1b. Drive the gain function K_x:

Given Data: $\dot{m}_i = r_{x,i}\,\bar{u}_i - (\mu + \theta_{m,i})\,m_i$

$\dot{m}_i$: derivate with respect to time

$r_{x,i}\,\bar{u}_i$: specific rate of transcription of gene i.

$\bar{u}_i$: promotor activity function $(0 \le \bar{u}_i \le 1)$

$m^* = k_x(g,\theta)\,\bar{u}(I,k)$ — pseudo steady state

K_x: gain function

g: lacZ gene abundance

I: inducer abundance

$\bar{u}$: promotor function

At pseudo steady state: $\dot{m}_i = 0$

then: $0 = r_{x,i}\,\bar{u}_i - (\mu + \theta_{m,i})\,m_i$

for lacZ mRNA $m_i = m^*$

then: $0 = r_{x,i}\,\bar{u}_i - (\mu + \theta_{m,i})\,m^* \Rightarrow r_{x,i}\,\bar{u}_i = (\mu + \theta_{m,i})\,m^*$

$\Rightarrow m^* = \dfrac{r_{x,i}\,\bar{u}_i}{\mu + \theta_{m,i}} = \dfrac{r_{x,i}}{\mu + \theta_{m,i}}\,\bar{u}_i \Rightarrow \boxed{K_x = \dfrac{r_{x,i}}{\mu + \theta_{m,i}}}$

(k_x = promotor fun.; dilution; degradation)

From lecture notes:

$$r_{x,i} = K_{E,i}\,R_{x,T}\left(\frac{g_i}{\tau_{x,i}\,K_{x,i} + (\tau_{x,i}+1)\,g_i}\right)$$

$$\therefore \boxed{K_x = \frac{K_{E,i}\,R_{x,T}\left(\dfrac{g_i}{\tau_{x,i}\,K_{x,i} + (\tau_{x,i}+1)\,g_i}\right)}{\mu + \theta_{m,i}}}$$

$\therefore K_x = f(g, \text{constants})$

$$\bar{u} = \frac{w_1 + w_2 f_I}{1 + w_1 + w_2 f_I}, \text{ where } f_I = \frac{I^n}{k_0^n + I^n}, \quad w = \exp\left(\frac{-g}{RT}\right)$$

$\therefore \bar{u} = f(I, \text{constants})$