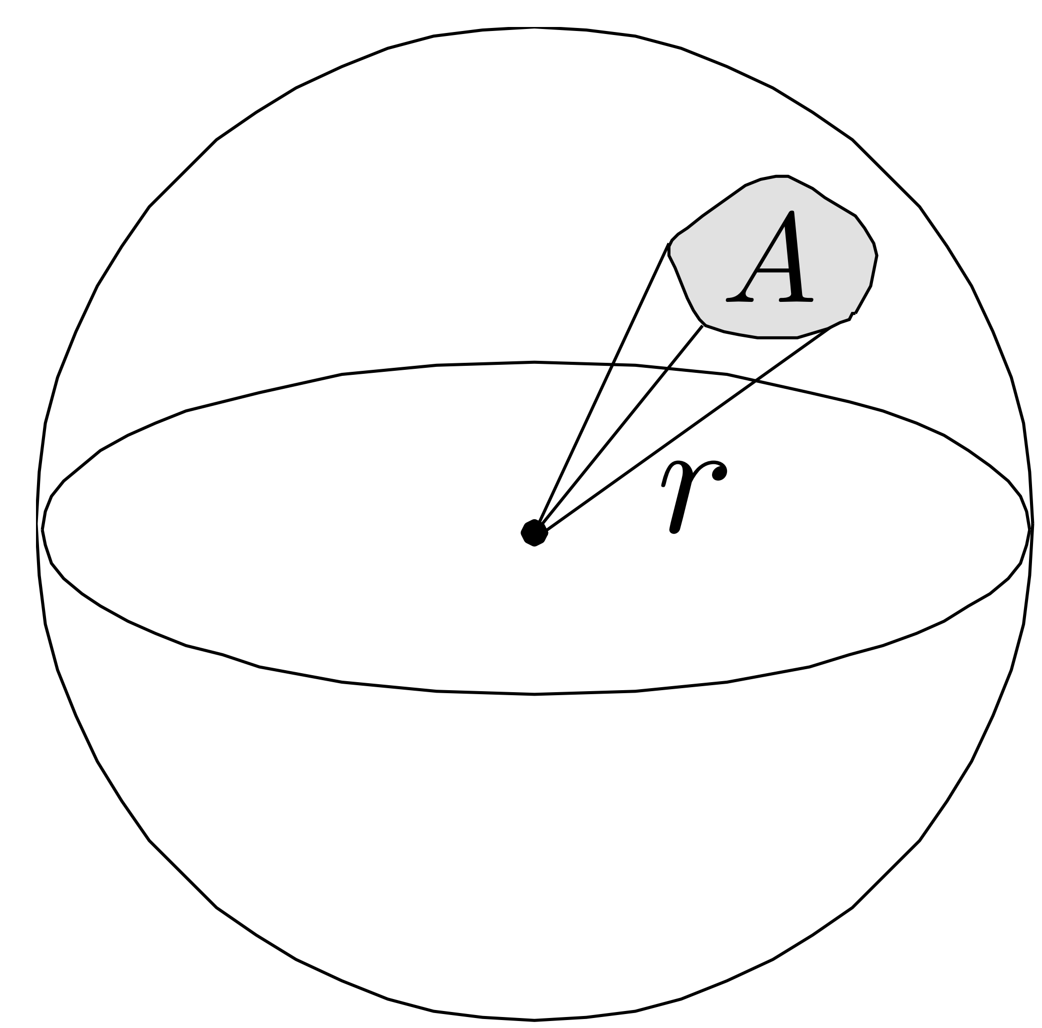
$$\frac{E_2}{E_1} = \frac{r_1^2}{r_2^2}$$

Angles and solid angles

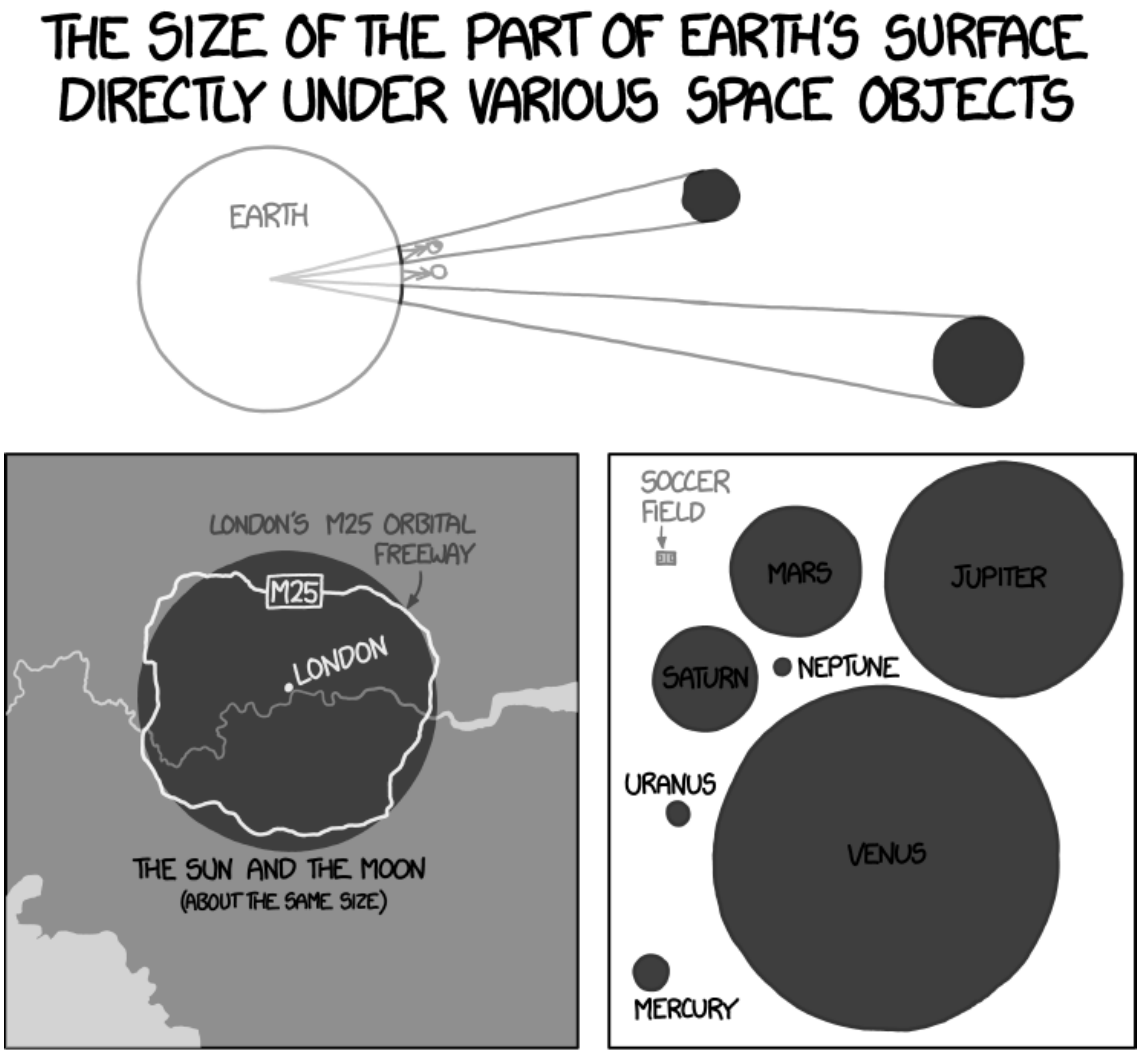
- **Angle: ratio of subtended arc length on circle to radius**
  - $\theta = \frac{l}{r}$
  - **Circle has 2π radians**



- **Solid angle: ratio of subtended area on sphere to radius squared**
  - $\Omega = \frac{A}{r^2}$
  - **Sphere has 4π steradians**



Solid angles in practice

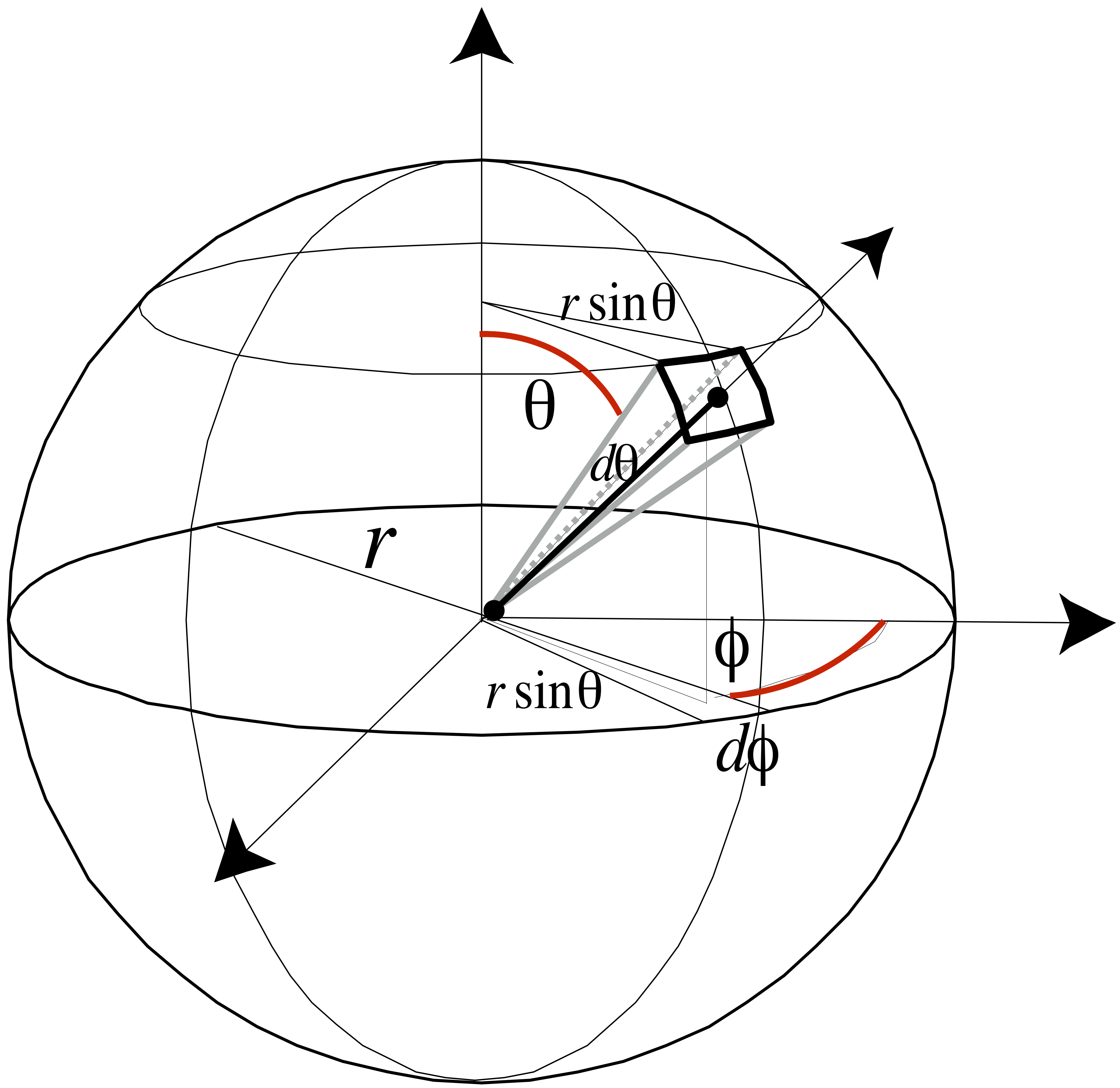


http://xkcd.com/1276/

- **Sun and moon both subtend ~60μ sr as seen from earth**
- **Surface area of earth: ~510M km²**
- **Projected area:**

$$510\text{Mkm}^2 \frac{60\mu\text{sr}}{4\pi\text{sr}} = 510\frac{15}{\pi}$$
$$\approx 2400\text{km}^2$$

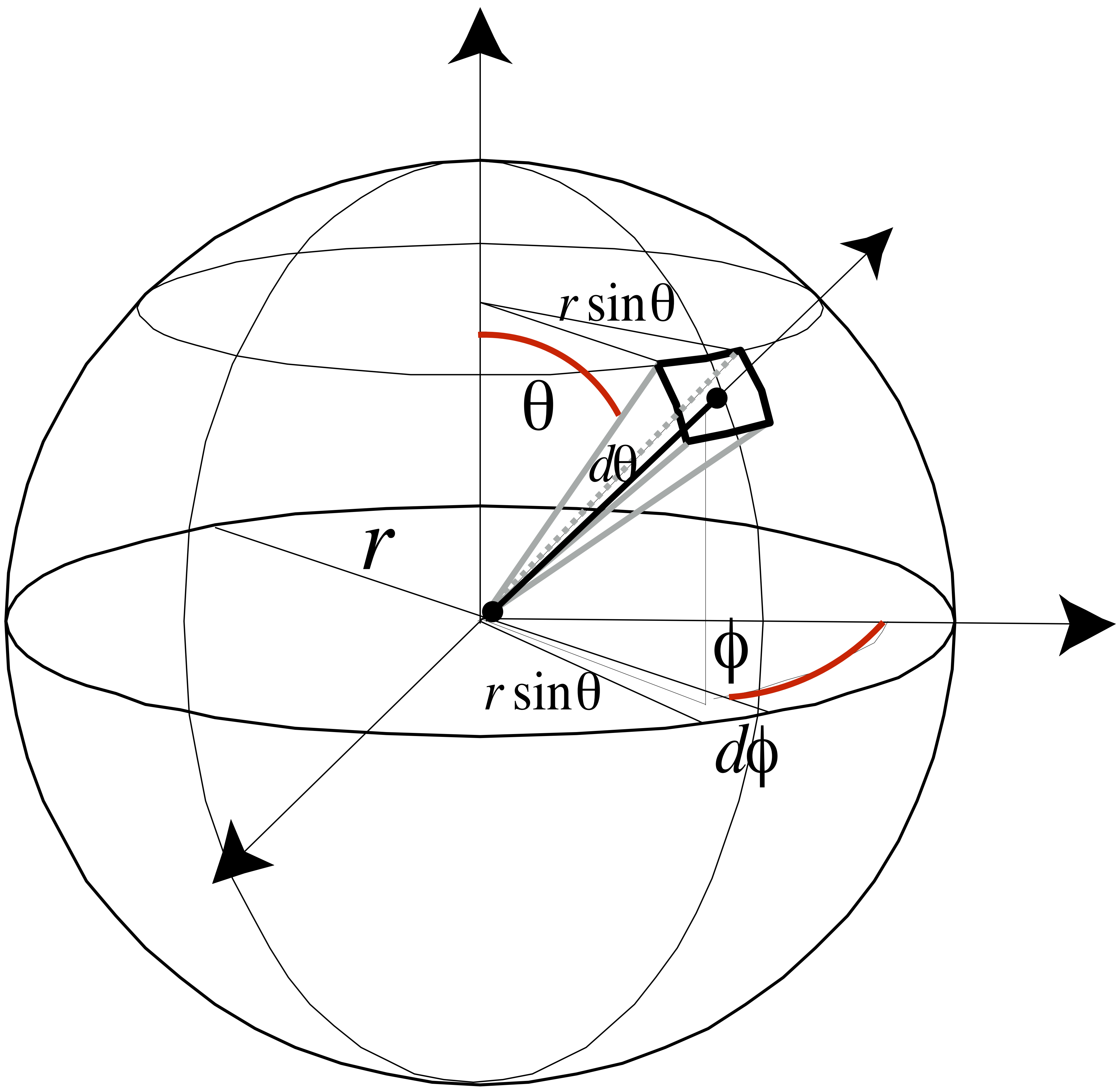
Differential solid angle



$$\mathrm{d}A = (r\,\mathrm{d}\theta)(r\sin\theta\,\mathrm{d}\phi)$$
$$= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$\mathrm{d}\omega = \frac{\mathrm{d}A}{r^2} = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

Differential solid angle

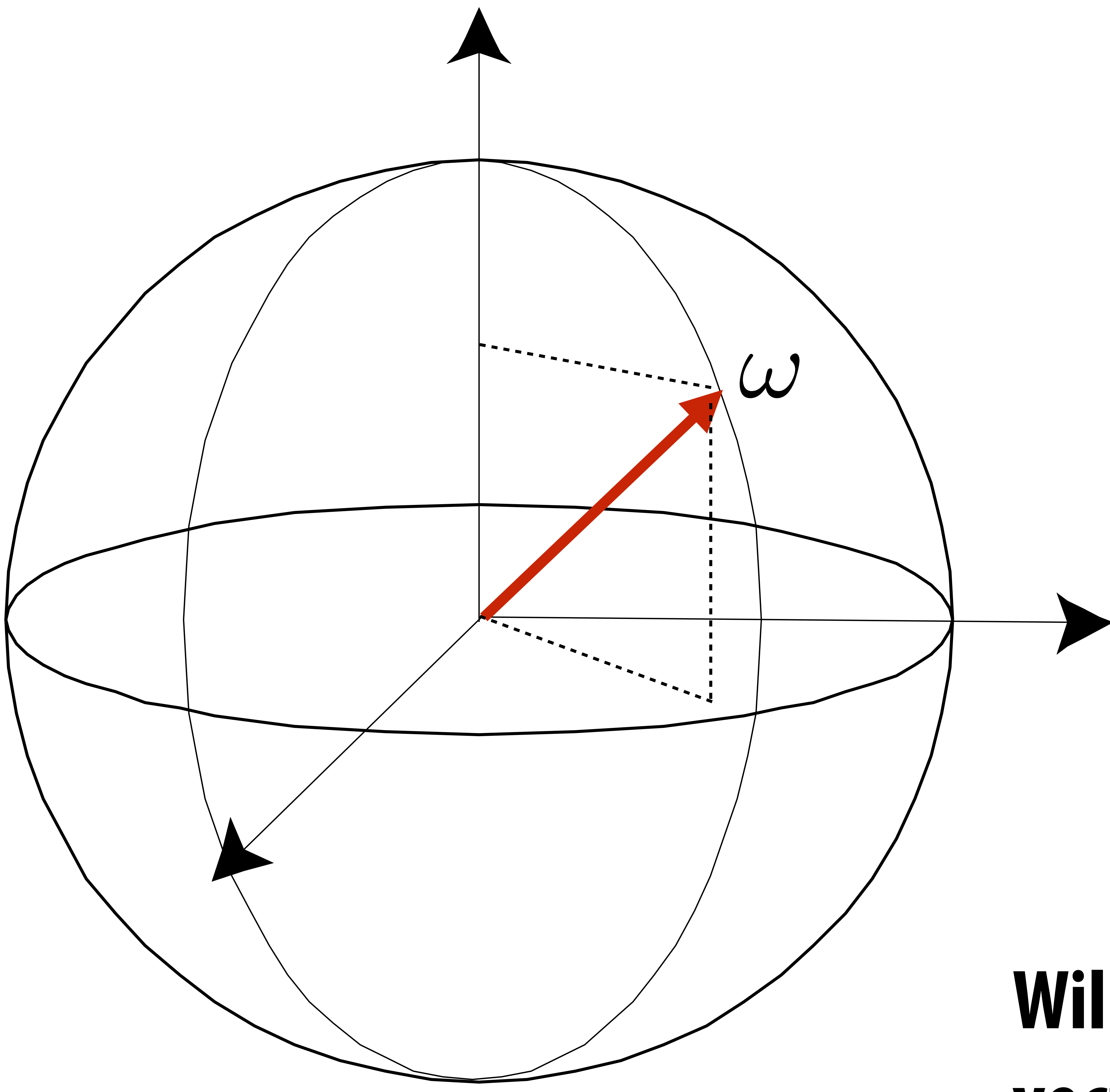


$$\Omega = \int_{S^2} \mathrm{d}\omega$$

$$= \int_0^{2\pi} \int_0^{\pi} \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

$$= 4\pi$$

ω as a direction vector



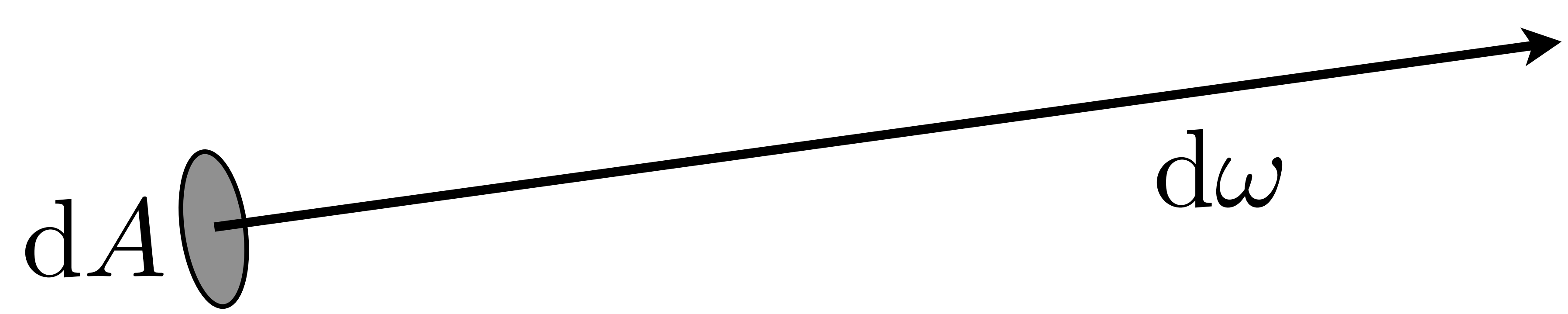
Will use ω to donate a direction vector (unit length)

Measuring illumination: radiance

- **Radiance is the solid angle density of irradiance**

$$L(\mathrm{p},\omega) = \lim_{\Delta\to 0} \frac{\Delta E_\omega(\mathrm{p})}{\Delta\omega} = \frac{\mathrm{d}E_\omega(\mathrm{p})}{\mathrm{d}\omega} \quad \left[\frac{\mathrm{W}}{\mathrm{m}^2\,\mathrm{sr}}\right]$$

where E_ω denotes that the differential surface area is oriented to face in the direction ω



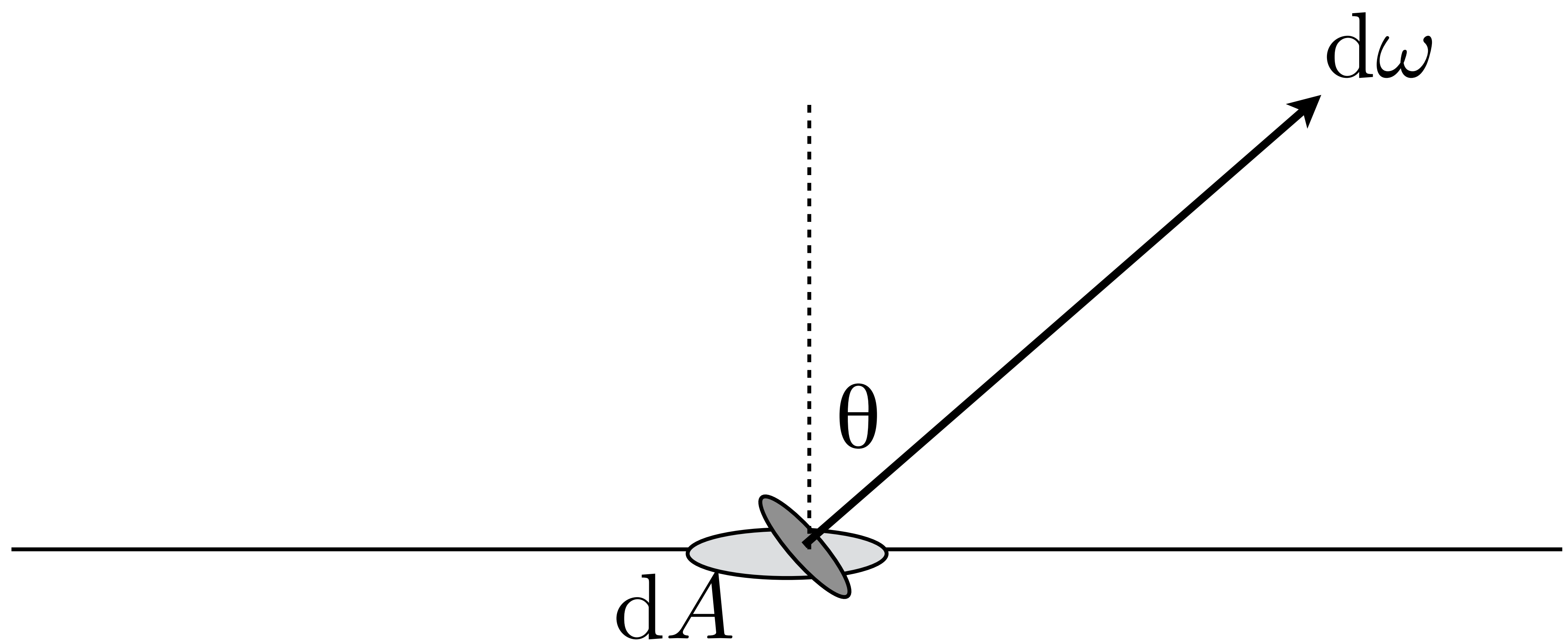
In other words, radiance is energy along a ray defined by origin point *p* and direction ω

Radiance functions

- **Equivalently,**

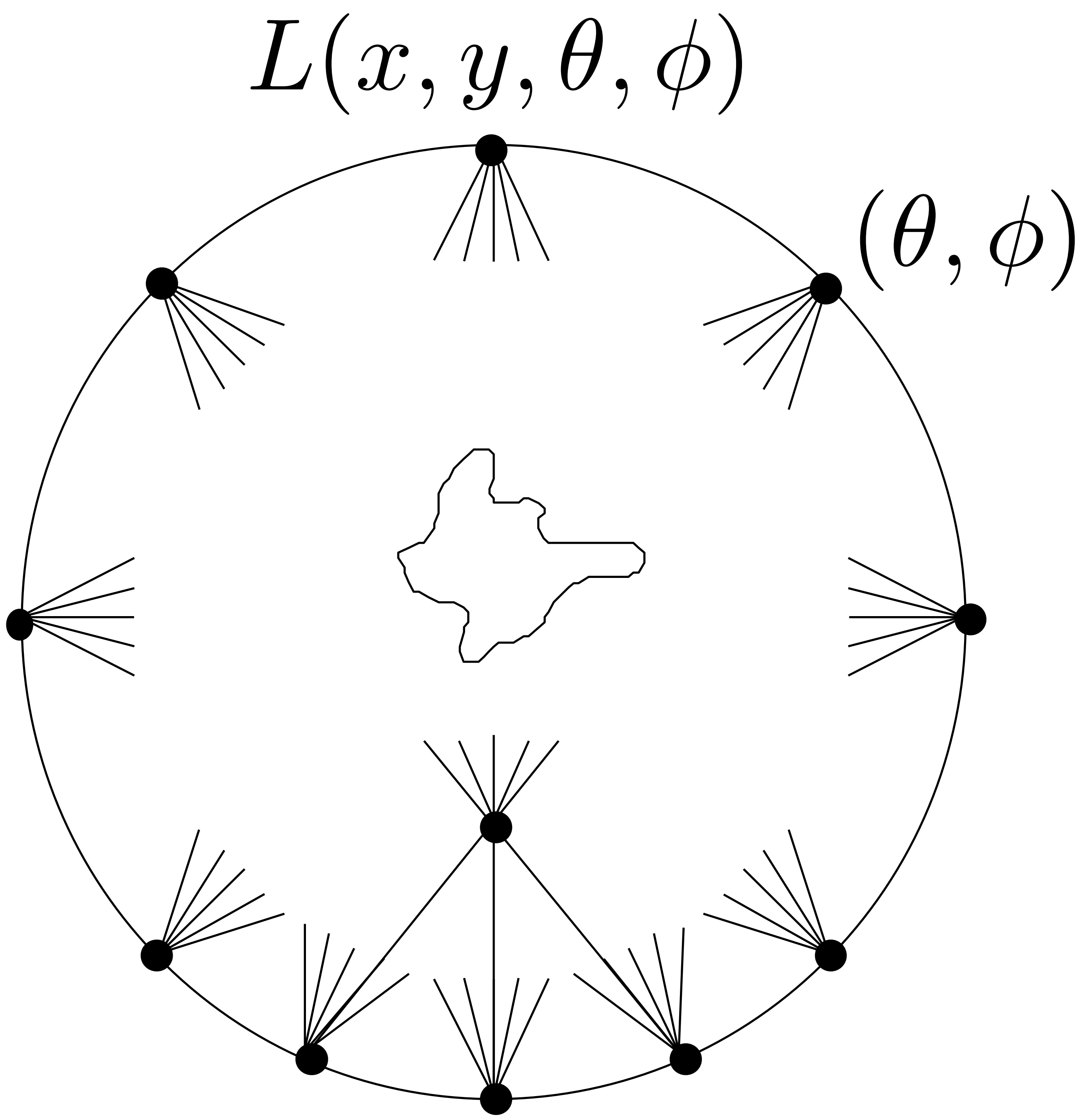
$$L(\mathrm{p}, \omega) = \frac{\mathrm{d}E(\mathrm{p})}{\mathrm{d}\omega \cos\theta} = \frac{\mathrm{d}^2\Phi(\mathrm{p})}{\mathrm{d}A\,\mathrm{d}\omega\cos\theta}$$

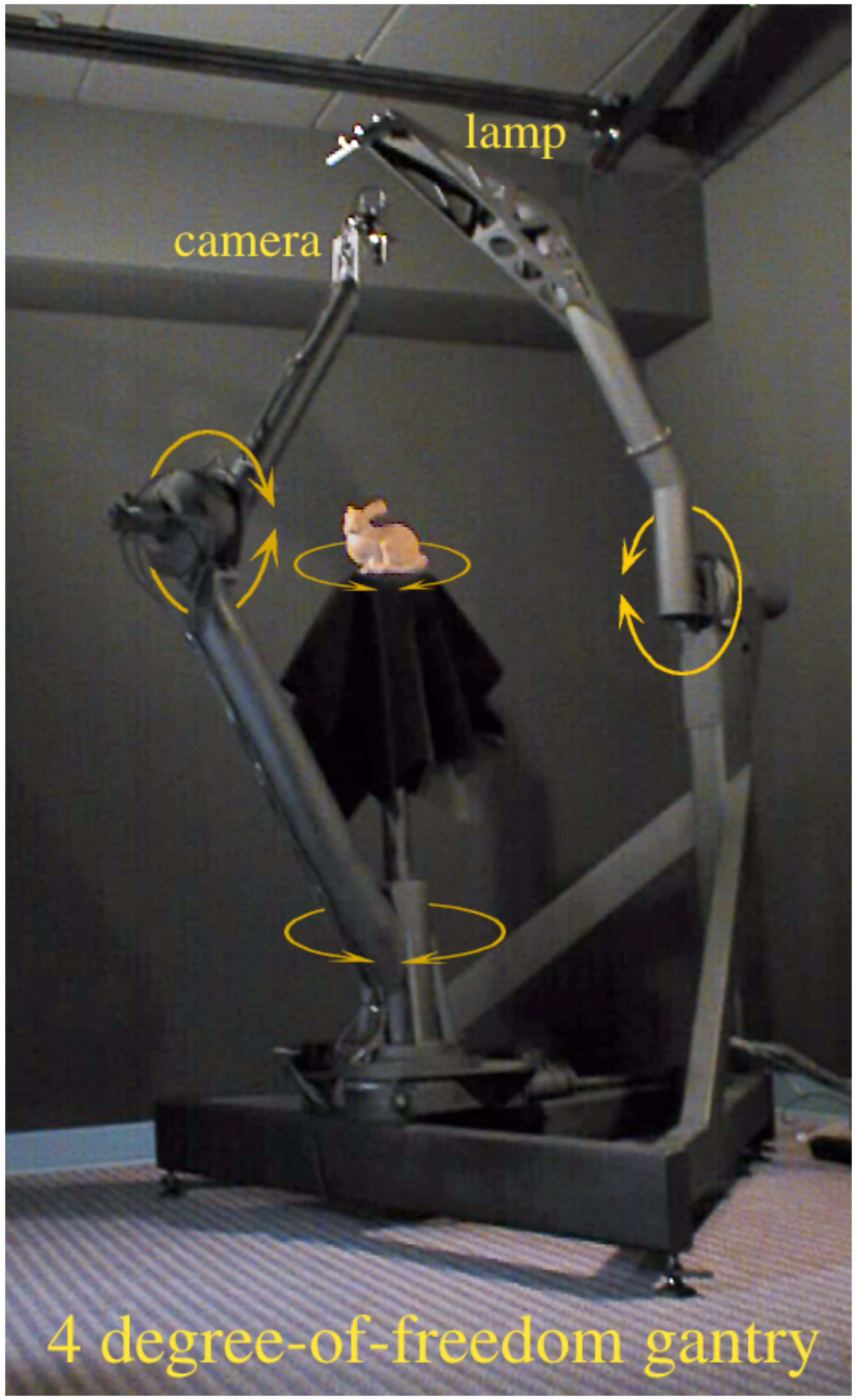
- **Previous slide described measuring radiance at a surface oriented in ray direction**
  - **cos(theta) accounts for different surface orientation**



Field radiance: the light field

- Light field = radiance function on rays
- Radiance is constant along rays *
- Spherical gantry: captures 4D light field (all light leaving object)

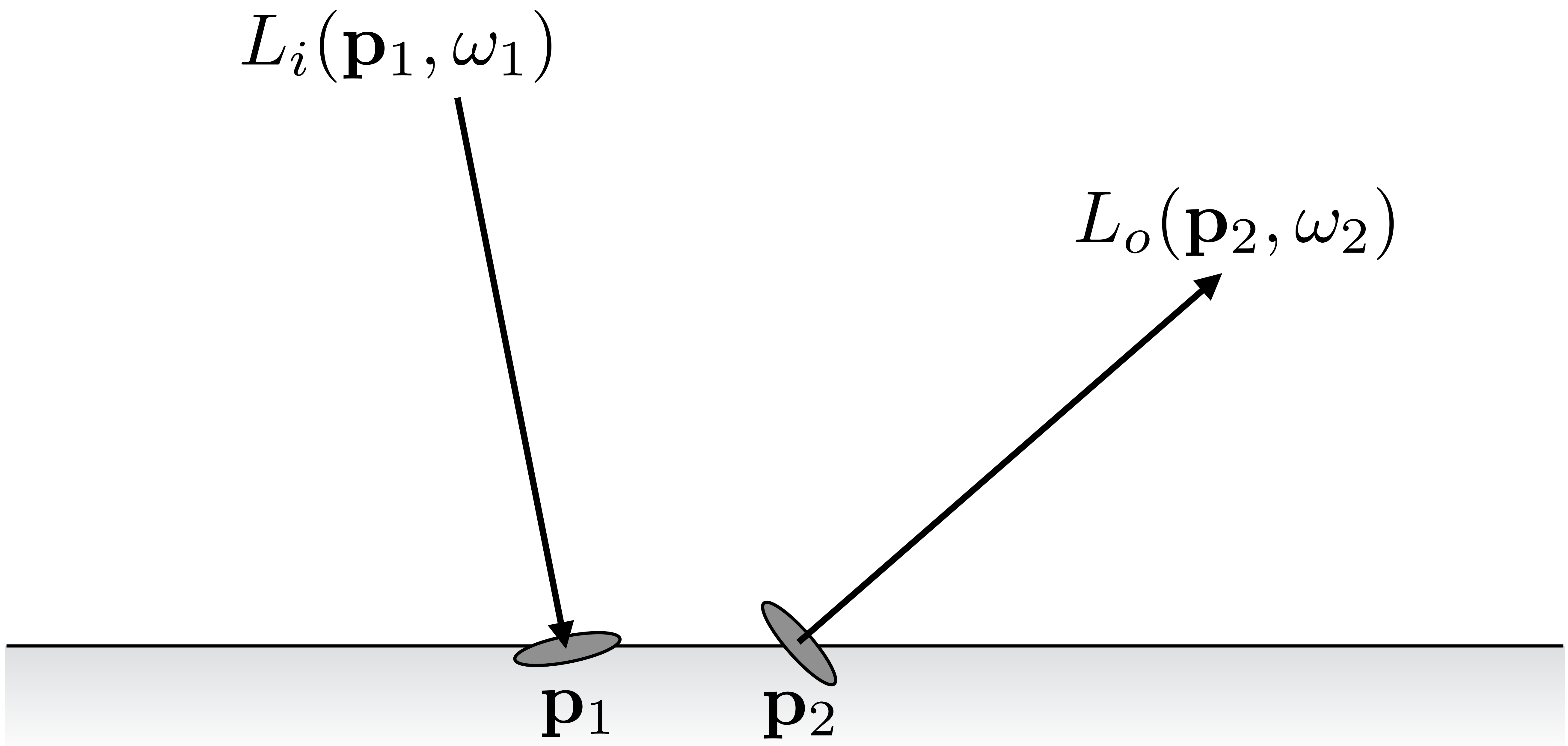




* in a vacuum

Surface radiance

- **Need to distinguish between incident radiance and exitant radiance functions at a point on a surface**



In general: $L_i(\mathbf{p}, \omega) \neq L_o(\mathbf{p}, \omega)$

Properties of radiance

- Radiance is a fundamental field quantity that characterizes the distribution of light in an environment
  - Radiance is the quantity associated with a ray
  - Rendering is all about computing radiance

- Radiance is invariant along a ray in a vacuum

- A pinhole camera measures radiance

Irradiance from the environment

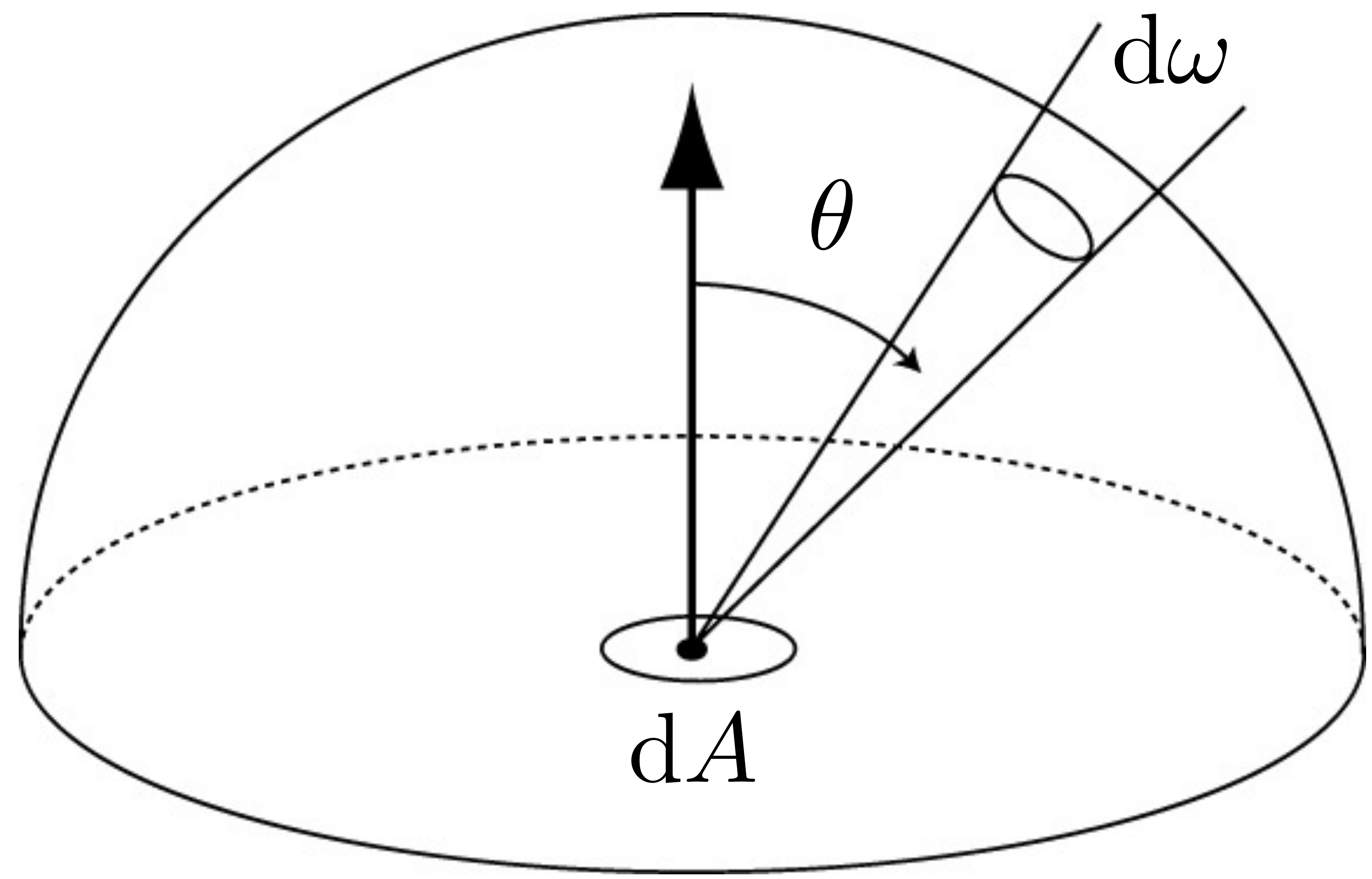
Computing flux per unit area on surface, due to incoming light from all directions.

$$dE(\mathrm{p},\omega) = L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

Contribution to irradiance from light arriving from direction ω

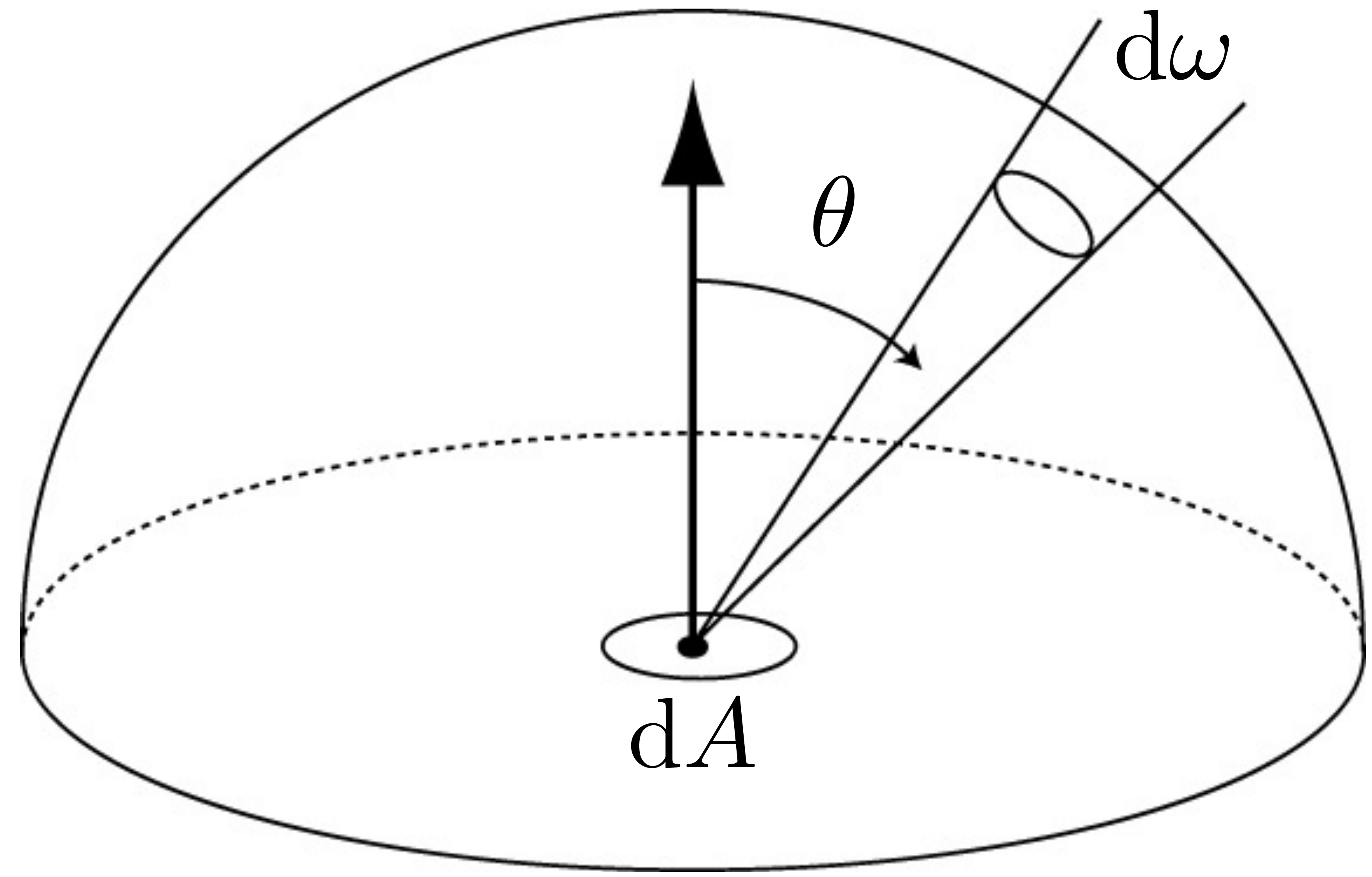
$$E(\mathrm{p},\omega) = \int_{H^2} L_i(\mathrm{p},\omega)\cos\theta\,\mathrm{d}\omega$$

Light meter



Simple case: irradiance from uniform hemispherical source

$$
\begin{aligned}
E(\mathrm{p}) &= \int_{H^2} L \, \mathrm{d}\omega \\
&= L \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \\
&= L\pi
\end{aligned}
$$



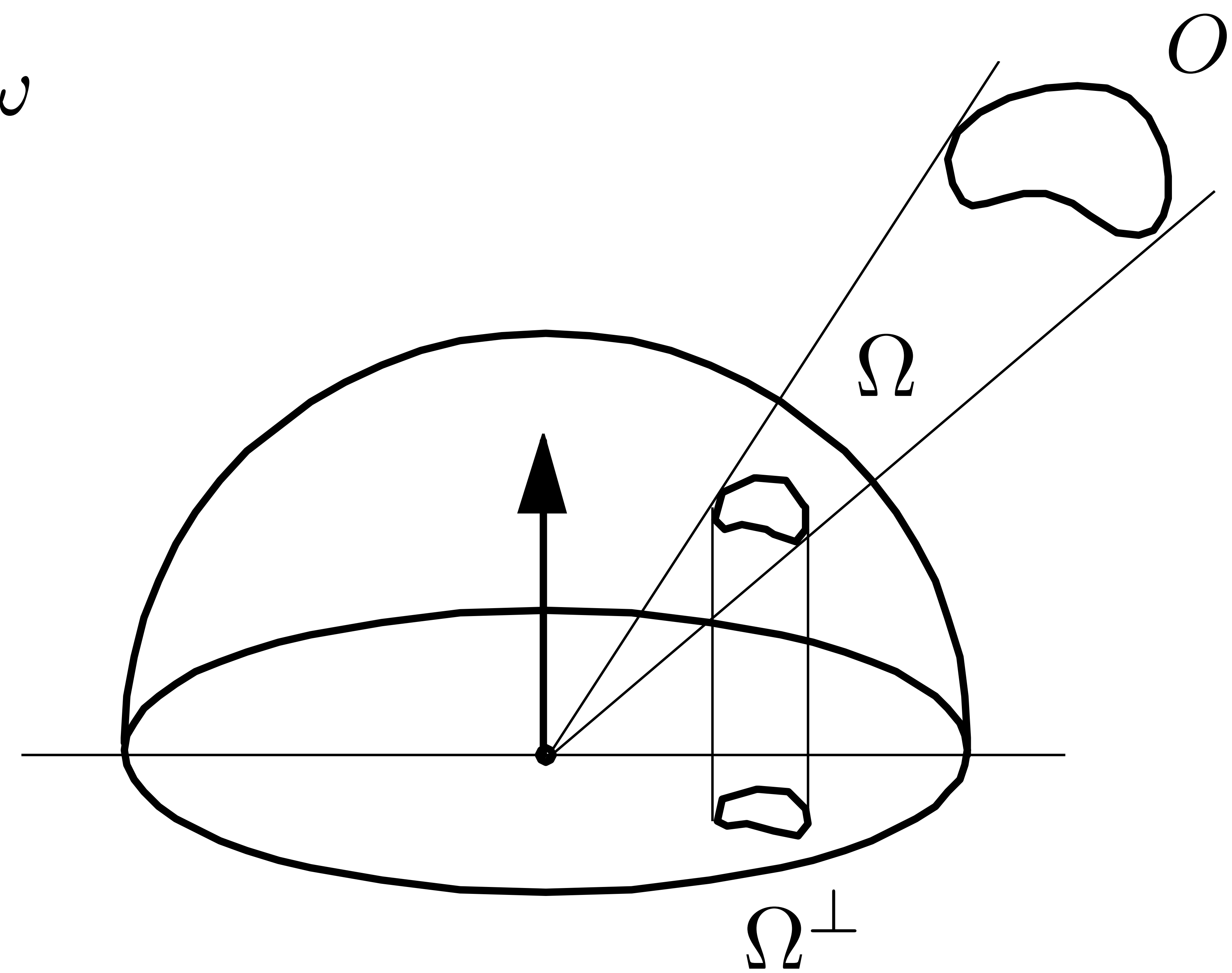
Irradiance from a uniform area source

(source emits radiance L)

$$E(\mathrm{p}) = \int_{H^2} L(\mathrm{p},\omega)\,\cos\theta\,\mathrm{d}\omega$$

$$= L\int_{\Omega} \cos\theta\,\mathrm{d}\omega$$

$$= L\Omega^{\perp}$$



Projected solid angle:

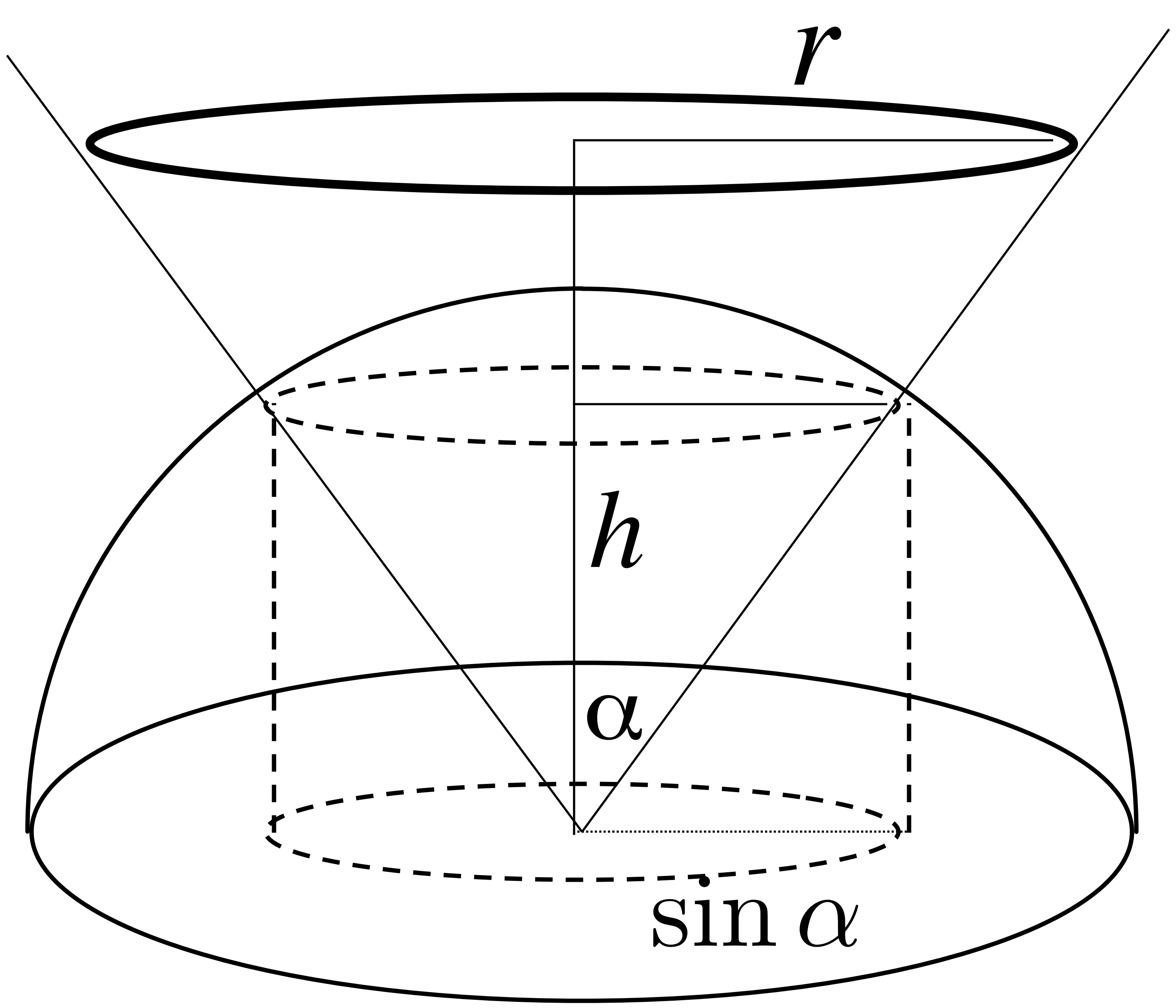
- **Cosine-weighted solid angle**
- **Area of object *O* projected onto unit sphere, then projected onto plane**

$$\mathrm{d}\omega^{\perp} = |\cos\theta|\,\mathrm{d}\omega$$

Uniform disk source (oriented perpendicular to plane)

Geometric Derivation

(using projected solid angle)



$$\Omega^{\perp} = \pi \sin^2 \alpha$$

Algebraic Derivation

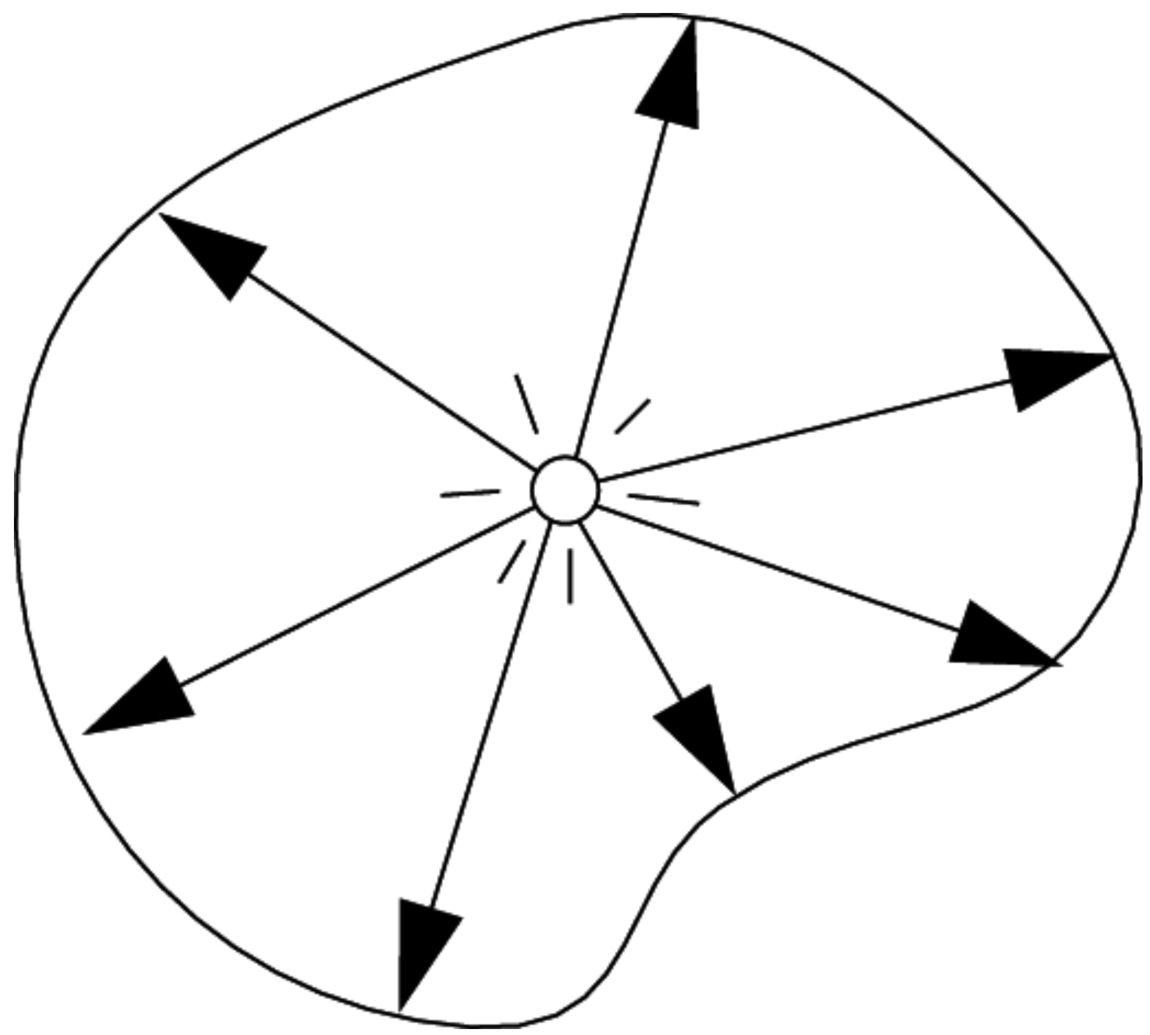
$$\Omega^{\perp} = \int_0^{2\pi} \int_0^{\alpha} \cos\theta \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

$$= 2\pi \frac{\sin^2 \theta}{2} \Big|_0^{\alpha}$$

$$= \pi \sin^2 \alpha$$

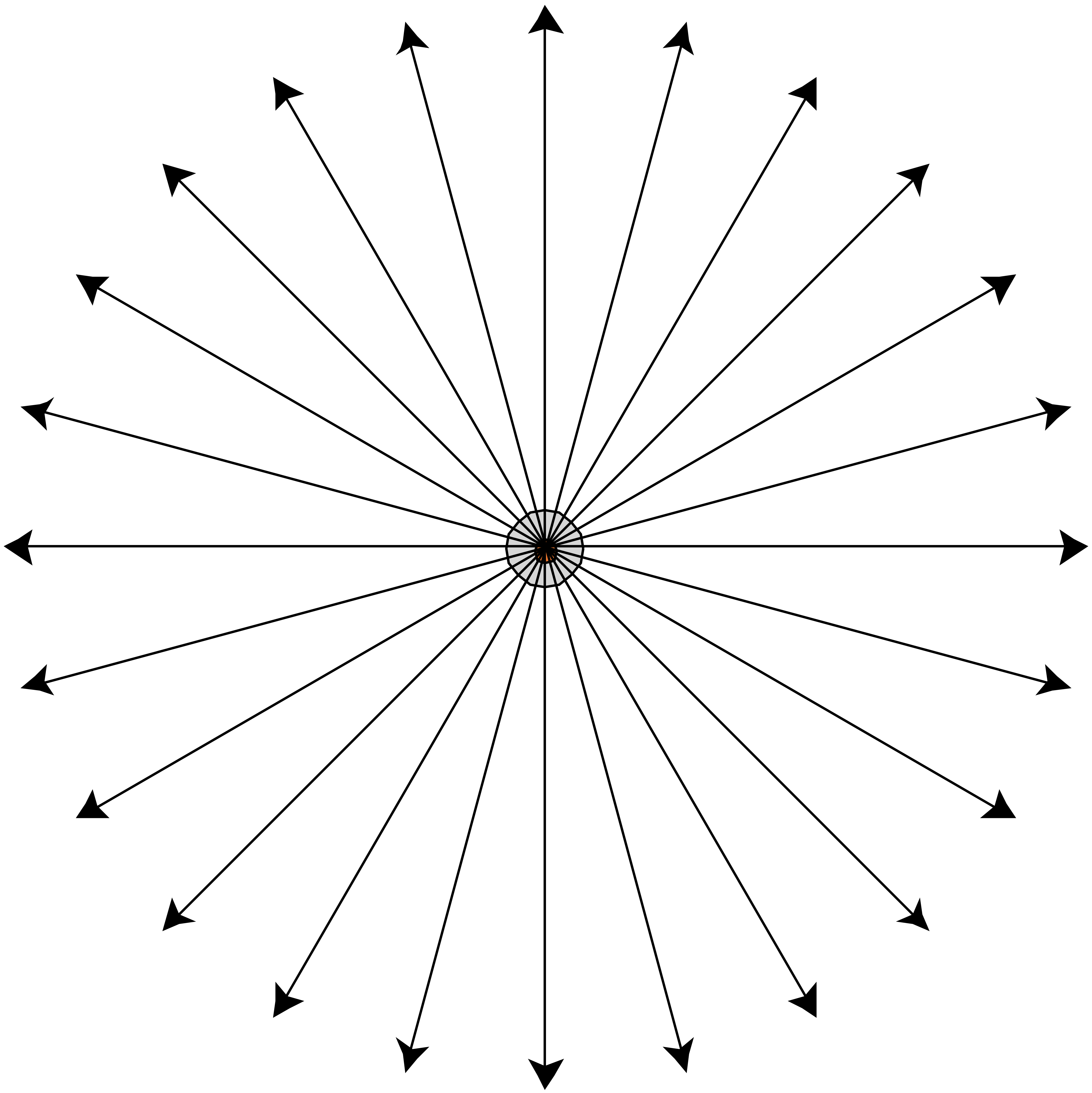
Measuring illumination: radiant intensity

- **Power per solid angle emanating from a point source**



$$I(\omega) = \frac{\mathrm{d}\Phi}{\mathrm{d}\omega} \quad \left[\frac{\mathrm{W}}{\mathrm{sr}}\right]$$

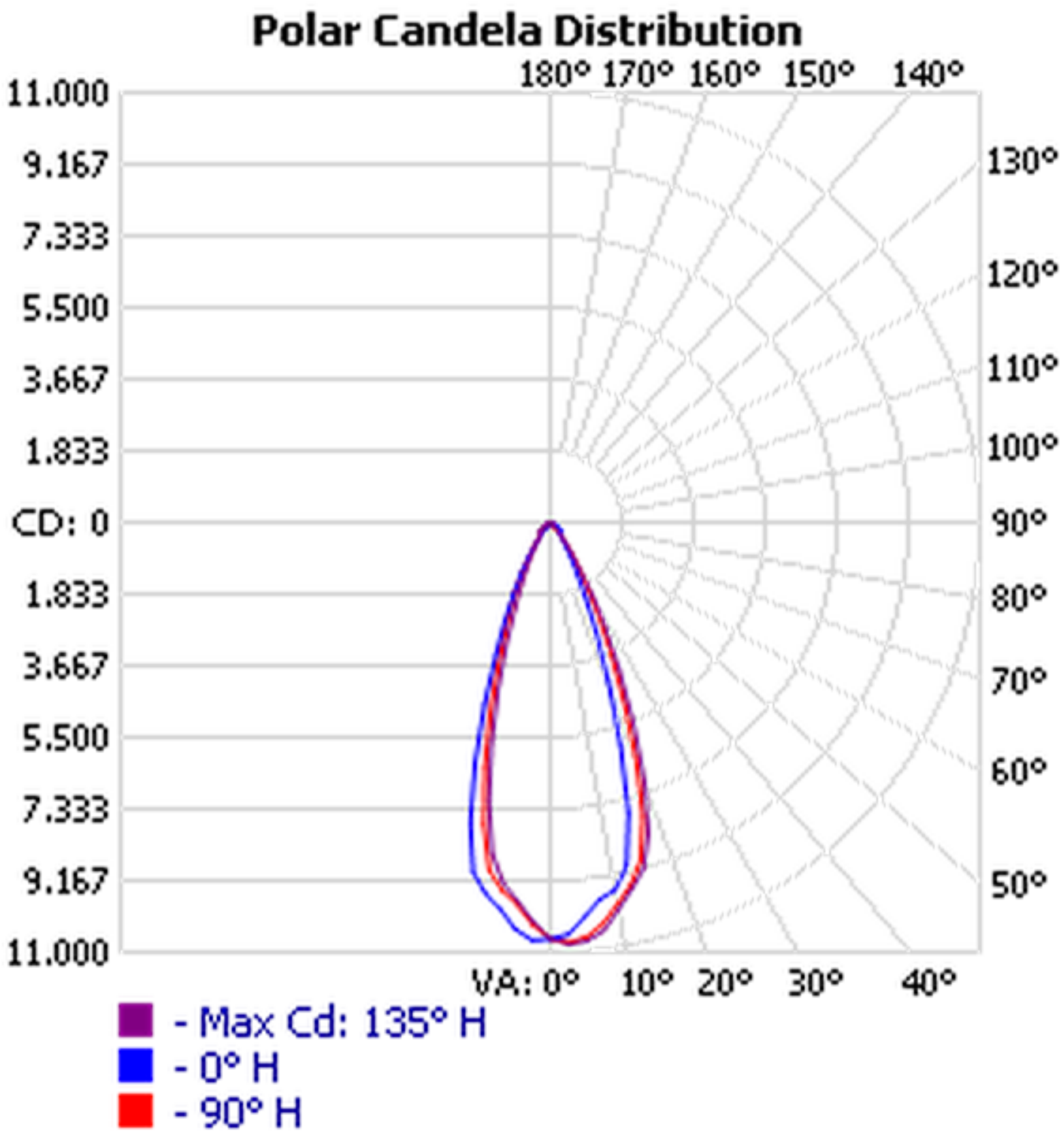
Isotropic point source

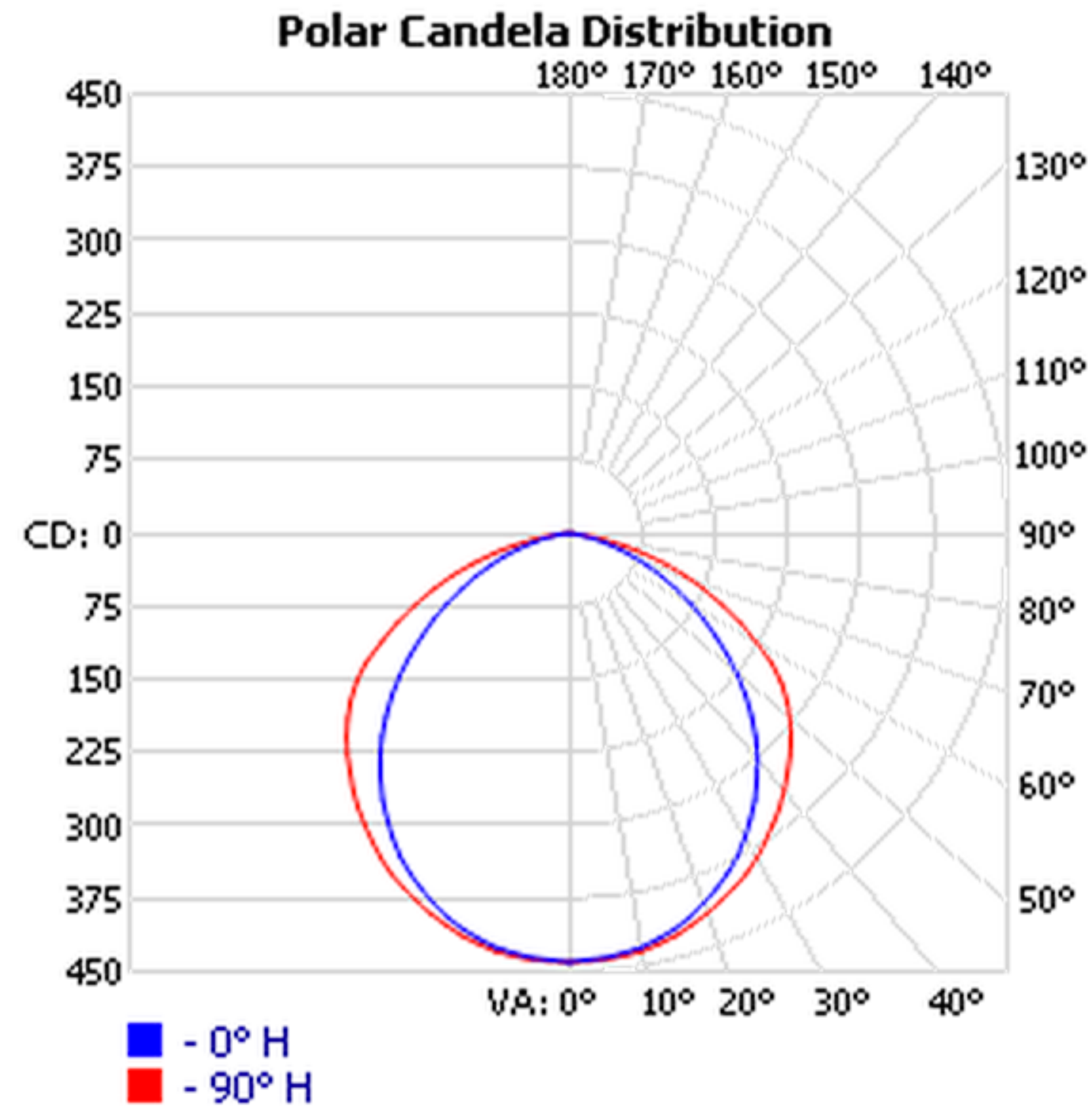


$$\Phi = \int_{S^2} I \, \mathrm{d}\omega$$
$$= 4\pi I$$

$$I = \frac{\Phi}{4\pi}$$

Goniometric diagrams





http://www.visual-3d.com/tools/photometricviewer/

Rendering with goniometric diagrams

Luminaire 1 - Louis Poulsen 94-02511

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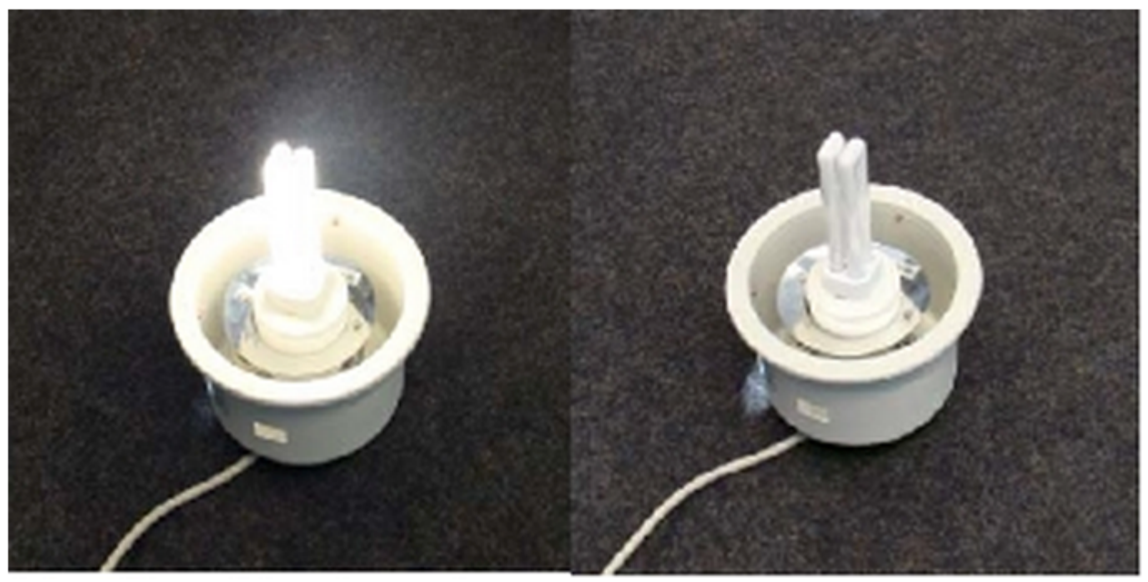

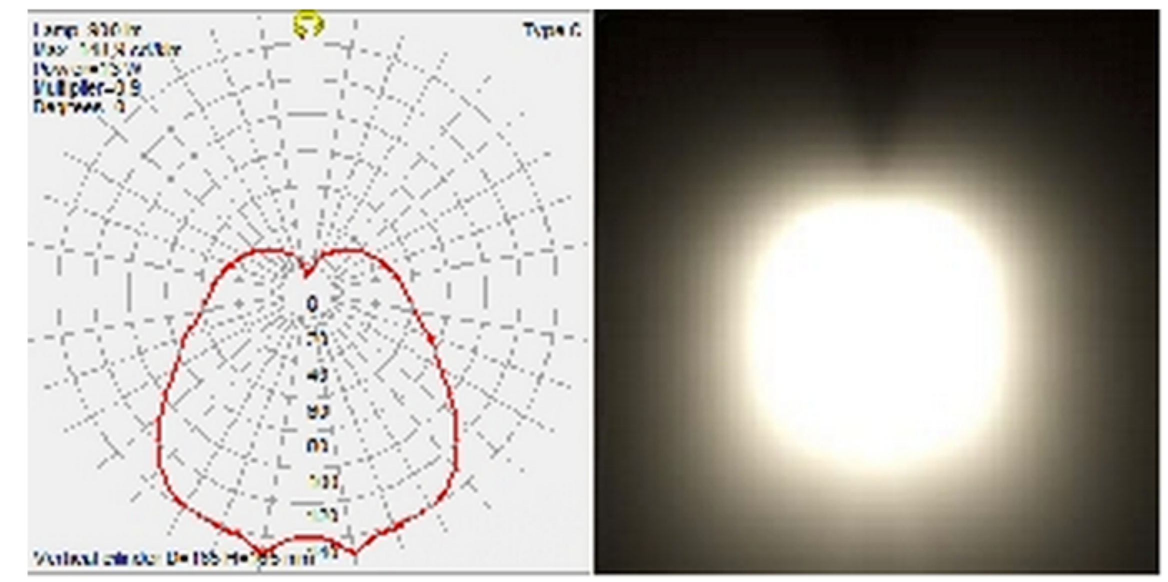

Luminaire 2 - Siemens 5LJ180 7-1CD2 EVG

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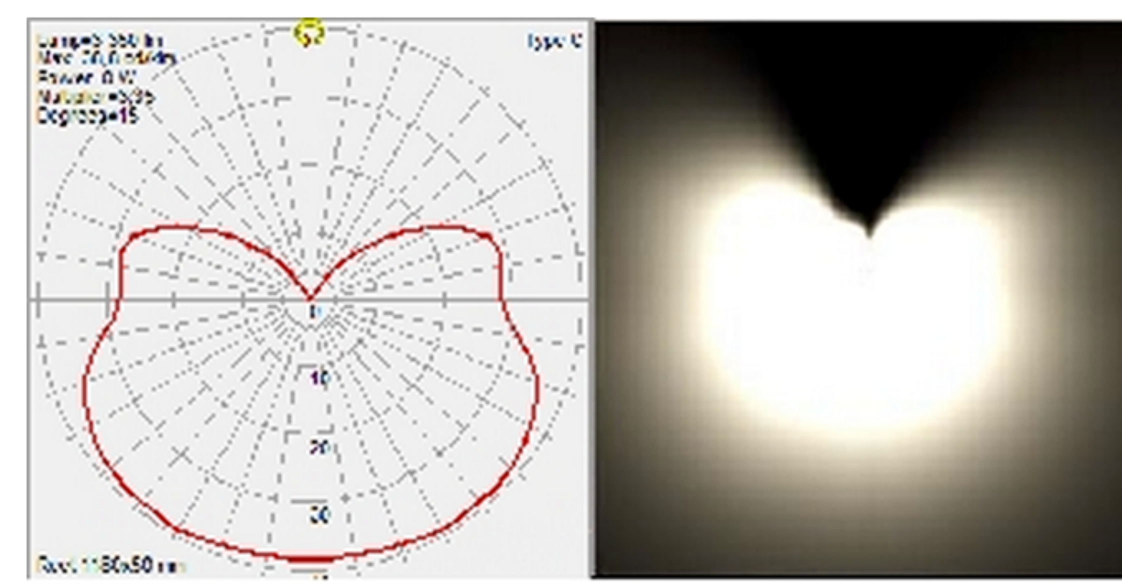
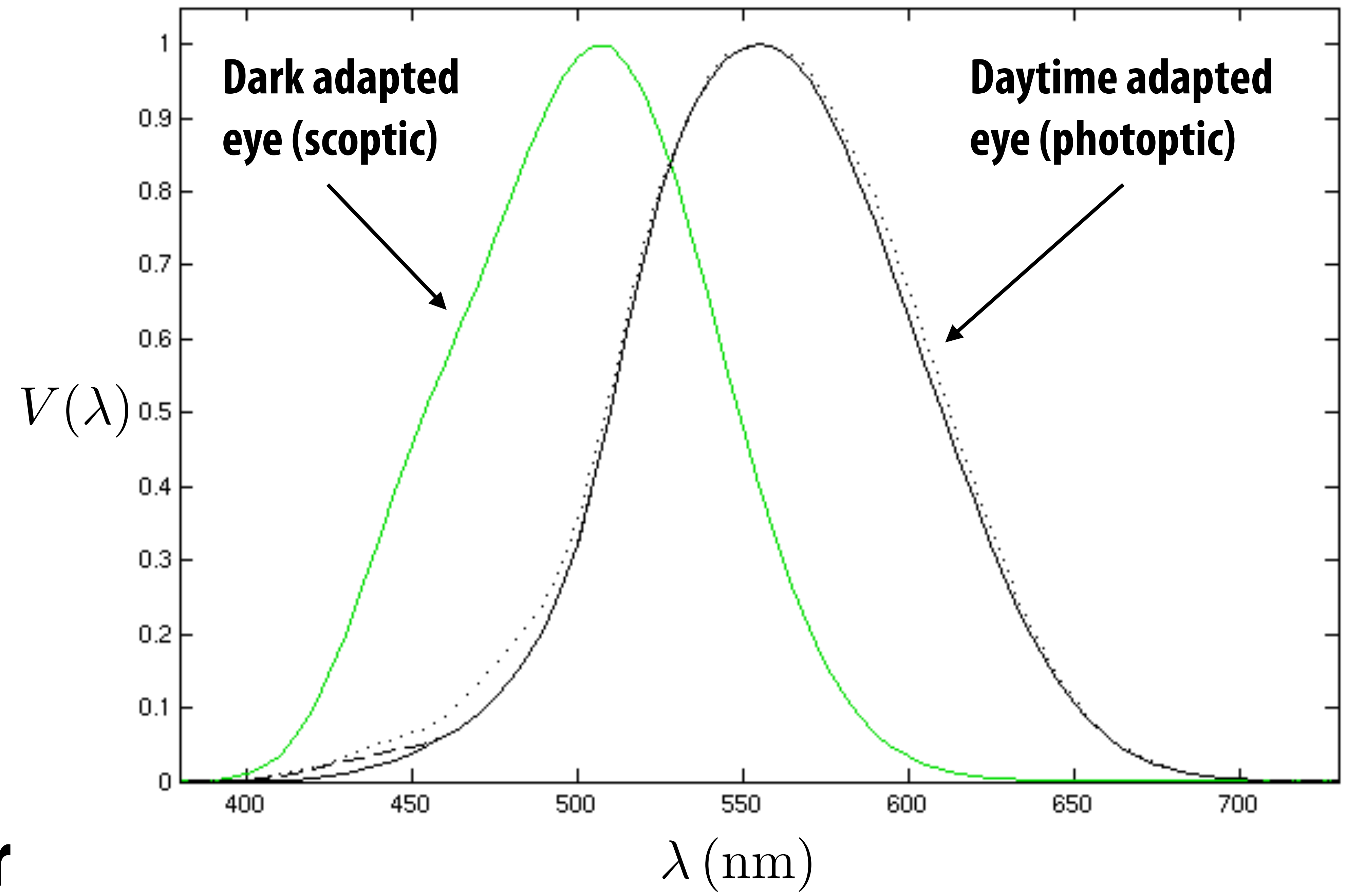


http://www.mpi-inf.mpg.de/resources/mpimodel/v1.0/luminaires/index.html

Photometry

- All radiometric quantities have equivalents in photometry
- Photometry: accounts for response of human visual system to electromagnetic radiation
- Luminance (Y) is photometric quantity that corresponds to radiance: integrate radiance over all wavelengths, weight by eye's luminous efficiacy curve, e.g.:



$$Y(\mathrm{p},\omega) = \int_0^\infty L(\mathrm{p},\omega,\lambda)\,V(\lambda)\,\mathrm{d}\lambda$$

https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png

Radiometric and photometric terms

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance (incoming) Radiosity (outgoing)	Illuminance (incoming) Luminosity (outgoing)
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Photometric Units

Photometry	MKS	CGS	British
Luminous Energy	Talbot	Talbot	Talbot
Luminous Power	Lumen	Lumen	Lumen
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit, Apostlib, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela	Candela	Candela

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?" —*James Kajiya*

Modern LED light

Input power: 11 W

Output: 815 lumens

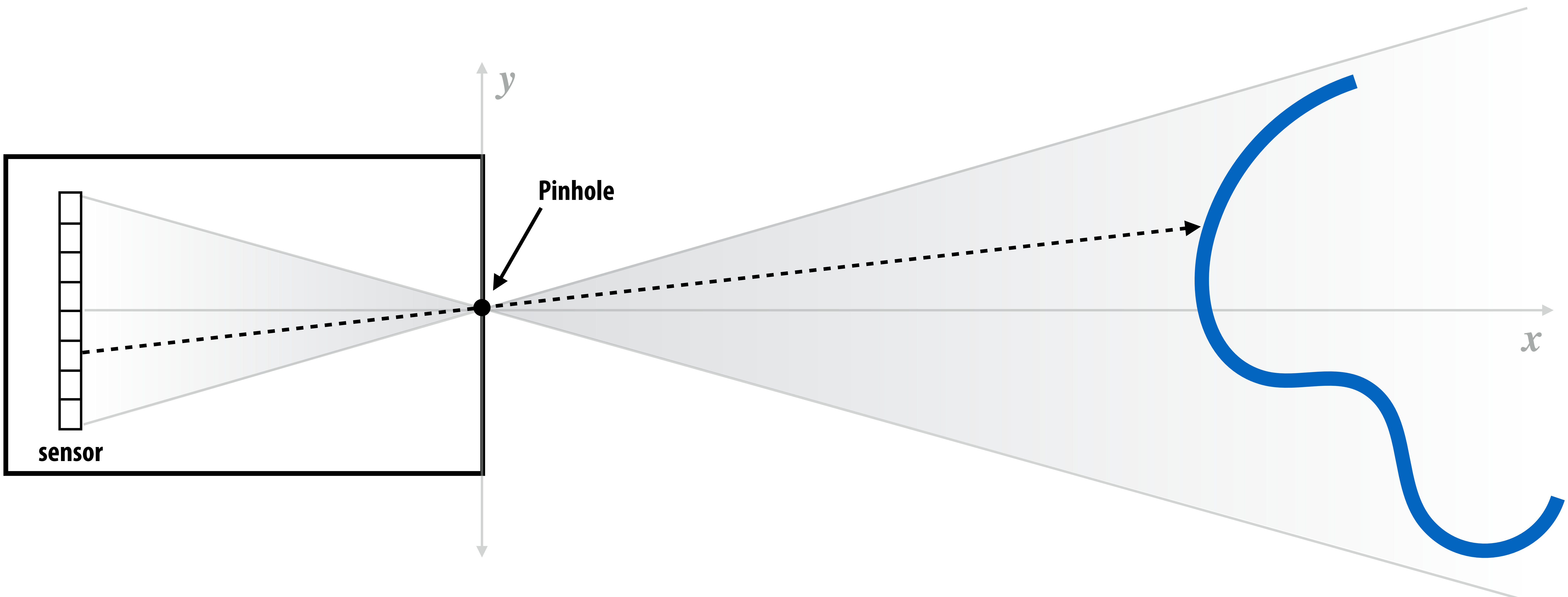
(~ 80 lumens / Watt)

Incandescent bulbs:
~15 lumens / Watt)

Cameras

Pinhole camera

Infinitesimally small aperture

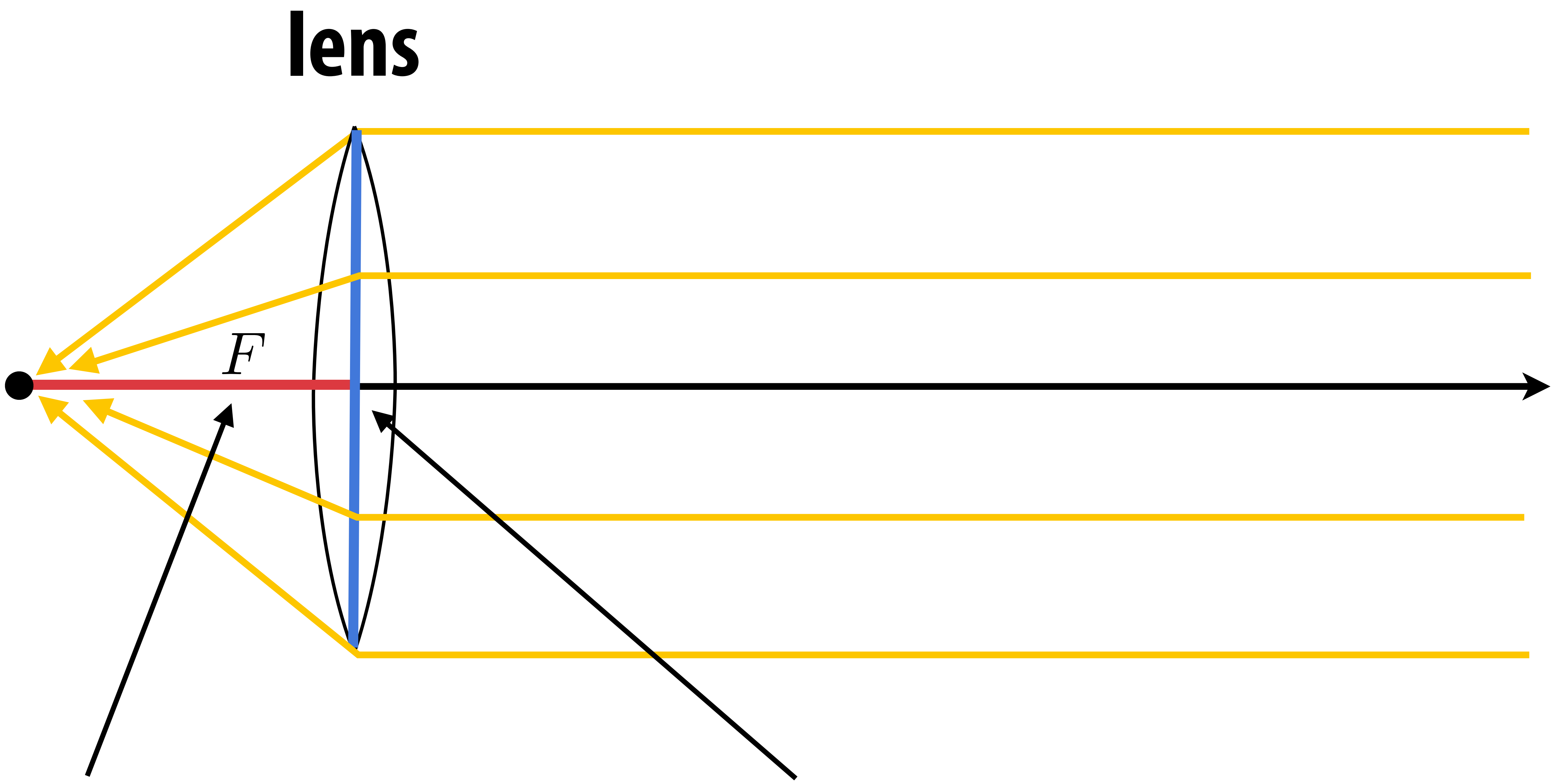


Real camera has lens with finite aperture

Sensor plane: (X,Y)

Image credit: Canon (EOS M)

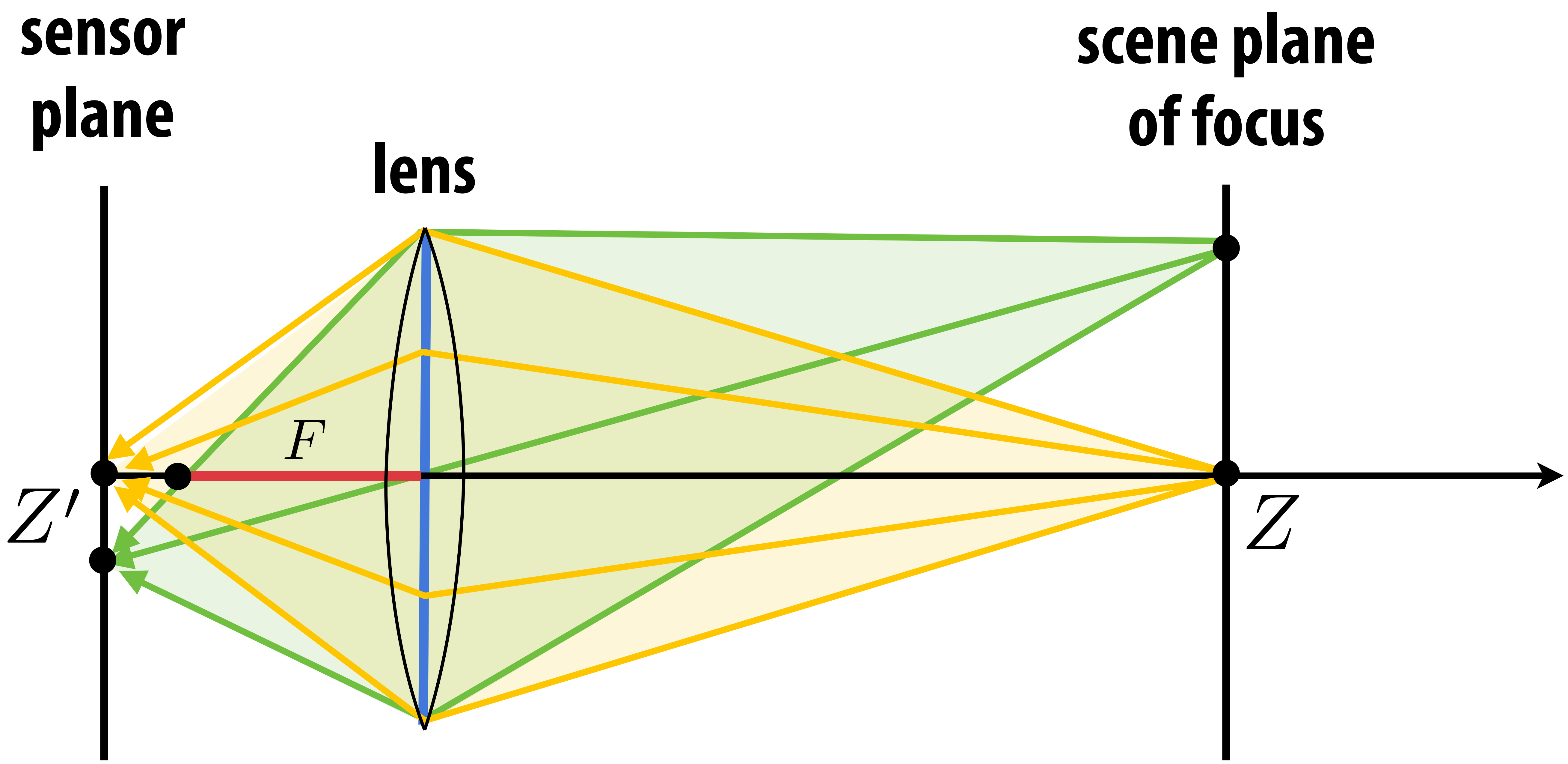
Thin lens model



Focal length, *F*, is the distance behind the lens where parallel rays focus

Lens diameter is *d*=F/n, where *n* is the lens' *f*-number

Gaussian thin lens model



Points at distance *z* focus at distance *z'* behind the lens given by

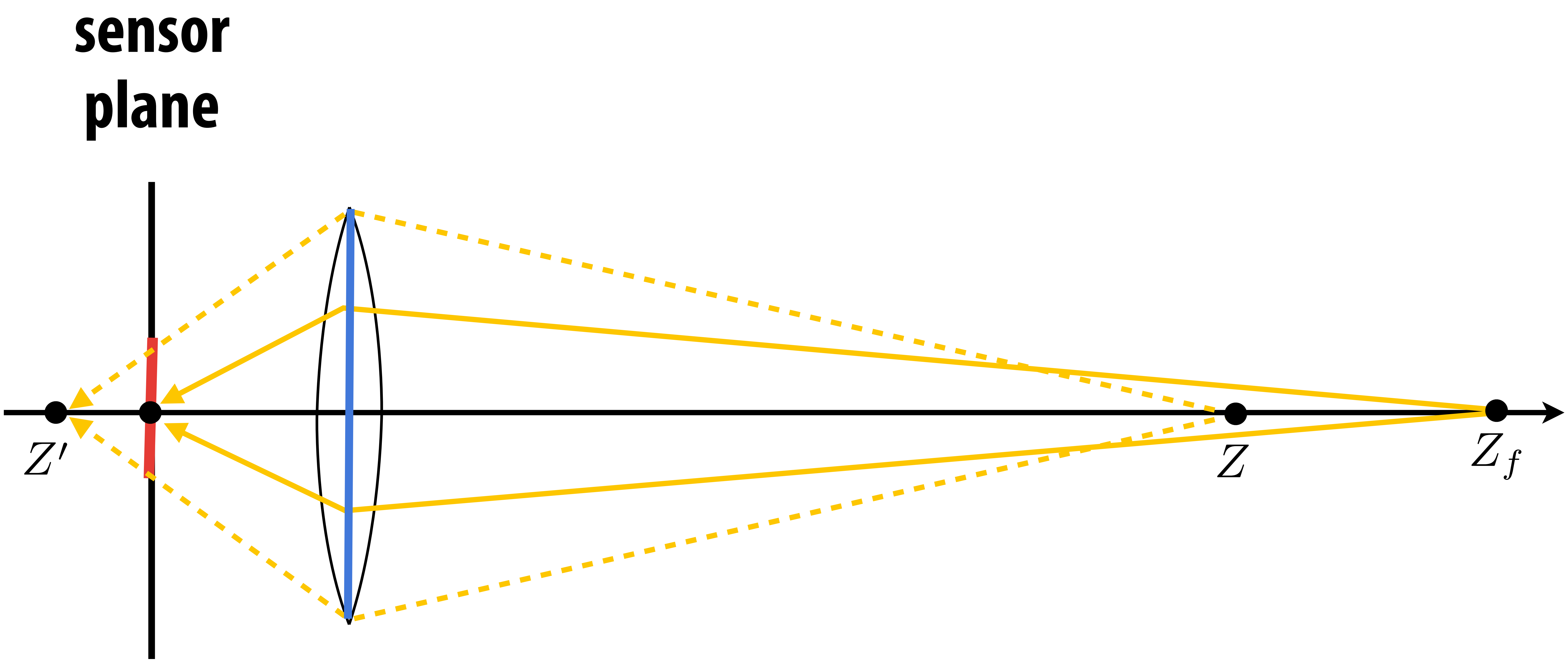
$$\frac{1}{F} = \frac{1}{Z} + \frac{1}{Z'}$$

Lenses are focused by moving them closer to/farther from the sensor plane

Defocus blur

Image credit: Mike Stobe/Getty Images for the USTA
(link: https://www.bostonglobe.com/news/bigpicture/2015/09/09/open-tennis/NKLwD3zIKkrKWkYQIcrM3J/story.html)

Defocus blur



Lens is focused at depth z_f; points at other depths image to an area (not a single point) on the sensor plane: this area is called the circle of confusion

How big is the circle of confusion?

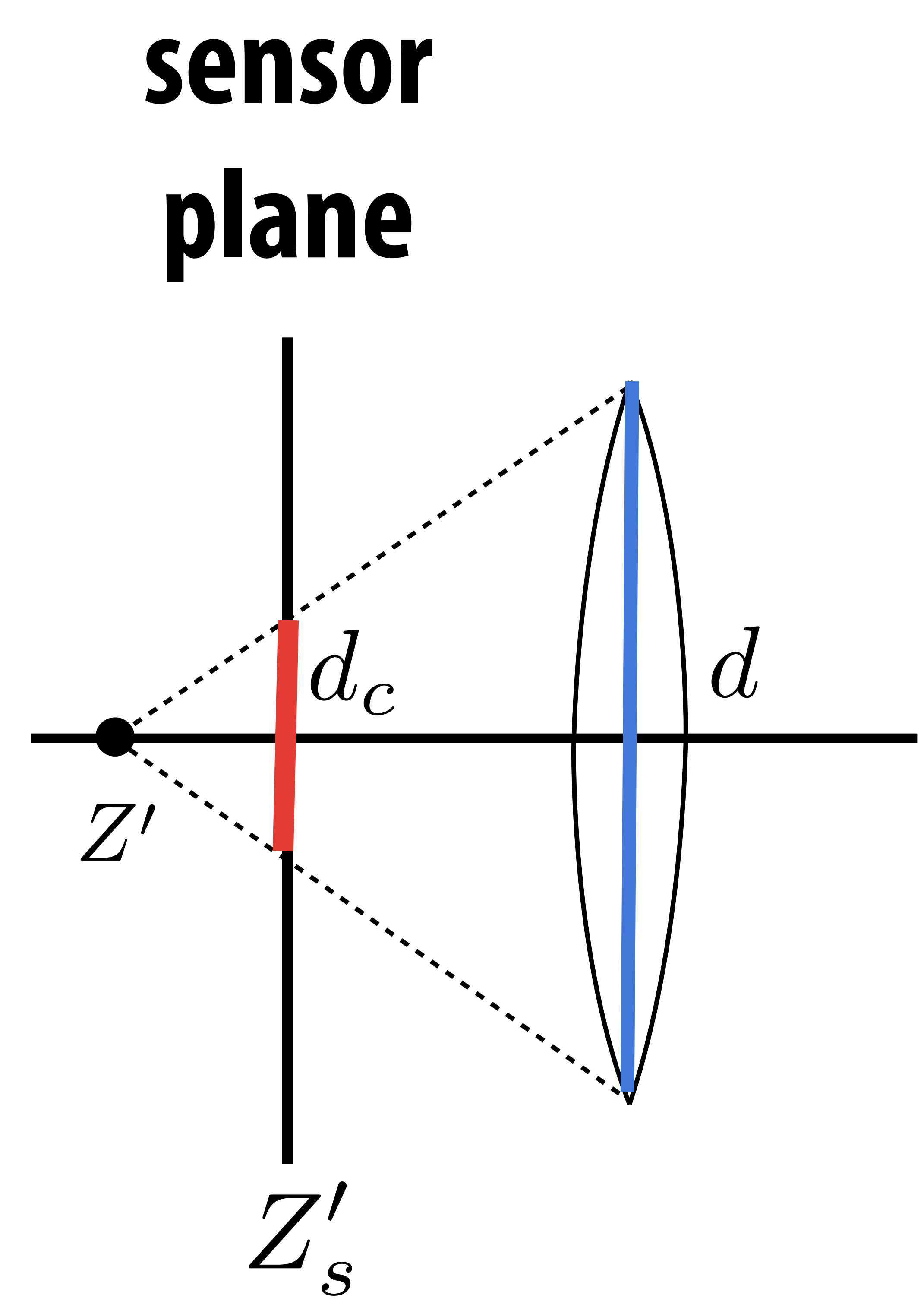
What is d_c, the diameter of the circle of confusion of a scene point at depth z?

We know z'_s, the sensor plane position, and the lens diameter d

Compute z′ from F and z: $$\frac{1}{F} = \frac{1}{Z} + \frac{1}{Z'}$$

Similar triangles: $$\frac{d_c}{|Z'_s - Z'|} = \frac{d}{Z'}$$

$$d_c = d\,|Z'_s - Z'| \left(\frac{1}{F} - \frac{1}{Z} \right)$$



Depth of field is inversely proportional to aperture

Plane of focus

Image credit: http://damienfournier.co/dof-and-aperture/

Motion blur

Camera records light over finite aperture and finite amount of time

Cook et al. 1984

Film and sensors

- Camera sensors (film, CCD, ...) effectively count the number of photons that hit them over small areas of the image
- Want to determine how much energy arrives over regions of the sensor (pixels) over a given period of time (exposure time)
  - Integrate irradiance over pixel area to get power (flux)
  - Integrate flux over time to get energy (joules)

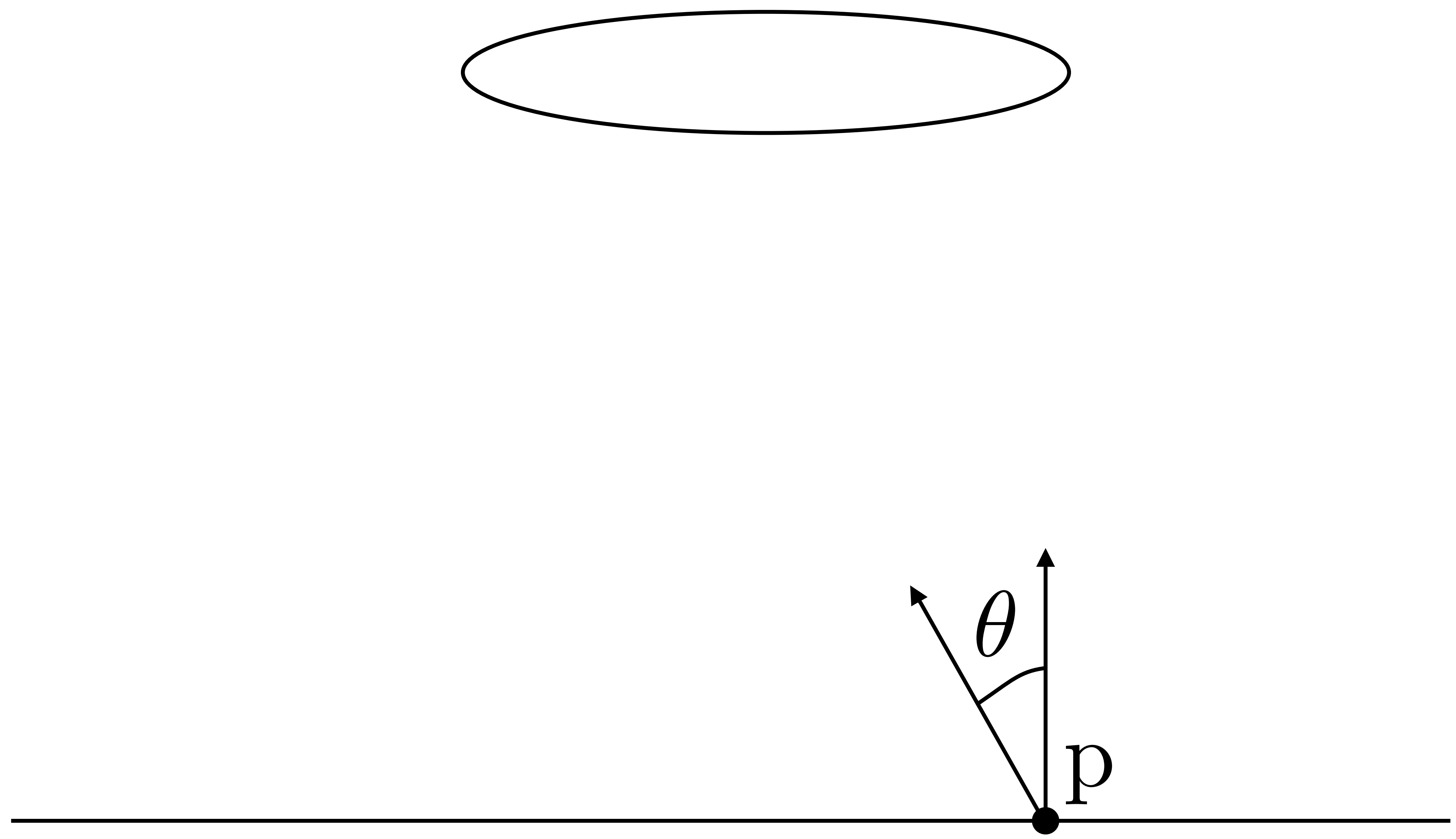
$$Q = \int_{t_0}^{t^1} \int_A E(\mathrm{p}', t')\, \mathrm{d}p'\, \mathrm{d}t'$$

- How do we compute irradiance at a point on the image plane?

The measurement equation

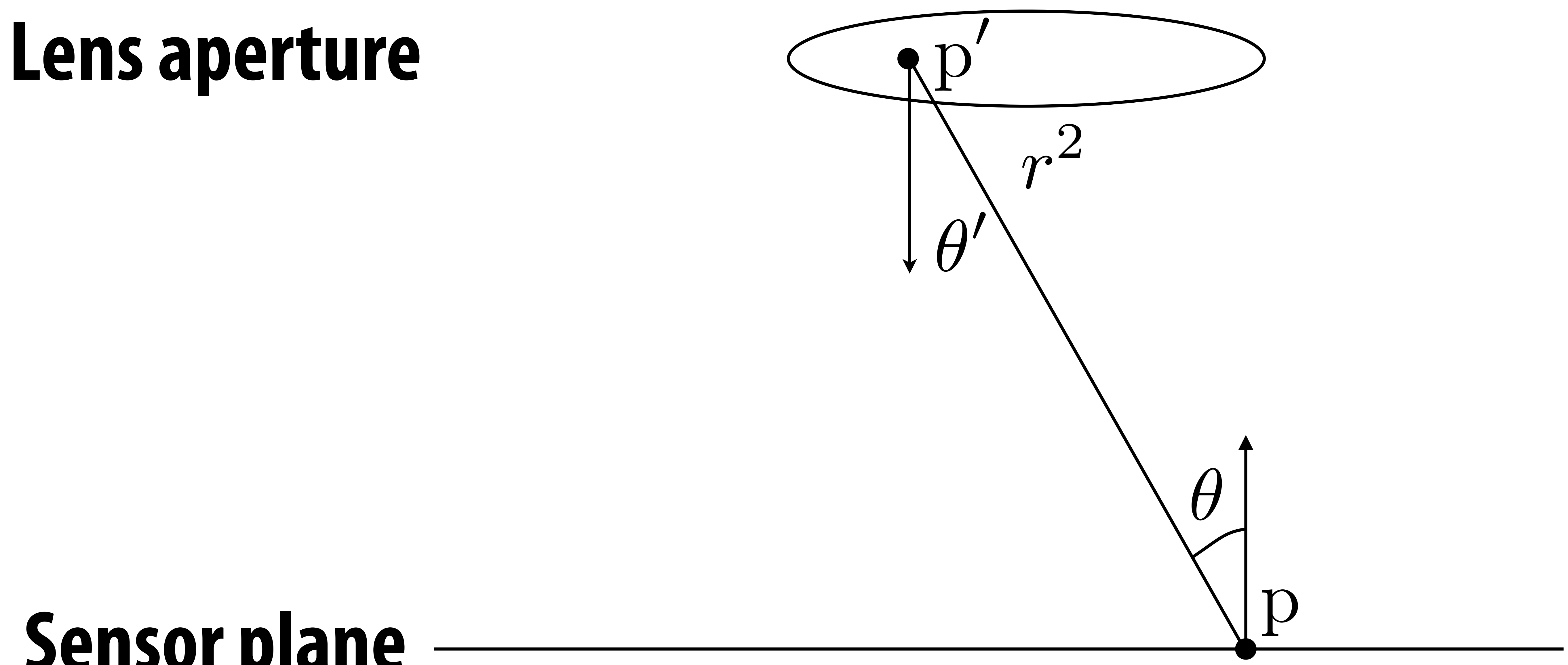
Lens aperture

Sensor plane



$$E(\mathrm{p}, t) = \int_{H^2} L_i(\mathrm{p}, \omega', t)\, \cos\theta \, \mathrm{d}\omega'$$

The measurement equation



$$E(\mathrm{p}, t) = \int_A L(\mathrm{p}' \to \mathrm{p}, t) \frac{\cos\theta \cos\theta'}{||\mathrm{p}' - \mathrm{p}||^2} \, \mathrm{d}A'$$

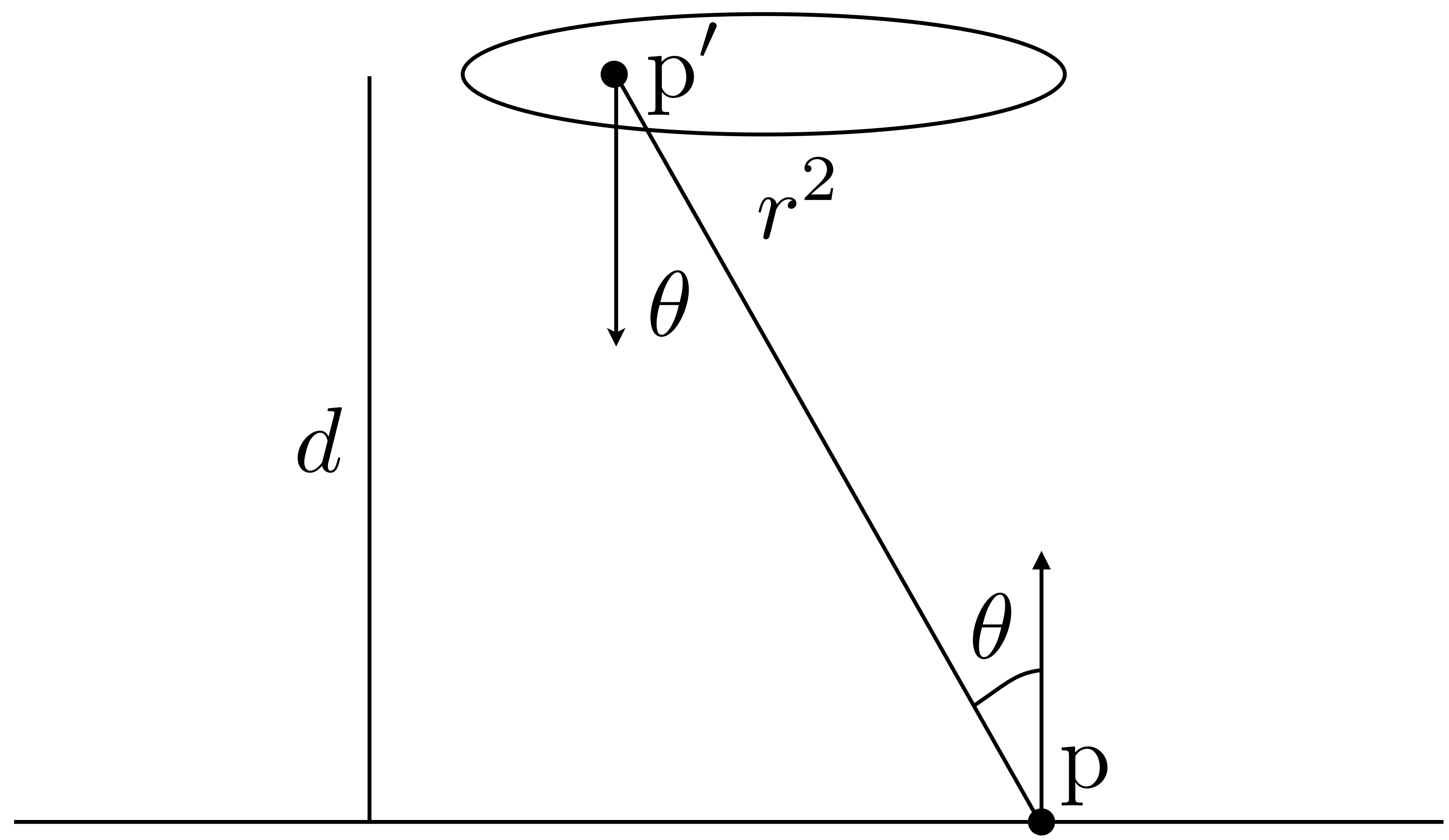
Transform integral over solid angle to integral over lens aperture

$$= \int_A L(\mathrm{p}' \to \mathrm{p}, t) \frac{\cos^2\theta}{||\mathrm{p}' - \mathrm{p}||^2} \, \mathrm{d}A'$$

Assume aperture and film plane are parallel: $\theta = \theta'$

The measurement equation

Lens aperture



$$||\mathrm{p}' - \mathrm{p}|| = \frac{d}{\cos\theta}$$

Sensor plane

$$E(\mathrm{p}, t) = \int_A L(\mathrm{p}' \to \mathrm{p}, t) \frac{\cos^2\theta}{||\mathrm{p}' - \mathrm{p}||^2} \, \mathrm{d}A'$$

$$= \frac{1}{d^2} \int_A L(\mathrm{p}' \to \mathrm{p}, t) \, \cos^4\theta \, \mathrm{d}A'$$

$$Q = \frac{1}{d^2} \int_{t_0}^{t^1} \int_{A_{\mathrm{lens}}} \int_{A_{\mathrm{film}}} L(\mathrm{p}' \to \mathrm{p}, t) \cos^4\theta \, \mathrm{d}p \, \mathrm{d}p' \, \mathrm{d}t'$$

Summary

- **Today:**
  - How to describe/measure light (radiant energy)
  - How light is measured by real sensors

- **Next time:**
  - How to compute all these integrals

- **Next, next time:**
  - Materials: how to describe the interaction of light with surfaces