

On optimal upgrade strategy for second-hand multi-component systems sold with warranty

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Reliability improvement strategies such as upgrade, reconditioning and remanufacturing have been extensively adopted by dealers of second-hand systems to improve the system reliability and reduce the warranty servicing cost. However, most existing studies on this topic do not consider the multi-component structures of complex second-hand systems, and either treat them as black-box systems by ignoring their internal structures or simply deal with individual components. In this paper, a new upgrade model is developed for complex second-hand systems sold with a non-renewing free repair/replacement warranty, by explicitly considering their multi-component configurations. Two types of components, i.e. repairable and non-repairable components, are taken into account. During the upgrade process, non-repairable components can be upgraded only by replacement (if necessary), while repairable ones may be imperfectly upgraded with various degrees. The main objective of the dealer is to determine which components to upgrade and the corresponding upgrade degrees, to minimise the total expected servicing cost. In view of the problem structure, a marginal analysis based algorithm is presented. It is shown that the proposed upgrade strategy contains the 'no upgrade' strategy and the 'component-level perfect upgrade/replacement' strategy as special cases, and outperforms them. Finally, several extensions of the proposed upgrade model are discussed.

Keywords: second-hand systems; series configuration; warranty; upgrade; maintenance management; cost analysis

1. Introduction

In recent years, due to environment pollution and resource shortage, many countries have enacted laws regarding the reuse/recycling of used or end-of-life products, pushing manufacturers or dealers to remarket their second-hand products (Darghouth, Chelbi, and Ait-kadi 2017). On the other hand, some companies or individuals purchase second-hand equipment or products due to limited budgets, consciousness of sustainability, and/or personal preference, etc. This boosts the development of second-hand markets, especially for electronics, home appliances, automobiles, and heavy equipment, among others. For example, in 2016, the sales volume of second-hand vehicles in the USA increased for the third consecutive year to 38.5 million, which was approximately 2.2 times of the new vehicle sales (Edmunds 2017). Similarly, the trade volume of second-hand vehicles in China rose to 10.39 million in 2016, which accounted for about 40% of the new vehicle sales (Statista 2017).

Despite its steady growth, one issue that hinders a further development of the second-hand markets is the quality and reliability of the products. When prospective consumers are interested in certain second-hand products, the uncertainty of products' quality and reliability is one of the main factors that affect their purchase decisions, due to the lack of sufficient information on past usage and maintenance history (Lo and Yu 2013; Zhu et al. 2016). Faced with this situation, product warranty, an essential after-sales service (Xie and Ye 2016; Gupta, De, and Chatterjee 2017; He et al. 2017; Chen and Ye 2017a), has been widely adopted by dealers to signal the quality and reliability of their second-hand products, and to protect consumers against premature product failures. According to the report of the Car Internet Research Program II (Navarre et al. 2007), when selecting certified second-hand vehicles, consumers regard inspection process as the most important factor, followed by warranty coverage.

However, offering warranty to second-hand products incurs additional cost to the dealers resulting from the servicing of consumer claims (Aksezer 2011; Chari et al. 2016). Generally speaking, second-hand products tend to result in higher warranty servicing expenses to the dealers, since they are statistically inferior to new ones. In practice, many dealers are employing reliability improvement strategies, such as upgrade, reconditioning and remanufacturing, to improve the reliability of their second-hand products and thus reduce the warranty servicing cost (Diallo et al. 2017; Wang and Xie 2018). For

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instance, in most countries, every Ford certified pre-owned vehicle must pass a 172-point inspection process. The technicians recondition any components that do not meet programme standards, or replace them with authorised new or remanufactured components. Likewise, New Holland's certified pre-owned farm machinery undergoes a thorough inspection and testing process, as well as a reconditioning process to repair or replace any necessary components. With such inspection and upgrade programme, the reliability and performance of second-hand systems can be improved, and the warranty expenses can be reduced accordingly.

In practice, many complex second-hand systems, such as home appliances, vehicles, lanes, and mechatronic systems, have multiple components. Thus, decisions on their upgrade actions should be made by taking into account their multi-component configurations. This paper investigates the optimal upgrade strategy for second-hand series systems sold with a free repair/replacement warranty. The complex systems of interest consist of both repairable and non-repairable components. During the upgrade process, some repairable components are imperfectly upgraded with specific efficiencies, while a proportion of non-repairable components are replaced by new ones, with which the reliability of the entire system is improved. The main objective of the dealer is to determine (1) which components to upgrade and (2) to what degrees the selected components should be upgraded, in order to minimise the total expected servicing cost. Furthermore, several extensions of the proposed model are discussed to improve its applicability in different practical scenarios.

The remainder of this paper is organised as follows. Section 2 briefly reviews the related literature. Section 3 deals with the modelling and optimisation of the proposed upgrade strategy. Special cases of the upgrade model are discussed in Section 4. Numerical examples are provided in Section 5 to demonstrate the applicability of the upgrade model. Section 6 discussed several ways of extending the original model. The last section concludes this paper.

2. Literature review

This work is closely related to two streams of research, i.e. reliability improvement strategies for second-hand products/systems under warranty contracts and warranty cost analysis for multi-component systems.

In the first stream of research, efforts have been devoted to modelling and analysing various reliability improvement strategies for second-hand products/systems sold with warranty contracts. Shafiee et al. (2011) developed a stochastic cost-benefit analysis model for investment made in upgrade programmes for second-hand products sold with a free repair warranty. Lo and Yu (2013) proposed a threshold-based upgrade strategy and derived the optimal upgrade level and warranty period to maximise the dealer's expected profit. Moreover, Yazdian, Shahanaghi, and Makui (2016) jointly optimised the price of returned used products (cores), the remanufacturing degree, the selling price and the warranty period for end-of-life products to maximise the remanufacturer's expected profit. Su and Wang (2016a, 2016b) investigated the optimal upgrade strategies for used products sold with two-dimensional warranty policies by adopting bivariate and univariate failure modelling approaches, respectively. Meng et al. (2017) proposed a quality-driven recovery decision-making model for used products by combining conditional remaining useful life with cost-benefit analysis. For more research on this topic, interested readers are referred to Shafiee, Finkelstein, and Chukova (2011), Yeh, Lo, and Yu (2011), and Kim, Lim, and Park (2015), etc.

Recently, some studies investigated the worthiness of implementing both pre-sale upgrade and post-sale preventive maintenance (PM) actions from the dealer's perspective. Su and Wang (2014) jointly derived the optimal upgrade level and age-based PM policy so that the dealer's expected profit is maximised. Darghouth, Chelbi, and Ait-kadi (2017) attempted to identify the optimal upgrade level for second-hand production equipment when not performing and when performing PM during the warranty period, respectively. Khatab, Diallo, and Sidibe (2017) developed a mathematical model to jointly determine the optimal acquisition age, upgrade level, and reliability-based PM strategy for a second-hand system. Furthermore, Wang et al. (2017) studied the worthiness of reliability improvement, including both upgrade and PM actions, for repairable second-hand products sold with a two-dimensional warranty.

The studies above have made important contributions to this research area. However, most of them either treat second-hand systems as black-box systems by ignoring their internal structures or simply deal with individual components (Diallo et al. 2017). In practice, many complex second-hand systems contain multiple components. On the one hand, modelling upgrade actions for complex multi-component systems by using black-box approach, as is done in the existing literature, may be over simplistic. On the other hand, optimising upgrade strategies for individual components separately may lead to suboptimal upgrade decisions at the system level. Unlike the existing research, this paper attempts to model and optimise upgrade strategies for complex second-hand systems by explicitly considering their multi-component structures.

The second stream of research is on the warranty cost analysis for multi-component systems. Bai and Pham (2004) developed discounted warranty cost models for repairable series systems under free repair warranty and pro-rata warranty, respectively, by considering minimal repairs upon failures. Bai and Pham (2006) presented warranty cost models for multi-component systems, under which failed components or subsystems will be replaced by new identical ones.

Park and Pham (2010) investigated the warranty cost analysis for multi-component systems based on quasi-renewal processes (to model imperfect repair) and exponential distribution. Liu, Wu, and Xie (2015) dealt with the warranty cost analysis for multi-component systems under renewing free replacement warranty with consideration of failure interactions among components. Recently, Zhang, Fouladirad, and Barros (2018) examined the short-run warranty costs and profits of a two-component system with type I stochastic dependence.

This paper is different from the second stream of research in the sense that (i) we focus on the optimisation of upgrade strategy for second-hand multi-component systems, rather than pure warranty cost analysis for new multi-component systems; (ii) the aforementioned literature predominately considers multi-component systems with either repairable or non-repairable components, while in this work the systems contain both repairable and non-repairable components, which is more practical. To our knowledge, this paper represents the early attempt to investigate the optimal upgrade strategies for second-hand multi-component systems sold with a non-renewing free repair/replacement warranty, which is a rather common scenario in the industry when such decisions are to be made.

3. Model development

3.1. Assumptions

To facilitate our model description, the assumptions needed are given below.

- (1) The system has multiple (say, n) independent components connected in series. That is, if one of these components fails, the system as a whole stops working.
- (2) Among the n components, k of them are repairable, and the other $n-k$ are non-repairable. Without loss of generality, we suppose that components $1, 2, \dots, k$ are repairable, and components $k+1, k+2, \dots, n$ are non-repairable.
- (3) During the earlier usage of the system, all of its components either have no failures or have been maintained through minimal repairs (for repairable ones) up to age x . After a minimal repair, the failed component is restored to an operational state, but its failure rate remains unchanged.
- (4) At age x , the system is subject to an upgrade programme. The upgrade actions for repairable and non-repairable components are different, namely, the repairable components can be imperfectly upgraded with various degrees, while the non-repairable ones can be upgraded only through replacements with new identical ones.
- (5) After the upgrade, the second-hand system is sold with a non-renewing free repair/replacement warranty. During the warranty period, the dealer adopts two types of corrective rectifications, i.e. minimal repairs for failures of repairable components, and replacements for failures of non-repairable components.
- (6) Failures of all components are statistically independent, and whenever a component failure occurs within the warranty period, it results in an immediate warranty claim.
- (7) The durations of all upgrade actions and repair/replacement actions are sufficiently small compared to the mean time to failure, and are thus negligible.

3.2. Modelling the component reliability

Suppose that the lifetime T_i of a new component i can be modelled by a distribution function $F_i(t)$, and its reliability function is $R_i(t) = 1 - F_i(t)$. Then, the corresponding failure rate function $\lambda_i(t)$ of the time to first failure can be derived as

$$\lambda_i(t) = f_i(t)/R_i(t), \quad i = 1, 2, \dots, n, \quad (1)$$

where $f_i(t) = dF_i(t)/dt$ is the probability density function of T_i .

Based on the assumption (3), given component i has survived for x time units, the conditional distribution function and conditional failure rate function of its lifetime T_i , before the first maintenance, can be derived as

$$F_i(t|x) = P\{T_i \leq x+t | T_i > x\} = \frac{F_i(x+t) - F_i(x)}{1 - F_i(x)}, \quad i = 1, 2, \dots, n, \quad (2)$$

and

$$\lambda_i(t|x) = dF_i(t|x)/[1 - F_i(t|x)] = \lambda_i(x+t), \quad i = 1, 2, \dots, n, \quad (3)$$

respectively.

3.3. Modelling the upgrade programme

In this study, different upgrade actions are considered for repairable and non-repairable components, that is, imperfect upgrade for repairable components, and replacements for non-repairable components. The two types of upgrade actions will be described in Sections 3.3.1 and 3.3.2, respectively.

3.3.1. Upgrade actions for repairable components

For repairable components, their upgrade actions (if necessary) can be imperfect from the reliability perspective. Basically, an imperfect upgrade action improves the component reliability to an intermediate level, i.e. between as-bad-as-old and as-good-as-new. In this study, the effect of imperfect upgrade action on the component reliability is modelled by the virtual age reduction approach. That is, an imperfect upgrade action results in a restoration of repairable component i so that its virtual age is effectively reduced. The *virtual age* concept was firstly introduced in Kijima (1989), and then widely adopted to describe the effect of various reliability improvement actions; see Tong et al. (2014), Zhou et al. (2015), Wang and Su (2016), Duan et al. (2018), and Zhao, He, and Xie (2018), for example.

During the upgrade process, some repairable components are imperfectly upgraded and restored to better states, and others not, depending on the cost–benefit analysis. Let $d_i \in \{0, 1\}$ denote a binary decision variable, namely, $d_i = 1$, if component i is upgraded; $d_i = 0$, otherwise. For repairable component i , $i = 1, \dots, k$, its conditional failure rate after upgrade can be modelled as

$$\lambda_i((1 - d_i\delta_i)x + t) = \begin{cases} \lambda_i(x + t) & \text{if } d_i = 0, \\ \lambda_i((1 - \delta_i)x + t) & \text{if } d_i = 1, \end{cases} \quad (4)$$

where $\delta_i \in [0, 1]$ is the upgrade degree (age reduction factor) of component i .

From (4), if repairable component i , $i = 1, \dots, k$, is not upgraded ($d_i = 0$), its conditional failure rate remains $\lambda_i(x + t)$; if it is subject to an imperfect upgrade ($d_i = 1$) with degree δ_i , its conditional failure rate becomes $\lambda_i((1 - \delta_i)x + t)$. It can be seen that the component's virtual age immediately after upgrade, i.e. $(1 - \delta_i)x$, decreases as δ_i increases. When $\delta_i = 0$, there is no upgrade effect on component i ; when $\delta_i = 1$, component i is perfectly upgraded to its original state; while the general case of $\delta_i \in (0, 1)$ corresponds to an imperfect upgrade.

It is reasonable to assume that the cost of an imperfect upgrade action includes fixed and variable costs (Lo and Yu 2013; Su and Wang 2016b; Wang et al. 2017). The fixed cost is related to the disassembly, assembly, cleaning, lubricating, testing, and so on, which are to be performed on the component if upgrade is needed; while the variable cost depends on the upgrade degree δ_i and the initial age x . If component i is imperfectly upgraded with degree δ_i , then its upgrade cost can be modelled as follows:

$$c_{mi}(\delta_i) = c_{si} + (c_{vi} - c_{si})\delta_i^{\varphi\xi_i(x)}, \quad i = 1, 2, \dots, k, \quad (5)$$

where c_{si} and $(c_{vi} - c_{si})$ are the fixed and variable cost elements, respectively, φ is a positive parameter, and $\xi_i(x)$ is a characteristic function that determines the exact relationship between the upgrade cost and the initial age x . Following Khatab, Diallo, and Sidibe (2017), $\xi_i(x)$ is formulated as the ratio of the mean residual life $MRL_i(x)$ of component i and its initial age x :

$$\xi_i(x) = \frac{MRL_i(x)}{x} = \frac{\int_x^\infty R_i(t) dt}{xR_i(x)}. \quad (6)$$

Expressions (5) and (6) enable the characterisation of the fact that the upgrade cost of a younger component is less than that of an older component, under the same upgrade degree; see Figure 1 for demonstration. More specifically, if component i is relatively younger, i.e. its initial age x is less than its mean residual life $MRL_i(x)$, then its corresponding $\xi_i(x)$ is larger, and because $0 \leq \delta_i \leq 1$ its upgrade cost from (5) is smaller. In contrast, if a component is older, i.e. its initial age x is higher than its $MRL_i(x)$, then $\xi_i(x)$ is smaller, and its upgrade cost is significantly larger.

Notice that, the upgrade cost of component i in (5) increases with δ_i , from the lower bound c_{si} (at $\delta_i = 0$) to the upper bound c_{vi} (at $\delta_i = 1$). In this manner, c_{si} can be regarded as the minimal upgrade cost, while c_{vi} as the perfect upgrade/replacement cost of component i . Any imperfect upgrade action with an intermediate degree $\delta_i \in (0, 1)$ would incur an upgrade cost between the two extremes.

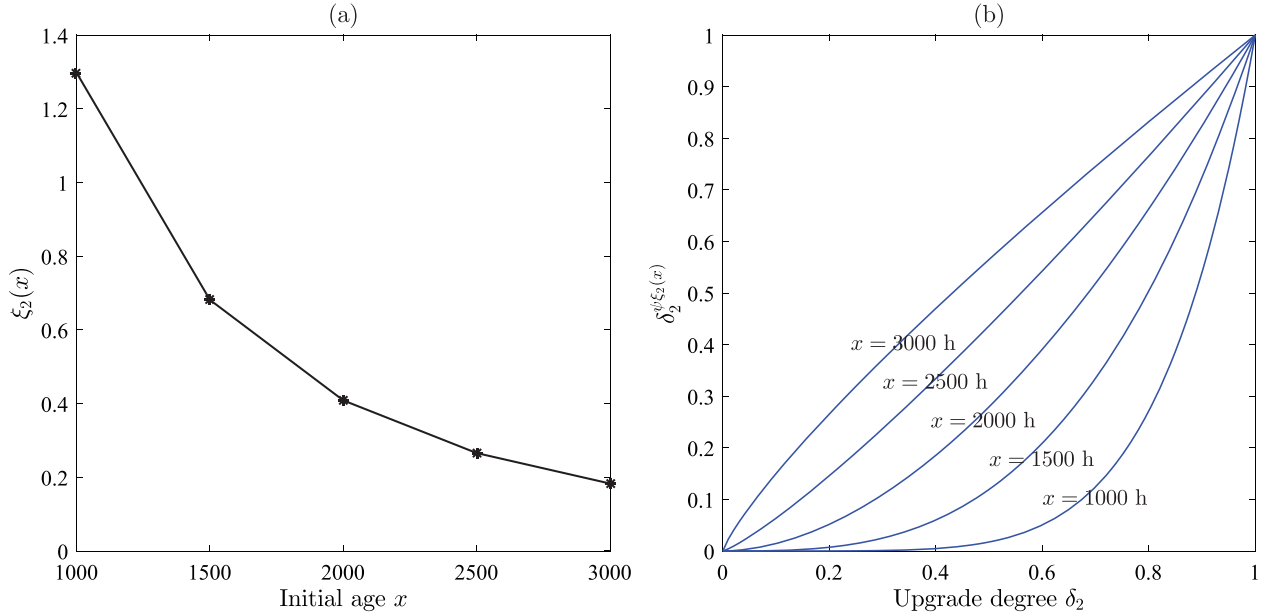


Figure 1. The characteristic function $\xi_2(x)$ and the term $\delta_2^{\varphi\xi_2(x)}$ for component 2 (in the numerical example in Section 5) with $\eta_2 = 2400$, $\beta_2 = 2.5$, and $\varphi = 4.5$.

3.3.2. Upgrade actions for non-repairable components

The upgrade actions for non-repairable components (if necessary) are component replacements by new identical ones. Similarly, which non-repairable components should be replaced during the upgrade process depends on the cost–benefit analysis.

For non-repairable component i , $i = k+1, \dots, n$, its conditional failure distribution function after upgrade can be expressed as

$$F_i(t|x, d_i) = \begin{cases} F_i(t|x) & \text{if } d_i = 0, \\ F_i(t) & \text{if } d_i = 1, \end{cases} \quad (7)$$

where $F_i(t)$ is the failure distribution of a new component i , and $F_i(t|x)$ is given by (2).

Expression (7) means that if non-repairable component i , $i = k+1, \dots, n$, is not upgraded ($d_i = 0$), its conditional failure distribution remains $F_i(t|x)$; if it is replaced ($d_i = 1$), its conditional failure distribution becomes $F_i(t)$. This implies that the upgrade degree for non-repairable component i can be set as $\delta_i = d_i$ (either one or zero). Furthermore, if non-repairable component i is subject to an upgrade action, its upgrade cost would be $c_{mi}(\delta_i = 1) = c_{vi}$, $i = k+1, \dots, n$.

3.3.3. Total upgrade cost

By combining the upgrade costs of all components, the total upgrade cost of the entire second-hand system is

$$\begin{aligned} C_u(\mathbf{d}, \boldsymbol{\delta}|x) &= c_a I_{\{l \geq 1\}} + \sum_{i=1}^k d_i c_{mi}(\delta_i) + \sum_{i=k+1}^n d_i c_{mi}(\delta_i = 1) \\ &= c_a I_{\{l \geq 1\}} + \sum_{i=1}^k d_i (c_{si} + (c_{vi} - c_{si}) \delta_i^{\varphi\xi_i(x)}) + \sum_{i=k+1}^n d_i c_{vi}, \end{aligned} \quad (8)$$

where $\mathbf{d} = (d_1, d_2, \dots, d_n)$ and $\boldsymbol{\delta} = (\delta_1, \delta_2, \dots, \delta_n)$ are vectors of decision variables, c_a is the set-up cost of the upgrade programme, which may include the administration cost, access cost, disassembly and assembly cost, among others, and $l = \sum_{i=1}^n d_i$ is the number of components to upgrade.

In (8), $I_{\{l \geq 1\}}$ is an indicator function, i.e. $I = 1$ if $l \geq 1$; $I = 0$, otherwise. This implies that the set-up cost c_a is incurred only when there is at least one component to upgrade. It should be noted that upgrading multiple components requires only one set-up, and these components share the set-up cost. In this sense, the multiple components of this second-hand system are economically dependent.

3.4. Modelling the expected warranty cost

After the upgrade process, the second-hand system is released to the marketplace, bundled with a non-renewing free repair/replacement warranty of length w . Under this warranty policy, any failures within the warranty period will be rectified by the dealer at no cost to the consumer. In this study, we assume that the dealer adopts two types of corrective rectifications, i.e. minimal repairs for failures of repairable components and replacements with new identical ones for failures of non-repairable components.

Given that all failures of repairable component i , $i = 1, \dots, k$, are minimally repaired with negligible durations, it is well known that component failures over time occur according to a non-homogenous Poisson process (NHPP) with intensity function $\lambda_i((1 - d_i\delta_i)x + t)$. Thus, the expected number of failures of repairable component i during the warranty period is

$$E[N_i(d_i, \delta_i|x, w)] = \int_0^w \lambda_i((1 - d_i\delta_i)x + t)dt, \quad i = 1, \dots, k. \quad (9)$$

On the other hand, given that all failures of non-repairable component i , $i = k + 1, \dots, n$, are rectified through replacements with new identical ones, then component failures over time occur according to a modified renewal process with the first failure given by $F_i(t|x, d_i)$ and subsequent failures by $F_i(t)$ (Chattopadhyay and Murthy 2000). Hence, by assuming negligible replacement durations, the expected number of failures of non-repairable component i during the warranty period is given by

$$\begin{aligned} E[N_i(d_i, \delta_i|x, w)] &= F_i(w|x, d_i) + \int_0^w M_i(w - t)dF_i(t|x, d_i) \\ &= F_i(w|x, d_i) + \int_0^w F_i(w - t|x, d_i)dM_i(t), \quad i = k + 1, \dots, n, \end{aligned} \quad (10)$$

where $M_i(t)$ is the renewal function associated with $F_i(t)$. The renewal function can be efficiently evaluated by the so-called Riemann-Stieltjes (RS) method in Xie (1989).

Since all components in the second-hand system are independent, the expected warranty servicing cost of the entire system can be obtained by simply summing the expected warranty costs of all of its components, as follows:

$$\begin{aligned} E[C_w(\mathbf{d}, \boldsymbol{\delta}|x, w)] &= \sum_{i=1}^k c_{fi}E[N_i(d_i, \delta_i|x, w)] + \sum_{i=k+1}^n c_{ri}E[N_i(d_i, \delta_i|x, w)] \\ &= \sum_{i=1}^k c_{fi} \int_0^w \lambda_i((1 - d_i\delta_i)x + t)dt \\ &\quad + \sum_{i=k+1}^n c_{ri} \left[F_i(w|x, d_i) + \int_0^w F_i(w - t|x, d_i)dM_i(t) \right], \end{aligned} \quad (11)$$

where c_{fi} and c_{ri} are the average minimal repair cost and the replacement cost, respectively, for component i in the event of a failure. It is worth noting that in practice, we should have $c_{si} < c_{fi}$ and $c_{vi} < c_{ri}$, since scheduled upgrade actions should be less costly than unscheduled corrective maintenance actions.

3.5. The optimisation model

For a second-hand system with initial age x and warranty period w , the dealer's total expected servicing cost, including the upgrade cost and the expected warranty cost, can be calculated as

$$\begin{aligned}
 C_T(\mathbf{d}, \boldsymbol{\delta}|x, w) &= C_u(\mathbf{d}, \boldsymbol{\delta}|x) + E[C_w(\mathbf{d}, \boldsymbol{\delta}|x, w)] \\
 &= c_a I_{\{t \geq 1\}} + \sum_{i=1}^k d_i (c_{si} + (c_{vi} - c_{si}) \delta_i^{\varphi_{\xi_i}(x)}) + \sum_{i=k+1}^n d_i c_{vi} \\
 &\quad + \sum_{i=1}^k c_{fi} \int_0^w \lambda_i((1 - d_i \delta_i)x + t) dt \\
 &\quad + \sum_{i=k+1}^n c_{ri} \left[F_i(w|x, d_i) + \int_0^w F_i(w - t|x, d_i) dM_i(t) \right].
 \end{aligned} \tag{12}$$

The dealer's optimisation problem is to identify which components to upgrade, i.e. $\mathbf{d}^* = (d_1^*, d_2^*, \dots, d_n^*)$, and the corresponding upgrade degrees, i.e. $\boldsymbol{\delta}^* = (\delta_1^*, \delta_2^*, \dots, \delta_n^*)$, in order to minimise the total expected servicing cost. Mathematically, the optimisation problem can be expressed as

$$\begin{aligned}
 &\min C_T(\mathbf{d}, \boldsymbol{\delta}|x, w) \\
 &s.t. d_i \in \{0, 1\}, \quad i = 1, 2, \dots, n \\
 &\quad \delta_i \in [0, 1], \quad i = 1, 2, \dots, k \\
 &\quad \delta_i = d_i, \quad i = k + 1, k + 2, \dots, n.
 \end{aligned} \tag{13}$$

It is quite difficult, if not impossible, to obtain the analytical solution to this optimisation problem. In view of the problem structure, a marginal analysis based algorithm is presented to solve this problem, which will be elaborated in the next subsection.

3.6. Solution procedure

Since the objective function (12) is additively separable, marginal analysis can be a simple and efficient method to solve this problem. In marginal analysis, a marginal value is defined based on the cost-benefit trade-off and then adopted to determine whether a specific component should be upgraded or not. Here, the marginal value of component i is constructed according to the following optimisation rule: *for a specific component i , an upgrade action is worthwhile only when the reduction in the expected warranty cost exceeds the additional upgrade cost incurred.*

Hence, for repairable component i , $i = 1, \dots, k$, the marginal value with upgrade degree δ_i can be defined as

$$\begin{aligned}
 \Delta_i(\delta_i) &= c_{fi} (E[N_i(d_i = 0, \delta_i|x, w)] - E[N_i(d_i = 1, \delta_i|x, w)]) - c_{mi}(\delta_i) \\
 &= c_{fi} \int_0^w [\lambda_i(x + t) - \lambda_i((1 - \delta_i)x + t)] dt - c_{si} - (c_{vi} - c_{si}) \delta_i^{\varphi_{\xi_i}(x)}, \quad i = 1, \dots, k.
 \end{aligned} \tag{14}$$

The maximal marginal value of repairable component i , resulting from the imperfect upgrade, attains at $\delta_i^* = \max \{\delta_i | \Delta_i(\delta_i); \delta_i \in [0, 1]\}$, $i = 1, \dots, k$. Since δ_i is defined within the finite support $[0, 1]$, simple numerical search methods can be efficient to obtain δ_i^* . Then, the imperfect upgrade action of repairable component i is worthwhile at the component level only when $\Delta_i(\delta_i^*) > 0$, $i = 1, \dots, k$. In other words, if $\Delta_i(\delta_i^*) > 0$, then $d_i^* = 1$; otherwise, $d_i^* = 0$.

Similarly, for non-repairable component i , $i = k + 1, \dots, n$, the marginal value can be defined as

$$\begin{aligned}\Delta_i &= c_{ri}(E[N_i(d_i = \delta_i = 0|x, w)] - E[N_i(d_i = \delta_i = 1|x, w)]) - c_{vi} \\ &= c_{ri} \left[F_i(w|x) + \int_0^w F_i(w - t|x) dM_i(t) \right] \\ &\quad - c_{ri} \left[F_i(w) + \int_0^w F_i(w - t) dM_i(t) \right] - c_{vi}, \quad i = k + 1, \dots, n.\end{aligned}\quad (15)$$

Likewise, only when $\Delta_i > 0$, the upgrade action of non-repairable component i , $i = k + 1, \dots, n$, is worthwhile at the component level. In other words, if $\Delta_i > 0$, then $d_i^* = \delta_i^* = 1$; otherwise, $d_i^* = \delta_i^* = 0$.

Based on the component-level marginal analysis for both repairable and non-repairable components, we can tentatively set $\mathbf{d}^* = (d_1^*, \dots, d_k^*, d_{k+1}^*, \dots, d_n^*)$. Notice that, \mathbf{d}^* is tentative in the sense that the set-up cost c_a of the upgrade programme has not been incorporated into the component-level marginal analysis. Further taking c_a into account, the upgrade programme of the entire second-hand system is worthwhile only when $\sum_{i=1}^k d_i^* \Delta_i(\delta_i^*) + \sum_{i=k+1}^n d_i^* \Delta_i > c_a$, i.e. the summation of net cost savings of all individual components due to this upgrade programme exceeds the set-up cost c_a . Based on the marginal analysis above, Algorithm 1 summarises the overall structure of the solution procedure.

Algorithm 1. Determining the optimal \mathbf{d}^* and $\mathbf{\delta}^*$.

Input: $x, w, \varphi, c_a, \eta_i, \beta_i, c_{si}, c_{vi}, c_{fi}, c_{ri}$, for $i = 1, 2, \dots, n$.

Output: $\mathbf{d}^*, \mathbf{\delta}^*, l^*$ and $C_T(\mathbf{d}^*, \mathbf{\delta}^*|x, w)$

1. Search $\delta_i^* = \max \{ \delta_i | \Delta_i(\delta_i) > 0, \delta_i \in [0, 1] \}$, for $i = 1, \dots, k$.
 2. For each $i \leq k$: if $\Delta_i(\delta_i^*) > 0$, then set $d_i^* = 1$; otherwise, set $d_i^* = 0$.
For each $i > k$: if $\Delta_i > 0$, then set $d_i^* = 1$; otherwise, set $d_i^* = 0$.
 3. Check: If $\sum_{i=1}^n d_i^* \geq 1$, go to 4; otherwise, go to 6.
 4. Check: If $\sum_{i=1}^k d_i^* \Delta_i(\delta_i^*) + \sum_{i=k+1}^n d_i^* \Delta_i > c_a$, go to 5; otherwise, go to 6.
 5. $\mathbf{d}^* \leftarrow (d_1^*, d_2^*, \dots, d_n^*)$, $l^* \leftarrow \sum_{i=1}^n d_i^*$ and $\mathbf{\delta}^* \leftarrow (d_1^* \delta_1^*, \dots, d_k^* \delta_k^*, d_{k+1}^*, \dots, d_n^*)$.
 6. $\mathbf{d}^* \leftarrow (0, 0, \dots, 0)$, $l^* \leftarrow 0$ and $\mathbf{\delta}^* \leftarrow (0, 0, \dots, 0)$.
 7. Compute $C_T(\mathbf{d}^*, \mathbf{\delta}^*|x, w)$.
-

4. Special cases

The proposed upgrade model is quite flexible in the sense that it can characterise different component repairability and/or upgrade strategies. In this section, special cases of the upgrade model are discussed to illustrate its flexibility.

4.1. Component repairability

The proposed upgrade model assumes that k components of the second-hand system are repairable, and the other $n-k$ are non-repairable. In practice, for some complex systems such as production systems, all of its critical components can be regarded as repairable, i.e. $k = n$. In this case, the model (12) is reduced to

$$C_T(\mathbf{d}, \mathbf{\delta}|x, w) = c_a I_{\{l \geq 1\}} + \sum_{i=1}^n d_i (c_{si} + (c_{vi} - c_{si}) \delta_i^{\varphi_{\xi_i}(x)}) + \sum_{i=1}^n c_{fi} \int_0^w \lambda_i((1 - d_i \delta_i)x + t) dt. \quad (16)$$

On the other hand, for certain simple products, either all of its components are non-repairable or it is more economical to directly replace the failed components, instead of repairing them. In this scenario, $k = 0$, and the model (12) is simplified to

$$C_T(\mathbf{d}, \mathbf{\delta}|x, w) = c_a I_{\{l \geq 1\}} + \sum_{i=1}^n d_i c_{vi} + \sum_{i=1}^n c_{ri} \left[F_i(w|x, d_i) + \int_0^w F_i(w - t|x, d_i) dM_i(t) \right]. \quad (17)$$

4.2. Upgrade strategies

Here, two special cases of the proposed upgrade strategy are considered, i.e. no upgrade actions for all components, and component-level perfect upgrade/replacement for selected components.

No upgrade. When no upgrade actions are performed for all components, i.e. $\mathbf{d} = \mathbf{0}$ and $\mathbf{\delta} = \mathbf{0}$, the total expected servicing cost to the dealer, denoted by C_{T0} , is given by

$$C_{T0} = C_T(\mathbf{d} = \mathbf{0}; \mathbf{\delta} = \mathbf{0} | x, w) \\ = \sum_{i=1}^k c_{fi} \int_0^w \lambda_i(x+t) dt + \sum_{i=k+1}^n c_{ri} \left[F_i(w|x) + \int_0^w F_i(w-t|x) dM_i(t) \right]. \quad (18)$$

In practice, C_{T0} can be regarded as a *benchmark cost*. That is to say, any upgrade decision leading to a total servicing cost higher than C_{T0} would be infeasible.

Component-level perfect upgrade/replacement. In this case, if a component is selected to upgrade ($d_i = 1$), then it will be perfectly upgraded or replaced by a new identical one, no matter whether it is repairable or non-repairable; otherwise, it remains unaltered. By setting $\delta_i = d_i$ for $i = 1, 2, \dots, k$, the dealer's total expected servicing cost (12) can be revised to

$$C_T(\mathbf{d}, \mathbf{\delta} | x, w) = C_u(\mathbf{d}, \mathbf{\delta} | x) + E[C_w(\mathbf{d}, \mathbf{\delta} | x, w)] \\ = c_a I_{\{l \geq 1\}} + \sum_{i=1}^n d_i c_{vi} + \sum_{i=1}^k c_{fi} \int_0^w \lambda_i((1-d_i)x+t) dt \\ + \sum_{i=k+1}^n c_{ri} \left[F_i(w|x, d_i) + \int_0^w F_i(w-t|x, d_i) dM_i(t) \right]. \quad (19)$$

Therefore, the optimisation problem in this case can be expressed as

$$\min C_T(\mathbf{d}, \mathbf{\delta} | x, w) \quad s.t. \delta_i = d_i \in \{0, 1\}, i = 1, 2, \dots, n. \quad (20)$$

Hereafter, we refer to the proposed upgrade strategy as *Strategy 1*, the no upgrade strategy as *Strategy 2*, and the component-level perfect upgrade/replacement strategy as *Strategy 3*.

5. Numerical examples

In this section, numerical examples are provided to demonstrate the applicability and effectiveness of the proposed upgrade model and solution algorithm. The system under consideration is a complex mechatronic system which consists of five components that are connected in series, namely, (1) control, (2) power, (3) transmission, (4) sensing, and (5) tool (Figure 2). Among them, we suppose that the first three components are repairable, while the remaining two are non-repairable. The reliability of each component can be modelled by a Weibull distribution due to its ability and flexibility in describing various shapes of component failure rates. Thus, the failure distribution function and failure rate function, respectively, of a new component i are given by

$$F_i(t) = 1 - \exp \left\{ - \left(\frac{t}{\eta_i} \right)^{\beta_i} \right\} \quad \text{and} \quad \lambda_i(t) = \frac{\beta_i}{\eta_i} \left(\frac{t}{\eta_i} \right)^{\beta_i-1},$$

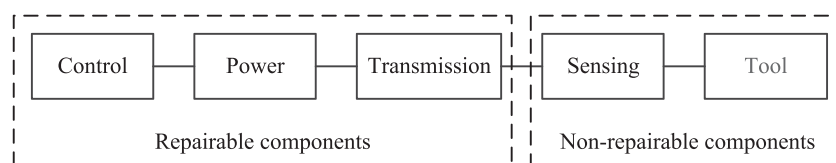


Figure 2. Illustration of the structure of the mechatronic system.

Table 1. Parameters of the five components in this example.

Index (<i>i</i>)	Component	η_i	β_i	MTBF _{<i>i</i>} (h)	c_{ri} (\$)	c_{fi} (\$)
1	Control	1300	1.8	1155	180	54
2	Power	2400	2.5	2127	240	72
3	Transmission	2600	3.2	2326	400	120
4	Sensing	3800	3.1	3395	320	-
5	Tool	2000	3.1	1787	260	-

where η_i and β_i are the scale and shape parameters, respectively, which can be estimated from field failure data and/or warranty claims data (Wu 2013; Chen and Ye 2017b).

In this study, the units for time and money are hours (h) and US dollar (\$), respectively. Table 1 lists the values of η_i , β_i , c_{ri} and c_{fi} for all the five components, which are directly adopted from Tsai, Wang, and Tsai (2004). Note that, since the components 4 and 5 are treated as non-repairable, c_{r4} and c_{r5} are not applicable in our example. Other parameters are arbitrarily set as $c_a = \$100$, $\varphi = 4.5$, $x = 2000$ h, $w = 2000$ h, $c_{vi} = 0.7 \times c_{ri}$, $i = 1, \dots, 5$, and $c_{si} = 0.5 \times c_{fi}$, $i = 1, \dots, 3$.

5.1. Optimisation of the upgrade strategies

Based on the parameter setting above, Table 2 shows the details of the marginal analysis and the optimal upgrade decision for each component. According to Algorithm 1, the first step of the marginal analysis is to numerically search the tentative upgrade degree δ_i^* , $i = 1, 2, 3$, such that the marginal value for each repairable component is maximised (see Figure 3). It is shown that the maximal marginal values for repairable components 1, 2 and 3 are achieved at $\delta_1^* = 1$, $\delta_2^* = 0.61$, and $\delta_3^* = 0.68$, respectively. The identified δ_i^* and $\Delta_i(\delta_i^*)$ for all repairable components are listed in the second and third columns of Table 2, respectively. Moreover, the marginal values of the two non-repairable components are -53.10 and 7.92 , respectively.

Afterwards, we can observe that the (maximal) marginal values of components 1, 2, 3, and 5 are positive, while that of component 4 is negative. This leads to $\mathbf{d}^* = (1, 1, 1, 0, 1)$, $\mathbf{\delta}^* = (1, 0.61, 0.68, 0, 1)$ and $l^* = 4$. In other words, components 1, 2, 3, and 5 should be upgraded based on the component-level marginal analysis. Furthermore, $\sum_{i=1}^3 d_i^* \Delta_i(\delta_i^*) + \sum_{i=4}^5 d_i^* \Delta_i = \$208.19 > c_a$ indicates that the upgrade programme of the entire system is beneficial to the dealer. In this case, the component failure rates (without failures) over time is depicted in Figure 4. At age $x = 2000$ h, components 1, 2, 3, and 5 are upgraded to mitigate the rapid growth of their failure rates. Among them, components 2 and 3 are imperfectly upgraded to intermediate states between as-bad-as-old and as-good-as-new.

With the optimal upgrade strategy, the total expected servicing cost of the second-hand system to the dealer is \$1395.53. The expected servicing costs for each component without upgrade are presented in the last column of Table 2. It can be seen that the reduction amount of the expected warranty servicing cost due to the upgrade programme is \$776.44 ($= \$1503.72 - \727.28). While the additional upgrade cost incurred is \$668.25 ($= \$568.25 + \100). Therefore, carrying out this upgrade programme is worthwhile, and the overall net benefit of the upgrade programme for the second-hand system is \$108.19.

Interestingly, let us consider the case of $x = 2000$ h and $w = 1500$ h; see Table 3. In this case, we can observe that $\Delta_1^*(\delta_1^*)$, $\Delta_3^*(\delta_3^*)$, and Δ_5^* are positive, indicating that it is worth upgrading them based on the component-level cost–benefit analysis. However, $\Delta_1^*(\delta_1^*) + \Delta_3^*(\delta_3^*) + \Delta_5^* = \$70.31 < c_a$ implies that the cost reduction attained by upgrading components 1, 3, and 5 is less than the set-up cost of the upgrade programme. In other words, the upgrade programme is not beneficial to the dealer at the system level. This demonstrates that optimising upgrade strategies for individual components separately may lead to sub-optimal upgrade decisions at the system level (as mentioned in Section 2).

Table 2. The details of the marginal analysis ($x = 2000$ h and $w = 2000$ h).

Index (<i>i</i>)	δ_i^*	Δ_i^*	d_i^*	$d_i^* \delta_i^*$	$d_i^* \Delta_i^*$	$d_i^* c_{mi}$	$E[C_{wi}(d_i^*, \delta_i^*, x, w)]$	Total cost	$E[C_{wi}(0, 0; x, w)]$
1	1.00	47.80	1	1.00	47.80	126.00	117.26	243.26	291.06
2	0.61	23.73	1	0.61	23.73	89.19	99.64	188.83	212.56
3	0.68	128.74	1	0.68	128.74	171.06	124.66	295.72	424.45
4	-	-53.10	0	0.00	0.00	0.00	212.05	212.05	212.05
5	-	7.92	1	1.00	7.92	182.00	173.67	355.67	363.60
					208.19	568.25	727.28	1395.53 ^a	1503.72

^aThis cost includes the set-up cost c_a of the upgrade programme.

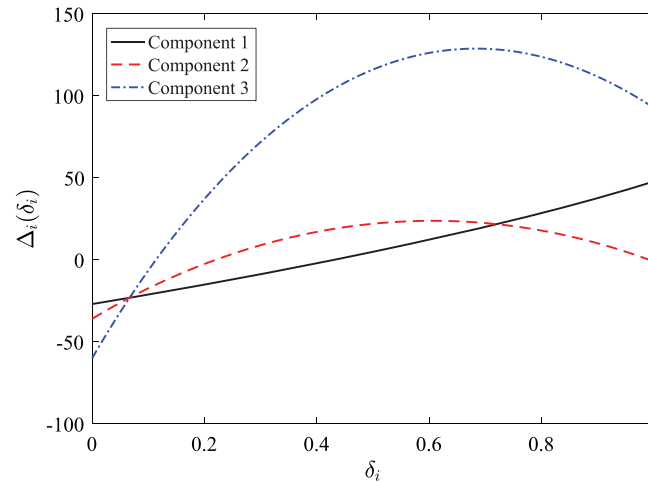


Figure 3. The marginal value $\Delta_i(\delta_i)$ versus upgrade degree δ_i , $i = 1, 2, 3$.

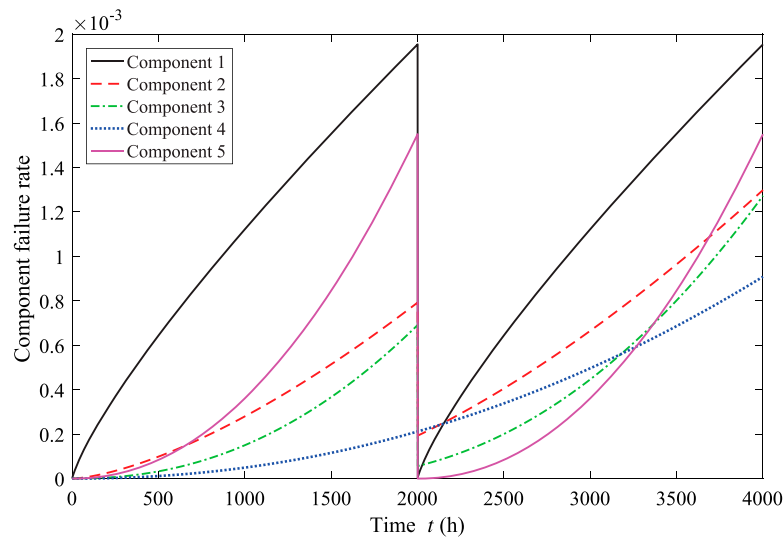


Figure 4. Evolution of component failure rates over time.

5.2. Sensitivity analyses

Now, we are ready to explore some managerial insights about the upgrade strategies through sensitivity analyses. The sensitivity analyses are performed by varying one or two parameters at a time while keeping the others unchanged.

Table 4 summarises the optimal upgrade decisions and the corresponding total expected servicing costs for the proposed upgrade strategy (*Strategy 1*), the no upgrade strategy (*Strategy 2*), and the component-level perfect upgrade/replacement strategy (*Strategy 3*) under various combinations of initial age x and warranty period w . As can be seen, when initial age x and/or warranty period w increases, the following three quantities show upward trends, i.e. (1) the optimal number of components subject to upgrade, l^* , (2) the optimal upgrade degrees of the targeted components, δ^* , and (3) the dealer's total expected servicing cost. This is to be expected since larger x and/or w would result in higher probability of system failures within the warranty period, and thus increase the dealer's total expected servicing expenses.

Moreover, Table 4 shows that the total expected servicing costs for Strategy 1, which allows different types of upgrade actions for different components, are always lower than or identical to those for the other two strategies (see also Figure 5). This is not surprising since the other two strategies are special cases of Strategy 1. In other words, their feasible domains are subsets of that of Strategy 1, thus their optimal solutions cannot be better than that of Strategy 1. Also, as we can observe from Figure 6, Strategy 1 reduces to Strategy 2 when x and w are small (zone I); while it simplifies to Strategy 3 when x and w are large (zone III). This demonstrates the superiority and flexibility of the proposed upgrade strategy.

Table 3. The details of the marginal analysis ($x = 2000$ h and $w = 1500$ h). The overall net benefit of the upgrade programme is \$70.31, which is less than the set-up cost.

Index (i)	δ_i^*	Δ_i^*	d_i^*	$d_i^* \delta_i^*$	$d_i^* \Delta_i^*$	$d_i^* c_{mi}$	$E[C_{wi}(d_i^*, \delta_i^*; x, w)]$	Total cost	$E[C_{wi}(0, 0; x, w)]$
1	1.00	7.96	1	1.00	7.96	126.00	69.87	195.87	203.82
2	0.44	-4.83	0	0.00	0.00	0.00	139.27	139.27	139.27
3	0.51	34.27	1	0.51	34.27	126.69	97.87	224.56	258.83
4	-	-88.83	0	0.00	0.00	0.00	152.66	152.66	152.66
5	-	28.08	1	1.00	28.08	182.00	89.20	271.20	299.28
				70.31		434.69	548.87	1083.56 ^a	1053.86

^aThis cost includes the set-up cost of the upgrade programme.

Furthermore, the effects of upgrade cost structures on the optimal upgrade decisions and the total expected servicing costs under the three upgrade strategies are shown in Table 5. As we can observe, the optimal number of components subject to upgrade, l^* , and the optimal upgrade degrees of the selected components, δ^* , decrease as the cost of perfect upgrade/replacement c_{vi} increase, as to be expected. Besides, the dealer's total expected servicing cost increases with c_{vi} and/or c_{si} , however, c_{vi} has more significant effect on the total servicing cost than c_{si} . This is because c_{vi} can affect the upgrade costs for both repairable and non-repairable components, while c_{si} only impacts the upgrade costs for repairable components.

What's more, Table 5 also shows that Strategy 1 always leads to the lowest total expected servicing cost among the three upgrade strategies (see Figure 7), which is consistent with the observation from Table 4 and Figure 5. Likewise, Strategy 1 reduces to Strategy 2 [Strategy 3] when c_{vi} is large [c_{vi} is small]. This demonstrates the robustness of the two findings.

6. Extensions to the model

The assumptions in Section 3.1 are certainly not applicable to all types of second-hand multi-component systems in practice. In this section, we discuss several extensions/modifications that might be important for the application of the upgrade model, by relaxing some of the original assumptions.

Table 4. Optimal upgrade decisions under various combinations of x and w .

x (h)	w (h)	Strategy 1				Strategy 2 C_{T0}	Strategy 3		
		d^*	l^*	δ^*	$C_T(d^*, \delta^*)$		d^*	l^*	$C_T(d^*, \delta^*)$
1000	1000	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	359.80	359.80	(0, 0, 0, 0, 0)	0	359.80
	1500	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	625.17	625.17	(0, 0, 0, 0, 0)	0	625.17
	2000	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	938.43	938.43	(0, 0, 0, 0, 0)	0	938.43
	2500	(1, 1, 1, 0, 0)	3	(0.61, 0.65, 0.68, 0, 0)	1313.02	1318.66	(0, 0, 0, 0, 0)	0	1318.66
	3000	(1, 1, 1, 0, 0)	3	(0.67, 0.68, 0.72, 0, 0)	1678.61	1775.36	(0, 0, 0, 0, 0)	0	1775.36
1500	1000	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	523.87	523.87	(0, 0, 0, 0, 0)	0	523.87
	1500	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	836.36	836.36	(0, 0, 0, 0, 0)	0	836.36
	2000	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	1211.57	1211.57	(0, 0, 0, 0, 0)	0	1211.57
	2500	(1, 1, 1, 0, 0)	3	(1.00, 0.62, 0.67, 0, 0)	1547.37	1668.35	(1, 0, 1, 0, 0)	2	1661.33
	3000	(1, 1, 1, 0, 0)	3	(1, 0.69, 0.76, 0, 0)	1920.67	2202.07	(1, 1, 1, 0, 0)	3	2001.91
2000	1000	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	685.98	685.98	(0, 0, 0, 0, 0)	0	685.98
	1500	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	1053.87	1053.87	(0, 0, 0, 0, 0)	0	1053.87
	2000	(1, 1, 1, 0, 1)	4	(1, 0.61, 0.68, 0, 1)	1395.53	1503.71	(1, 0, 1, 0, 1)	3	1455.37
	2500	(1, 1, 1, 0, 1)	4	(1, 0.76, 0.85, 0, 1)	1720.04	2040.32	(1, 1, 1, 0, 1)	4	1738.60
	3000	(1, 1, 1, 0, 1)	4	(1, 0.91, 1, 0, 1)	2052.13	2653.68	(1, 1, 1, 0, 1)	4	2053.53
2500	1000	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	848.66	848.66	(0, 0, 0, 0, 0)	0	848.66
	1500	(1, 0, 1, 0, 1)	3	(1, 0, 0.73, 0, 1)	1221.23	1285.43	(1, 0, 1, 0, 1)	3	1238.42
	2000	(1, 1, 1, 0, 1)	4	(1, 0.94, 0.95, 0, 1)	1493.59	1816.12	(1, 1, 1, 0, 1)	4	1494.65
	2500	(1, 1, 1, 0, 1)	4	(1, 1, 1, 0, 1)	1770.92	2435.35	(1, 1, 1, 0, 1)	4	1770.92
	3000	(1, 1, 1, 0, 1)	4	(1, 1, 1, 0, 1)	2079.84	3135.5	(1, 1, 1, 0, 1)	4	2079.84
3000	1000	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	1018.27	1018.27	(0, 0, 0, 0, 0)	0	1018.27
	1500	(1, 1, 1, 0, 1)	4	(1, 1, 0.97, 0, 1)	1285.40	1534.00	(1, 1, 1, 0, 1)	4	1285.66
	2000	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	1509.56	2149.44	(1, 1, 1, 1, 1)	5	1509.56
	2500	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	1773.64	2857.14	(1, 1, 1, 1, 1)	5	1773.64
	3000	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	2088.49	3654.29	(1, 1, 1, 1, 1)	5	2088.49

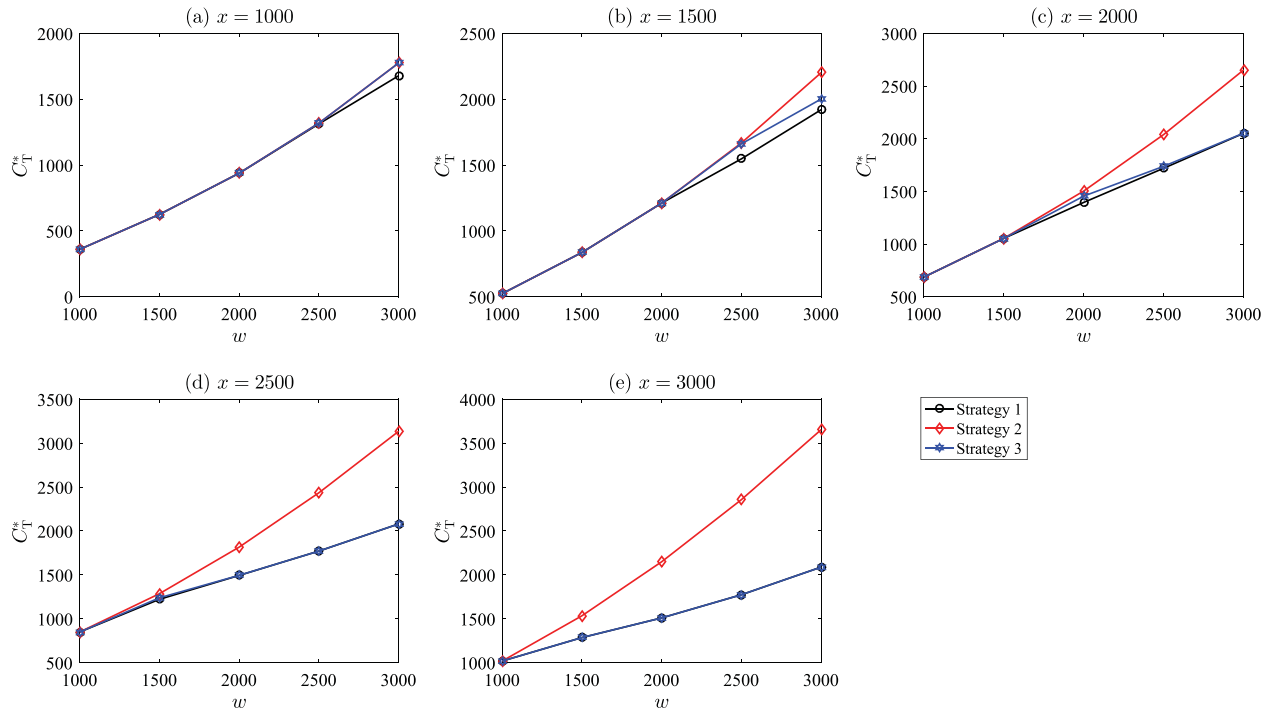


Figure 5. Minimal total expected servicing costs under various combinations of x and w .

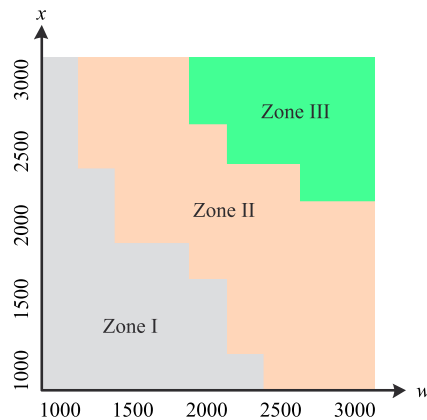


Figure 6. Illustration of the three zones of optimal upgrade strategies (with a step of 250 h). Zones I, II, and III correspond to the no upgrade strategy, the proposed upgrade strategy, and the component-level perfect upgrade/replacement strategy, respectively.

6.1. Component-specific initial ages

This study assumed, according to the assumption (3), that the (virtual) ages of all components at age x are identical. In practice, however, a specific component i may be imperfectly or perfectly maintained one or more times during its initial life. In this case, its virtual age at actual age x would be smaller than x , i.e. $x_i < x$. Thus, if the virtual ages are component-specific, the term x in the proposed model should be replaced by x_i ($x_i \leq x$), $i = 1, 2, \dots, n$. Nevertheless, this requires the dealer to have sufficient information about the maintenance history of each component in the used system, which is not easy in reality.

6.2. Component-specific warranty periods

In practice, it is not uncommon that dealers provide different types or lengths of warranty contracts for different components in a second-hand system. For example, the warranty period for the powertrain of a second-hand vehicle is often much longer than that of the other components; also, most of the vehicle components are covered by a free repair/replacement warranty, while the tires are usually protected by a pro-rata warranty (Wang and Xie 2018).

Table 5. Optimal upgrade decisions under various upgrade cost structures.

ρ_1	ρ_2	Strategy 1				Strategy 2 C_{T0}	Strategy 3		
		\mathbf{d}^*	l^*	δ^*	$C_T(\mathbf{d}^*, \delta^*)$		\mathbf{d}^*	l^*	$C_T(\mathbf{d}^*, \delta^*)$
0.1	0.1	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	669.56	1503.71	(1, 1, 1, 1, 1)	5	669.56
	0.4	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	669.56	1503.71	(1, 1, 1, 1, 1)	5	669.56
	0.7	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	669.56	1503.71	(1, 1, 1, 1, 1)	5	669.56
	1.0	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	669.56	1503.71	(1, 1, 1, 1, 1)	5	669.56
0.4	0.1	(1, 1, 1, 1, 1)	5	(1, 0.82, 0.84, 1, 1)	1077.19	1503.71	(1, 1, 1, 1, 1)	5	1089.56
	0.4	(1, 1, 1, 1, 1)	5	(1, 0.96, 0.94, 1, 1)	1088.37	1503.71	(1, 1, 1, 1, 1)	5	1089.56
	0.7	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	1089.56	1503.71	(1, 1, 1, 1, 1)	5	1089.56
	1.0	(1, 1, 1, 1, 1)	5	(1, 1, 1, 1, 1)	1089.56	1503.71	(1, 1, 1, 1, 1)	5	1089.56
0.7	0.1	(1, 1, 0, 0, 1)	3	(1, 0.51, 0.61, 0, 1)	1350.54	1503.71	(1, 0, 1, 0, 1)	3	1455.37
	0.4	(1, 1, 1, 0, 1)	4	(1, 0.58, 0.66, 0, 1)	1385.02	1503.71	(1, 0, 1, 0, 1)	3	1455.37
	0.7	(1, 1, 1, 0, 1)	4	(1, 0.67, 0.73, 0, 1)	1414.73	1503.71	(1, 0, 1, 0, 1)	3	1455.37
	1.0	(1, 1, 1, 0, 1)	4	(1, 0.77, 0.81, 0, 1)	1438.09	1503.71	(1, 0, 1, 0, 1)	3	1455.37
1.0	0.1	(0, 1, 1, 0, 0)	2	(0, 0.36, 0.47, 0, 0)	1460.81	1503.71	(0, 0, 0, 0, 0)	0	1503.71
	0.4	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	1503.71	1503.71	(0, 0, 0, 0, 0)	0	1503.71
	0.7	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	1503.71	1503.71	(0, 0, 0, 0, 0)	0	1503.71
	1.0	(0, 0, 0, 0, 0)	0	(0, 0, 0, 0, 0)	1503.71	1503.71	(0, 0, 0, 0, 0)	0	1503.71

Note: $\rho_1 = c_{vi}/c_{ri}$, $\rho_2 = c_{si}/c_{fi}$.

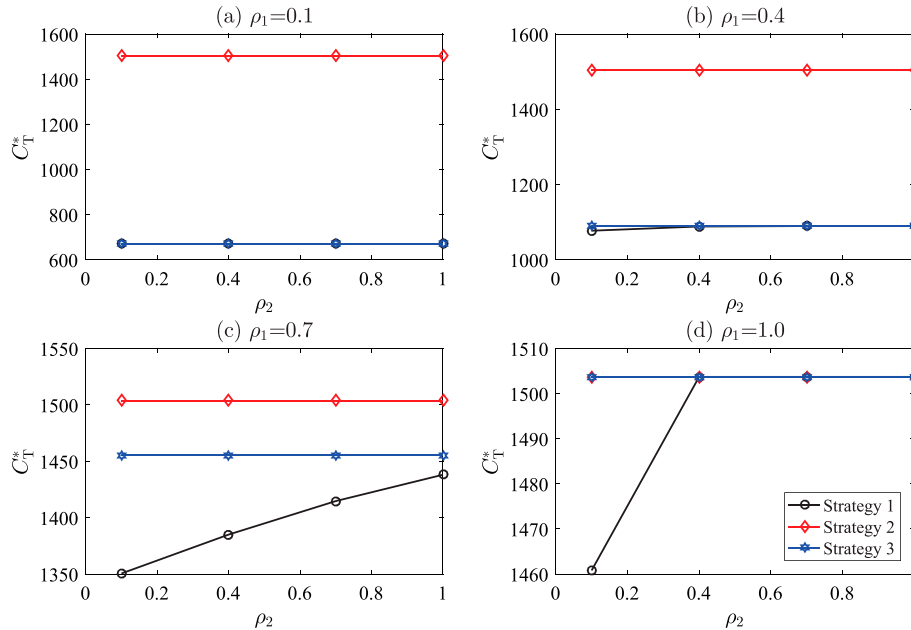


Figure 7. Minimal total expected servicing costs under various upgrade cost structures.

Nevertheless, it is quite easy to incorporate different warranty contracts for different components into our original model. For instance, by considering the diversity of warranty lengths, the warranty period w in our model should be replaced by w_i for component i , $i = 1, 2, \dots, n$. With consideration of component-specific initial ages and warranty periods, the expected warranty cost (11) should be revised as

$$\begin{aligned}
 E[C_w(\mathbf{d}, \delta | \mathbf{x}, \mathbf{w})] &= \sum_{i=1}^k c_{fi} E[N_i(d_i, \delta_i | x_i, w_i)] + \sum_{i=k+1}^n c_{ri} E[N_i(d_i, \delta_i | x_i, w_i)] \\
 &= \sum_{i=1}^k c_{fi} \int_0^{w_i} \lambda_i((1 - d_i \delta_i)x_i + t) dt + \sum_{i=k+1}^n c_{ri} \left[F_i(w_i | x_i, d_i) + \int_0^{w_i} F_i(w_i - t | x_i, d_i) dM_i(t) \right],
 \end{aligned} \tag{21}$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)$ and $\mathbf{w} = (w_1, w_2, \dots, w_n)$.

6.3. Complex system configurations

The upgrade model in Section 3 only considered second-hand systems with a series configuration. In reality, the structures of many systems can be much more complex, say, series-parallel, parallel-series, or k -out-of- n configurations (Park and Pham 2010; Xie, Liao, and Jin 2014). The proposed upgrade modelling framework can obviously be extended to these scenarios. In this subsection, we briefly introduce the upgrade modelling procedure for a series-parallel system.

We consider a series-parallel system containing n subsystems connected in series, and each subsystem has m_i , $i = 1, 2, \dots, n$, independent, and possibly, non-identical components connected in parallel; see Figure 8. Let $F_{ij}(t)$, $R_{ij}(t)$, and $\lambda_{ij}(t)$ represent the failure distribution function, the reliability function, and the failure rate function of the j th component in the i th subsystem, respectively, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m_i$. Without loss of generality, we suppose that subsystems $1, 2, \dots, k$ are repairable, and subsystems $k+1, k+2, \dots, n$ are non-repairable. For each subsystem, no matter it is repairable or non-repairable, it breaks down only when all of its components have failed. Once a subsystem fails, the whole system has to be stopped for necessary maintenance.

Here, the assumptions (2)–(7) in Section 3.1 are also adopted to characterise the subsystems. For repairable subsystem i , $i = 1, \dots, k$, the conditional failure rate of its component j , after the imperfect upgrade process at age x , becomes $\lambda_{ij}((1 - d_{ij}\delta_{ij})x + t)$, where d_{ij} and δ_{ij} are the decision variables as defined before. Considering the parallel structure of repairable subsystem i , its conditional reliability, $R_{si}(t|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i)$, after upgrade is

$$\begin{aligned} R_{si}(t|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i) &= 1 - \prod_{j=1}^{m_i} [1 - R_{ij}(t|x, d_{ij}, \delta_{ij})] \\ &= 1 - \prod_{j=1}^{m_i} \left[1 - \exp \left\{ - \int_0^t \lambda_{ij}((1 - d_{ij}\delta_{ij})x + v) dv \right\} \right], \quad i = 1, \dots, k, \end{aligned} \quad (22)$$

where $\tilde{\mathbf{d}}_i = (d_{i1}, d_{i2}, \dots, d_{im_i})$ and $\tilde{\delta}_i = (\delta_{i1}, \delta_{i2}, \dots, \delta_{im_i})$.

Define the conditional failure rate of subsystem i , after upgrade, as $\lambda_{si}(t|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i) = -dR_{si}(t|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i)/[R_{si}(t|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i)dt]$. With the assumption of minimal repairs upon subsystem failures, the failures of a repairable subsystem will occur according to an NHPP. In this case, the expected number of failures of repairable subsystem i during the warranty period is given by

$$\begin{aligned} E[N_{si}(\tilde{\mathbf{d}}_i, \tilde{\delta}_i; x, w)] &= \int_0^w \lambda_{si}(t|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i) dt \\ &= \ln R_{si}(0|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i) - \ln R_{si}(w|x, \tilde{\mathbf{d}}_i, \tilde{\delta}_i), \quad i = 1, \dots, k. \end{aligned} \quad (23)$$

On the other hand, the conditional failure distribution of the j th component in non-repairable subsystem i , $i = k+1, \dots, n$, after the upgrade process at age x , becomes $F_{ij}(t|x, d_{ij})$. Analogously, $F_{ij}(t|x, d_{ij})$ equals to $F_{ij}(t|x)$ if $d_{ij} = 0$, $F_{ij}(t)$ if $d_{ij} = 1$. Then, the conditional failure distribution of non-repairable subsystem i after upgrade can be expressed as $F_{si}(t|x, \tilde{\mathbf{d}}_i) = \prod_{j=1}^{m_i} F_{ij}(t|x, d_{ij})$, $i = k+1, \dots, n$.

With the assumption of replacements upon subsystem failures, the failures of a non-repairable subsystem will occur according to a modified renewal process. In this case, the expected number of failures of non-repairable subsystem i during the warranty period is

$$E[N_{si}(\tilde{\mathbf{d}}_i, \tilde{\delta}_i|x, w)] = F_{si}(w|x, \tilde{\mathbf{d}}_i) + \int_0^w F_{si}(w-t|x, \tilde{\mathbf{d}}_i) dM_{si}(t), \quad i = k+1, \dots, n, \quad (24)$$

where $M_{si}(t)$ is the renewal function associated with $\prod_{j=1}^{m_i} F_{ij}(t)$, and $\tilde{\delta}_i = \tilde{\mathbf{d}}_i$.

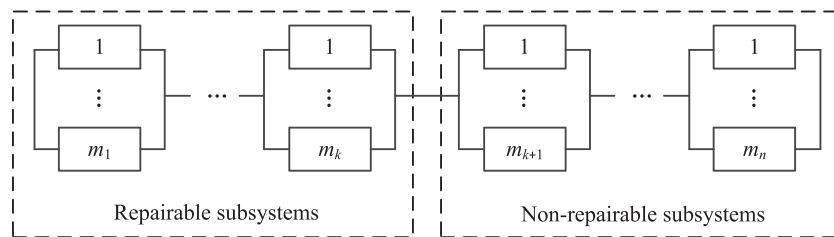


Figure 8. The structure of a general series-parallel system.

Therefore, the total expected servicing cost to the dealer, including the upgrade cost and the expected warranty cost, can be derived as

$$\begin{aligned}
 C_T(\tilde{\mathbf{d}}, \tilde{\mathbf{\delta}}|x, w) &= C_u(\tilde{\mathbf{d}}, \tilde{\mathbf{\delta}}|x) + \sum_{i=1}^k \tilde{c}_{fi} E[N_{si}(\tilde{\mathbf{d}}_i, \tilde{\mathbf{\delta}}_i; x, w)] + \sum_{i=k+1}^n \tilde{c}_{ri} E[N_{si}(\tilde{\mathbf{d}}_i, \tilde{\mathbf{\delta}}_i; x, w)] \\
 &= c_a I_{\{\tilde{l} \geq 1\}} + \sum_{i=1}^k \sum_{j=1}^{m_i} d_{ij} (\tilde{c}_{sij} + (\tilde{c}_{vij} - \tilde{c}_{sij}) \delta_{ij}^{\varphi_{\xi_{ij}(x)}}) + \sum_{i=k+1}^n \sum_{j=1}^{m_i} d_{ij} \tilde{c}_{vij} \\
 &\quad + \sum_{i=1}^k \tilde{c}_{fi} [\ln R_{si}(0|x, \tilde{\mathbf{d}}_i, \tilde{\mathbf{\delta}}_i) - \ln R_{si}(w|x, \tilde{\mathbf{d}}_i, \tilde{\mathbf{\delta}}_i)] \\
 &\quad + \sum_{i=k+1}^n \tilde{c}_{ri} \left[F_{si}(w|x, \tilde{\mathbf{d}}_i) + \int_0^w F_{si}(w-t|x, \tilde{\mathbf{d}}_i) dM_{si}(t) \right],
 \end{aligned} \tag{25}$$

where $\tilde{l} = \sum_{i=1}^n \sum_{j=1}^{m_i} d_{ij}$, $\tilde{\mathbf{d}} = (\tilde{\mathbf{d}}_1, \tilde{\mathbf{d}}_2, \dots, \tilde{\mathbf{d}}_n)$, $\tilde{\mathbf{\delta}} = (\tilde{\mathbf{\delta}}_1, \tilde{\mathbf{\delta}}_2, \dots, \tilde{\mathbf{\delta}}_n)$, and \tilde{c}_{fi} and \tilde{c}_{ri} are the average minimal repair cost and the replacement cost of subsystem i , respectively.

The optimisation problem for the second-hand system with a series-parallel structure can be developed in a similar way to (13). However, the marginal analysis based algorithm may be no longer effective to solve this complex problem, and heuristic algorithms may be needed.

6.4. Failure interactions among components

The proposed upgrade model assumed that all component failures in the system are statistically independent. However, for many real-life multi-component systems, the failure of a component may have interacting effects on one or more of the other components. This phenomenon is referred to as *failure interaction* (Liu, Wu, and Xie 2015; Olde Keizer, Flapper, and Teunter 2017). In the literature, Murthy and Nguyen (1985) proposed three types of failure interactions that have been investigated by many subsequent studies. For example, Type I failure interaction assumes that a failure of component i (called *nature failure*) can induce a simultaneous failure of component j (termed *induced failure*) with probability p , and has no effect on component j with probability $1 - p$. In this case, upgrading component i will improve its reliability, which further reduces the probability of induced failures of component j . Nevertheless, failure interactions among components could complicate the warranty cost modelling and the optimisation of upgrade strategies. Further investigations need to be conducted in this aspect to make our model more useful for real-world applications.

7. Concluding remarks

For second-hand series systems sold with a free repair/replacement warranty, this paper develops a new upgrade model to assist the dealers in making the optimal upgrade decisions. In this model, the complex second-hand system contains two types of components, i.e. repairable and non-repairable, which require different kinds of upgrade actions and corrective maintenance actions. The main objective of the dealer is to determine (i) which components to upgrade, and (ii) to what degrees the selected components should be upgraded, in order to minimise the total expected servicing cost. Numerical results show that the proposed upgrade strategy contains the no upgrade strategy and the component-level perfect upgrade/replacement strategy as its special cases, and it is always superior to, or at least identical to, these two strategies in terms of warranty cost reduction.

This work represents the early attempt to investigate the optimal upgrade strategy for second-hand multi-component systems sold with warranty. We have discussed how the proposed model may be extended in several different ways to capture different aspects of the real-world applications. Our future research is to investigate the optimal upgrade strategies for second-hand multi-component systems with complex configurations and failure interactions among components. Moreover, some capital-intensive products, such as automobiles and heavy machinery, are often sold with a two-dimensional warranty contract considering both age and usage (Ye and Murthy 2016; Wang and Xie 2018). The optimal upgrade strategies for second-hand complex systems sold with a two-dimensional warranty is also an open question.

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