Original Article

Proc IMechE Part O:
J Risk and Reliability
2014, Vol. 228(5) 518–528

DOI: 10.1177/1748006X14537250
pio.sagepub.com

Optimizing upgrade level and preventive maintenance policy for second-hand products sold with warranty

Chun Su and Xiaolin Wang

Abstract

With the steady growth of second-hand products' market, offering warranty for second-hand products is becoming popular for dealers in order to meet customers' requirement. During that process, upgrade actions and maintenance policies are of great importance. In this article, an optimization model for second-hand items sold with non-renewing free repair warranty is proposed from the view of dealers, in which a second-hand product is subjected to upgrade action at the end of its past life and during the warranty period, preventive maintenance actions are carried out when the age of the product reaches a pre-specified threshold value. Optimal upgrade level and preventive maintenance policy are jointly derived so that the dealer's expected profit can be maximized. The structural properties of the optimal policies are investigated. Finally, a numerical example and sensitivity analysis are provided to demonstrate the proposed approach.

Keywords

Second-hand product, warranty, upgrade action, preventive maintenance, threshold value

Date received: 21 February 2014; accepted: 30 April 2014

Introduction

Warranty is a contractual obligation incurred by a manufacturer (vendor or seller) in connection with the sale of a product. In broad terms, the purpose of warranty is to establish liability in the event of a premature failure of an item or the inability of the item to perform its intended function.[1] From the perspective of manufacturers or dealers, the warranty (1) provides assurance to customers, (2) signals product reliability and (3) differentiates the product from those of competitors.[2] Nowadays, warranty is becoming increasingly important in consumer and commercial transactions, and more and more manufactures begin offering warranties for their products.

Research on warranty has received extensive attention from various disciplines for decades, and a diverse range of issues have been dealt with. Murthy and Djamaludin[3] gave a detailed review on new product warranty. The status and progress of warranty data analysis can be found in Wu.[4] Yun et al.[5] aimed to optimize burn-in time so as to minimize the total mean servicing cost of a system sold with cumulative free replacement warranty. Su and Shen[6] derived expected warranty cost and profit models for two extended warranty policies, and three different corrective repair options were considered.

In recent years, the importance of second-hand/used product market has been on the rise. For example, in the United States, approximately 11.58 million new cars were sold in 2010, compared with the sale of 36.88 million used cars from both dealers and end users. Furthermore, the used-to-new car ratio in the United States has increased from 2.4 to 3.2 since 2000; it shows a steady growth of used car sales by volume.[7] Similar to warranty for new products, nowadays, warranty for used ones is also becoming increasingly popular. In line with this, the length of the warranty period of used products offered by dealers has been steadily

Department of Industrial Engineering, School of Mechanical Engineering, Southeast University, Nanjing, China

Corresponding author:
Chun Su, Department of Industrial Engineering, School of Mechanical Engineering, Southeast University, Jiangning District, Nanjing 211189, China.
Email: suchun@seu.edu.cn

increasing. For instance, in the United Kingdom, the warranty period for used automobiles was only 6 months/6000 miles in the early 1990s; nowadays, it has increased to as much as 3 years/36,000 miles.[8] As we know, offering warranty will incur additional cost for dealers. Different from the warranty cost for new items, warranty cost for second-hand products usually changes greatly from item to item due to different age, usage and maintenance history.[9] Therefore, it is crucial to study the warranty policies and to estimate the corresponding warranty cost for second-hand products from the view of dealers.

In contrast to the vast literature on warranty policies for new items, up to now, study on warranty policies for second-hand items receives less attention. Chattopadhyay and Murthy[9] discussed many aspects of one-dimensional free repair/replacement warranty (FRW) and pro-rata warranty (PRW) policies for second-hand products. Chattopadhyay and Murthy[10] proposed three kinds of cost-sharing warranty policies for second-hand products. Aksezer[11] studied failures of used vehicles, and the dealer's expected warranty cost was calculated by taking into account the age, usage and maintenance data of the vehicles.

In the above researches,[9–11] it is assumed that there are no reliability improvement actions at the end of second-hand products' past life. In fact, reliability improvement actions such as upgrade actions are particularly important for warranted second-hand products. The research of Chattopadhyay and Murthy[12] is the first reported work on this topic. Besides, considering the age at the end of first life as stochastic, Lo and Yu[13] proposed a profit model, where the optimal upgrade level and warranty length were jointly derived. Moreover, some other work has been done by Shafiee and his co-authors. The reader can refer to Shafiee and Chukova[14] and the references therein on their latest development in this area.

In the above-mentioned researches[9–14], maintenance policy for second-hand products during the warranty period was limited to corrective repairs. In fact, implementing preventive maintenance (PM) can effectively reduce the warranty cost for products.[15–18] To the best of the authors' knowledge, there are two reported researches on the development of PM policies for second-hand products. Yeh et al.[19] proposed two periodical age reduction PM models to decrease the high failure rate of the second-hand products. Kim et al.[20] studied the optimal periodic PM policies of a second-hand item following the expiration of warranty. Therefore, developing PM policies for second-hand products still needs further researches.

From the perspective of the dealer, it is worthwhile to carry out upgrade and/or PM actions only when the reduction of warranty servicing cost exceeds the additional cost incurred by upgrade and/or PM programs. Therefore, considering the effects of upgrade and PM actions on warranty cost is of vital importance to dealers. But up to now, integrated consideration of both concerns has received little attention. In this article, considering second-hand products sold with non-renewing free repair warranty (NFRW), a stochastic model is proposed to optimize the upgrade level and PM policy, with the aim of maximizing the dealer's expected profit.

It should be noted that the proposed model is suitable to the repairable products which are relatively expensive and not so easily affected by fashion, for instance, large-screen color TVs, household appliances, electric tools, etc. For this kind of products, many customers would be willing to purchase second-hand ones due to limited budget, and the products' functions are much more important than their styles. Moreover, the two-dimensional extension of our model shows the potential to be applied to automobiles under two-dimensional warranty.

The rest of this article is structured as follows. Section "Model formulation" presents the elements of the mathematical model, including upgrade and PM models, cost functions and expected profit function. The properties of the optimal upgrade level and PM policy are investigated in Section "Model optimization". Section "Numerical example" presents a numerical example to illustrate the proposed approach, and sensitivity analysis is also conducted. Conclusion and future research directions are stated in the last section.

Model formulation

In this section, for a second-hand product sold with NFRW, a mathematical optimization model is derived to maximize the dealer's expected profit. To do this, first we need to model the failure process of the second-hand product. After that, the effect of upgrade and PM actions on product reliability is modeled. Then, four kinds of cost functions, sale price and the expected profit are proposed.

Model assumptions

The following assumptions are required in the model formulation:

1. Second-hand products have aging property, and they are repairable.
2. Product failures are statistically independent.
3. Product failures can be detected immediately, which result in immediate claims. It is also supposed that all claims are valid.
4. The dealer will rectify all failures occurring during warranty period $[0, w]$ at no cost to buyers. We consider warranty policy of NFRW starting at the time of purchase by a new owner.
5. All the repairs throughout product past life up to age x and corrective repairs under warranty are assumed to be minimal. Minimal repair restores the failed product to operational state, but the failure rate remains unchanged.

6. At age x, the second-hand product is subjected to an upgrade action at no cost to the buyers. And it is supposed that the upgrade actions are instantaneous.
7. PM actions are carried out by the dealer within the warranty period at no cost to the buyers.
8. All repairs and maintenance actions are instantaneous.
9. Buyers are homogeneous in their risk attitudes toward the warranty price and uncertain repair cost.

Product failure model

Generally, modeling of an item's failure can be done at two different levels, that is, system level and component level. In either case, the modeling can be done by viewing the item as a black-box or white-box.[21] Here, we confine our attention to modeling product failures at the system level, and the black-box approach is used.

It is supposed that the lifetime of product T is a random variable, and it can be defined by a cumulative distribution function (CDF) $F_0(t)$. Then, $f_0(t)$ and $h_0(t)$ are the corresponding probability density function (PDF) and failure rate (hazard) function, respectively. Therefore, we have

$$f_0(t) = \frac{dF_0(t)}{dt} \text{ and } h_0(t) = \frac{f_0(t)}{1 - F_0(t)} \tag{1}$$

In this article, we assume that all repairs throughout product's past life up to age x are minimal. Given an item has survived for x time units, let $F_x(t)$ and $f_x(t)$ denote the conditional CDF and PDF of its lifetime, respectively. Then, we have

$$\begin{aligned} F_x(t) &= P(T \leqslant x + t | T > x) \\ &= \frac{F_0(x + t) - F_0(x)}{1 - F_0(x)}, \text{ for all } t \geqslant 0 \end{aligned}$$

and

$$f_x(t) = \frac{dF_x(t)}{dt} = \frac{f_0(x + t)}{1 - F_0(x)}$$

Therefore, the subsequent failures before the first maintenance action will occur according to a nonhomogeneous Poisson process (NHPP) with conditional failure rate function given by

$$h_x(t) = \frac{f_x(t)}{1 - F_x(t)} = h_0(x + t) \tag{2}$$

Note that subsequent failures of the product depend on the type of corrective maintenance (CM) actions and/or PM actions.

Purchase price from an end user

Let C_p denote the price that the dealer pays to an end user for a second-hand product of age x ($x > 0$). We assume that the dealer's purchase price equals the salvage value of the used product, which is defined as market value of the product at the end of its past life.[22] The salvage value can be estimated from the depreciation schedule of the product. In this article, we consider a purchase price function incorporating the economic effects of product deterioration, which is given by[22]

$$C_p = \frac{P_0}{\eta[\rho_1 h_0(x) + \rho_2]^x} \tag{3}$$

where P_0 is the sale price of new product "with no warranty"; η, ρ_1 and ρ_2 are positive constant parameters which are market driven.

Note that the salvage value function used here is a monotonically decreasing function with past age x, which is convinced in practice. Functional depreciation[22] of the product and tax effects of the system disposal are ignored.

Model for the effect of upgrade action

We assume that a used product is subjected to an upgrade action with level u at age x. In this article, we adopt the age reduction approach[14] to analyze the effect of upgrade action on product reliability. In this approach, age reduction factor p ($0 \leqslant p \leqslant 1$) is used to describe the level of an upgrade action. It is assumed that after upgrade action at age x, the product's virtual age is expressed as $(1-p)x$. Therefore, the upgrade level is $u = x - (1-p)x = px$, $0 \leqslant u \leqslant x$, as shown in Figure 1. In other words, an upgrade action with level px will leave the second-hand product with past age x at its virtual age $(1-p)x$.

Therefore, given an item has survived for x time units, after upgrade action its corresponding failure rate is given by

$$h_u(t) = h_0((1 - p)x + t) \tag{4}$$

Note that $p = 0$ means no improvement; $p = 1$ implies restoring the item back to new.

It is reasonable to assume that the cost of an upgrade action could be divided into a setup cost (displacement, etc.) and a variable cost modeled by the increasing power function of the age reducing factor p. Thus, the upgrade cost C_u is given by[14]

$$C_u = c_s + c_1 p^{\xi_1} x^{\xi_2} \tag{5}$$

where c_s is the setup cost; c_1, ξ_1 and ξ_2 are positive real parameters, respectively. This implies that the cost of an upgrade action increases as x and/or p increases.

Model for the effect of PM actions

In order to reduce the risk of failure, PM actions are carried out by the dealer during the warranty period. Here, we use the same modeling framework considered by Yeh et al.[23,24] to model the effect of PM actions.

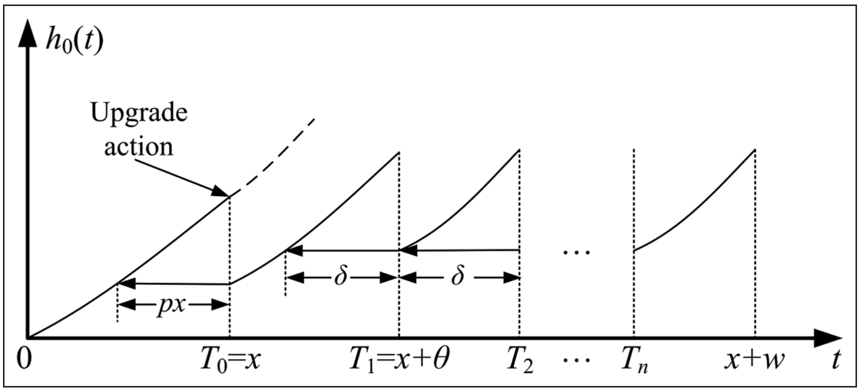

Figure 1. Scheme for upgrade and PM policies for second-hand products.

Consider a second-hand product with past age x which is sold with a NFRW with period w. Within the warranty period, if the age of second-hand product reaches a pre-specified threshold value θ, the imperfect PM action is carried out. Age reduction method is used to describe the effect of each imperfect PM action. That is, when PM action is performed with degree δ, the age of the second-hand product will be δ units of time younger than before. Meanwhile, any failures between two successive PM actions are rectified minimally at no cost to the buyer. It is supposed that the time required for performing all minimal repairs and PMs are negligible.

As shown in Figure 1, the maintenance scheme is described in terms of the failure rate function. In Figure 1, when the threshold value θ and PM degree δ are given, n and T_i for $i=$ 1, 2, …, n can be easily obtained. Here, n represents the number of PM actions, and $0 = T_0 \leqslant T_1 \leqslant \ldots \leqslant T_n \leqslant W$ means the time instants for performing these PM actions. When minimal repairs are performed, failure process of the product in interval $[T_i, T_{i+1})$ is a NHPP with intensity function $h_u(t-i\delta)$.

Assume that the cost of each PM action is $c_{pm}(\delta)$. In practice, each PM cost is a non-negative and non-decreasing function of PM degree δ. Here, we assume that the cost of each PM is a linear function of PM degree δ, that is

$$C_{pm} = nc_{pm}(\delta) = n(c + d\delta) \tag{6}$$

where c and d represent the fixed and variable cost of PM action, respectively. They can be estimated through the past PM history.

Warranty servicing cost

In this subsection, we derive the expected warranty cost for second-hand product with past age x under NFRW. Under NFRW, after sale and during warranty period $[0, w]$, the dealer agrees to rectify all failures with free of charge to the new buyer, and the warranty does not renew. As stated before, the failure process between two successive PM actions is a NHPP with the assumption of minimal repairs at failures.

When a second-hand product with past age x is subjected to pre-sale upgrade action and n post-sale PM actions, let $E[N]$ denote the expected number of failures during the warranty period. Then, we have

$$\begin{aligned} E[N] &= \sum_{i=0}^{n} \int_{T_i}^{T_{i+1}} h_0(x(1-p) + t - i\delta)dt \\ &= \sum_{i=0}^{n} \{H_0(x(1-p) + T_{i+1} - i\delta) \\ &\quad - H_0(x(1-p) + T_i - i\delta)\} \end{aligned} \tag{7}$$

where $T_0 = 0$, $T_{n+1} = w$ and $H_0(t) = \int_0^t h_0(x)dx$.

From Figure 1, we know that $T_1 = \theta$, $T_i = T_1 + (i-1)\delta$ ($i = 1,2, \ldots, n$) and the boundary conditions $\delta \leqslant \theta \leqslant w$ and $(w-\theta)/n \leqslant \delta \leqslant w$. Hence

$$\begin{aligned} E[N] = n[&H_0(x(1-p) + \theta) - H_0(x(1-p) + \theta - \delta)] \\ &+ H_0(x(1-p) + w - n\delta) - H_0(x(1-p)) \end{aligned} \tag{8}$$

Suppose that during the warranty period, each failure incurs average repair cost c_r. Therefore, the dealer's expected warranty servicing cost over warranty period w is given by $EC_w = c_r E[N]$.

Sale price

We assume that the sale price of a second-hand product is a function of its past age, length of warranty period and upgrade level. The conditions for modeling the sale price include the following: (1) The sale price increases as warranty length and/or upgrade level increases. (2) It allows for the non-zero price when no warranty and/or no upgrade action are offered. In this article, the original Cobb–Douglas-type function is modified to make it suitable to model the sale price of a second-hand product.[14] Hence, the sale price S can be modeled by

$$S = k_0 C_p (w + k_w)^a (p + k_p)^b \tag{9}$$

where k_0 is an amplitude factor, C_p is the purchase price given by equation (3), k_w and k_p are positive constant parameters of warranty length/age reducing displacement which are used to satisfy the condition (2) and $a > 0$ and $b > 0$ are parameters of "warranty length elasticity" and "age reducing elasticity", respectively. The value of a and b can be obtained from the dealer and by considering the economic and engineering factors.[14]

Profit function

Let J denote the dealer's expected profit per second-hand item sold. Taking the effect of upgrade and PM actions into account, J can be given by

$$J = S - C_p - C_u - C_{pm} - c_r E[N] \tag{10}$$

where C_p is given by equation (3), C_u by (5), C_{pm} by (6), $E[N]$ by (8), S by (9). Substituting equations (3), (5), (6), (8) and (9) into (10), we can obtain

$$\begin{aligned} J = {} & \frac{P_0}{\eta[\rho_1 h_0(x) + \rho_2]^x}\left[k_0(w + k_w)^a(p + k_p)^b - 1\right] \\ & - \left(c_s + c_1 p^{\xi_1} x^{\xi_2}\right) \\ & - n(c + d\delta) \\ & - c_r\left\{\begin{array}{l} n[H_0(x(1-p) + \theta) - H_0(x(1-p) + \theta - \delta)] \\ + H_0(x(1-p) + w - n\delta) - H_0(x(1-p)) \end{array}\right\} \end{aligned} \tag{11}$$

This is an optimization problem with the objective function being the expected profit per item sold. We need to find out the optimal values of the four variables p, δ, n and θ, so as to maximize the objective function. In the next section, the structural properties of the optimal policy are investigated.

Model optimization

Based on the objective function of equation (11), the structural properties of optimal upgrade level and PM policy are obtained in this section. First, we use the same optimization frame given by Yeh et al.[23,24] to find out θ^* and δ^*. Then, n^* is searched through enumerative search method, and p^* is obtained by evaluating J for discrete values of p over the search interval.

With no upgrade and PM actions, that is, $p = \delta = n = 0$ and $\theta = w$, equation (11) can be written as

$$\begin{aligned} J_0 = {} & \frac{P_0}{\eta[\rho_1 h_0(x) + \rho_2]^x}\left[k_0(w + k_w)^a(k_p)^b - 1\right] \\ & - c_s - c_r\{H_0(x + w) - H_0(x)\} \end{aligned} \tag{12}$$

Obviously, J_0 provides the lower bound for dealer's expected profit. Note that this case is denoted as "Scheme 0" in the following sections.

Next, we investigate the following three schemes for upgrade and/or PM actions: (1) upgrade and PM, (2) upgrade only and (3) PM only.

Scheme 1: Upgrade and PM

In this scheme, optimal upgrade and PM policies are considered simultaneously so as to maximize the dealer's expected profit. In order to investigate the optimal age threshold value θ^*, first we consider the case when n, δ and p are pre-specified. Taking the first partial derivative of equation (11) with respect to θ, we have

$$\frac{\partial J}{\partial \theta} = -nc_r[h_0(x(1-p) + \theta) - h_0(x(1-p) + \theta - \delta)] \tag{13}$$

Clearly, equation (13) plays an important role in obtaining the optimal age threshold value θ^*.

Given any $p > 0$, $n > 0$ and $\delta > 0$, when $h_0(t)$ is a decreasing function of t, we have $h_0(t_1) > h_0(t_2)$ for $0 \leqslant t_1 < t_2$. From equation (13), we know that $\partial J/\partial\theta > 0$ in this case. This implies that J is an increasing function of θ. Since the boundary condition for θ is $\delta \leqslant \theta \leqslant w$, we have the optimal threshold value $\theta^* = w$. In other words, the optimal PM number n^* within the warranty period should be zero in the case of decreasing failure rate. In this case, there is no need for performing PM, which is not the case discussed here.

On the other hand, when $h_0(t)$ is an increasing function of t, from equation (13), the following theorem shows that there exists a unique optimal age threshold value θ^* where the expected profit can be maximized.

Theorem 1. When $h_0'(t) > 0$ for all $t > 0$, the optimal age threshold value θ^* is δ for any $p > 0$, $n > 0$ and $\delta > 0$.

Proof. Given any $p > 0$, $n > 0$ and $\delta > 0$, when $h_0'(t) > 0$ for all $t > 0$, we have $\partial J/\partial\theta < 0$ in equation (13). This implies that J is a decreasing function of θ. Therefore, under the boundary condition $\delta \leqslant \theta \leqslant w$, the optimal age threshold value of PM is $\theta^* = \delta$.

From Theorem 1, we know that the optimal age threshold value θ^* and the PM degree δ are equal in the case of increasing failure rate. In other words, the second-hand product should be restored to its original status after each PM action.

On the basis of Theorem 1 and equation (11), the expected total cost becomes

$$\begin{aligned} J|_{\theta^* = \delta} = {} & \frac{P_0}{\eta[\rho_1 h_0(x) + \rho_2]^x} \\ & \left[k_0(w + k_w)^a(p + k_p)^b - 1\right] - \left(c_s + c_1 p^{\xi_1} x^{\xi_2}\right) \\ & - n(c + d\delta) \\ & - c_r\left\{\begin{array}{l} n[H_0(x(1-p) + \delta) - H_0(x(1-p))] \\ + H_0(x(1-p) + w - n\delta) - H_0(x(1-p)) \end{array}\right\} \end{aligned} \tag{14}$$

and the boundary condition for δ becomes $w/(n+1) \leqslant \delta \leqslant w$.

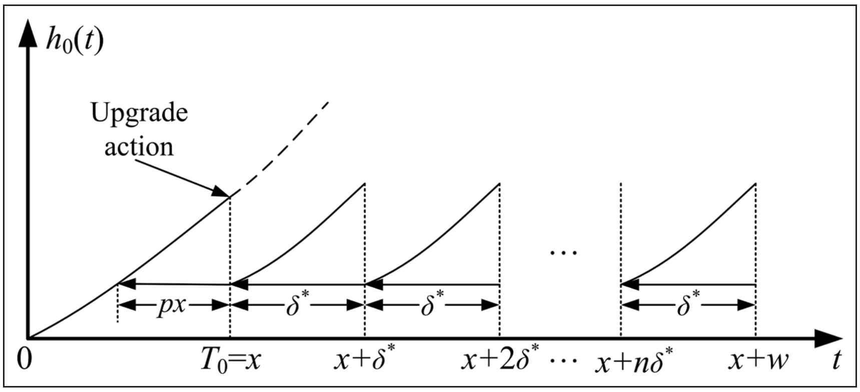

Figure 2. Optimal PM policies for second-hand product under warranty.

Then, in order to investigate the optimal PM degree δ^*, we consider the case when n and p are pre-specified. Taking the first partial derivative of equation (14) with respect to δ, then we have

$$\frac{\partial J}{\partial \delta} = -nc_r[h_0(x(1-p)+\delta) - h_0(x(1-p)+w-n\delta)] - nd \tag{15}$$

and the following theorem holds.

Theorem 2. When $h_0'(t)>0$ for all $t>0$, the optimal PM degree δ^* is $w/(n+1)$ for any $p>0$ and $n>0$.

Proof. Given any $p>0$ and $n>0$, when $h_0'(t)>0$ for all $t>0$, obviously we have $\partial J/\partial\theta < 0$ in equation (15), which means that J is a decreasing function of δ. Since the boundary condition for δ is $w/(n+1) \leqslant \delta \leqslant w$, the optimal PM degree is $\delta^* = w/(n+1)$.

According to Theorems 1 and 2, we know that the proposed PM policy is simplified to periodical PM policy. Meanwhile, the periodical PM actions should be performed at time $i\delta^*$ ($i = 1, 2, \ldots, n$) as shown in Figure 2. The rest of the decision variables in the expected profit function include the number of PM actions n and upgrade level p.

Now, we need to determine the optimal number of PM actions. Note that the maximum reduction in the expected CM cost is given by

$$EC_0 = c_r E\left[N|_{n=0}\right] = c_r[H_0(x+w) - H_0(x)]$$

The cost of PM actions is greater than cn. As a result, $n < n_m$, where $n_m = \lfloor EC_0/c \rfloor$ is the integer part of EC_0/c. This implies that an enumerative search for n varying from 0 to n_m will yield the global optimal n^*.

Using the properties of Theorems 1 and 2, and the upper bound of n^*, the expected total cost from equation (14) becomes

$$\begin{aligned} J|_{n=n^*,\theta^*=\delta^*=w/(n^*+1)} &= \frac{P_0}{\eta[\rho_1 h_0(x)+\rho_2]^x} \\ &\left[k_0(w+k_w)^a(p+k_p)^b - 1\right] - \left(c_s + c_1 p^{\xi_1} x^{\xi_2}\right) \\ &- n^*\left(c + \frac{dw}{n^*+1}\right) - c_r(n^*+1) \\ &\left[H_0\left(x(1-p) + \frac{w}{n^*+1}\right) - H_0(x(1-p))\right] \end{aligned} \tag{16}$$

The optimal p is obtained by maximizing $J|_{n=n^*,\theta^*=\delta^*=w/(n^*+1)}$. Taking the first partial derivative of equation (16) with respect to p, we have

$$\begin{aligned} \frac{\partial J|_{n=n^*,\theta^*=\delta^*=w/(n^*+1)}}{\partial p} &= \frac{bk_0P_0}{\eta[\rho_1 h_0(x)+\rho_2]^x} \\ &(w+k_w)^a(p+k_p)^{b-1} - \xi_1 c_1 p^{\xi_1-1} x^{\xi_2} \\ &+ xc_r(n^*+1)\left[h_0\left(x(1-p) + \frac{w}{n^*+1}\right) - h_0(x(1-p))\right] \end{aligned} \tag{17}$$

From equation (17), we can obtain the optimal upgrade level p^*, by setting the first derivative of $J|_{n=n^*,\theta^*=\delta^*=w/(n^*+1)}$ to zero. Unfortunately, it is too complex to derive analytical results from equation (17). However, as the value of $p^* \in [0, 1]$, we can easily obtain p^* by evaluating $J|_{n=n^*,\theta^*=\delta^*=w/(n^*+1)}$ for discrete values of p over the search interval.

Scheme 2: Upgrade only

In this scheme, there is only upgrade action and no PM actions, which means $n = 0$. As a result, the dealer's expected profit is given by

$$\begin{aligned} J|_{n=0} &= \frac{P_0}{\eta[\rho_1 h_0(x)+\rho_2]^x} \\ &\left[k_0(w+k_w)^a(p+k_p)^b - 1\right] \\ &- \left(c_s + c_1 p^{\xi_1} x^{\xi_2}\right) \\ &- c_r\{H_0(x(1-p)+w) - H_0(x(1-p))\} \end{aligned} \tag{18}$$

From equation (18), we can obtain optimal upgrade level p^*, by setting the first derivative of $J|_{n=0}$ to zero. Similar to Scheme 1, it is too complex to obtain an analytical expression for p^* and a computational method is needed.

Scheme 3: PM only

In this scheme, there is only PM actions and no upgrade action, that is, $p = 0$. As a result, the dealer's expected profit is given by

$$\begin{aligned} J|_{p=0} = & \frac{P_0}{\eta[\rho_1 h_0(x) + \rho_2]^x} \\ & \left[k_0(w + k_w)^a (p + k_p)^b - 1\right] - c_s - n(c + d\delta) \\ & - c_r\{n[H_0(x + \theta) - H_0(x + \theta - \delta)] \\ & + H_0(x + w - n\delta) - H_0(x)\} \end{aligned} \tag{19}$$

The analysis of this scheme is similar to that of Scheme 1. The optimal age threshold and degree of PM are given by $\theta^* = \delta^* = w/(n^* + 1)$, and the optimal number of PM n^* can be obtained by enumerative search.

Numerical example

In this section, a numerical example is given to illustrate the proposed model. After that, sensitivity analysis is conducted to show how the model parameters affect the optimal upgrade level, PM policy and the corresponding expected profit of the dealer.

Owning to its flexibility by changing parameter's values, Weibull distribution is chosen to describe the lifetime for the corresponding second-hand products. Failure rate function $h_0(t) = \lambda\beta(\lambda t)^{\beta-1}$ is a linearly increasing function with scale parameter $\lambda = 0.5$/year and shape parameter $\beta = 2.0$. Besides, the unit of money is US dollar ($) in this article. The other parameters in the model are listed as in Table 1.

Table 1. Numerical values of parameters in the numerical example.

Parameters	Values
Purchase price	P_0 = \$15,000, η = 1.0, ρ_1 = 0.2, ρ_2 = 1.2
Past age of second-hand product	x = 2 years
Warranty period	w = 2 years
Expected cost of each rectification	c_r = \$200
Upgrade cost	ξ_1 = 1.15, ξ_2 = 0.2, c_m = \$100, c_1 = \$500
PM cost	c= \$10, d = \$10
Sale price	k_0 = 1.2, k_w = 0.1, k_p = 1.1, a = 0.2, b = 0.04

General results

Without upgrade and PM actions (Scheme 0), the dealer's expected profit can be calculated by equation (12). In this case, the dealer's expected profit per item sold (J_0) is $2340.34, which provides the lower bound for expected profit.

Next, we search for the optimal upgrade level p^*, optimal number of PM actions n^* by using the grid search method. Furthermore, the optimal threshold value of age and optimal PM degree are $\theta^* = \delta^* = w/(n^* + 1)$. Therefore, the dealer's expected profit can be maximized, which is given by J^*. The increased profit by performing optimal upgrade and PM actions is evaluated by the percentage growth of expected profit, which is given by $\psi = (J^* - J_0)/J_0$.

Next, three schemes for upgrade and/or PM actions will be considered, that is, (1) upgrade and PM, (2) upgrade only and (3) PM only.

Scheme 1: Upgrade and PM. In this case, we can obtain that the optimal upgrade level $p^* = 0.76$, the optimal number of PM actions $n^* = 3$, and the optimal age threshold and degree of PM $\theta^* = \delta^* = 0.5$. The dealer's expected profit per item sold corresponding to this policy is $2557.49, which is increased by 9.28% compared with J_0. Under this optimal policy, the upgrade action makes the second-hand product 1.52 years younger than before. Within the warranty period, PM actions with degree 0.5 should be carried out three times with time instants $T_1 = 0.5$ year, $T_2 = 1.0$ year and $T_3 = 1.5$ year, respectively.

Scheme 2: Upgrade only. In this scheme, the optimal upgrade level $p^* = 0.76$, which is the same as that in Scheme 1. The corresponding expected profit is $2452.49, which is 4.79% greater than J_0.

Scheme 3: PM only. In this scheme, the optimal number of PM actions $n^* = 3$, and the optimal age threshold and degree of PM are $\theta^* = \delta^* = 0.5$, which is the same as those of Scheme 1. The expected profit corresponding to this policy is $2445.34, which is 4.49% greater than J_0.

Table 2. Comparison of different schemes.

Scheme	Optimal policy	J^* (\$)	ψ (%)
Scheme 1	p^* = 0.76, n^* = 3, $\theta^* = \delta^*$ = 0.5	2557.49	9.28
Scheme 2	p^* = 0.76	2452.49	4.79
Scheme 3	n^* = 3, $\theta^* = \delta^*$ = 0.5	2445.34	4.49
Scheme 0	–	2340.34	–

The optimal upgrade and/or PM policies and the corresponding expected profits under the above schemes are summarized in Table 2.

From Table 2, we can observe that with performing either upgrade or PM actions only, the total expected profit for the dealer is higher than Scheme 0. When both upgrade and PM actions are carried out, the dealer's expected profit is even higher.

Sensitivity analysis

As we know, the uncertainty of model parameters could affect the optimal policies significantly. In this section, sensitivity analysis is conducted to illustrate the effect of model parameters on the results by varying one parameter at a time while keeping the others unchanged.

Effect of second-hand product's past age x. The effect of second-hand product's past age on upgrade and PM policies and the corresponding expected profit is shown in Table 3.

As can be seen, as for Scheme 1, when the past age of second-hand product x increases, the optimal upgrade level p^* and percentage growth of expected profit ψ also increase, but the dealer's corresponding expected profit decreases, which is reasonable in practice. However, n^* and $\delta^*(\theta^*)$ remain unchanged. The reason is that when $\beta = 2$, the failure rate function is $h_0(t) = 2\lambda^2 t$, that is, the failure rate function is a linear function of t. Therefore, x has no significant effect on optimal PM policy in this case.

Additionally, the optimal upgrade level p^* in Scheme 2 and the optimal PM policy (n^*, δ^*) in Scheme 3 are the same as those in Scheme 1, but the expected profit in each scheme is lower than Scheme 1.

Effect of warranty length w. Table 4 indicates the effect of warranty length on the optimal policies and the corresponding expected profit.

As shown in Table 4, p^*, n^*, J^* and ψ increase significantly as w increases in Scheme 1. Furthermore, the optimal policies in Schemes 2 and 3 are the same as those in Scheme 1, but the expected profit in either Scheme 2 or Scheme 3 is lower. The results imply that upgrade and PM actions should be performed for second-hand products with long warranty period.

Table 3. Effect of x on optimal policies.

	Scheme 1					Scheme 2			Scheme 3				Scheme 0
x	p^*	n^*	δ^*	J^* ($)	ψ (%)	p^*	J^* ($)	ψ (%)	n^*	δ^*	J^* ($)	ψ (%)	J_0 ($)
1.0	0.77	3	0.50	4318.56	5.74	0.77	4213.56	3.17	3	0.50	4188.90	2.57	4083.90
1.5	0.74	3	0.50	3420.36	6.92	0.74	3315.36	3.63	3	0.50	3304.08	3.28	3199.08
2.0	0.76	3	0.50	2557.49	9.28	0.76	2452.49	4.79	3	0.50	2445.34	4.49	2340.34
2.5	0.85	3	0.50	1777.33	14.39	0.85	1672.33	7.63	3	0.50	1658.74	6.76	1553.74
3.0	1.00	3	0.50	1110.51	28.29	1.00	1005.51	16.16	3	0.50	970.65	12.13	865.65

Table 4. Effect of w on optimal policies.

	Scheme 1					Scheme 2			Scheme 3				Scheme 0
w	p^*	n^*	δ^*	J^* ($)	ψ (%)	p^*	J^* ($)	ψ (%)	n^*	δ^*	J^* ($)	ψ (%)	J_0 ($)
1.0	0.16	1	0.50	1416.47	1.68	0.16	1406.47	0.96	1	0.50	1403.10	0.72	1393.11
1.5	0.39	2	0.50	2052.99	4.65	0.39	2007.99	2.35	2	0.50	2006.80	2.29	1961.80
2.0	0.76	3	0.50	2557.49	9.28	0.76	2452.49	4.79	3	0.50	2445.34	4.49	2340.34
2.5	1.00	4	0.50	3002.58	15.73	1.00	2812.58	8.41	4	0.50	2784.51	7.32	2594.51
3.0	1.00	5	0.50	3385.13	22.80	1.00	3085.13	11.92	5	0.50	3056.58	10.88	2756.58

Table 5. Effect of d on optimal policies.

	Scheme 1					Scheme 2			Scheme 3				Scheme 0
d	p^*	n^*	δ^*	J^* ($)	ψ (%)	p^*	J^* ($)	ψ (%)	n^*	δ^*	J^* ($)	ψ (%)	J_0 ($)
0	0.76	3	0.50	2572.49	9.92	0.76	2452.49	4.79	3	0.50	2460.34	5.13	2340.34
10	0.76	3	0.50	2557.49	9.28	0.76	2452.49	4.79	3	0.50	2445.34	4.49	2340.34
20	0.76	3	0.50	2542.49	8.64	0.76	2452.49	4.79	3	0.50	2430.34	3.85	2340.34
30	0.76	3	0.50	2527.49	8.00	0.76	2452.49	4.79	3	0.50	2415.34	3.20	2340.34
40	0.76	2	0.67	2512.49	7.36	0.76	2452.49	4.79	2	0.67	2400.34	2.56	2340.34
50	0.76	2	0.67	2499.16	6.79	0.76	2452.49	4.79	2	0.67	2387.01	1.99	2340.34

Table 6. Effect of b on optimal policies.

	Scheme 1					Scheme 2			Scheme 3				Scheme 0
b	p^*	n^*	δ^*	J^* ($)	ψ (%)	p^*	J^* ($)	ψ (%)	n^*	δ^*	J^* ($)	ψ (%)	J_0 ($)
0.01	0.13	3	0.50	2423.75	4.93	0.13	2318.75	0.39	3	0.50	2414.81	4.54	2309.81
0.02	0.30	3	0.50	2452.75	5.72	0.30	2347.75	1.20	3	0.50	2424.98	4.52	2319.98
0.03	0.52	3	0.50	2497.19	7.17	0.52	2392.19	2.66	3	0.50	2435.16	4.51	2330.16
0.04	0.76	3	0.50	2557.49	9.28	0.76	2452.49	4.79	3	0.50	2445.34	4.49	2340.34
0.05	1.00	3	0.50	2632.91	12.01	1.00	2527.91	7.55	3	0.50	2455.54	4.47	2350.54
0.06	1.00	3	0.50	2715.24	15.02	1.00	2610.24	10.57	3	0.50	2465.75	4.45	2360.75

Effect of variable cost of PM action d. Table 5 illustrates the effect of the variable cost of PM on the optimal policies and the corresponding expected profit.

The results show that n^*, J^* and ψ decrease as d increases, as is to be expected. This can be explained that when d is high, the reduction of warranty cost is lower than the additional PM cost incurred by increasing the PM degree, so it is beneficial to reduce the PM degree. Note that d has no influence on p^* in Schemes 1 and 2, since it has impact only on the expected cost of PM.

Effect of "age-reducing elasticity" parameter b. The effect of "age-reducing elasticity" parameter on optimal policies and the corresponding expected profit is shown in Table 6.

We can observe from Table 6 that b has significant impact on both p^* and J^*. As b increases, p^* and J^* increase rapidly, as expected. This result can be explained by the fact that as the "age-reducing elasticity" increases, the sale price is more sensitive to the upgrade level. In this case, the increase of sale price can exceed the additional upgrade cost incurred by increasing upgrade level. Therefore, it is beneficial to increase the upgrade level. However, a variation in b does not have any significant effect on PM policy. Note that b also has impact on the expected profit in Scheme 3, since b can affect sale price even though in the case of $p = 0$, which can be seen in equation (19).

In addition, from Tables 2–6, it is interesting to observe that the percentage growth of expected profit ψ in Scheme 1 is the sum of those in Schemes 2 and 3. It means that for dealers the benefit by performing both optimal upgrade and PM actions is a simple summation of those by carrying out optimal upgrade only and optimal PM only. Note that, the reason is also that when $\beta = 2$, failure rate function is linearly increasing with t, as discussed earlier. However, when $\beta \neq 2$, this phenomenon would not hold again.

Effect of repair cost c_r under different β. The optimal upgrade level p^*, optimal number of PM actions n^*, the corresponding expected profit J^* and the percentage of profit growth ψ when repair cost c_r varies from \$100 to \$300 under $\beta = 1.5$, 2.0 and 2.5 are depicted in Figures 3–6, respectively.

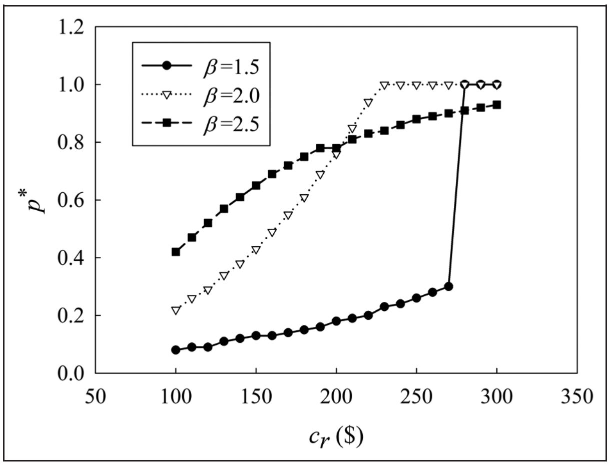

Figure 3. Effect of repair cost on optimal upgrade level.

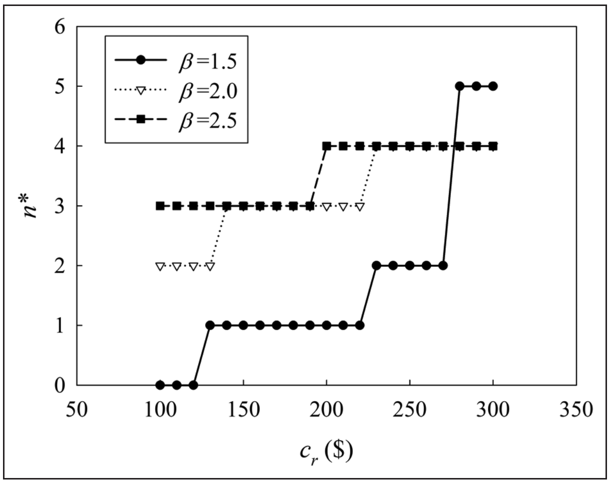

Figure 4. Effect of repair cost on optimal number of PM actions.

As shown in Figure 3, the optimal upgrade level p^* tends to approach one in all cases. However, the less β is, the faster the optimal upgrade level p^* increases.

From Figure 4, we can observe that the optimal number of PM actions n^* increases with repair cost c_r, no matter what β is. However, the less β is, the faster n^* increases.

From Figure 5, we can find that the corresponding expected profit J^* decreases with repair cost c_r, which is reasonable in practice.

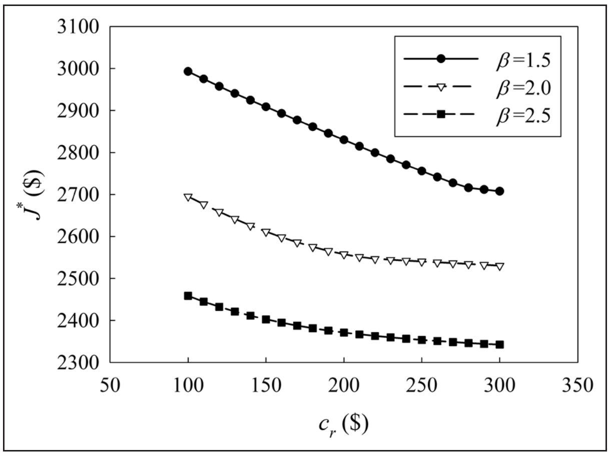

Figure 5. Effect of repair cost on dealer's expected profit.

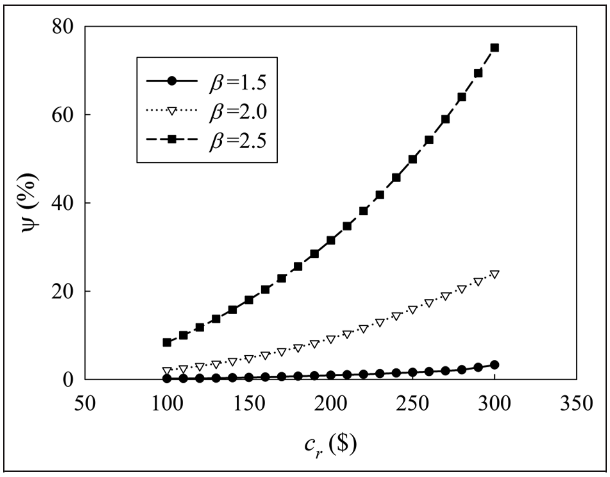

Figure 6. The percentage growth of expected profit under fixed repair cost.

As shown in Figure 6, the percentage of profit growth ψ increases with repair cost c_r, no matter what β is. And the larger β is, the faster ψ increases. Additionally, ψ will increase with β under fixed c_r. This implies that applying upgrade and PM actions for unreliable second-hand products can significantly increase the dealer's expected profit, especially when the failure repair cost is high.

From the sensitivity analysis above, it can be found that the decision on upgrade level and PM policy depends on various factors related to second-hand products, including product failure rate, past age, warranty length, repair cost and sale price.

Conclusion and topics for future research

In this article, we study second-hand products sold with NFRW, and a stochastic optimization model is proposed to determine the optimal upgrade level and PM policy to maximize dealer's expected profit. The structural properties of the optimal policies are also investigated. Numerical results show that warranty length, past life of second-hand product, upgrade and PM policies all have significant effect on the dealer's expected profit. Furthermore, when warranty period is relatively long and/or the product is relatively unreliable, the dealer should pay more attention to upgrade and PM actions since the expected profit can be increased significantly.

This article is contributory in the sense that it offers a joint determination of optimal upgrade and PM policies for the dealers to maximize their profit. However, there are still some limitations in this study, and some possible research directions are as follows:

1. In this article, we confine our analysis to the FRW policy. In fact, the other types of warranty policies (e.g. PRW) are also worthy of studying.
2. Statistical estimation of model's parameters based on real data has not been discussed in this article. That is a topic of interest from both theoretical and practical points of view.
3. Developing stochastic reliability improvement (upgrade and PM) models for second-hand products sold with warranty is an open problem.
4. It is also meaningful to model the failures and warranty cost for second-hand products sold with two-dimensional warranty, especially for the products such as automobiles.

Acknowledgement

The authors are grateful to the anonymous referees for their valuable comments and suggestions that improved the quality of this paper.

Declaration of conflicting interests

The authors declare that there is no conflict of interest.

Funding

This research was partially supported by the National Natural Science Foundation of China under contract no. 50405021 and the Fundamental Research Funds for the Central Universities and the Graduate Training Innovative Projects Foundation of Jiangsu Province under contract no. CXLX12_0081.

References

1. Blischke WR and Murthy DNP. Product warranty management—I: A taxonomy for warranty policies. *Eur J Oper Res* 1992; 62: 127–148.
2. Jack N, Iskandar BP and Murthy DNP. A repair–replace strategy based on usage rate for items sold with a two-dimensional warranty. *Reliab Eng Syst Safe* 2009; 94: 611–617.
3. Murthy DNP and Djamaludin I. New product warranty: a literature review. *Int J Prod Econ* 2002; 79: 231–260.
4. Wu S. Warranty data analysis: A review. *Qual Reliab Eng Int* 2012; 28: 795–805.

5. Yun WY, Lee YW and Ferreira L. Optimal burn-in time under cumulative free replacement warranty. *Reliab Eng Syst Safe* 2002; 78: 93–100.
6. Su C and Shen J. Analysis of extended warranty policies with different repair options. *Eng Fail Anal* 2012; 25: 49–62.
7. US Department of Transportation, Bureau of Transportation Statistics. National Transportation Statistics, http://www.bts.gov/publications/national_transportation_statistics (2013, accessed 18 January 2014).
8. BCA Group, produced in association with Sewells Information and Research. The used car market report, http://www.sewells.co.uk (2007, accessed 20 January 2014).
9. Chattopadhyay G and Murthy DNP. Warranty cost analysis for second-hand products. *Math Comput Model* 2000; 31: 81–88.
10. Chattopadhyay G and Murthy DNP. Cost sharing warranty policies for second-hand products. *Int Trans Oper Res* 2001; 8: 47–60.
11. Aksezer CS. Failure analysis and warranty modeling of used cars. *Eng Fail Anal* 2011; 18: 1520–1526.
12. Chattopadhyay G and Murthy DNP. Optimal reliability improvement for used items sold with warranty. *Int J Reliab Appl* 2004; 5: 47–58.
13. Lo H-C and Yu R-Y. A study of quality management strategy for reused products. *Reliab Eng Syst Safe* 2013; 119: 172–177.
14. Shafiee M and Chukova S. Optimal upgrade strategy, warranty policy and sale price for second-hand products. *Appl Stoch Model Bus* 2013; 29: 157–169.
15. Chen J-A and Chien Y-H. Renewing warranty and preventive maintenance for products with failure penalty post-warranty. *Qual Reliab Eng Int* 2007; 23: 107–121.
16. Wu S and Zuo MJ. Linear and nonlinear preventive maintenance models. *IEEE T Reliab* 2010; 59: 242–249.
17. Wu J, Xie M and Adam Ng TS. On a general periodic preventive maintenance policy incorporating warranty contracts and system ageing losses. *Int J Prod Econ* 2011; 129: 102–110.
18. Chang WL and Lin J-H. Optimal maintenance policy and length of extended warranty within the life cycle of products. *Comput Math Appl* 2012; 63: 144–150.
19. Yeh RH, Lo H-C and Yu R-Y. A study of maintenance policies for second-hand products. *Comput Ind Eng* 2011; 60: 438–444.
20. Kim D-K, Lim J-H and Park D-H. Optimal maintenance policies during the post-warranty period for second-hand item. In: *Proceedings of the 2011 international conference on quality, reliability, risk, maintenance, and safety engineering*, Xi'an, China, 17–19 June 2011, pp.446–450. New York: IEEE.
21. Jaturonnatee J, Murthy DNP and Boondiskulchok R. Optimal preventive maintenance of leased equipment with corrective minimal repairs. *Eur J Oper Res* 2006; 174: 201–215.
22. Monga A and Zuo MJ. Optimal design of series–parallel systems considering maintenance and salvage value. *Comput Ind Eng* 2001; 40: 323–337.
23. Yeh RH, Chang WL and Lo H-C. Optimal threshold values of age and two-phase maintenance policy for leased equipments using age reduction method. *Ann Oper Res* 2010; 181: 171–183.
24. Yeh RH, Chang WL and Lo H-C. Optimal length of lease period and maintenance policy for leased equipment with a control-limit on age. *Math Comput Model* 2011; 54: 2014–2019.