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A two-dimensional preventive maintenance strategy for items sold with warranty

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Conventional preventive maintenance (PM) strategies under two-dimensional (2D) warranties are usually age-based or usage-based, which means that the implementation of PM activities is based solely on item age or usage. In this paper, a new PM strategy, called 2D PM strategy, is proposed for items sold with a 2D warranty. Under this strategy, the item is preventively maintained every K units of age or L units of usage, whichever occurs first. The marginal approach is used to describe the effect of age and usage on item reliability by treating usage as a random function of age. Besides, the effect of PM is characterised by the reduction of virtual age. The objective of this study is to identify the optimal 2D PM strategy under fixed warranty terms so as to minimise the total expected warranty servicing cost from the manufacturer's perspective. A numerical example is provided to demonstrate the effectiveness of the proposed PM strategy. It is shown that the 2D PM strategy contains the age-based and usage-based strategies as special cases, and outperforms them in terms of warranty servicing cost. Finally, how to implement the proposed PM strategy in practice is discussed with an illustrative case.

Keywords: two-dimensional warranty; preventive maintenance; age reduction; warranty cost analysis; optimisation

1. Introduction

Nowadays, beyond excellent performance, competitive price and high quality, attractive after-sale services including warranty and maintenance have played an increasingly significant role in product marketing. Warranty is a contractual obligation incurred by a manufacturer in connection with the sale of a product. Its primary purpose is to offer post-sale remedy for customers when the products fail to fulfil their intended function during the warranty period (Blischke and Murthy 1992). Product warranty has been widely used to serve different proposes, and extensive attention has been paid to warranty management from various disciplines (Huang, Lo and Ho 2008; Tong et al. 2014; Xie, Liao and Zhu 2014).

Up to now, various warranty policies have been proposed and studied in the literature, see Blischke and Murthy (1992) for a good taxonomy for warranty policies. Based on the number of variables that are used to define the policy, warranty can be broadly divided into two categories, i.e. one-dimensional (1D) and two-dimensional (2D). A 1D policy is defined in terms of a single variable, e.g. age or usage; while a 2D policy is represented by a region in a 2D plane usually with one dimension denoting age and the other one representing usage. A new automobile is usually covered by a 2D warranty, e.g. free repair for 3 years or 60,000 km, whichever occurs first.

Offering attractive warranty terms to customers can promote item sales. However, this will incur additional cost to the manufacturer resulting from the servicing of warranty claims. Depending on the products and manufacturers, warranty servicing cost can range from 2% to as much as 15% of net sales (Murthy and Djamaludin 2002). For example, according to the 2014 General Motors annual report, the firm's total revenue was US\$ 155.9 billion and the warranty servicing cost on sold automobiles was estimated to be US\$ 4.3 billion, which is about 2.8% of the revenue (General Moter 2014). Besides, the total warranty servicing cost of US-based automobile manufacturers has reached US\$ 15.65 billion in 2014, which was the highest since 2003 (Warranty Week 2015). As a consequence, many manufacturers, especially automakers, give high priority to warranty cost reduction programmes.

One effective way of reducing the warranty servicing cost is to incorporate appropriate preventive maintenance (PM) actions into the warranty policy. Unlike corrective maintenance activities which are intended to restore failed items to an operational state, PM activities are scheduled activities in order either to reduce the likelihood of item failures or to prolong the item's lifetime (Wu and Zuo 2010). In both cases, PM activities can lead to effective warranty cost reduction for manufacturers. From the view of customers, PM activities can reduce the number of failures and increase the items' availability. Therefore, PM activities under warranty may be attractive for both manufacturers and customers.

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Although PM activities have multiple benefits, they are also an extra expenditure. Obviously, from the cost-benefit point of view, it is worthwhile for a manufacturer to adopt a PM programme only when the reduction of warranty servicing cost exceeds the additional cost incurred by the PM programme (Djamaludin, Murthy and Kim 2001; Kim, Djamaludin and Murthy 2004).

Since the pioneering research of Barlow and Hunter (1960), the modelling and optimisation of PM strategies have aroused the interests from many researchers; see Chang and Lo (2011), Khatab, Ait-Kadi and Rezg (2014), Zhou et al. (2015) and Tarakci (2016), for example. The present study focuses on the optimisation of PM strategies incorporating warranty contracts through mathematical modelling and analysis. Up to now, the related researches predominantly deal with PM problems under one-dimensional warranties; see, for instance, Djamaludin, Murthy and Kim (2001), Yeh and Lo (2001), Jack and Murthy (2002), Kim, Djamaludin and Murthy (2004), Chen and Chien (2007), Wu, Xie and Adam Ng (2011), Bouguerra, Chelbi and Rezg (2012), Chang and Lin (2012), Shafiee, Finkelstein and Zuo (2013), and Su and Wang (2014). Among them, Kim, Djamaludin and Murthy (2004) developed a framework for the cost analysis linking warranty and PM policy from a life-cycle perspective. In which, the PM activities are carried out at discrete time instants, and the effect of PM is characterised by the reduction of virtual age. A variation of the modelling framework in Kim, Djamaludin and Murthy (2004) is adopted in this study to serve the research purpose.

Moreover, previous studies have revealed that warranty policies should be developed based on a 2D framework which considers both the age and usage of items to increase the competitive advantage and customer satisfaction (Huang and Yen 2009). On the one hand, 2D warranty specifies a usage limit in addition to an age limit, thus it can protect the manufacturer from experiencing high warranty expense due to customers' heavy usage (Moskowitz and Chun 1994; Su and Shen 2012). On the other hand, 2D warranty provides the customer with potential flexibility to select one policy among available options that fits his/her needs and/or usage pattern best (Moskowitz and Chun 1994; Manna, Pal and Sinha 2006). Despite the importance of both 2D warranty and PM, integrated studies of both concerns have gained rather limited attention. To maximise the manufacturer's profit, Huang and Yen (2009) derived optimal 2D warranty terms by considering periodic PM activities, which are expected to be conducted by the customers. Shahanaghi et al. (2013) developed a mathematical model to identify the optimal PM strategy during 2D extended warranty contracts so as to minimise the warranty provider's servicing cost. Huang, Chen and Ho (2013) investigated the optimal 2D warranty terms with consideration of a reliability-based PM strategy. In Cheng et al. (2015), a warranty servicing strategy combining imperfect PM and minimal repair was proposed, where the PM actions were implemented in a special sub-region of warranty coverage and all failures were repaired minimally. Recently, Wang, Liu and Liu (2015) attempted to connect the 2D basic warranty and extended warranty, and determine the optimal number of PM activities performed within both basic and extended warranty periods.

In the above-mentioned references (Huang and Yen 2009; Shahanaghi et al. 2013; Cheng et al. 2015; Wang, Liu and Liu 2015), PM activities are performed by considering only the dimension of item age, e.g. PM activities are scheduled according to a specified age interval, irrespective of the item's usage intensity. We call it the age-based PM strategy in this study. Similarly, under the usage-based strategy, PM activities are implemented based solely on the dimension of item usage. Age-based and usage-based PM strategies have been widely employed in automobile industry. However, a shift in PM paradigm, from conventional 1D framework to new 2D framework, has occurred in some automakers. Under the new PM paradigm, the item is preventively maintained every K units of age or L units of usage, whichever occurs first. Here we refer it to as *2D PM strategy*, since the implementation of PM activities is based on a 2D framework, similar to a 2D warranty. For example, in Chinese market, SAIC-GM-Wuling Automobile Company provides their sold mini-commercial vehicles with free PM services every 4.5 months or 7500 km, whichever occurs first. Likewise, Chrysler recommends that individual vehicle owner should change oil and filter and perform some necessary inspections every 12 months or 16,000 km (i.e. 10,000 miles), whichever occurs first. For more information on free automobile maintenance programmes, please visit edmunds (www.edmunds.com).

In this paper, we investigate the imperfect 2D PM strategy for repairable items under 2D warranty. This PM strategy is imperfect in the sense that it can bring an item to somewhere between as good as new (AGAN) and as bad as old (ABAO). The optimal 2D PM strategy is determined to minimise the manufacturer's total expected servicing cost. Then the proposed PM strategy is compared with conventional age-based and usage-based strategies to illustrate its superiority. Moreover, some practical aspects of the proposed PM strategy are also discussed to help implement it. To the best of our knowledge, this paper represents the first attempt to model and analyse imperfect 2D PM strategy for items under 2D warranty, which is a valuable exploration in this area.

The rest of this paper is organised as follows. Section 2 formulates the model elements, including the item failure model and PM model. Section 3 focuses mainly on model analysis for the 2D PM strategy. The age-based and usage-based PM strategies are regarded as two special cases of the proposed PM strategy. In Section 4, a numerical example is provided to demonstrate the proposed PM strategy. Finally, Section 5 concludes this paper.

2. Model formulation

2.1 *Notations*

The notations used throughout this paper are summarised as follows:

t, u	age and usage of the item
W, U	age and usage limits of 2D warranty
K, L	age and usage intervals of 2D PM strategy
η_1	$=U/W$
η_2	$=L/K$
r	usage rate ($=u/t$)
$G(r)$, $g(r)$	distribution and density functions of r
$r_{\min}$, $r_{\max}$	lower and upper bounds of r
$\lambda(t\|r)$	conditional intensity function given $R = r$
m	level of PM effort ($0 \le m \le M$)
$\delta(m)$	age reduction factor of PM with level m
$C_p(m)$	PM cost with level m
C_r	expected minimal repair cost
Ω	decision space = (K, L, m)

2.2 *Modelling item failures*

This research focuses primarily on the failure process of repairable item as a whole, which means that the failure process is modelled at the system level and by using the black-box approach. In the literature, three main approaches have been developed to model failures for items under 2D warranties, i.e. bivariate, composite scale and marginal (univariate) approaches; refer to Jack, Iskandar and Murthy (2009) and Wu (2014) for more discussions on these approaches.

In this study, the marginal approach is applied to model the item's failure process. Denote the age and total usage of the item by t and u, respectively, and $(t, u) = (0, 0)$ corresponds to the point of item purchase. An assumption adopted by the marginal approach is that the usage rate R varies randomly across the customer population but is constant over time for a specific customer. As a result, R can be modelled as a random variable with distribution function $G(r) = P(R \le r)$, $0 \le r < \infty$. Besides, it is supposed that the distribution $G(r)$ is known for the manufacturer, either through past history of similar items or from a customer survey (Ye et al. 2013). Conditional on $R = r$, the total usage u of an item at age t is given by $u = rt$.

Item failures can thus be modelled by a point process with an intensity function that is dependent on both age and usage. Given $R = r$, the conditional intensity function can be denoted as $\lambda(t|r) = \varphi(t, u)$, where $\varphi(\cdot)$ is a non-decreasing function of both t and u. Assume that all failures are minimally repaired with negligible durations and no PM is performed, then the failures over time occur according to a non-homogeneous Poisson process (NHPP) with conditional failure intensity function $\lambda(t|r)$.

For the numerical example, following Murthy and Wilson (1991), the conditional intensity function in this study is modelled as a polynomial function, and expressed as

$$\lambda(t|r) = \theta_0 + \theta_1 r + (\theta_2 + \theta_3 r)t \tag{1}$$

where θ_0, θ_1, θ_2 and θ_3 are positive constants, which can be estimated through the historical record of warranty claims and maintenance activities kept by the manufacturer.

As can be seen, by using the marginal approach, the 2D failure modelling problem is reduced to a 1D problem. It is worth mentioning that the polynomial intensity function has been widely used in the 2D warranty literature; see e.g. Iskandar and Murthy (2003), Huang and Yen (2009), Wang, Liu and Liu (2015) and Su and Wang (2016).

2.3 *Modelling preventive maintenance effect*

Generally speaking, a PM package corresponds to a set of maintenance tasks, including, for example, cleaning, systematic inspection, lubricating, adjusting and calibrating, replacing different sets of components, etc. (Ben Mabrouk, Chelbi and Radhoui 2016). For a manufacturer, appropriate PM activities can reduce the number of failures effectively, thus reduce the warranty cost and increase the customer satisfaction. In this study, a new 2D PM strategy is proposed for these purposes. As mentioned earlier, we adopt the modelling framework proposed by Kim, Djamaludin and Murthy (2004) to model the effect of PM activities.

Suppose that a series of PM activities of an item are scheduled at actual age $\tau_1, \tau_2, \ldots, \tau_j, \ldots$, with $\tau_0 = 0$. Here, the effect of PM is that it results in a restoration of the item so that the item's virtual age is effectively reduced. The concept of virtual age is introduced in Kijima, Morimura and Suzuki (1988), and then extended in Kijima (1989). In this study, we assume that the jth PM only compensates the damage accumulated during the time between the $(j-1)$th and the jth PM activities, which leads to an arithmetic reduction of virtual age (Martorell, Sanchez and Serradell 1999; Kim, Djamaludin and Murthy 2004). The virtual age right after performing the jth PM activity, i.e. v_j, is then given by

$$v_j = v_{j-1} + \delta(m)\left(\tau_j - \tau_{j-1}\right) \tag{2}$$

where m is the level of PM effort, and $\delta(m)$, $m = 0, 1, \ldots, M$, is the age reduction factor of PM with effort m.

Note that, the effect of PM depends on its level m, $0 \le m \le M$, and its relationship with the age is characterised by the age-reduction factor $\delta(m)$. Larger value of m represents greater PM effort, hence $\delta(m)$ is a decreasing function of m with $\delta(0) = 1$ and $\delta(M) = 0$. More specifically, if $m = 0$, then $v_j = \tau_j$, $j \ge 1$, which means that the item is restored to ABAO; if $m = M$, the item is restored back to AGAN; while in a more general case $m \in (0, M)$, the item is partially restored, i.e. the PM activity is imperfect. Here, we model $\delta(m)$ as an exponentially decreasing function of m, namely, $\delta(m) = (1 + m)e^{-m}$, as in Kim, Djamaludin and Murthy (2004).

In this study, it is assumed that the level of PM effort m keeps unchanged throughout the warranty period. Such assumption is frequently adopted in the existing PM models under warranty; see Kim, Djamaludin and Murthy (2004), Huang and Yen (2009), Wu, Xie and Adam Ng (2011), and Bouguerra, Chelbi and Rezg (2012), for example. Analogous to Huang and Yen (2009), Equation (2) can be recursively derived as

$$\begin{aligned} v_j &= v_{j-1} + \delta(m)\left(\tau_j - \tau_{j-1}\right) \\ &= v_{j-2} + \delta(m)\left(\tau_j - \tau_{j-2}\right) \\ &= \cdots \\ &= v_0 + \delta(m)\left(\tau_j - \tau_0\right) \end{aligned} \tag{3}$$

Since $\tau_0 = 0$ and $v_0 = 0$, we have $v_j = \delta(m)\tau_j$. This expression will be used in the next section to derive the expected number of failures when the 2D PM strategy is applied.

Remark 1. Expressions (2) and (3) show the effect of PM on the virtual age, however, one may wonder how the PM activities impact the item usage. Since the 2D failure modelling problem has been addressed as a 1D problem by using the relationship $u = rt$, this study implicitly assumes that the jth PM activity has the same influence (as factor $\delta(m)$) on virtual usage due to the reduction of virtual age.

3. Model analysis

Consider that an item is sold with a 2D warranty, which is characterised by a rectangular region as shown in Figure 1. Under this policy, the manufacturer agrees to repair all failures free of charge up to age limit W or up to usage limit U

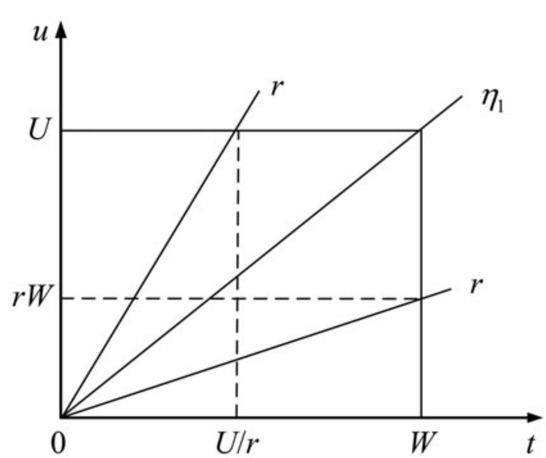

Figure 1. Two-dimensional warranty policy.

from the point of purchase, whichever occurs first. In other words, the customer is guaranteed a maximum coverage for W units of age and for U units of usage.

3.1 *Two-dimensional preventive maintenance strategy*

In order to reduce the number of failures within the warranty coverage, a 2D PM programme is offered and paid by the manufacturer. That is, the PM activities are scheduled based on a 2D framework, say every K units of age or every L units of usage, whichever occurs first.

Let $E[C(\Omega)]$ denote the manufacturer's total expected warranty servicing cost per unit item within the warranty coverage, where the decision space Ω is characterised by $\Omega = (K, L, m)$. The total warranty servicing cost consists of the PM cost and the minimal repair cost. Let $C_p(m)$ denote the cost for a PM activity with level m, and it increases as m increases. Assume that a minimal repair under warranty results in an average cost of C_r.

Define $\eta_1 = U/W$ and $\eta_2 = L/K$. To analyse the proposed 2D PM strategy in details, two cases are considered, i.e. $\eta_2 \leq \eta_1$ and $\eta_2 > \eta_1$.

3.1.1 *Case 1:* $\eta_2 \leq \eta_1$

In this case, we need to investigate the following three subcases: (a) $r \leq \eta_2$; (b) $\eta_2 < r \leq \eta_1$; (c) $r > \eta_1$, as illustrated in Figure 2.

3.1.1.1 *Subcase 1(a):* $r \leq \eta_2$. For an item with usage rate $r \leq \eta_2$, the warranty terminates when the age reaches W, with the corresponding usage rW, i.e. the pair of age and usage at the time of warranty termination is (W, rW). Under the proposed PM strategy, the time instant of performing the jth PM activity is $\tau_j = jK$, with the corresponding usage jrK; see Figure 2(a). In this case, the number of PM activities performed over the warranty period is

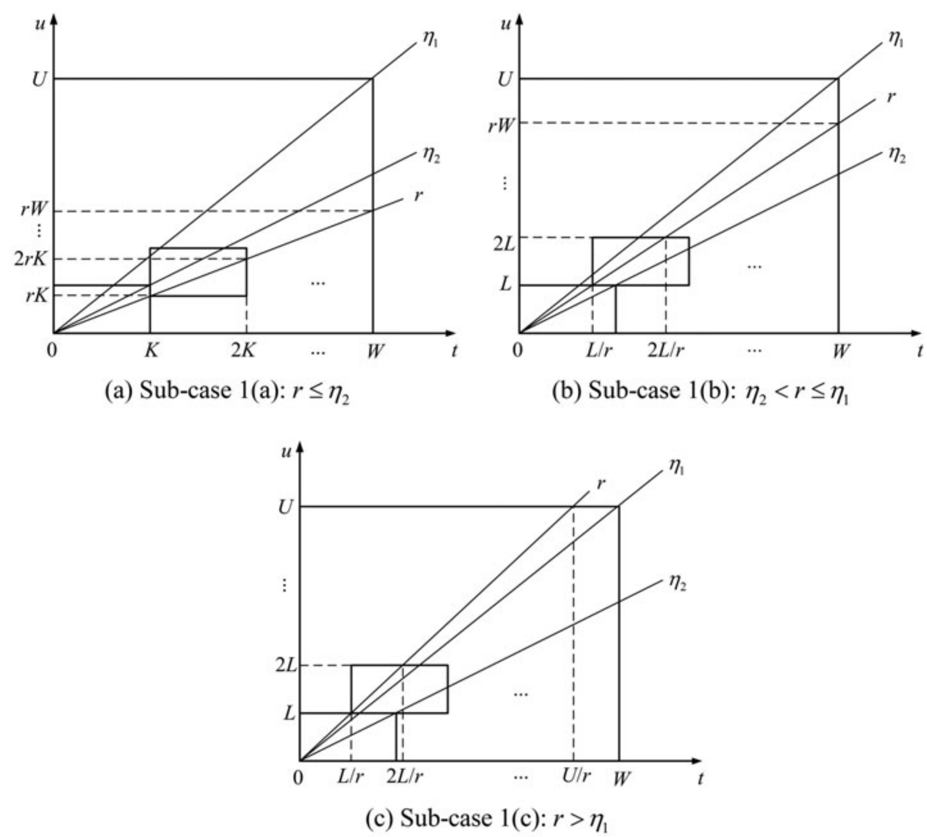

Figure 2. Graphical illustration of the three subcases of Case 1.

$$\begin{aligned} n_{11} &= \max\{j \mid jK < W \ \& \ jrK < rW,\ j \geq 0\} \\ &= \max\{j \mid jK < W,\ j \geq 0\} \end{aligned}$$

In other words, n_{11} PM activities are performed within the warranty period, and after the n_{11}th PM activity, the subsequent interval would end when the warranty terminates, since the manufacturer would not perform an additional PM activity.

It is assumed that any failures between two successive PM activities are minimally rectified with negligible durations. Thus, the failure process between two successive PM activities is also an NHPP. Using Equation (3), the conditional expected number of failures over the warranty period is given by

$$E\left[N_1(\Omega)\big|_{r \leq \eta_2}\right] = \sum_{j=0}^{n_{11}-1} \int_{j\delta(m)K}^{j\delta(m)K+K} \lambda(t|r)\mathrm{d}t + \int_{n_{11}\delta(m)K}^{n_{11}\delta(m)K+W-n_{11}K} \lambda(t|r)\mathrm{d}t \tag{4}$$

Combining the PM cost and minimal repair cost yields the expected warranty servicing cost per unit item conditional on $R = r \leq \eta_2$, which is

$$E\left[C_1(\Omega)\big|_{r \leq \eta_2}\right] = C_r E\left[N_1(\Omega)\big|_{r \leq \eta_2}\right] + n_{11} C_p(m) \tag{5}$$

3.1.1.2 *Subcase 1(b):* $\eta_2 < \mathrm{r} \leq \eta_1$. From Figure 2(b), we know that for an item with usage rate $\eta_2 < r \leq \eta_1$, the pair of age and usage at the time of warranty expiration is (W, rW). Under the proposed PM strategy, the time instant of carrying out the jth PM activity is $\tau_j = jL/r$, when the total usage reaches its limit jL. In this case, the number of PM activities carried out during the warranty period can be obtained as

$$n_{12} = \max\{j \mid jL/r < W,\ j \geq 0\}$$

Then the conditional expected number of failures during the warranty period is

$$E\left[N_1(\Omega)\big|_{\eta_2 < r \leq \eta_1}\right] = \sum_{j=0}^{n_{12}-1} \int_{j\delta(m)\frac{L}{r}}^{j\delta(m)\frac{L}{r}+\frac{L}{r}} \lambda(t|r)\mathrm{d}t + \int_{n_{12}\delta(m)\frac{L}{r}}^{n_{12}\delta(m)\frac{L}{r}+W-n_{12}\frac{L}{r}} \lambda(t|r)\mathrm{d}t \tag{6}$$

Therefore, given $\eta_2 < R = r \leq \eta_1$, the conditional total expected warranty servicing cost is given by

$$E\left[C_1(\Omega)\big|_{\eta_2 < r \leq \eta_1}\right] = C_r E\left[N_1(\Omega)\big|_{\eta_2 < r \leq \eta_1}\right] + n_{12} C_p(m) \tag{7}$$

3.1.1.3 *Subcase 1(c):* $\mathrm{r} > \eta_1$. For an item with usage rate $r > \eta_1$, as shown in Figure 2(c), the pair of age and usage at the time of warranty expiration is (U/r, U). Under the proposed PM policy, the pair of item age and usage at the jth PM activity is (jL/r, jL). In this case, the number of PM activities applied within the warranty period is given by

$$n_{13} = \max\{j \mid jL < U, \quad j \geq 0\}$$

Then the conditional expected number of failures within the warranty period is

$$E\left[N_1(\Omega)\big|_{r > \eta_1}\right] = \sum_{j=0}^{n_{13}-1} \int_{j\delta(m)\frac{L}{r}}^{j\delta(m)\frac{L}{r}+\frac{L}{r}} \lambda(t|r)\mathrm{d}t + \int_{n_{13}\delta(m)\frac{L}{r}}^{n_{13}\delta(m)\frac{L}{r}+\frac{U}{r}-n_{13}\frac{L}{r}} \lambda(t|r)\mathrm{d}t \tag{8}$$

As a result, given $R = r > \eta_1$, the conditional total expected warranty servicing cost is given by

$$E\left[C_1(\Omega)\big|_{r > \eta_1}\right] = C_r E\left[N_1(\Omega)\big|_{r > \eta_1}\right] + n_{13} C_p(m) \tag{9}$$

Finally, removing the conditioning on R, the total expected warranty servicing cost to the manufacturer in Case 1 can be calculated by

$$E[C_1(\Omega)] = \int_0^{\eta_2} E\left[C_1(\Omega)\big|_{r \le \eta_2}\right] g(r)\mathrm{d}r + \int_{\eta_2}^{\eta_1} E\left[C_1(\Omega)\big|_{\eta_2 < r \le \eta_1}\right] g(r)\mathrm{d}r + \int_{\eta_1}^{\infty} E\left[C_1(\Omega)\big|_{r > \eta_1}\right] g(r)\mathrm{d}r \tag{10}$$

3.1.2 *Case 2:* $\eta_2 > \eta_1$

Following similar procedures, in this case, we then need to investigate the following three subcases: (a) $r \le \eta_1$; (b) $\eta_1 < r \le \eta_2$; (c) $r > \eta_2$, as illustrated in Figure 3.

Since the derivation process for Case 2 is quite similar to that for Case 1, here we present the mathematical formulations without detailed explanations.

3.1.2.1 *Subcase 2(a):* $r \le \eta_1$

$$E\left[N_2(\Omega)\big|_{r \le \eta_1}\right] = \sum_{j=0}^{n_{21}-1} \int_{j\delta(m)K}^{j\delta(m)K+K} \lambda(t|r)\mathrm{d}t + \int_{n_{21}\delta(m)K}^{n_{21}\delta(m)K+W-n_{21}K} \lambda(t|r)\mathrm{d}t \tag{11}$$

$$E\left[C_2(\Omega)\big|_{r \le \eta_1}\right] = C_r E\left[N_2(\Omega)\big|_{r \le \eta_1}\right] + n_{21} C_p(m) \tag{12}$$

where

$$n_{21} = \max\{j | jK < W, j \ge 0\}$$

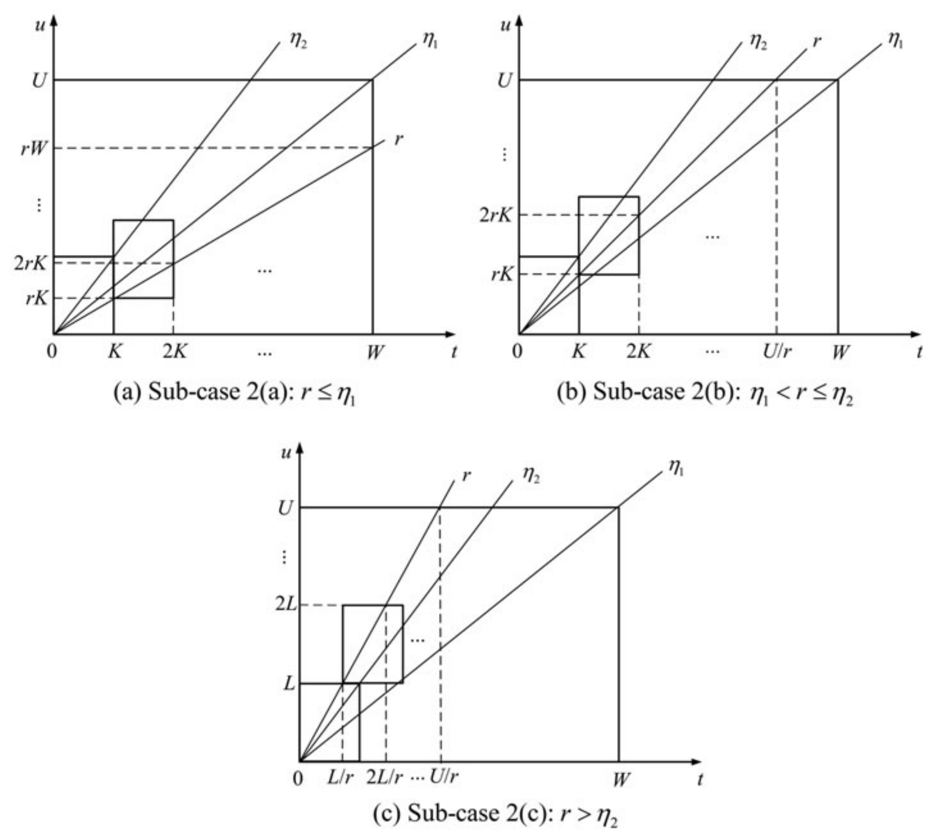

Figure 3. Graphical illustration of the three subcases of Case 2.

3.1.2.2 *Subcase 2(b):* $\eta_1 < r \leq \eta_2$

$$E\left[N_2(\Omega)\big|_{\eta_1<r\leq\eta_2}\right] = \sum_{j=0}^{n_{22}-1}\int_{j\delta(m)K}^{j\delta(m)K+K}\lambda(t|r)\mathrm{d}t + \int_{n_{22}\delta(m)K}^{n_{22}\delta(m)K+\frac{U}{r}-n_{22}K}\lambda(t|r)\mathrm{d}t \tag{13}$$

$$E\left[C_2(\Omega)\big|_{\eta_1<r\leq\eta_2}\right] = C_r E\left[N_2(\Omega)\big|_{\eta_1<r\leq\eta_2}\right] + n_{22}C_p(m) \tag{14}$$

where

$$n_{22} = \max\{j|jK < U/r,\ j \geq 0\}$$

3.1.2.3 *Subcase 2(c):* $r > \eta_2$

$$E\left[N_2(\Omega)\big|_{r>\eta_2}\right] = \sum_{j=0}^{n_{23}-1}\int_{j\delta(m)\frac{L}{r}}^{j\delta(m)\frac{L}{r}+\frac{L}{r}}\lambda(t|r)\mathrm{d}t + \int_{n_{23}\delta(m)\frac{L}{r}}^{n_{23}\delta(m)\frac{L}{r}+\frac{U}{r}-n_{23}\frac{L}{r}}\lambda(t|r)\mathrm{d}t \tag{15}$$

$$E\left[C_2(\Omega)\big|_{r>\eta_2}\right] = C_r E\left[N_2(\Omega)\big|_{r>\eta_2}\right] + n_{23}C_p(m) \tag{16}$$

where

$$n_{23} = \max\{j|jL < U,\ j \geq 0\}$$

Finally, removing the conditioning on R gives the manufacturer's total expected warranty servicing cost in Case 2, which is

$$\begin{aligned}E[C_2(\Omega)] = \textstyle\int_0^{\eta_1} E\left[C_2(\Omega)\big|_{r\leq\eta_1}\right]g(r)\mathrm{d}r + \int_{\eta_1}^{\eta_2} E\left[C_2(\Omega)\big|_{\eta_1<r\leq\eta_2}\right]g(r)\mathrm{d}r \\ + \textstyle\int_{\eta_2}^{\infty} E\left[C_2(\Omega)\big|_{r>\eta_2}\right]g(r)\mathrm{d}r\end{aligned} \tag{17}$$

3.1.3 Discussions on the optimal PM strategies

The optimisation problem of this work is to identify the optimal PM strategy (K *, L*, m*) such that the manufacturer's warranty servicing cost is minimised. However, it is quite difficult, if not impossible, to obtain analytical forms of the optimal solutions. Some simple observations of the cost structure can help reduce the complexity of the optimisation problem. Not surprisingly, the cost function $E[C(K, L, m)]$ is not continuous in K and/or L (as number of PM activities is not continuous in K and/or L). One interesting property regarding the optimal PM intervals can be described as follows.

Property 1. Consider that the customers' usage rates range from $r_{\min}$ to $r_{\max}$, i.e. the lower and upper bounds of customers' usage rates are $r_{\min}$ and $r_{\max}$, respectively. Let K* and L* denote the optimal PM age and usage intervals, respectively. Then, no matter which distribution the usage rates obey, we have $L^*/K^* \in [r_{\min}, r_{\max}]$.

Proof. This property is straightforward and intuitive, so the proof is omitted.

It is noteworthy that the age and usage intervals of 2D PM strategy are usually discrete in practice (e.g. K = 1, 2, … months; L = 1, 2, … × 10^3 km), but most previous studies fail to recognise this fact by making an oversimplified assumption of continuity. The discrete PM policy is more convenient to implement, although its continuous counterpart may result in the lowest warranty cost. In this study, for the sake of efficient maintenance management, we search for the best, not necessarily optimal, solution $\Omega^* = (K^*, L^*, m^*)$ over a coarse grid with K and L varying within (0, W] and (0, U], respectively.

3.2 *Age-based and usage-based preventive maintenance strategies*

Age-based and usage-based PM strategies are very common in the automobile market. Under the age-based strategy, PM activities are scheduled every K units of age, regardless of the item's usage rate; whereas under the usage-based strategy, PM activities are scheduled every L units of usage, regardless of the item's usage rate. In this subsection, age-based and usage-based PM strategies are briefly investigated, with the purpose of comparison with the proposed 2D PM strategy.

For the 2D PM strategy, we have the following observations:

- For an item with usage rate $r \le L/K$, the PM activities are scheduled with constant age interval K, i.e. the 2D PM strategy is reduced to an age-based strategy;
- For an item with usage rate $r > L/K$, the PM activities are performed every L units of usage (or equivalently every L/r units of age), i.e. the 2D PM strategy is reduced to a usage-based strategy.

That is to say, the 2D PM strategy is developed at the *population* level. While at the *individual* level, the 2D PM strategy always reduces to either age-based or usage-based strategy for an individual item.

Remark 2. The age-based and usage-based PM strategies can be considered as special cases of the proposed 2D PM strategy. More specifically, the age-based PM strategy is characterised by (K, $L = \infty$), while the usage-based PM strategy by ($K = \infty$, L).

Based on the remark above, model analyses of age-based and usage-based PM strategies can be proceeded as follows.

For the age-based PM strategy, we have that $\eta_2 = L/K = \infty$. In this case, $\eta_1 < \eta_2$, i.e. Case 2, always holds. Then, only two sub-cases need to be considered, namely, $r \le \eta_1$ and $\eta_1 < r \le \eta_2 = \infty$. Detailed analysis follows directly from Subcases 2(a) and 2(b).

For the usage-based PM strategy, we know that $\eta_2 = L/K = 0$. In this case, $\eta_2 < \eta_1$, i.e. Case 1, always holds. Then, only two sub-cases should be considered, namely, $0 = \eta_2 < r \le \eta_1$ and $r > \eta_1$. Detailed analysis follows directly from Subcases 1(b) and 1(c).

4. Numerical example

In this section, a numerical example is given to demonstrate the applicability and superiority of the proposed PM strategy. Suppose that the item under consideration is a repairable automobile component sold with a 2D free repair warranty, with $W = 3$ years and $U = 3 \times 10^4$ km. Hence, we have $\eta_1 = U/W = 1$.

The original conditional intensity function is $\lambda(t|r) = \theta_0 + \theta_1 r + (\theta_2 + \theta_3 r)t$, with the parameter values given by $\theta_0 = 0.1$, $\theta_1 = 0.2$, $\theta_2 = 0.7$, $\theta_3 = 0.7$. Besides, the usage rate R is assumed to be uniformly distributed over $[r_{\min}, r_{\max}]$, with the density function given by $g(r) = 1/[r_{\max} - r_{\min}]$, for $r_{\min} \le r \le r_{\max}$. In this study, we consider three sets of values for $r_{\min}$ and $r_{\max}$, which broadly correspond to light, medium and heavy usage intensities, respectively. Following Iskandar and Murthy (2003), the values and the corresponding mean usage rates ($E[R]$) are listed in Table 1.

Moreover, we consider $\delta(m) = (1 + m)e^{-m}$ for $m = 0, 1, \ldots, 5$ and the corresponding PM cost is $C_p(0) = \$0$, $C_p(1) = \$10$, $C_p(2) = \$30$, $C_p(3) = \$60$, $C_p(4) = \$100$ and $C_p(5) = \$160$, which is consistent with that of Kim, Djamaludin and Murthy (2004).

As mentioned earlier, the optimisation problem is to identify the optimal PM decision such that the manufacturer's warranty servicing cost is minimised. In this work, for each of the three usage types, we search for the best solution $\Omega^* = (K^*, L^*, m^*)$ over a coarse grid with K and L varying within $(0, W]$ and $(0, U]$, respectively. For the sake of efficient maintenance management, the search steps of K and L are 0.0833 year (i.e. one month) and 10^3 km, respectively.

Under the three usage types, the optimal decisions and corresponding minimal expected warranty servicing costs for age-based, usage-based and 2D PM strategies are summarised in Table 2. For instance, under the medium usage intensity with $C_r = \$250$, the optimal 2D PM strategy is $(K^*, L^*, m^*) = (10, 8, 3)$ and the corresponding expected warranty cost is $E[C^*(\Omega)] = \$945.7$, as illustrated in Figure 4. This means that within the 2D warranty period, the item should be preventively maintained with level $m^* = 3$, every 10 months or 8000 km, whichever occurs first.

Table 1. Three usage types under uniform distribution.

Usage intensity	$r_{\min}$	$r_{\max}$	$E[R]$
Light	0.1	0.9	0.5
Medium	0.7	1.3	1.0
Heavy	1.1	2.9	2.0

Table 2. Optimal PM decisions under the three usage types (K^* in months, L^* in 10^3 km).

		2D PM				Age-based PM			Usage-based PM		
Usage type	C_r	K^*	L^*	m^*	$E[C^*]$	K^*	m^*	$E[C^*]$	L^*	m^*	$E[C^*]$
Light	50	19	14	2	226.3	19	2	226.3	9	2	241.9
	100	13	9	3	401.8	13	3	402.0	5	2	431.0
	150	13	9	3	542.7	13	3	542.9	5	3	591.8
	200	13	9	3	683.7	13	3	683.9	4	3	734.4
	250	10	7	3	809.9	10	3	810.4	4	3	870.0
	300	10	5	3	935.2	10	3	936.5	3	3	994.6
	350	10	7	4	1050.1	10	4	1050.8	3	3	1115.7
	400	10	7	4	1157.3	10	4	1158.1	3	3	1236.8
	450	10	5	4	1262.5	10	4	1265.4	3	3	1357.9
	500	10	5	4	1365.7	8	4	1366.7	3	4	1462.8
Medium	50	19	15	2	265.5	19	2	267.4	15	2	267.5
	100	13	11	3	462.7	13	3	470.2	10	3	467.8
	150	13	11	3	634.0	13	3	645.2	10	3	641.7
	200	10	8	3	792.5	10	3	804.9	8	3	803.3
	250	10	8	3	945.7	8	3	955.3	6	3	951.6
	300	10	8	4	1086.7	8	3	1101.5	6	3	1095.6
	350	10	8	4	1217.8	8	4	1236.5	6	4	1231.4
	400	10	8	4	1349.0	8	4	1359.9	6	4	1352.1
	450	8	6	4	1469.0	8	4	1483.2	6	4	1472.8
	500	8	6	4	1587.1	8	4	1606.5	6	4	1593.5
Heavy	50	16	15	2	162.3	11	1	166.8	15	2	162.3
	100	11	15	2	294.2	8	2	300.4	15	2	294.7
	150	11	15	3	414.3	8	2	422.6	15	3	415.4
	200	11	11	3	524.9	8	3	531.2	11	3	525.1
	250	8	11	3	625.3	8	3	636.1	11	3	626.4
	300	8	11	3	723.9	7	3	738.2	11	3	727.7
	350	8	11	3	822.6	6	3	835.1	8	3	823.8
	400	8	8	3	915.5	5	3	927.5	8	3	915.8
	450	6	8	3	1007.4	5	3	1017.9	8	3	1007.8
	500	6	8	3	1097.6	5	3	1108.4	8	3	1099.8

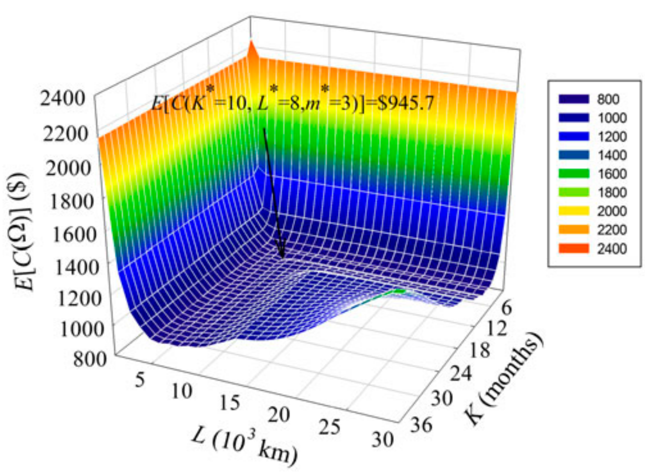

Figure 4. Illustration of the existence of optimal 2D PM strategy under $m^* = 3$.

From Table 2, the following findings can be summarised:

(1) For each PM strategy and under each usage type, K^* and L^* tend to decrease (with some exceptions), but m^* and $E[C^*(\Omega)]$ tend to increase as C_r increases. This finding is in line with intuition. In fact, when the minimal repair cost increases, the manufacturer should enhance the PM effort and/or reduce the PM intervals so as to

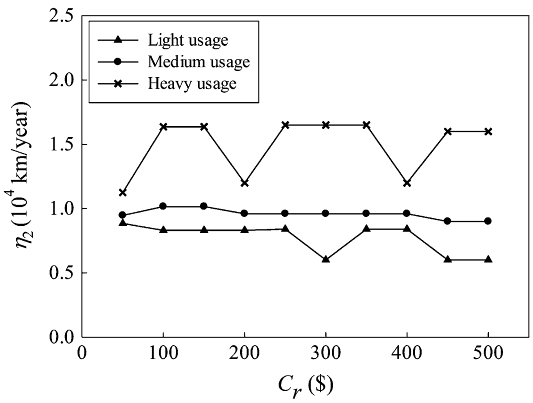

Figure 5. Comparison of η_2 for 2D PM strategy under the three usage types.

compensate the increase of warranty cost. One thing particularly noteworthy is that under light usage intensity, when C_r increase from \$300 to \$350, the PM level for 2D PM strategy jumps from $m^* = 3$ to $m^* = 4$; however, it is unexpected that L^* jumps from 5 back to 7. This observation shows that when C_r increases, the manufacturer can adopt a mixed strategy of enhancing the PM effort and slightly increasing the PM intervals at the same time.

(2) For the 2D PM strategy, as shown in Figure 5, the ratio of PM usage interval to age interval, i.e. $\eta_2 = L^*/K^*$, varies across different usage types (here, the unit of η_2 has been implicitly transformed into 10^4 km/year). More specifically, η_2 tends to increase as customer usage intensity increases; while it fluctuates with slight downtrend as the minimal repair cost increases. In addition, from Figure 5, the result presented in Property 1, i.e. $L^*/K^* \in [r_{\min}, r_{\max}]$ can be observed for the three usage types.

(3) It is interesting to note that for all the three PM strategies, the warranty servicing cost under medium usage intensity is higher than those under light and heavy usage intensities; see Figure 6 for the case of 2D PM strategy. This is due to a unique property of the rectangular warranty region, i.e. this warranty region tends to favour customers with medium usage intensities. Similar observations and detailed explanations can be found in Iskandar, Murthy and Jack (2005), Wang, Liu and Liu (2015), and Su and Wang (2016), etc. However, this pattern will change depending on the parameter values and the structure of the conditional intensity function.

(4) As can be seen from Table 2, warranty servicing costs for the 2D PM strategy are always lower than, or identical to, those for the age-based and usage-based PM strategies, and this demonstrates its cost advantage.

Moreover, as shown in Table 2, the warranty costs for the 2D PM strategy are slightly lower than those for age-based or usage-based PM strategies. This is mainly due to our parameter setting. Considering a wider range of usage rate ($r_{\min} = 0.1$, $r_{\max} = 2.9$), Table 3 summarises the optimal PM decisions versus different warranty limits with $C_r = \$250$. As can be observed, warranty cost reductions resulting from the implementation of 2D PM strategy are more

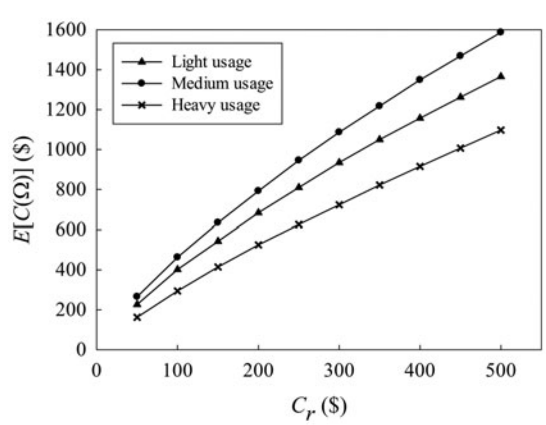

Figure 6. Minimal warranty cost for 2D PM strategy under the three usage types.

Table 3. Optimal PM decisions versus various warranty limits (K^* in months, L^* in 10^3 km).

		$U = 2 \times 10^4$ km				$U = 4 \times 10^4$ km				$U = 6 \times 10^4$ km			
		K^*	L^*	m^*	$E[C^*]$	K^*	L^*	m^*	$E[C^*]$	K^*	L^*	m^*	$E[C^*]$
$W = 2$ years	2D PM	13	10	3	414.4	9	14	3	656.7	9	15	3	731.1
	Age-based PM	7	–	2	424.4	9	–	3	671.3	9	–	3	735.1
	Usage-based PM	–	7	2	435.9	–	10	3	689.1	–	10	3	767.8
$W = 4$ years	2D PM	11	10	3	558.6	10	14	4	1033.9	10	12	4	1364.7
	Age-based PM	10	–	3	575.5	10	–	4	1047.2	10	–	4	1380.8
	Usage-based PM	–	5	3	618.8	–	8	3	1126.3	–	10	4	1488.3
$W = 6$ years	2D PM	11	10	3	643.2	11	14	4	1223.8	11	12	4	1702.7
	Age-based PM	11	–	4	658.9	11	–	4	1243.2	11	–	4	1731.7
	Usage-based PM	–	5	3	718.7	–	8	4	1357.9	–	9	4	1881.9

attractive. Nevertheless, bear in mind that the annual warranty servicing costs of most world-leading automakers can run into billions of dollars, and General Motors is a typical example. In this context, any reduction in the expected warranty servicing cost per unit item, achieved by adopting the proposed 2D PM strategy, will account in significant savings for the manufacturers.

Furthermore, in practice, some manufacturers do not cover the expenses of regular PM activities within the warranty period, and the customers must afford these PM activities by themselves. This will increase the customers' burdens and hinder the customers' enthusiasm for investing in maintenance activities, which in turn increase the risk of item failures, and thus increase the expected warranty cost. One compromise scheme is PM cost sharing, namely, the PM expenses of an item may be shared between the manufacturer and customer on a pro-rata basis, similar to the idea of pro-rata warranty (Blischke and Murthy 1996).

It is quite easy to incorporate this consideration into our original model. Take Subcase 1(b) as an example. In this subcase, the warranty terminates at time W. For the jth PM activity carried out at time instant $\tau_j = jL/r$, the pro-rata PM cost for the manufacturer can be specified as

$$C_{pj}(m) = C_p(m)\left(1 - \frac{jL}{rW}\right) \tag{18}$$

For the manufacturer, the total PM cost within the warranty period is thus

$$\sum_{j=1}^{n_{12}} C_p(m)\left(1 - \frac{jL}{rW}\right) = C_p(m)\left[n_{12} - \frac{L}{rW}\frac{n_{12}(n_{12}+1)}{2}\right] \tag{19}$$

The other five subcases can be proceeded similarly.

With the same parameter setting as in Table 3, under the PM cost sharing scheme, the optimal PM decisions versus various warranty limits are listed in Table 4. As can be seen, the total expected warranty costs under the PM cost sharing

Table 4. Optimisation results under PM cost sharing scheme (K^* in months, L^* in 10^3 km).

		$U = 2 \times 10^4$ km				$U = 4 \times 10^4$ km				$U = 6 \times 10^4$ km			
		K^*	L^*	m^*	$E[C^*]$	K^*	L^*	m^*	$E[C^*]$	K^*	L^*	m^*	$E[C^*]$
$W = 2$ years	2D PM	7	8	3	354.0	7	11	4	561.0	7	11	4	617.4
	Age-based PM	5	–	3	356.8	7	–	4	565.3	7	–	4	620.4
	Usage-based PM	–	6	3	379.8	–	6	3	594.5	–	8	4	656.8
$W = 4$ years	2D PM	7	13	4	472.0	7	11	4	849.8	7	11	4	1130.9
	Age-based PM	7	–	4	472.9	7	–	4	854.1	6	–	4	1138.6
	Usage-based PM	–	4	3	529.4	–	6	4	926.1	–	6	4	1220.3
$W = 6$ years	2D PM	8	13	4	533.1	7	11	4	1008.6	7	11	4	1408.8
	Age-based PM	7	–	4	534.1	7	–	4	1012.9	7	–	4	1417.2
	Usage-based PM	–	5	4	617.2	–	5	4	1118.4	–	5	4	1542.1

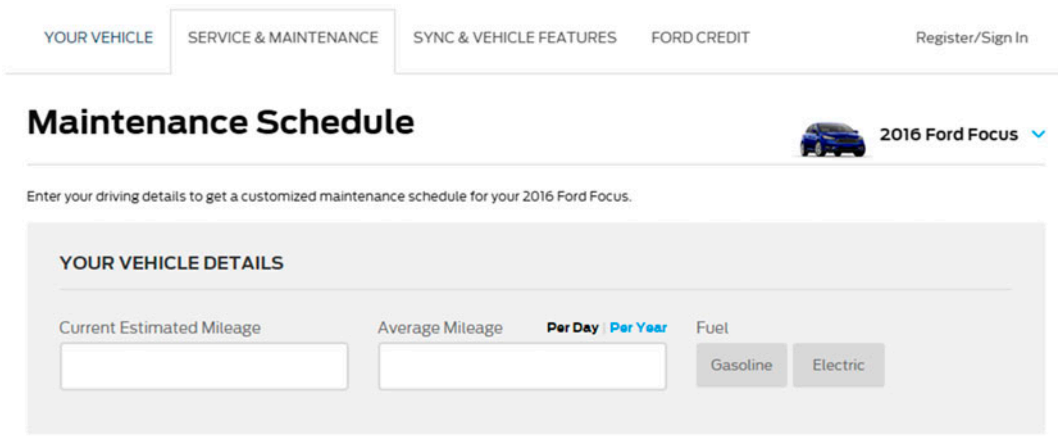

Figure 7. A real interface for PM scheduling from Ford's official website (www.ford.com).

scheme are much lower than those in Table 3. This observation is intuitive since the total PM costs are shared by manufacturer and customer. In line with this, the PM intervals become shorter and/or the PM levels become higher.

5. Discussions

The numerical results above demonstrate the cost advantage of the proposed 2D PM strategy; nevertheless, the practical aspects of this strategy also deserve to be discussed. As mentioned earlier, the 2D PM strategy is mainly employed by automobile manufacturers. In general, there are two complementary ways to help implement the 2D PM strategy in practice.

The first way is through the 'Owner's Manual'. Nowadays, almost all vehicles are sold with an Owner's Manual, in which the corresponding PM schedule is detailed. However, it is generally the owners' responsibility to make sure that all of the scheduled PM activities are performed, and the PM actions are usually carried out by authorised maintenance centres. Failure to perform scheduled PM as specified will invalidate warranty coverage on parts impacted by the lack of maintenance. Some manufacturers even require maintenance crew to return the PM details, such as vehicle age, mileage, actions performed, and maintenance cost, etc., to their databases. This way has an obvious drawback for owners, i.e. if a vehicle owner delays one PM action due to certain reasons, all subsequent failures affected by the lack of maintenance are not covered by warranty.

While the second way is via the Internet. Several automobile manufacturers have developed interfaces for PM scheduling embedded in their official websites. Figure 7 illustrates a real interface of Ford (www.ford.com). As can be seen, vehicle owners can enter their driving details, such as vehicle age, mileage, mileage accumulation rate, etc., to obtain customised PM schedules for their vehicles. By this means, the manufacturers can collect more data about customers' usage patterns, and these data can in turn help them improve PM decision-makings.

Furthermore, in order to increase the customer satisfaction, the manufacturer could even send mobile messages or emails to an owner, say one or two weeks ahead of the (expected) due date, if permitted, informing the owner to prepare for the coming PM activity.

Here, we show how to implement the two ways through an illustrative case. Suppose that the manufacturer has determined an optimal 2D PM strategy for all customers, say every 10 months or every 8000 km, whichever occurs first (see Figure 4). This PM schedule has also been elaborated in the Owner's Manual. Consider an individual vehicle with current age and mileage, say 15 months and 10,000 km, respectively. Its average usage rate is thus 666.7 km/month, which is smaller than 800 km/month. In this case, the 2D PM strategy reduces to the age-based PM strategy, that is, the vehicle should be preventively maintained every 10 months. The (expected) next PM time instant will be 5 months later. The manufacturer could inform the owner of this vehicle in advance via mobile messages or emails, if permitted.

6. Concluding remarks

The primary contribution of this paper is that it is the first one, to the authors' knowledge, in the warranty-oriented literature to investigate the imperfect 2D PM strategy. Under this strategy, the imperfect PM activities are scheduled every K units of age or L units of usage, whichever occurs first. The main objective of this study is to determine the optimal

PM strategy to minimise the manufacturer's total expected warranty servicing cost. It is shown that the proposed 2D PM strategy contains the age-based and usage-based PM strategies as special cases. Meanwhile, it is always superior, or at least identical, to the other two strategies in terms of warranty servicing cost. Finally, how to implement the proposed PM strategy in practice is discussed with an illustrative case.

This study can be extended in the following directions.

(1) In this study, the customers' usage rates are assumed to be uniformly distributed, which is not always convincing. Weibull, lognormal, gamma and some other distributions are of potential interest.
(2) Besides, this study assumes that the PM cost relates only to the PM level. It is more practical to consider the PM cost as a function of item age and usage, in addition to the PM level. In this case, the PM cost model of Shahanaghi et al. (2013) can be useful.
(3) Furthermore, for deteriorating items, a non-periodic 2D PM policy with decreasing PM intervals would be more appropriate.
(4) Finally, designing flexible warranty contracts with consideration of the proposed 2D PM is another valuable direction for future research.

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