FRAGMENTS OF THE HISTORY OF SHEAF THEORY

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It is the task of history to explain how modern concepts arose and why they are interesting and relevant. It is the task of modern mathematics to explain the same thing independent of and irrespective of history.

Sheaf theory, not really being a subject, cannot properly be said to have a history. Rather, it is an octopus spreading itself throughout everyone else's history. Of course, "everyone" is an exaggeration since sheaf theory is a part of geometry; namely, that part concerned with the passage from local properties to global properties. For instance, in complex analysis there is a sharp distinction between studying convergent power series at a point and investigating the Riemann surface of a whole "analytic configuration". Similarly, in algebraic topology, chain complexes with supports in a topological space have similar aspects, but this dichotomy was not so evident until Cartan clarified it and provided the major tool - cohomology with coefficients in a sheaf - which ever since has mediated the passage from local to global.

The description of a sheaf as an etale space (due to Lazard), developed by Cartan in 1950 , was and is adequate for nearly all of the geometric uses of sheaf theory. Analytic geometry, differential geometry, and differential equations are all perfectly satisfied with this formulation. It was only the work of Grothendieck in algebraic geometry that demanded a more general notion, that of a topos, i.e. sheaves on a site; which, via the Giraud characterization theorems, led to the development of elementary topoi by Lawvere and Tierney and the consequent geometrization of logic. As touchstones one has a pair of well known slogans; Grothendieck's in SGA4 saying " ... il semble raisonnable et légitime aux auteurs du present Séminaire de considérer que l'objet de la Topologie est l'étude des *topos* (et non seuls des espaces topologiques)", and a remark I heard attributed to M. Auslander saying sheaf theory is the subject in which you do topology horizontally and algebra vertically. He went on to wonder who would figure out to include logic in the third dimension. There is also an observation which Jon Beck attributes to H. Samelson to the effect that the word "sheaf" in English is much more descriptive of the concept it names than either "faisceau" in French or "Garbe" in German since it conveys both the idea of a sheaf of grain with its parallel stalks bound together and the idea of a sheaf of paper with its horizontal layered structure. Or perhaps best is the aside of Douady at Spencer's lectures in 1958/59, "sheafing sickness".

Sheaf theory probably played a fundamental role in the development of the modern mathematical consciousness. It is clear from reading the early papers that an era somehow ended with the reports by Chern and Zariski and the books of Hirzebruch and Godement. During the first half of the 1950's mathematicians learned how to say what they were talking about. In more sociological terms, a paradigm was established for the correct way to formulate definitions of abstract mathematical concepts, a paradigm which has served us very well for twenty years. Papers before the Second World War in algebraic topology and several complex variables dealt with very concrete notions. After the war came a period of groping for new formulations. The first paper of Leray in 1945 is very hard to read; even Cartan's first paper on sheaves in complex analysis in 1950 deals only with very concrete sheaves of ideals. But shortly thereafter everybody began speaking a language which we now still easily comprehend. This clear, precise dialect was probably necessary for the subsequent categorical reformulation in the second half of the 1950's of these new ideas, a linguistic change that was much smaller than the one that occurred during the preceding half decade and which was undoubtedly greatly facilitated by the existence of categories of sheaves where arguments involving elements seemed inappropriate.

Sheaf theory was not recognized as a topic to be indexed by Mathematical Reviews before 1959. Between then and 1961 it was indexed under algebraic topology. The subject index then vanished until 1974. In the new index, sheaves appear in at least six places explicitly, and a number of others implicitly. However, many of the important papers were never reviewed and/or appeared in obscure places. Hence any bibliography is bound to be incomplete. Furthermore, there is considerable difficulty in determining what to include. For instance, where does one draw the line between bundles and sheaf theory? We have included a paper about the decomposition of locally free sheaves but not its source, which is phrased in terms of vector bundles. In some places the distinction between a bundle F and its sheaf of sections $\tilde{F}$ is totally obscured by haphazard treatment of the tilda (e.g., in Goldschmidt and Spencer). If one assumes that the category of coherent sheaves is the abelianization of the category of finite-dimensional vector bundles, then the distinction may be only terminological. In any case, we have tried to include only articles that deal explicitly with sheaf theory, and we were especially partial to those which describe some new sheaf or new construction for sheaves.

This paper is divided into five chapters, each with its own bibliography.

Chapter I on *algebraic topology* discusses the origins of the subject in the work of Leray and Cartan and continues through to the work on Borel-Moore homology theory and Poincaré duality in the 1960's .

Chapter II on *complex analysis* describes the other part of the origins of sheaf theory in the study of ideals of germs of holomorphic functions. The topics discussed here include coherent sheaves, complex manifolds, and deformation theory.

Chapter III on *algebraic geometry* begins with FAC and ends with SGA.

Chapter IV on *differential equations* treats a number of topics: (a) distributions, hyperfunctions and microfunctions; (b) abstract potential theory and the Spencer sequences; and (c) pseudogroups.

Chapter V on *category theory* and *topoi* is concerned with the interaction between category theory and sheaf theory, starting with Buchsbaum and going through Grothendieck and Giraud to Lawvere and Tierney.

Three important topics have been omitted from consideration because adequate bibliographies are to be found elsewhere in this volume; namely, (a) Representation of algebraic structures by sheaves : see Mulvey, Representations of rings and modules; (b) Logic : see Fourman and Scott, Sheaves and logic; and (c) Banach spaces : see Burden and Mulvey, Banach spaces in a category of sheaves.

References in what follows to entries in the bibliographies are written "name-date" or "name [n] date", if they are to the bibliography for the chapter in which the citation occurs; otherwise, a Roman numeral is included specifying the appropriate chapter. LNM stands for Springer-Verlag Lecture Notes in Mathematics; other abbreviations are standard.

CHAPTER I. SHEAVES AND ALGEBRAIC TOPOLOGY

Sheaf theory was originally a part of algebraic topology; namely, that part concerned with studying the various kinds of chain complexes, like the Alexander complex or the De Rham complex, which involve chains with supports in a topological space. The folklore, as I heard it, was that Leray invented sheaves and spectral sequences at the same time and that if you looked at his papers, it was hard to tell the two apart. As usual, the folklore misrepresents the facts but has a certain validity in its expression of the spirit of the original papers. The reason for pursuing the history of the subject back to the 1940's is that there was a dichotomy then which reflects itself in a current division between those who view topoi as categories of sheaves and those who prefer to think in terms of Heyting-algebra-valued logics. As we shall see, both of these aspects played an important role in the early period of terminological confusion in the works of Cartan and Leray. In addition there was another confusion as to whether one should use closed sets or open sets. In order to keep the terminology straight we shall use subscripts on the word sheaf (i.e., sheaf_1 , sheaf_2 , etc.) until we arrive at the final current use of the term in Godement 1958. "Sheaf" is the translation of the French word "faisceau" and we may sometimes use the French term with or without subscripts if it seems appropriate.

The paper that is usually cited as the origin of sheaf theory is Leray [1] 1945. It is the first part of a series of three papers reporting on a course of lectures delivered while Leray was a prisoner of war in Oflag XVII (as reported by H. Villat, dated Jan. 11, 1944). The papers were sent to H. Hopf in Zurich for publication and had a profound effect on French algebraic topology for the next decade. The word "faisceau" is not mentioned here in any form. Instead, one finds the notion of a "concrete complex" which is a chain complex (in the modern sense) in which each element is assigned as "support" a non-empty (sic) subset of a set $|C|$, subject to some properties which are difficult to interpret. A special type (see below) of concrete complex is called a "couverture"; in particular, supports are closed subsets of a topological space. There are examples based on particular kinds of cellular chains or "forms" and a number of explicit calculations are carried out. (I was unable to find any mention of differential forms as an example.) The main purpose of the work was, of course, to study fixed points and solutions of equations.

The original fundamental idea was that of a module (or ring, or algebra, etc.) equipped with a support function taking values in the *closed* subsets of a topological space, and satisfying suitable properties. As Leray acknowledges, this idea already existed in the literature; namely, in Alexander [1] 1936 , where there is an example of such a structure given by a "grating" and an explicit ring which was constructed to have supports in the cells of this grating. Apparently Leray did not know the subsequent long paper, Alexander [2] 1938 , which gives a thorough discussion of these rings, using the term "loci" instead of "supports" and stating their properties in a form that was not adopted by Leray until Leray [3] 1949 , where he still does not reference Alexander [2] . Alexander gave a complete account of his theory in Alexander [3] 1947 , after which he apparently dropped the subject.

To return to the main stream, in 1946 Leray published several notes in the Comptes Rendus, the most important of which for us is Leray [2] 1946. As far as I know, this is the first place in which the word "faisceau" is used with anything like its current mathematical meaning. A *faisceau*$_1$ (or *sheaf*$_1$) B (of modules) on a topological space E is defined to be a function assigning to each closed set $F \subset E$ a module B_F such that $B_\emptyset = 0$, together with a homomorphism from B_F to B_f whenever $f \subset F$ taking an element b_F in B_F to $b_F \cdot f$ in B_f , such that $f' \subset f \subset F$ implies $(b_F \cdot f) \cdot f' = b_F \cdot f'$. A faisceau$_1$ is called *normal* if for each $b_F \in B_F$ there is a closed neighbourhood V of F and $b_V \in B_V$ with $b_F = b_V \cdot F$, and if $b_F \in B_F$ satisfies $b_F \cdot f = 0$ for some $f \subset F$, then there is a closed neighbourhood v of f in F with $b_F \cdot v = 0$.

Leray says that the notion of local coefficients in Steenrod [1] 1942 and [2] 1943 is a very special case of this. What Steenrod considers is a family of groups $\{G_x\}_{x \in X}$ indexed by a topological space X together with maps from G_x to G_y for each homotopy class of paths from x to y satisfying the expected transitivity condition. Since Steenrod assumes X is locally simply connected, this does determine groups G_F for closed sets F belonging to a closed basis for the topology, but it is essential for Leray's purposes that G_F should be defined for all F , in particular for $F = X$. Nevertheless if we admit Steenrod as a psychological precursor at least of sheaf theory, then we must also admit his sources which he says are Whitney [1] 1940, Reidemeister [1] 1935, and most important De Rham [1] 1932. De Rham's theorem was a constant challenge to early sheaf theorists, and it is interesting that Steenrod cites this paper as influential in his thinking, although all it does is to suggest changing some signs when dealing with non-orientable manifolds. Maybe that really is one of the key insights.

Leray [2] 1946 must be one of the first instances of the French saying that they are going to take what they have done for spaces and generalize it to mappings: "Nous nous proposons d'indiquer sommairement comment les méthodes par lesquelles nous avons etudié la topologie d'un espace peuvent être adaptées à l'étude de la topologie d'une représentation." In any case, following this note there must have been considerable activity in Paris working out these ideas. In 1947, from June 26 to July 2, the XIIth Colloque International de Topologie Algébrique was held in Paris. One can only speculate about what went on there since the proceedings were not published until two years later, giving people a chance to change their minds and their terminologies. During the winter of 1947-48 Leray gave a course in algebraic topology which he wrote up for the proceedings of this colloquium as Leray [3] 1949. By now, Leray's ideas were clearly developed and he gave here almost the definitive descriptions of his versions of sheaves$_1$ and spectral sequences. Sheaf$_1$ is defined as in Leray [2] , but now written $B(F)$ instead of B_F . It is called *continuous* (instead of the earlier term normal) if $B(F) = \lim B(V)$, the limit denoting the direct limit over all closed neighbourhoods V of F . Subsheaves, quotient sheaves, and direct image sheaves are described. Similarly, the notion of a *complex* is also clearly stated; namely, it is a differential ring K with a support function S taking values in the closed subsets of a locally compact space X such that $S(0) = \emptyset$, $S(k-k') \subset S(k) \cup S(k')$, $S(kk') \subset S(k) \cap S(k')$, and $S(\delta k) \subset S(k)$. One is told how to construct a sheaf$_1$ B_K from such a complex; namely, if F is a closed subset of X , then $B_K(F) = K/S_F^{-1}(\emptyset)$, where S_F is the new support function on K defined by $S_F(k) = S(k) \cap F$. Fine complexes are defined in terms of partitions of unity and we are told that this is a special case of Cartan's notion of a "carapace", but that does not seem to fit with Cartan's later use of this term. Couvertures are defined as special complexes. Finally, for any differential sheaf, B , on X , one defines the cohomology ring of X relative to B to be $H(\chi \circ B)$ where χ is any fine couverture and $\circ$ is a graded tensor product.

This use of fine couvertures is one of the central ideas of sheaf theory. There were subsequently many related notions; for instance, homotopically fine Cartan [4] 1950/51, flasque (flabby) and mou (soft) in Godement 1958, and ultimately injective in Grothendieck [5] 1957. All of them are concerned with what was regarded as the main concern of sheaf theory - that of extending partial sections to global sections. Their original use was the same as their later use: to construct resolutions of the sheaves in which one is interested by homologically trivial sheaves. Isomorphism theorems and duality theorems usually were proved by showing that some known resolutions were fine, flasque, or mou, etc.

Leray's ideas were further refined during a course given in 1949/50 which was published as Leray [4] 1950, his final paper on sheaf theory and spectral sequences. It clears up some of the ambiguities of the preceding paper but does not essentially change any of the ideas concerning sheaves. Cartan II [5] 1953 identifies this paper as the one defining cohomology with coefficients in a sheaf rather than the preceding paper, presumably because even then Leray [3] was not easily available. One might remark that on pages 96ff there are diagrams with arrows representing mappings.

Meanwhile, in 1948/49 between Leray's two series of lectures, Cartan held his first Seminaire at the Ecole Normale Supérieure (Cartan [3] 1948/49). This was devoted to algebraic topology and contained five chapters on "Théorie des Faisceaux" which were never published. In Cartan [1] 1949, which is Cartan's report on his talk "Carapaces" at the XIIth Colloque International referred to above, he says (in rough translation): "The ideas which I proposed in 1947 under this title have noticeably evolved since this date, if not in essential principles, at least in presentation. Further, their range of applicability has been notably extended. One can understand that, two years later, the author would prefer not to have a text printed which does not completely correspond to his present views." (For the new uses, see Cartan II [2] 1950.) He goes on to say that the point is to establish uniqueness theorems which lie outside the format of Eilenberg-Steenrod because these give De Rham's theorem and the duality theorems of Poincare, Alexander, Pontryagin and others. Finally he acknowledges Leray [1] as the origin of his researches but adds that he also recognized a relationship between Leray's work and a proof of De Rham's theorem proposed by A. Weil in an unpublished letter of February 1947. He then refers to Cartan [2] 1949 for an account of the things he doesn't want to talk about anymore. See Weil 1952 .

Cartan [2] 1949 is lecture notes for a course at Harvard University held during the spring of 1948. The notes were prepared by George Springer and Henry Pollak, who thank Paul Olum, Maxwell Rosenlicht and Lawrence Marcus for helping with details of proofs. All of them must have been a bit dismayed by the radical change represented by Cartan's next publication on the subject. In the Harvard notes, Cartan describes "gratings" which are essentially the same as Leray's "complexes" described above. He or the editors refer to Alexander [1] and [2] (which is how I found [2] .)

During the third year of the Cartan seminars, Cartan [4] 1950/51, the theory of sheaves was completely reformulated. The current definition of what is now called (following Godement) an "espace étalé" is given, except it is called a faisceau. Namely, a *faisceau*$_2$ *(sheaf*$_2$*)* on a regular topological space χ is a map p from a set F to χ such that (1) for each $x \in \chi$, $p^{-1}(x) = F_x$ is a K-module; and (2) F has a (non-separated) topology such that the algebraic operations of F are continuous, and p is a local homeomorphism. This description is credited to Lazard. The collection of sections of F over an open set $X \subset \chi$ is denoted by $\Gamma(F,X)$; and if $s \in \Gamma(F,X)$, then the closed set $\{x \mid s(x) \neq 0\}$ is called the support of s . The restriction maps $\Gamma(F,Y) \to \Gamma(F,X)$ for $X \subset Y$ are noted, and the stalk $F_x = \lim \Gamma(F,X)$ is defined. Conversely, from modules F_X given for each *open* set $X \subset \chi$ with transitive maps $F_Y \to F_X$ when $X \subset Y$, one is told how to construct a sheaf$_2$ F by the now standard procedure of defining the stalks to be the direct limit $F_x = \lim_{x \in X} F_X$ and topologizing their union $F = \bigcup_{x \in \chi} F_x$ by taking the sets $\{ s_x(\sigma) \mid x \in \chi \}$ as a basis, where $s_x(\sigma)$ is the image of $\sigma \in F_X$ under the canonical map to F_x , when $x \in \chi$. Many of the usual constructions are described (e.g., the reciprocal image, but not the direct image), and the example of germs of holomorphic functions on a complex manifold is mentioned.

The succeeding chapters discuss fine sheaves$_2$, families of supports, and the axiomatic theory of cohomology with supports, existence being shown by means of fine resolutions although injectives are mentioned. There is a very brief discussion of the Cech procedure for constructing cohomology groups. The lectures then turn to the study of "carapaces". These seem to be a mixture of Leray's continuous sheaves_1 and his complexes; namely, a *carapace* on a topological space X is a K-module A and, for each $x \in X$, a homomorphism $A \to A_x$ ("sur un module-quotient", which seems wrong) such that

(i) if $\phi_x(a) = 0$, then there is a neighbourhood V of x such that for all $y \in V$, $\phi_y(a) = 0$; and

(ii) if $\phi_x(a) = 0$ for all x, then $a = 0$.

By definition, the support of $a \in A$ is the closed set $\sigma(a) = \{x \mid \phi_x(a) \neq 0\}$. A carapace clearly determines a sheaf_2 whose stalks are the A_x's. Conversely, it is asserted that for any sheaf_2 F, the family $\{\Gamma(F) \to F_x\}$ is a carapace, although these are certainly not surjective maps in general. The following two chapters deal with the "fundamental theorems" of sheaf theory, which turn out to be theorems about spectral sequences and cohomology (I recall being disappointed when I first saw that). Finally one gets the intended applications to De Rham type theorems and duality theorems.

A notable feature of this work is the change in emphasis from closed sets to open sets, although that does not become completely explicit until Godement [1] 1958. Supports are still important because of cohomology with given (e.g., compact) supports; but it is clear that Leray's idea of assigning a module $B(F)$ to each closed set F has been supplanted by the modules $\Gamma(-,X)$ of sections over an open set X together with the stalks at each point x, where a point now may or may not be a closed set.

A nice brief account of the theory as it was at this time can be found in Thom's thesis, Thom [1] 1952. The introduction reviews sheaves_2 and cohomology with supports as in Cartan [4] and then gives a different definition of carapace which is almost identical with Leray's description of a complex or Alexander's description of loci; namely, a carapace is a graded differential K-module A with a support function σ taking values in the closed sets of a topological space such that

(i) $\sigma(a) = \emptyset$ if and only if $a = 0$;

(ii) if a and b are homogeneous of different degrees, then $\sigma(a+b) = \sigma(a) \cup \sigma(b)$;

(iii) $\sigma(a-b) \subset \sigma(a) \cup \sigma(b)$; and

(iv) $\sigma(\delta a) \subset \sigma(a)$.

A thorough discussion of Leray's notion of couverture (a special kind of complex or carapace) can be found in Fary [2] 1954. Fary used the term "faisceau" in the Leray sense in Fary [1] 1952, [2] 1956 (which has a long chapter on the subject) and [4] 1957. In Fary [5] 1958, written in English, he translates "faisceau_1" as "stack". I have not been able to find any other uses of couvertures or carapaces except in the textbook Bourgin [1] 1963, which uses "grating" for Thom's sense of "carapace", but the chapter on this appears to have been written long before the book was published. There is no mention of the notions in Godement [1] 1958. Another case in point is the famous set of lectures Borel [1] 1951 from the ETH in Zurich. In the first edition Borel faithfully follows Leray's terminology; in the second edition in 1957 he follows Fary's terminology for couvertures but switches to Cartan's (i.e., Lazard's) definition of faisceau_2. He also uses the term "prefaisceau" here. This work taught so many of us that it justly had a third edition in 1964 as the second volume of the Springer Lecture Notes in Mathematics.

It remained for a functional analyst, R. Godement, to write the definitive account of this sort of sheaf-theoretic approach to algebraic topology. The book Godement [1] 1958 actually existed much earlier as mimeographed notes of lectures given by him at the University of Illinois in 1954/55, which were presumably available to selected people. The only copy I ever saw was shown to me by D.C. Spencer sometime during the years 1957-59. As far as I remember, it differed substantially from the published work. Amongst its many virtues, this book standardized the terminology once and for all. A *presheaf* is a contravariant set (or module or ring, etc.) valued functor defined on the category of open subsets of a topological space, and a sheaf is a special kind of presheaf. A faisceau_2 in the sense of Cartan et al. is now called an "espace étalé" (spread space or etale space). The book makes full use of categories, functors, derived functors, and all of the machinery of homological algebra. It does use "flasque" resolutions instead of injective ones (although they are briefly treated) but, other than that, it is still the standard reference for the algebraic topology aspects of sheaf theory that it covers.

The main result in sheaf theory of a topological nature missing from Godement is any version of *Poincaré duality*. This is a theorem asserting that, for a suitable sort of n-dimensional manifold X and a sheaf F of K-modules, there is an isomorphism

$$H^p_\phi(X, O \otimes F) \simeq H^\phi_{n-p}(X, F) \quad .$$

Here ϕ is an appropriate family of supports, and O is an orientation sheaf. The main problem is to describe the homology groups on the right hand side. Theorems of this sort are to be found both in Leray and in Cartan where sheaves and carapaces are all mixed together. The same thing happens in the more modern version in Borel [2] 1957, and even in Borel [3] 1960 one still finds gratings in the account of homology. They disappear finally in Borel-Moore 1962, which, without using the term, actually constructs a differential cosheaf whose dual differential sheaf gives the appropriate homology groups. A complete account with explicit cosheaves can be found in Bredon [1] 1967. See also Bredon [3] 1968. Similar, independent treatments of homology groups using cosheaves explicitly can be found in Luft 1959, Kultze 1959 and Kawada [1] and [2] 1960. Presumably the phrase "Borel-Kawada-Kultze-Luft-Moore homology theory" is too long, but it certainly seems justified. Borel-Moore homology theory has played an important role in applications of sheaf theory to transformation groups, Borel [3] 1960, and to analytic spaces, Borel and Haefliger II 1961, and most recently, Douady and Verdier II [2] 1976. The ultimate expression of Poincare duality is probably the version developed by Verdier and Zisman using derived categories. A brief account of this is in Zisman [1] 1968. There are thorough accounts of derived categories in Hartshorne [1] 1966 and Verdier [1] 1963. See Verdier-Zisman 1967 for a more thorough discussion of Poincare duality and the Lefschetz formula.

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CHAPTER II. SHEAVES AND COMPLEX ANALYSIS

The definition in Leray [2] 1946 of a normal (or continuous) faisceau_1 immediately suggests looking at the module B_x assigned to a (closed) point $x \in X$. Continuity implies that it is the (direct) limit of the modules B_V, where V ranges over the closed neighbourhoods of x. This situation is familiar from another part of mathematics. In complex analysis, a germ of a holomorphic function is described in almost exactly the same way, except that a holomorphic function is defined on an open set. One says that two holomorphic functions determine the same germ at a point x if they agree on some open neighbourhood of x. The idea of a germ of a holomorphic function certainly goes back to K. Weierstrass (1815-1897); his development in the 1840's of the theory of analytic continuation of function elements represented by convergent power series was based on ideas of his teacher C. Gudermann (1798-1858) (see Weierstrass [1] 1894). I do not know if Weierstrass used the term "Keim" or, if not, who was the first to use the term "germ". Cartan refers to it as being "classical", which may mean prior to World War II.

In any case, Cartan presumably made the connection between Leray's definition and the notion of germs, subject to the proviso that things be defined for open sets rather than closed sets. At least in the first of his papers on the connection between faisceaux and complex analysis, Cartan [2] 1950, he defines a faisceau_3 (sheaf_3) to be a function assigning to an open set X in $\mathbb{C}^n$ a submodule F_X of O_X^n (the n-fold product of the ring O_X of holomorphic functions on X) such that if $X \subset Y$ then the module generated by F_Y in X is contained in F_X. He then observes that for any subset A of $\mathbb{C}^n$ one can define F_A to be the (directed) union of the F_X for all open X containing A. This determines the "point modules" F_x, and a result is proved asserting that faisceaux_3 F satisfying the condition (i) that $f \in O_X^n$ belongs to F_X if and only if $f \in F_x$ for all $x \in X$, are in bijective correspondence with collections of point modules $\{F_x\}$ satisfying the condition (ii) that, if a function f which is holomorphic in a neighbourhood of x belongs to F_x, then it belongs to F_y for all y in a neighbourhood of x. (In modern terms, subsheaves of the sheaf of holomorphic functions are in bijective correspondence with open etale subspaces of the etale space of germs of holomorphic functions.) In Oka [1] 1950, the same notion is called an "idéal holomorphe de domaines indéterminés".

There were essentially two different methods developed to study the sheaves that arose in complex analysis. One of them was based on the notion of *coherence*, which in the early years was almost exclusively used by the Paris school; while the other was based on *harmonic forms and integrands* and was used in Princeton although its basic theorem came from France (Dolbeault [1] 1953). They will be discussed in turn.

1. COHERENCE. The idea of a coherent family of ideals was introduced in Cartan [1] 1944 and described in more sheaf-theoretic terms in Cartan [2] 1950. It is clear from reading Cartan [3] 1950, which never mentions the word "faisceau", that a given family of ideals $\{I_x \subset O_x\}$ was coherent if the ideals were stalks of a subsheaf of O which was finitely generated by sections. The reason for studying coherence is that if suitable properties hold at a point, then they hold in a neighbourhood of the point; thus, theorems can be proved just by proving them at a point.

The definition in Cartan [1] 1944 reads: "Let E be a subset of the space of n complex dimensions and let q be an integer ≥ 1 given once and for all. Suppose that to each point x of E there has been attached a module M_x (of dimension q) of functions holomorphic at the point x. We say that the point modules M_x form a coherent system if every point a of E possesses a neighbourhood V on which there exists a module (of dimension q) which at every point x of the intersection $E \cap V$ generates the point module M_x ". The definition in Cartan [2] 1950 reads: "An A-faisceau$_3$ F is called coherent at a point a of A if a possesses an open neighbourhood X such that not only does F_X generate F_a at the point a, but also F_X generates F_x at all points x sufficiently near to a. (Under these conditions, for every sufficiently small open neighbourhood Y of a, F_Y generates F_x at every point $x \in Y$.) "

A systematic study of coherent sheaves was carried out in Cartan [4] 1951/52. Here faisceau is used in the sense of faisceau$_2$ as in Cartan I [4] 1950/51, and a subfaisceau$_2$ of $O^q(E)$ is called coherent (in Expose 15) at $x \in E$ if there is an open neighbourhood U of x and a finite system of elements $u_i \in O^q_U$ such that at every point $y \in U$, the submodule of O^q_y generated by the u_i's is F_y. Briefly, it is a finitely generated submodule of $O^q(E)$. This description is deceptive since it only works for submodules of O^q and does not generalize. The main result of Expose 15, § 5, is the theorem of Oka: the sheaf of relations for a finite sequence of elements of O^q_E is coherent. This implies among other results that the sheaf of holomorphic functions on a complex manifold is itself coherent, although it is not clear just from reading the statement of the theorem that this is so. In Expose 16 it is shown that the sheaf of ideals of an analytic subvariety is coherent, which was the other fundamental result on coherence at that time.

In Expose 18 coherence is generalized, and an analytic faisceau$_2$ F is now called coherent if every $x \in X$ has an open neighbourhood U such that the restriction of F to U is isomorphic to $O^p(U)/R$ for some p and some coherent subsheaf$_2$ R of $O^p(U)$. Using this definition it is shown that the kernel, image, and cokernel of a map between coherent sheaves$_2$ are coherent. Then one finds the two fundamental theorems concerning a coherent sheaf F on a Stein manifold X which played an important role for several years:

THEOREM A. The stalks of F are generated by global sections.

THEOREM B. $H^q(X,F) = 0$ for $q \geq 1$.

This second result clearly implies that the first Cousin problem is solvable on a Stein manifold; i.e., that there is always a meromorphic function having given principal parts. In Exposé 20, Serre gives a number of other applications. In particular, he shows that Theorems A and B characterize Stein manifolds and that the second Cousin problem (the existence of a meromorphic function with a given divisor) is classified by $H^2(X,Z)$.

These results are essentially repeated in Cartan [5] 1953 and Serre [2] 1953, except that here coherence is defined by saying that every point x has an open neighbourhood U such that $F \mid U$ is the cokernel of a map $f : O^p|U \to O^p|U$. As became clear later, this description works in general precisely when the sheaf of rings O is itself coherent as a sheaf of O-modules. It is remarked that Theorems A and B extend to real analytic submanifolds of $\mathbb{R}^n$ and real coherent sheaves. Another fundamental result is proved in Cartan-Serre [1] 1953, where it is shown that if F is a coherent sheaf on a compact complex manifold X , then the groups $H^q(X,F)$, $q \geq 0$, are finite dimensional complex vector spaces, which can be regarded as a generalization of the theorem of Liouville for $X = S^2$. This result also appears in Cartan [8] 1953/54, where it is remarked that Kodaira proved it for $q = 1$ using Dolbeault [1] 1953. (See below.)

Exposé 19 of Cartan [8] 1953/54 by Serre studies coherent sheaves, F , on complex projective space, X , and it is shown that there is an integer $n_0(F)$ such that for all $n \geq n_0(F)$ one has:

THEOREM A'. The stalks of $F(n)$ are generated by global sections.

THEOREM B'. $H^q(X,F(n)) = 0$ for $q \geq 1$.

Here $F(n)$ is given by "twisting" F by the isomorphisms $(z_i/z_j)^n$. At the end of this Exposé one finds the theorem of Chow: every analytic subset of projective space is algebraic. Finally Exposé 20 studies sheaves of automorphic functions.

The proper description of coherent sheaves was not settled until FAC (Serre [6] 1955) appeared. The first chapter of this famous and seminal work is concerned with sheaves in general and coherent sheaves in particular. The well known pair of conditions characterizing sheaves as special presheaves first occurs here. The definition of coherence is also the standard one currently in use. Namely, a sheaf

F of A-modules is of *finite type* if it is a quotient of a sheaf A^p for some finite p. A sheaf F of finite type is called *coherent* if for every open set U, every integer q and every sheaf morphism $f : A^q|U \to F|U$, the kernel of f is also of finite type. There is a section on Čech cohomology and some special conditions adapted to the algebraic situation for the Čech cohomology to be the cohomology of a covering.

2. HARMONIC FORMS AND INTEGRANDS. Meanwhile, a similar intensive study of complex manifolds was being carried out in Princeton, the main participants being Borel, Hirzebruch, Kodaira, Spencer, and later Atiyah, Grauert, and Serre. Besides these central figures there was a constant parade through the Institute for Advanced Study during the 1950's of others interested in and working on various aspects of sheaf theory. (I was there during the years 1957-59.)

The first published paper of this school is Kodaira-Spencer [1] 1953. Its purpose was "to prove the equality $p_a(M_n) = P_a(M_n)$ in full generality by means of the theory of faisceaux". Here M_n is "an irreducible non-singular algebraic variety of dimension n imbedded in a complex projective space", $P_a(M_n)$ is "the virtual dimension of the canonical system on M_n increased by $1 - (-1)^n$" and $p_a(M_n)$ is "$(-1)^n$ times the dimension of the complete linear system of effective divisors which are equivalent to 0". This sounds very algebraic and its subject foreshadows a continued interest in Riemann-Roch theorems in Princeton, but the techniques used involve studying the cohomology of the sheaf of germs of currents of type (p,q) on a complex analytic manifold and the sheaf of germs of meromorphic r-forms. Reference is made to Cartan I [4] 1950/51 and Cartan [4] 1951/52 for the theory of sheaves and to Dolbeault [1] 1953 for forms of type (p,q).

This paper is followed by several more within the next year, Kodaira-Spencer [2] 1953, [3] 1953, [4] 1953, Spencer [1] 1953, and Kodaira [1] 1953, [2] 1953, and [3] 1954. In Kodaira [1] it is shown that the groups $H^q(V,\Omega^p(F))$ are finite dimensional vector spaces, where V is a compact complex analytic variety and $\Omega^p(F)$ is the sheaf of germs of holomorphic p-forms with coefficients in the analytic vector bundle F (cf. Cartan-Serre [1] 1953). In Kodaira [2] there is a fundamental result about the vanishing of $H^q(V,\Omega^p(F))$. (See Hodge-Atiyah [1] 1955.) Both in this paper and in Kodaira-Spencer [4], the term "stack" is used as the translation of "faisceau"; Hodge-Atiyah [1] also uses it, remarking that "sheaf" has been used before in mathematics. Spencer [1] does not contain anything called a theorem but concludes by stating: "The above remarks are merely intended to suggest the manner in which cohomology with coefficients in a faisceau may be applied to obtain Riemann-Roch theorems". These papers contain a wealth of results too numerous to mention; for instance, Kodaira-Spencer [3] 1953 proves that every complex line bundle on a non-singular projective variety can be represented by a divisor.

At this same time the first of two papers of Hirzebruch appeared ([1] 1953 and [3] 1954 in the same journal) making essential use of the results of Kodaira and Spencer. In the second of these, Hirzebruch points out that the main theorem, which expresses the Euler-Poincaré characteristic of a non-singular complex projective variety V, with coefficients in the sheaf of germs of holomorphic sections of a complex analytic bundle W, as an explicit polynomial in $c_1(V)$, the Pontryagin classes of V, and the Chern classes of W, is a result which was conjectured by Serre in a letter to Kodaira and Spencer, thus nicely documenting the close cooperation (and competition presumably) between the two schools. Another paper, Hirzebruch [3] 1954 appeared simultaneously, concerned with the resolution of singularities; it was reviewed in Cartan [7] 1953.

In the summer of 1954 the Second Summer Institute on several complex variables was held at the University of Colorado and the reports from this conference, Chern [1] 1956 and Zariski III [1] 1956, provide an accurate picture of the situation at that time. It is interesting that Chern thanks Borel, Kodaira, Spencer, Wang, and Weil for reading his manuscript while Zariski thanks Lang, Igusa, Serre, and Spencer for reading his. The participants are not all identified, but one can deduce the presence of Baily, Bremermann, Gunning, Igusa, Lang, Washnitzer and presumably several of those who were thanked; so a large proportion of those working in the field attested to the accuracy of the reports. Both papers, which will not be summarized here, still make interesting mathematical reading, and Zariski's report contains some historical information. Chern in his report mentions Stein manifolds and the Hirzebruch-Riemann-Roch theorem, just managing to squeeze in a reference to Hirzebruch [5] 1956 at the end.

This book by Hirzebruch was really the final summary and exposition of the work of the Princeton school, although of course for many people it was the beginning of their knowledge of the subject. Hirzebruch visited the Institute for Advanced Study from 1952 to 1954 where he worked with Borel, Kodaira, and Spencer. He translates "faisceau" into German as "Garben" and gives the name "Garbendatum" to what is now called a presheaf, which may very well be the first published name for this concept. The term "presheaf" is used in Nickerson-Spencer [1] 1954, but that reference was never published and is virtually unobtainable, presumably being inhibited by Godement I 1958. Hirzebruch gives a very careful discussion of Cech cohomology on a paracompact space; I certainly learned it there, and I imagine everyone else did too. His proof of De Rham's theorem was and is the slickest and most convincing argument there is for the use of sheaf theory in geometry. Of course the book contains many things besides sheaf theory, namely, all of the ingredients that went into the Riemann-Roch theorem; but describing all of that would be a project for a separate report.

3. MORE RECENT TRENDS. After the middle of the 1950's a new era began in the use of sheaf theory in complex analysis. Two trends had a dominant influence on the development of the subject during the next two decades. One was the rise of the German school (which had always been active in classical complex analysis), best represented by the work of Grauert on the direct images of coherent sheaves. The seminal papers are those of Grauert and Remmert, [4] 1958 and [5] 1958. In [5] they credit Cartan with introducing the term "ringed space", which he did in Cartan [9] 1955, a report on the work of Grauert. The work on direct images was motivated by Grothendieck's vast generalization of the Hirzebruch-Riemann-Roch theorem to proper maps between algebraic varieties in the sense of Serre (FAC), reported on in Borel-Serre [1] 1958, which required the coherence of (the derived sheaves of) the direct image of a coherent sheaf under a proper map. This is what had to be proved for complex manifolds in order to have a similar Riemann-Roch theorem. In the algebraic case it is not difficult, but it is very involved in the analytic case. For an account of the situation in 1958, see Hirzebruch [6] 1958. Grauert gave the first (but by no means the last) proof in Grauert [11] 1960. Providing simpler proofs and generalizing the class of admissible mappings has been a constant challenge up to the present. A brief list in chronological order of the papers involved in this work includes: Grauert [13] 1968, Knorr [1] 1968/69, Kuhlmann [2] 1969, Narasimhan [2] 1969, Siu [9] 1970, Knorr [3] 1970, Siu [11] 1970/71, Foster-Knorr [1] 1971, Kiehl-Reinhardt 1971, Kiehl 1972 (these last two are described in Douady [4] 1971/72), Kiehl-Verdier 1971, Foster-Knorr [2] 1972, Banica [2] 1972, Flondor-Jurchescu 1972, Houzel 1973, and Kuhlmann [3] 1974. This theorem is a generalization of Cartan-Serre 1953, which is the special case of the theorem for a map onto a point, to a family parametrized by the codomain of the given map. For instance, Kiehl and Verdier use Fréchet modules over Fréchet algebras and quasinuclear homomorphisms, while Forster and Knorr need only the Banach open mapping theorem since they work directly with power series.

The other trend that will be mentioned briefly here is that of deformations of structure, to which Kodaira and Spencer turned in the middle 1950's. Their papers, Kodaira-Spencer [5] 1957, [6] 1958, [7] 1959, [8] 1960, Spencer [4] 1960, [5] 1962, and [6] 1969, exhibit a much more imaginative use of sheaf theory than had heretofore been the case, which certainly helped inspire many other imaginative uses. The subject of deformations has had a long development which still continues. Many of the most important papers involve little or no explicit sheaf theory and so are not recorded here; however, see Donin [2], Douady [1], [2], [5], W. Fisher, Frölicher, Kobayashi and Nijenhuis, Grauert [14], Griffiths [1], Kerner [1], Kodaira, Nirenberg and Spencer 1958, Kuranishi [2], [3], [4], [5] 1962/71 and Trautmann [5] 1974/75, [6] 1975 and [7] 1976. Deformations of other pseudogroup structures have also received considerable attention; for this see Gerstenhaber V, Gray V [1], Guillemin and Sternberg IV, Kupera and Spencer IV [2] , Spencer [5] , IV [3] , and Pommaret IV.

The subject of several complex variables from the viewpoint of sheaf theory had jelled enough by the end of the 1950's and early 1960's for a number of expositions and textbooks to appear. For instance, Behnke and Grauert 1960 contains a thorough discussion of the sheaf-theory aspects of complex analysis and its connections with classical analysis up to the time of the Brussels colloquium (1953); Hodge 1961 reviews the early work of Kodaira and Spencer in the direction of the Riemann-Roch theorems. Grauert, in his paper in the Bombay collection (Grauert [10] 1960), reviews the work of Kodaira and Spencer on deformations and discusses moduli in some generality. The first real textbook is the Tata lecture notes volume, Dowker 1957, which gives a very careful detailed treatment of sheaf theory with many counterexamples; furthermore, it contains an excellent account of coherent sheaves and a proof of the one-dimensional case of Oka's theorem (probably the first comprehensible account in print). For a modern proof see Kultze I [3] 1970 or Grauert and Remmert [8] 1977. As sheaf theory became more and more a standard part of the subject, the number of lecture notes and textbooks grew. In chronological order, there are at least the following works: Malgrange [4] 1958, Rossi [1] 1960, Norguet [6] 1962 (which has nice diagrammatic presentations of the relations between various properties), Fuks [1] 1962, Bers 1963, Hervé [1] 1963, Katznelson 1963/64 (which is one of the first places to discuss Frechet sheaves explicitly), Frenkel [2] 1965, Gunning and Rossi 1965 (the first "American" style textbook), Narasimhan [1] 1966, Chern [2] 1967, Sorani 1969, Morrow and Kodaira 1971, Whitney 1972, Hormander 1973, Wells 1973, Field [1] 1974, and G. Fischer [2] 1976 (which is probably the best survey of the subject). During the 1960's and 1970's the subject grew much too vast to conveniently summarize (as indicated by the inordinate length of our bibliography), and the interested reader will have to turn to books like Fischer's and other sources for an account of the present state of the field. A review of some of the literature between 1964 and 1973 can be found in Onishchik 1975 which lists some 135 references.

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CHAPTER III. SHEAVES AND ALGEBRAIC GEOMETRY

The history of the use of sheaf theory in algebraic geometry is very simple, and the major works are now - thanks to Springer-Verlag - easily available. There were two stages: first algebraic varieties and then schemes.

1. ALGEBRAIC VARIETIES. The fundamental paper in the sheaf-theoretic study of algebraic varieties is FAC , here listed as Serre II [6] 1955. According to Zariski 1956, preliminary versions were available in 1954 (there was a Bourbaki seminar, Serre II [3] 1954), which were studied at the Boulder meeting that summer and which were the basis for the Igusa-Zariski seminar at Harvard in the fall - the first of what must have been a long line of such seminars. I recall a similar seminar at Columbia in 1960/61, though by then it clearly was old stuff. The main result of Chapter II of FAC is that affine varieties are the analogues of Stein manifolds in the complex category, so that Theorems A and B (of Chapter II) are valid here. The main result of Chapter III (omitting the number-theoretic aspects of the subject) is that projective algebraic varieties are analogous to complex projective analytic varieties, so that Theorems A' and B' (of Chapter II) are valid. This beautiful paper, which one still reads with pleasure, was followed a year later by GAGA , here listed as Serre II [7] 1955/56, which showed that, for a non-singular complex algebraic variety, the algebraic and analytic theories coincided. This is a fundamental result without which the subject would be hopelessly chaotic. The topic was studied further by Grothendieck in [6] 1956/57 and [10] 1960/61; see also Houzel 1960/61. The most definitive statements are in Hakim 1972, Chapter VII, entitled "Equivalence algébrique-analytique". In the year following GAGA came Grothendieck's fantastic breakthrough in the Riemann-Roch theorem (written up in Borel-Serre II 1958), which not only laid the foundations for future work on the Riemann-Roch theorem, K-theory, and Atiyah-Singer index theorems (not discussed here) but also laid to rest the sheaf-theoretic study of algebraic varieties for over a decade after three earth-shaking papers. In the 1970's interest in varieties has revived; see for instance Baum, Fulton and MacPherson 1975 on the Riemann-Roch theorem for singular varieties, or Hartshorne [6] 1975 on DeRham cohomology of algebraic varieties, which uses hypercohomology extensively.

2. SCHEMES. The second stage belongs,almost to the end, essentially to Grothendieck who, after abortive attempts by Chevalley and others, found the correct generalization of varieties - that of schemes. This is not the place to attempt a detailed account of the fifteen years that culminated in Deligne's proof of the Weil conjectures which were the motivation for the whole effort. The beginning stages of this study are clearly described in Grothendieck [7] 1958, where three main topics for cohomological investigations in algebraic geometry are identified:

(i) "Weil cohomology of an algebraic variety", via connections between sheaf-theoretic cohomology and cohomology of Galois groups on the one hand, and the classification of unramified coverings of a variety on the other;

(ii) cohomology theory of algebraic coherent sheaves, with

(a) general finiteness and asymptotic behaviour theorems,

(b) duality theorems, including (respecively identical with) a cohomological theory of residues,

(c) Riemann-Roch theorem, including the theory of Chern classes for algebraic coherent sheaves, and

(d) some special results, concerning mainly abelian varieties; and

(iii) application of the cohomological methods to local algebra.

In a detailed discussion of (ii) above, Spec(A) is defined as well as the category of schemes over a fixed ground scheme. "Most of the notions and results of usual Algebraic Geometry can now be stated and proved in this new context, provided essentially that in some questions one sticks to noetherian schemata and to morphisms which are of finite type."

As to the development of this program, we shall merely sketch in broad outline the major sources. A detailed review covering the years 1960-1971 and containing 514 references can be found in Dolgachev 1974. There are four collections that contain nearly all of Grothendieck's work on the subject together with the work of many of his collaborators. They are:

1. Grothendieck [11] 1962, which contains the Bourbaki seminars of Grothendieck up to 1962;

2. The later Bourbaki seminars together with papers from the IHES, cited as Grothendieck [13], [14], [15], and [16], and found in "Dix exposés sur la cohomologie des schémas", North Holland 1968;

3. The eight volumes of "Eléments de Géométrie Algébrique" 1960/67, written in collaboration with Dieudonné and laying the foundations for the subject; and

4. The eight books of the "Séminaire de Géométrie Algébrique du Bois Marie" (cited SGA), of which thirteen volumes volumes presently exist, and which contain most of the deeper aspects of the study in considerable detail.

The specific references to SGA are as follows:

SGA1 (1960/61)	= LNM <u>224</u> - Grothendieck [18] 1971
SGA2 (1962)	= North Holland 2 = Grothendieck [19] 1969
SGA3 (1962/64)	= LNM <u>151</u>, <u>152</u>, <u>153</u> = Demazure and Grothendieck 1970
SGA4 (1963/64)	= LNM <u>269</u>, <u>270</u>, <u>305</u>
	= Artin, Grothendieck and Verdier 1972/73

SGA4$\frac{1}{2}$	=	LNM 569	=	Deligne 1977
SGA5 (1965/66)	=	LNM 589	=	Illusie [5] 1977
SGA6 (1966/67)	=	LNM 225	=	Berthelot, Grothendieck and Illusie 1977
SGA7 I (1967/69)	=	LNM 288	=	Grothendieck [20] 1972/73
SGA7 II (1967/69)	=	LNM 340	=	Deligne and Katz 1973
SGA8(?)	=	LNM 407	=	Berthelot 1974 .

From our point of view, an interesting feature of these works is the shifting emphasis from coherent sheaves to sheaves for the etale cohomology. Graham White in reading this report has offered the following comments on this situation: "Weil cohomology is fundamentally *different* from the cohomology of coherent sheaves, which is the cohomology of things which look like sheaves of germs of algebraic functions. Weil cohomology is the cohomology of locally constant sheaves (e.g., the constant sheaves $\mathbb{Z}$ or $\mathbb{Z}/n\mathbb{Z}$). Again, in this case, we want the sets of a covering to fit together; so we need a theorem that says that, for suitable small open sets, all the cohomology groups vanish. This is not quite possible. What we can prove is that if we take a cohomology class in some higher cohomology group of a space, we can kill *that class* by restricting it to small enough "open sets". However, "open sets" is here interpreted in the sense of the etale topology: it means here an etale covering (i.e., merely a "covering space" in the topological sense) rather than an open subset; and (another however) we have to use torsion sheaves. With these modifications we can still construct a cohomology theory which is very precisely analogous to classical singular cohomology with coefficients in, say, $\mathbb{Z}/n\mathbb{Z}$. (If we are working over a field of positive characteristic, n here has to be prime to the characteristic.) Taking a prime ℓ , say, we can take inverse limits of cohomologies with coefficients in $\mathbb{Z}/\ell^r\mathbb{Z}$ and end up with a cohomology with coefficients in $\mathbb{Z}_\ell$, the ℓ-adics. There are good arithmetic reasons why we cannot get a theory with coefficients in $\mathbb{Z}$."

The etale cohomology was first described in Artin [1] 1962. In the introduction to the Springer edition of SGA , Grothendieck comments that the general idea of a topos developed while SGA1 was being written. The etale topology does not appear explicitly until SGA4 , which is a general treatise on Grothendieck topologies. SGA4$\frac{1}{2}$ is devoted to a simpler exposition of the etale topology and ℓ-adic cohomology, which is treated more fully in SGA7 . SGA4$\frac{1}{2}$ also contains Verdier's paper on derived categories. SGA5 concerns a number of topics, one of which is a duality theory for the etale topology and constructive torsion sheaves, which is formally analogous to the duality of Hartshorne [1] 1966 for the Zariski topology and coherent sheaves. In SGA6 the Riemann-Roch theorem is proved for suitable noetherian schemes and suitable morphisms using derived categories and the cotangent complex. (For further information on this last topic, see also Grothendieck [17] 1968 and Illusie [1] 1971 and [3] 1972 .) Finally, SGA3 is about group schemes, which are treated more fully in Gabriel and Demazure 1970; see also Voigt 1977. For an account of the final work on the Weil conjectures, see Mazur [3] 1975 as well as the original papers of Deligne (not cited).

Recently, there have been further generalizations of schemes. Certain functors on the category of schemes are not representable, but become so if more general objects called algebraic spaces are allowed. See Knutson 1971, Artin [4] 1969, [5] 1969, and [6] 1969/70 as well as Gabriel-Demazure 1970. Furthermore, sheaves themselves have again been replaced by stacks in Artin [9] 1974, although these are in fact suitable sheaves for the etale topology. Yet another generalization of a category of sheaves is that of an etendu as found in SGA4 and Lawvere V [11] 1975.

The sheaf-theoretic aspects of algebraic geometry have now stabilized enough for textbooks to begin to appear. For some time, the only one was MacDonald 1968, which gives a brief, simple account of schemes and varieties, including an account of the Riemann-Roch theorems but no material from SGA . In Shafarevitch 1974 there is a brief mention of schemes, identifying Kahler [2] 1958 as the origin of this notion. Most recently, Hartshorne 1977, in a tour de force of under 500 pages, carries the reader from basic commutative algebra to an understanding of the Weil cohomology. Approximately half of the book is devoted to schemes and their cohomology, and the bibliography contains over 200 items.

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CHAPTER IV. SHEAVES AND DIFFERENTIAL EQUATIONS

The study of differential equations frequently involves two steps, first establishing local existence theorems and then extending local solutions to global solutions which are often required to satisfy some additional conditions. The great success of sheaf theory in the case of the Cauchy-Riemann equations and the seemingly obvious sheaf-theoretic nature of the problem would suggest that sheaf theory has much to contribute to the study of differential equations. Actually, there has been less interplay than one might expect. We shall discuss three topics where sheaf theory has been of some use: distributions and their generalizations; elliptic operators, the Spencer sequences, etc.; and Pfaffian forms, pseudo-groups, etc.

1. DISTRIBUTIONS. The idea of solving differential equations by admitting more general entities than functions as solutions was systematically developed by Schwartz in Schwartz 1950. This was the first extended treatment of the theory of distributions, and one finds, as Theoreme IV, the following principle of "recollement des morceaux":

"Soit $\{\Omega_i\}$ un famille finie ou infinie d'ouverts, de réunion Ω; soit d'autre part $\{T_i\}$ une famille de distributions dépendent du même ensemble d'indices I. La distribution T_i est définie dans l'ouvert Ω_i; on suppose de plus que, si Ω_i et Ω_j ont une intersection non vide, T_i et T_j coincident dans cette intersection. Alors il existe une distribution et une seule, T, definie dans Ω, qui coincide avec T_i dans chaque ouvert Ω_i."

It is scarcely to be believed that Cartan and/or Serre was not somehow involved in this absolutely clear statement of the fundamental defining property of a sheaf. However, Schwartz does not use the word "faisceau", and the only obvious use made of this principle is to show that the support of a distribution is well defined. It should be mentioned that functional analytic properties of the spaces of distributions play an important role in the development of the theory. The sheaf property is mentioned again in Silva 1955, which is an axiomatic treatment of distributions. Sheaves are still not mentioned, but what is shown amounts to the assertion that the sheaf of distributions is characterized as being the unique sheaf on $\mathbb{R}^n$, containing the sheaf of continuous functions, which is closed under partial derivatives and such that every section s is locally an iterated partial derivative of a continuous function f in such a way that, if s is independent of x_i, then so is f !

Although sheaf theory has never played any real role in the theory of distributions, it is essential in the study of hyperfunctions of several variables as introduced in the second part of Sato [1] 1959. In the one dimensional case, for $\mathbb{R} \subset \mathbb{C}$, hyperfunctions on $\mathbb{R}$ are intuitively "differences of boundary values of complex analytic functions"; more precisely, if U is an open set in $\mathbb{C}$ and $V = \mathbb{R} \cap U$, then $B(V) = O(U-V)/O(U)$ is the space of hyperfunctions on V. In general, if M is an n-dimensional real analytic manifold and if U is a complex analytic neighbourhood of M, then the sheaf of hyperfunctions on M is the relative cohomology sheaf $B = H^n(U, U-M, O)$, where O is the sheaf of holomorphic functions on U. This is essentially independent of U and contains the sheaf of distributions as a subsheaf closed under differentiation. One of the main advantages of hyperfunctions over distributions is that B is a flabby sheaf. For details, see Sato [1] 1959, Verley 1966/67, Schapira [4] 1970, Komatsu [2], [5] 1973, and Cerezo, Charazain and Piriou 1975. Furthermore, hyperfunction solutions to differential equations are readily derived from the corresponding analytic results; see, e.g., Harvey [1] 1966, [2] 1969, Komatsu [4] 1973, Sato [2] 1969, and Schapira [1] 1967, [2] 1967/68, and [7] 1971.

However, this is not the whole story, since the sheaf of microfunctions plays an important role in extensions of the theory to pseudo-differential operators. Microfunctions are described as follows: Let A denote the sheaf of real analytic functions on M. There is a (not completely obvious) embedding of A into B, and the quotient sheaf is of interest. To describe it in a useful way, let S^* denote the conormal sphere bundle to M in U with projection $\pi : S^* \to M$. Then A lifts to a sheaf A^* on S^* which is a subsheaf of π^*B, and the quotient sheaf (on S^*) is called the sheaf C of microfunctions. Its direct image π_*C is the quotient of B by A. The projection $B \to \pi_*C$ is called the spectrum and assigns to a function its "singularities", which can be regarded as a microfunction (on S^*). This was introduced in Sato [3] 1970, where for instance there are necessary and sufficient conditions for the well-known (generally unsolvable) Levy differential equation to have a microlocal solution. (Actual solutions are much more recent; see Kohn 1977). For details, see Morimoto [1] 1970, [2] 1972, Schapira [3] 1970, [6] 1970/71 and, most importantly, Sato, Kawai, and Kashiwara 1973.

2. ELLIPTIC OPERATORS. There are two separate places where sheaves have been used (or at least mentioned) in the study of elliptic operators and related notions. The first is in abstract potential theory and concerns generalizations of harmonic functions. In all treatments these have been taken to be a subsheaf of the sheaf of continuous functions. This is first stated explicitly in Bauer [2] 1962; it is taken as part of the definition in Boboc, Constantinescu and Cornea [1] 1963 and [2] 1965. A curious aspect of [2] is the discussion of "abstract supports" for a vector lattice, this being a function K from the lattice L to the compact subsets of a topological space Y satisfying the properties (i) $K(a) = \emptyset$ iff $a = 0$; (ii) if $a \leq b$, then $K(a) \subset K(b)$; and (iii) if $Y = G_1 \cup G_2$, where G_1 and G_2 are non-empty open sets, and if $a \in L$, then $a = a_1 + a_2$ where $K(a_i) \subset G_i$. This is of course very reminiscent of the original definition in Leray I [1] 1945

and the current definitions in logic. The book Constantinescu and Cornea 1972 provides a complete account of the subject. While it makes heavy use of the word "sheaf", it makes almost no use of properties of sheaves. The historical remark on page 34 is misleading in that practically none of the papers mentioned in it use sheaves in any way.

The second and much more significant use of sheaves in this field centers around the Spencer sequences. The first account of this is in Bott 1963, which presumably led to Quillen 1964. Both of these are unobtainable in libraries. Spencer's own account is in Spencer 1968, where two sequences are described first for a vector bundle and then more generally for a differential operator between vector bundles. The first sequence is:

$$0 \to \overline{E} \to \overline{J_k(E)} \to \overline{T^* \otimes J_{k-1}(E)} \to \dots \to \overline{\Lambda^n T^* \otimes J_{k-n}(E)} \to 0 ,$$

where E is a vector bundle, $J_k(E)$ is the bundle of k-jets of E, T^* is the cotangent bundle, and bars denote sheaves of sections. The second sequence is:

$$0 \to \overline{E} \to C_k^o \to C_k^1 \to \dots \to C_k^n \to 0 ,$$

where the i-th term is a suitable quotient of the i-th term of the first sequence. Now let E and F be vector bundles and $D : \overline{E} \to \overline{F}$ a k-th order differential operator; i.e., a sheaf homomorphism which factors through the canonical map $j_k : \overline{E} \to \overline{J_k(E)}$ by $\phi : \overline{J_k(E)} \to \overline{F}$. The kernel of ϕ is denoted by R_k and is called the differential equation associated to D. It has prolongations $R_{k+\ell} \subset J_{k+\ell}(E)$. If θ denotes the sheaf of germs of solutions of R_k (i.e., $\theta = j_k^{-1}(R_k)$), then the first sequence for D is:

$$0 \to \theta \to R_m \to \overline{T^* \otimes R_{m-1}} \to \dots \to \overline{\Lambda^n T^* \otimes R_{m-n}} \to 0 ,$$

where $m \geq k+n$. The second sequence is:

$$0 \to \theta \to C^o \to C^1 \to \dots \to C^n \to 0 ,$$

where as before the i-th term of the second is a quotient of the i-th term of the first. An important role is played by the "symbols" of the different concepts mentioned here and the exact sequences involving them; see, e.g., Spencer [4] 1970. A more abstract treatment of some of this material is in Johnston [1], [2] 1971. In [2], for instance, it is shown that under appropriate hypotheses, for $k = \infty$, the Spencer cohomology groups are a suitable Ext. For current developments, see Goldschmidt [6] 1973 and Goldschmidt and Spencer 1976.

3. PFAFFIAN FORMS. The study of pseudogroups of transformations leaving invariant a suitable Pfaffian form and of the deformations of structures on manifolds defined by these pseudogroups has a long history going back to the original works of Lie in the 1890's and E. Cartan's in the 1920's (not cited). The modern theory owes much to the work of Kodaira and Spencer on deformations of complex structures (see

Chapter II), which is a special case. Some of the papers in the field are Goldschmidt [5] 1972/76 and [7] 1968/74, Gorbatenko 1973, Gray V [2] 1959, Guillemin and Sternberg 1966, Kumpera and Spencer [1] 1973, [2] 1974, Libermann 1958, Pommeret 1975, Reiffen and Vetter 1966, and Singer and Sternberg 1967.

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CHAPTER V. SHEAVES, CATEGORY THEORY, AND TOPOI

There has been a constant interplay between sheaf theory and category theory, each encouraging the development of the other. In the 1950's sheaves of modules over a sheaf of rings became important in algebraic and analytic geometry. At first it was not clear if one was to proceed by analogy with modules over a ring or if there was a common theory that subsumed both theories. For instance, was it a definition or a theorem that a sequence of sheaves was exact if and only if the sequence of stalks was exact at each point ? What was clear was that sheaves did not have elements in the same sense that modules have elements and that different, more intrinsic formulations were required. For many people, the realization that the category of sheaves of modules over a sheaf of rings was a "good" abelian category with enough injectives was probably the single most convincing argument for the continued development of a categorical approach to homological algebra and, in particular, for the abstract study of derived functors.

The first mention of this in print is in the introduction to Buchsbaum [1] 1955 which says: "Part III is devoted to the abstract treatment of the fundamental concepts in Cartan and Eilenberg, Homological Algebra, Princeton University Press, 1956 . Theorem 5.1 , however, is proved in its full generality so as to be applicable in the theory of sheaves. Part IV contains three applications of a purely algebraic nature. We desist from giving applications to (the) theory of sheaves as these would be fragmentary." There is no mention of sheaves in the body of this paper, but later in Buchsbaum [2] 1960 these applications were explicitly carried out. However, before that, the fundamental Tohoku paper, Grothendieck III [5] 1957, appeared, which in a certain sense covers much of the same material as Godement I 1958, but the approach is quite different. In any case, it, Godement, and Serre FAC were the indispensable references for the homological algebra of sheaves.

Once the questions about the homological algebra of sheaves of modules on a topological space were settled, it was appropriate to try to generalize the situation. Sheaves of modules on a topological space were thoroughly understood, but it was not clear (and still is not clear) how to treat sheaves of Fréchet spaces. (In what sense is there an associated-sheaf functor ?) Similarly, one would like to study things like bounded holomorphic functions, which form a "sheaf" only if coverings are required to be finite. The general question is thus to study functors (= presheaves) $F : C \to A$, where A has some sort of "algebraic" structure and C has some sort of "topological" structure, so that one can characterize the subclass of functors to be called sheaves. Usually one wants the subcategory of sheaves to be reflective and the reflection functor to be left exact. If C is the category of open sets of a topological space and sheaf has its usual meaning, then this was studied, in chronological order, in Gray [5] 1965 (preprint 1962), Heller and Rowe 1962, Mitchell 1965, Nishida 1969, Grillet [1] 1971, Ulmer 1971, and Felix 1975. The last reference gives a very good technical account of the different methods that have been used.

On the other hand, if the codomain category A is Sets but C is a general category, then one needs some notion of a "topology" on C in order to describe sheaves. The various descriptions all lead to what is called a Grothendieck topology on C . The first account is in Artin III [1] 1962, where a covering of an object $U \in C$ just means a family of maps $\{U_i \to U\}$. A *pretopology* on C is then a family $J(U)$ of coverings of U for each $U \in C$ satisfying three properties:

(i) $\{U \xrightarrow{1_U} U\} \in J(U)$;

(ii) if $\{U_i \to U\} \in J(U)$ and $V \to U$ is any map, then $\{V \times_U U_i \to V\} \in J(V)$ (that is, J is stable under pullbacks) ; and

(iii) $\{U_i \to U\} \in J(U)$ and for all i , $\{U_{ij} \to U_i\} \in J(U_i)$ implies $\{U_{ij} \to U_i \to U\} \in J(U)$ (that is, J has the "local property") .

A sheaf is a contravariant set valued functor F on C such that

$$F(U) \longrightarrow \Pi\, F(U_i) \rightrightarrows \Pi\, F(U_i \times_U U_j)$$

is an equalizer for all $\{U_i \to U\} \in J(U)$.

In Giraud III [2] 1963 this description was modified. One first defines a *crible* in a category C to be a right ideal, i.e., a family of morphisms S such that $u \in S$ and $u \circ v$ defined implies $uv \in S$. A topology on C then consists of a family $J(U)$ of cribles in C/U for each $U \in C$ such that:

(i) $C/U \in J(U)$;

(ii) J is stable under pullback = stable under change of base; and

(iii) J has the local property.

This description is further refined in Artin, Grothendieck, Verdier, III 1972/73 (written in 1963) (SGAA or SGA4), where it was observed that cribles in C/U correspond to subfunctors of the representable functor $C(-,U)$. These are in turn just subobjects of U , considered as an object in the category $\hat{C}$ of set-valued contravariant functors on U (i.e., $\hat{C}$ = presheaves). Thus, a topology on C is a family $J(U)$ of subobjects (in $\hat{C}$) of U for each $U \in C$ such that:

(i) $U \in J(U)$;

(ii) if $R \in J(U)$ and $u : V \to U$ is a map, then $u^{-1}(R) \in J(V)$; and

(iii) if $R \in J(U)$ and $R' \to U$ is a subobject such that for all $(u:V \to U) \in R$ one has $u^{-1}(R') \in J(V)$, then $R' \in J(U)$.

In this final version, $F \in \hat{C}$ is called a *sheaf* if for all $R \in J(U)$, the map $\hat{C}(U,F) \to \hat{C}(R,F)$ is an isomorphism. Using this formulation, it is indicated that there is a left exact, left adjoint functor to the inclusion functor of sheaves into presheaves, and the little Giraud theorem asserts that every such left-exact, reflective subcategory of $\hat{C}$ comes from a uniquely determined topology on C. A category equipped with a topology is called a *site*, and the category of sheaves for a site is called a *Grothendieck topos*. The big Giraud theorem says that a category E is a Grothendieck topos if and only if it satisfies four properties:

(i) E has finite limits;

(ii) E has arbitrary sums which are disjoint and universal;

(iii) E has universally effective equivalence relations; and

(iv) E has a small set of generators.

For proofs, see Barr 1971 or Schubert 1972.

The final form of this description of topologies is due to Lawvere, who observed that since $\hat{C}$ has a subobject classifier Ω, it follows (by axiom (ii)) that J is just a subobject of Ω subject to certain conditions. Or since subobjects correspond to maps to Ω, J corresponds to a map $j : \Omega \to \Omega$ and the axioms can be put in the form:

(i) $j \circ tr = tr$, where $tr : 1 \to \Omega$;

(ii) $j \circ j = j$;

(iii) $j \circ \wedge = \wedge \circ (j \times j)$, where $\wedge : \Omega \times \Omega \to \Omega$ corresponds to intersection.

Such a j can be regarded as a closure operator, from which it is clear what a dense subobject is. Finally F is called a sheaf if for all dense $X' \to X$, the map $\hat{C}(X,F) \to \hat{C}(X',F)$ is an isomorphism. This same description carries over to an elementary topos (where it actually was done first), and one shows that the category of sheaves is again a left-exact, reflective subcategory of the given topos and is itself a topos; see Freyd [1] 1972, Tierney [1] 1972 and Lambek and Rattray [2] 1974.

The history of elementary topoi is very short, as befits the most recent of our topics. During the 1960's Lawvere was working on two related questions, the categorical descriptions of *theories* and of *sets*. In his thesis, Lawvere [1] 1963, (finitary) algebraic theories and their models were treated in a completely categorical fashion. This was immediately generalized to (infinitary) equational theories in two different ways by Linton and Beck. However, Lawvere had a different generalization in mind, that of elementary theories. In algebraic theories there is a category of arities (in Bénabou's version, sorts are also included), but for elementary theories one needs in addition an object of *truth values* so that partially defined operations and relations can be described. A brief indication of what Lawvere had in mind can be found in Lawvere [3] 1966 and [4] 1967, and a full account is given in Volger [1] and [2] 1975. (See also Daigneault 1970.) On the other hand, in Lawvere [2] 1966, there is an account of the elementary theory of the category of sets. This obviously suggests looking for the elementary theory of other categories. Lawvere described categories themselves, and Schlomiuk described topological spaces, but neither of these descriptions was completely satisfactory. However, Bunge [1] 1966 treats categories of set-valued functors in an interesting and useful way. (Another version can be found in Gabriel-Ulmer 1971.) It seems now, and it seemed then, very natural to ask for a description of the elementary theory of categories of sheaves. Giraud's theorems characterizing Grothendieck topoi were known, but they were not elementary and depended heavily on set theory.

It was a brilliant inspiration to see that the answers to these two questions were the same: an elementary theory was slightly generalized to a cartesian closed category with a subobject classifier (= truth values object) thereby giving equally well the appropriate elementary notion of a category of sheaves. Lawvere has given his own account of how this inspiration took place - or at least how it should have taken place - in the Eilenberg volume, Lawvere [12] 1976. The basic theory was worked out in collaboration with Tierney in Halifax during 1969/70. Using notes from there, Bénabou produced the first systematic account in Paris, Bénabou, Celeyrette, and Jacob 1970/71. In the summer of 1970 Lawvere and Tierney gave talks in Zurich, a summary of which appeared in Gray [6] 1971. Finally, in the same year the quasi-textbook Kock and Wraith 1971 appeared with enough details to enable anyone who was interested (and there were many) to begin working on the theory. Shortly thereafter, Freyd [1] 1972 provided a deeper discussion of the theory, and Wraith [4] 1975 began treating the more global questions concerning the category of topoi. Finally, the book Johnstone [3] 1977 provides a complete account of nearly all aspects of the theory to date. Perhaps the main aspect which is difficult to document in published works is the connection with logic. The best sources are Lawvere [10] 1975 and [12] 1976, together with Reyes [1] 1974, [2] 1975 and [3] 1976, Lambek [2] and Makkai and Reyes [1] 1976 and [2] 1977, and also the articles in this volume. Unfortunately one of the most influential figures in this development, A. Joyal, has thus far not given us a written record of his work; however, see Labelle 1971. Besides topics discussed at the present meeting, future developments seem to be going in the directions of Bénabou [2] 1975 and [3] 1975, and recent unpublished work of Cole and Tierney on pseudolimits in the category of topoi.

The bibliography for this chapter includes two other topics of an algebraic nature as well as miscellaneous papers concerning sheaves and algebra. First, out of the vast literature on K-theory, a few papers concern themselves specifically with sheaves; e.g., Block [1] 1973 and [2] 1977, Brown and Gersten 1973, Gersten 1973, and Quillen [1] 1974 and [2] 1973. In the second place there have been a number of attempts to extend the definition of Spec to noncommutative rings. Some of the papers which describe a structure sheaf are Barnwell and Mewborn 1978, Golan 1975 Golan, Raynaud and van Ostaeyen 1976, Goldston and Mewborn 1977, Murdock and van Ostaeyen 1975, van Ostaeyen [1] 1975 and [2] 1977, and van Ostaeyen and Verschoren [1] 1976 and [2] 1976.

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