# Rental Threshold Calculator — Development Specification (v1.0)

## 1) Overview

**Goal:** Build a small calculator (CLI -compatible) that recommends **accept/reject** decisions for rental offers based on inventory, time left, costs, and price distribution. It must support **sub-cost acceptance** when needed to avoid end-of-horizon leftover risk (≤3 preferred; >5 = failure).

**Core idea:** Single-resource revenue management with a **threshold (bid-price) policy**. Baseline uses a **static cutoff**calibrated to expected arrivals; optional module provides a **dynamic (time/inventory) bid-price**. Includes a **pacing rule** to relax thresholds if behind target.

## 2) Scope

* **In scope (MVP):**  
  Static threshold selection from empirical prices; accept/reject decision; expected accepts/leftover; pacing checkpoint; penalty-aware recommendation; simple tables/exports.
* **Out of scope (MVP):** Multi-asset allocation, cross-price elasticity, strategic customer behavior, competitor reactions.

## 3) Users & Primary Use Cases

* **Ops manager:** fast go/no-go on each offer.
* **Planner/analyst:** choose horizon-wide cutoffs to hit sell-through goals (≤3 leftover; avoid >5).
* **Finance:** sensitivity on costs/penalties vs. cashflow.

## 4) Key Definitions & Symbols

* X: starting inventory
* T: number of periods (days/rounds)
* c: unit cost/base rent
* s: salvage per unit at T (default 0)
* N: expected number of offers over horizon (arrival rate  ×T)
* F(p): (empirical) CDF of offer prices
* **Static cutoff τ:** accept iff p≥max{c\_floor,τ} where τ solves N⋅(1−F(τ))≈X−L\\*.
* **Dynamic bid price:** accept iff p≥c+bt(x), where bt(x)=Vt+1(x)−Vt+1(x−1).
* **Leftover target L\\*** default 3 (soft), failure if leftover >5.
* **Terminal penalty:** convex penalty Φ(L) that grows sharply after 3 and strongly after 5

## 5) Functional Requirements

### F1. Inputs

* Required: X,T,c, (default 0), arrival rate (per period or total N), **empirical price list** (values & equal weights, or weighted).
* Risk controls:
  + Target leftover L\\*=3
  + Failure threshold Lfail=5
  + Optional holding cost per period h **or** explicit terminal penalty parameters for Φ(L).
* Offer stream mode:
  + **Batch planning:** compute thresholds before the horizon.
  + **Live decision:** enter a single offer p + current (t,x) → accept/reject.

### F2. Outputs

* Static cutoff τ and **operational cutoff**  τ\_use​=max{c\_floor​,τ} (when sub-cost allowed, set c\_floor​=−∞ or a user floor).
* Expected accepts E[A]=N⋅(1−F(τ\_use​)); expected leftover E[L]=max{X−E[A],0}.
* Conditional mean accepted price E[P∣P≥τ\_use].
* Expected margin and **penalty-adjusted** expected profit.
* **Decision** for live offer p: Accept or Reject; show margin p−c and rationale.
* **Pacing status** & recommendation (hold cutoff vs relax).

### F3. Pacing Rule (MVP)

* Mid-horizon checkpoint t=T/2. Let A\_t​ be accepted units to date.
  + If A\_T/2≥X−L\\* *×0.5* → maintain cutoff.
  + Else → relax cutoff by one step (e.g., from 65 → 62 in the provided example).

### F4. Penalty Handling

* Default **piecewise** terminal penalty:

Φ(L)=0, if 0≤L≤3

Φ(L)=α(L−3)^2, if 3<L≤5

Φ(L)=α(2)^2+β(L−5)^2, if L>5 (β≫α)

* Choose α,β so that β heavily discourages L>5.

### F5. Optional: Dynamic Program (V2)

* Backward induction over t=T−1…0, state x∈[0..X], price arrival & draw from empirical F.
* Terminal condition: V\_T(x)=s⋅x−Φ(x).

## 6) Algorithms (MVP)

### A1. Static Cutoff Selection

1. Compute N (arrivals).
2. For each candidate threshold qq in the **empirical price grid**, compute tail prob phat\_q=1−F(q)
3. Compute expected accepts E[A]\_q​=N⋅ phat\_q expected leftover E[L]\_q​=max{X−E[A]\_q​,0}.
4. Compute expected unit margin m\_q=E[P∣P≥q]−c.
5. Score: Score\_q=E[A]\_q⋅m\_q−E[Φ(L)] (use E[L]\_q​ for MVP).
6. Pick argmax q; set τ\_use=q (or enforce a user floor).
7. **Live decision:** accept iff p≥τ\_use ​.

### A2. Pacing Adjustment

* If behind target by mid-horizon, lower τuseτuse​ to next lower empirical level (e.g., 65→62).

## 7) UI/UX Requirements

### Web (simple, responsive)

* **Inputs panel:** X,T,c,; arrival rate/total N; empirical prices (textarea or CSV upload); L\\*,L\_fail​; penalty sliders.
* **Results cards:** cutoff, expected accepts/leftover, conditional mean price, expected (penalty-adjusted) profit.
* **Offer checker:** input one p → “Accept/Reject + why”.
* **Pacing widget:** shows on-track vs behind; one-click “relax cutoff”.
* **Tables/exports:** acceptance table by price; CSVor Excel export.