Problem 2: Logic gate structure and operation.

(a). @S-S: $\frac{d[B]}{dt} = V_1 - V_3 = 0$

From given data: $V_1 = \dfrac{V_{max,1}[A]}{(1+\frac{[I_1]}{k_{I1}})(k_{S1}+[A])}$; $V_3 = \dfrac{V_{max,3}[B]}{k_{S3}+[B]}$

$\Rightarrow \dfrac{V_{max,1}[A]}{(1+\frac{[I_1]}{k_{I1}})(k_{S1}+[A])} - \dfrac{V_{max,3}[B]}{k_{S3}+[B]} = 0$

Since $V_{max,1} = V_{max,2} = 5$ units; $V_{max,3} = V_{max,4} = 1$ units

$k_{S1} = k_{S2} = k_{S3} = k_{S4} = 5$ units; $k_{I1} = k_{I2} = 1$ units, $S_{tot} = 100$ units

$\Rightarrow \dfrac{5[A]}{(1+\frac{[I_1]}{1})(5+[A])} - \dfrac{[B]}{5+[B]} = 0$

$\Rightarrow \dfrac{5[A]}{5+[A]} - \dfrac{[B]}{5+[B]} = 0$

$\Rightarrow \dfrac{5[A]}{5+[A]} = \dfrac{[B]}{5+[B]}$ 　　　　(1)

@S-S: $\frac{d[C]}{dt} = V_2 - V_4$

From given data: $V_2 = \dfrac{V_{max,2}[A]}{(1+\frac{[I_2]}{k_{I2}})(k_{S2}+[A])}$, $V_4 = \dfrac{V_{max,4}[C]}{k_{S3}+[C]}$

$\Rightarrow \dfrac{V_{max,2}[A]}{(1+\frac{[I_2]}{k_{I2}})(k_{S2}+[A])} - \dfrac{V_{max,4}[C]}{k_{S3}+[C]} = 0$

$\Rightarrow \dfrac{5[A]}{(1+\frac{[I_2]}{1})(5+[A])} - \dfrac{[C]}{5+[C]} = 0$

$\Rightarrow \dfrac{5[A]}{(5+[A])} - \dfrac{[C]}{5+[C]} = 0$

$\Rightarrow \dfrac{5[A]}{(5+[A])} = \dfrac{[C]}{5+[C]}$ 　　　　(2)

Compare (1) & (2), we can tell $[C] = [B]$

Since $[A] + [B] + [C] = S_{tot} = 100$

$\Rightarrow [A] + 2[B] = 100$ 　　$\Rightarrow [A] = 100 - 2[B]$

$$\frac{5[A]}{5+[A]} = \frac{[B]}{5+[B]}$$

$$\Rightarrow \frac{5(100-2[B])}{5+(100-2[B])} = \frac{[B]}{5+[B]}$$

$$\Rightarrow 5(100-2[B]) \cdot (5+[B]) = (5+100-2[B]) \cdot [B]$$

$$(500-10[B])(5+[B]) = (105-2[B])[B]$$

$$2500 + 500[B] - 50[B] - 10[B]^2 - 105[B] + 2[B]^2 = 0$$

$$\underbrace{-8[B]^2}_{a} + \underbrace{345[B]}_{b} + \underbrace{2500}_{c} = 0$$

$$[B] = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-345 \pm \sqrt{119025 + 80000}}{-16} = \frac{-345 \pm 446.12}{-16} = 49.445 \text{ or } -6.32$$

for concentration, only consider positive value.

So $[B] = 49.445$

$[C] = [B] = 49.445$

$[A] = 100 - 2[B] = 1.11$