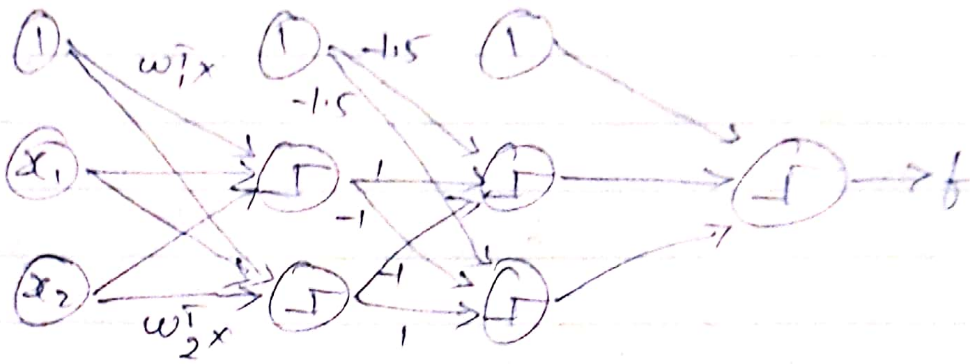
013.7.7.20228

Exercise 7.3:

$f(x) = \text{sign}\left[\text{sign}(h_1(x) - h_2(x) - \frac{3}{2}) - \text{sign}(h_1(x) - h_2(x) + \frac{3}{2}) + \frac{3}{2}\right]$, where

$h_1(x) = \text{sign}(w_1^T x)$ and $h_2(x) = \text{sign}(w_2^T x)$



$\Rightarrow$ we have. $h_1(x) = \text{Sign}(w_1^T x)$; $h_2(x) = \text{Sign}(w_2^T x)$

Now $A = (1.5)(1) + (1)(h_1(x)) + (-1)(h_2(x))$

$\therefore A(x) = \text{sign}(1.5 + h_1(x) - h_2(x))$

Similarly $B = (-1.5)(1) + (-1)(h_1(x)) + (1)(h_2(x))$

$= -1.5 - h_1(x) + h_2(x)$

$\therefore B(x) = \text{sign}(-1.5 - h_1(x) + h_2(x))$

for output layer $= (1.5)(1) + (1) A(x) + B(x)$

$\therefore f = \text{Sign}(1.5 + A(x) + B(x))$

substitute values of $A(x)$ and $B(x)$ in above.

$f = \text{sign}(1.5 + \text{sign}(1.5 + h_1(x) - h_2(x)) + \text{Sign}(h_2(x) - h_1(x) - 1.5))$

$f = \text{sign}\left(\text{sign}(h_1(x) - h_2(x) - \frac{3}{2}) - \text{Sign}(h_1(x) - h_2(x) + \frac{3}{2}) + \frac{3}{2}\right]$

hence proved ; when $h_1(x) = \text{Sign}(w_1^T x)$ & $h_2(x) = \text{Sign}(w_2^T x)$

Exercise 7.7

$E_{in}(w) = \frac{1}{N}\sum_{n=1}^{N} (\tanh(w^T x_n) - y_n)^2$ show that

$\nabla E_{in}(w) = \frac{2}{N}\sum_{n=1}^{N} (\tanh(w^T x_n) - y_n)(1-\tanh^2(w^T x_n))x_n$

Taking derivative with respect to 'w' of $E_{in}(w)$

$$\therefore \frac{\partial E_{in}(w)}{\partial w} = \frac{2}{N}\sum_{n=1}^{N} \frac{\partial(\tanh(w^T x_n) - y_n)}{\partial w}(\tanh(w^T x_n) - y_n)$$

$$= \frac{2}{N}\sum_{n=1}^{N} (1-\tanh^2(w^T x_n))\frac{\partial(w^T x_n)}{\partial w}.\tanh(w^T x_n - y_n)$$

$$= \frac{2}{N}\sum_{n=1}^{N} (1-\tanh^2(w^T x_n))(x_n)(\tanh(w^T x_n) - y_n)$$

$$= \frac{2}{N}\sum_{n=1}^{N} (\tanh(w^T x_n) - y_n)(1-\tanh^2(w^T x_n) - y_n).x_n$$

// hence proved

also if $w \to \infty \Rightarrow \lim_{w\to\infty} \nabla E_{in}(w)$

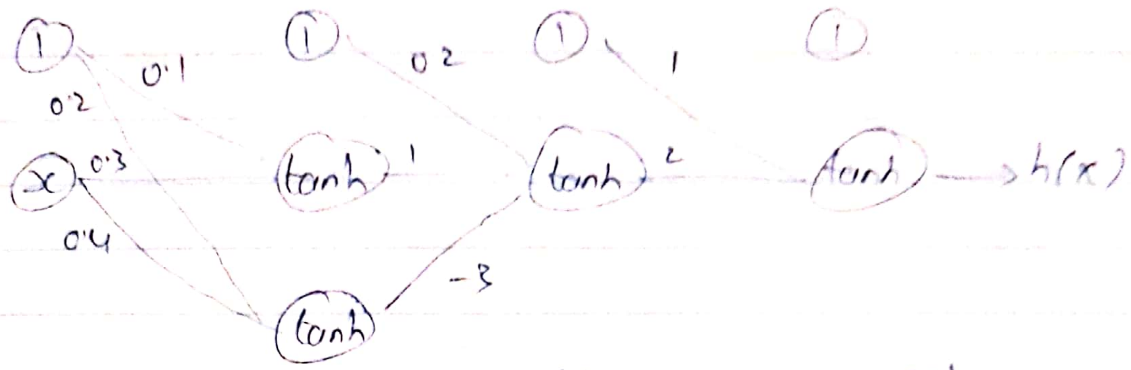
$$\therefore \lim_{w\to\infty} \nabla E_{in}(w) = \lim \frac{2}{N}\sum_{n=1}^{N} (\tanh(w^T x_n) - y_n)(1-\tanh^2(w^T x_n) - y_n)x_n$$

when $w \to \infty$, $\tanh^2(w^T x_n) \to 1$

$\therefore$ The quantity $(1-\tanh^2(w^T x_n)) - 1 - 1 = 0$.

Hence gradient $= 0$ no matter what and it will take long long time to converge as might go in an infinite loop as well as if this happens, perceptron will have difficult time to classify the data properly and will make things difficult to optimize.

Exercise - 7.8:



Compute $s^{(1)}$, $x^{(1)}$, $\delta^{(1)}$ and $\partial e / \partial w^{(1)}$

$$w^{(1)} = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{bmatrix} \quad w^{(2)} = \begin{bmatrix} 0.2 \\ 1 \\ -3 \end{bmatrix} \quad w^{(3)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$x = 2, y = 1$

$x(0)$	$s(1)$	$x^{(1)}$	$s^{(2)}$	$x^{(2)}$	$s^{(3)}$	$x^{(3)}$
$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$	$\begin{bmatrix} 0.7 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0.7 \\ 1 \end{bmatrix}$	$[-2.1]$	$\begin{bmatrix} 1 \\ -2.1 \end{bmatrix}$	$[-3.2]$	$[-3.2]$

$\therefore \delta^{(3)} = 2(x^{(2)} - 1)(1)$ as $\theta'(s) = 1$

$= 2(-3.2 - 1)$

$= 2(-4.2) = [-8.4]$ — (1)

$$\delta^{(2)} = \theta'(s^{(2)}) \otimes [w^{(3)} \delta^{(3)}]_1^{d(2)}$$

$= 1 \otimes [2 \times (-8.4)] = [-16.8]$ — (2)

$$\delta^{(1)} = \theta'(s^{(1)}) \otimes [w^{(2)} \delta^{(2)}]_1^{d(1)}$$

$= 1 \otimes \left[\begin{bmatrix} 1 \\ -3 \end{bmatrix} [-16.8] \right] = \begin{bmatrix} -16.8 \\ 50.4 \end{bmatrix}$

— (3)

$$\frac{\partial(l)}{\partial(w^{(1)})} = x^{0}\left(\delta^{(1)}\right)^{T} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} -16.8 & 50.4 \end{bmatrix}$$

$$= \begin{bmatrix} -16.8 & 50.4 \\ -33.6 & 100.8 \end{bmatrix}$$

$$\frac{\partial(l)}{\partial(w^{(2)})} = x^{1}\left(\delta^{(2)}\right)^{T} = \begin{bmatrix} 1 \\ 0.7 \\ 1 \end{bmatrix} \begin{bmatrix} -16.8 \end{bmatrix}$$

$$= \begin{bmatrix} -16.8 \\ -11.76 \\ -16.8 \end{bmatrix}$$

$$\frac{\partial(l)}{\partial(w^{(3)})} = x^{(2)}\left(\delta^{(3)}\right)^{T} = \begin{bmatrix} 1 \\ -2.1 \end{bmatrix} \begin{bmatrix} -8.4 \end{bmatrix}$$

$$= \begin{bmatrix} -8.4 \\ 17.64 \end{bmatrix} //$$