Problem 2: Logic gate structure and operation:

(a). @ S-S: $\frac{d[B]}{dt} = V_1 - V_3 = 0$

From given data: $V_1 = \frac{V_{max,1}[A]}{(1+\frac{[I_1]}{K_{I_1}})(K_{S_1}+[A])}$; $V_3 = \frac{V_{max,3}[B]}{K_{S_3}+[B]}$

$$\Rightarrow \frac{V_{max,1}[A]}{(1+\frac{[I_1]}{K_{I_1}})(K_{S_1}+[A])} - \frac{V_{max,3}[B]}{K_{S_3}+[B]} = 0$$

Since $V_{max_1} = V_{max_2} = 5$ units; $V_{max_3} = V_{max_4} = 1$ units

$K_{S_1} = K_{S_2} = K_{S_3} = K_{S_4} = 5$ units; $K_{I_1} = K_{I_2} = 1$ units, $S_{tot} = 100$ units

$$\Rightarrow \frac{5[A]}{(1+\frac{[I_1]}{1})(5+[A])} - \frac{[B]}{5+[B]} = 0$$

$$\Rightarrow \frac{5[A]}{5+[A]} - \frac{[B]}{5+[B]} = 0$$

$$\Rightarrow \frac{5[A]}{5+[A]} = \frac{[B]}{5+[B]} \qquad (1)$$

@ S-S: $\frac{d[C]}{dt} = V_2 - V_4$

From given data: $V_2 = \frac{V_{max,2}[A]}{(1+\frac{[I_2]}{K_{I_2}})(K_{S_2}+[A])}$, $V_4 = \frac{V_{max,4}[C]}{K_{S_3}+[C]}$

$$\Rightarrow \frac{V_{max,2}[A]}{(1+\frac{[I_2]}{K_{I_2}})(K_{S_2}+[A])} - \frac{V_{max,4}[C]}{K_{S_3}+[C]} = 0$$

$$\Rightarrow \frac{5[A]}{(1+\frac{[I_2]}{1})(5+[A])} - \frac{[C]}{5+[C]} = 0$$

$$\Rightarrow \frac{5[A]}{(5+[A])} - \frac{[C]}{5+[C]} = 0$$

$$\Rightarrow \frac{5[A]}{(5+[A])} = \frac{[C]}{5+[C]} \qquad (2)$$

Compare (1) & (2), we can tell $[C] = [B]$

Since $[A]+[B]+[C] = S_{tot} = 100$

$\Rightarrow [A] + 2[B] = 100 \quad \Rightarrow [A] = 100 - 2[B]$