Problem 8.2

$$X = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \quad Y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$$

optimal hyperplane

$(b^*, w^*) = ?$ & its margin

$\therefore (-1)(0(w_1) + 0(w_2) + b) \geq 1$

$\Rightarrow (-1)(0(w_1) + (-1)w_2 + b) \geq 1$

$\Rightarrow (+1)(-2(w_1) + (0)(w_2) + b) \geq 1$

① $-b \geq 1$; ② $w_2 - b \geq 1$; ③ $-2w_1 + b \geq 1$

by adding 1 and 3 $\Rightarrow -2w_1 \geq 2$

$\Rightarrow -w_1 \geq 1$

$\Rightarrow w_1 \leq -1$ //

Similarly from 1 and 2 $\Rightarrow -b \geq 1 - w_2$.

$\therefore 1 = 1 - w_2$

$\therefore w_2 = 0 \Rightarrow w_2 \geq 0$ //

$\Rightarrow$ we have $w_2 \geq 0$; $w_1 \leq -1$ & $-b \geq 1$

Taking the minimums, $w_2 = 0$, $w_1 = -1$ & $-b = 1 \Rightarrow b = -1$

$$\boxed{\therefore b^* = -1 \ \& \ w^* = [-1, 0]}$$

Margin $= \dfrac{1}{\|w^*\|} = \dfrac{1}{\sqrt{w_1^2 + w_2^2}} = \dfrac{1}{\sqrt{1+0}} = 1$ $\therefore$ margin $= 1$ //

Problem 8.4

$$X = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 2 & 0 \end{bmatrix} \quad y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix} \quad w = \begin{bmatrix} 1.2 \\ -3.2 \end{bmatrix}$$

$b = -0.5$ Solve dual problem and compute α^*

Dual problem: $\min_{\alpha \in \mathbb{R}} \frac{1}{2} \alpha^T Q_D \alpha - 1_N^T \alpha$

Subject to, $A_D \alpha \geq 0_{N+2} \longrightarrow C$

$Q_D = X_s X_s^T$ $\quad X_s = \begin{bmatrix} - y_1 x_1^T - \\ - y_2 x_2^T - \\ \vdots \\ - y_N x_N^T - \end{bmatrix}$

$A_D = \begin{bmatrix} y^T \\ -y^T \\ I_{N \times N} \end{bmatrix}$

$P = -1_N = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

Now putting the values to get Q_D, A_D, P, C

$$\therefore Q_D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 8 & -4 \\ 0 & -4 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad A_D = \begin{bmatrix} -1 & -1 & +1 \\ 1 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad P = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

Using 'CVXOPT' library's QP Solver.

$$\alpha^* = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

attached in the submission are more .ipynb file for these calculations done to get α^* with CVXOPT QPsolver

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