Probability of the sum of two independent variables

Let X and Y two independent random variables with possible values:

- X= {0,1,2,3}
- Y= {0,1,2,3}

with identical probabilities for all values P(0) = P(1) = P(2) = P(3) = 0.25

The possible values of the sum are: Z= {0,1,2,3,4,5,6} and the probabilities are:

- P(Z=0) = P(X =0) * P(Y =0)
- P(Z=1) = P(X =1) * P(Y =0) + P(X =0) * P(Y =1)
- P(Z=2) = P(X =2) * P(Y =0) + P(X =1) * P(Y =1) + P(X =0) * P(Y =2)
- Etc.

In other words, P(Z=k) = Sum of all P(X)*P(Y) where X + Y =k

In the continuous case

P(Z=k) = Sum of all P(X)*P(Y) where X + Y =k

Becomes a convolution

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y) dy$$