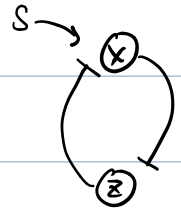
Problem 2



a). Because the system

the ans :

$$\boxed{\begin{aligned} \frac{d\breve{x}}{d\breve{t}} &= \frac{\breve{\alpha}_x + \breve{\beta}_x S}{1 + S + (\breve{z}/\breve{z}_x)^{n_{zx}}} - \breve{\delta}_x \breve{x} \\ \frac{d\breve{z}}{d\breve{t}} &= \frac{\breve{\alpha}_z}{1 + (\breve{x}/\breve{x}_z)^{n_{zx}}} - \breve{\delta}_z \breve{z} \end{aligned}}$$

b). $\boxed{\text{Error in paper : } t = \hat{t}\delta_x \text{ should be } t = \breve{t}\breve{\delta}_x}$

dimensionless : $\frac{dt}{\breve{\delta}_x} = d\breve{t}$, $\frac{\breve{\alpha}_z}{\breve{\delta}_x} dx = d\breve{x}$, $\frac{\breve{\alpha}_z}{\breve{\delta}_x} dz = d\breve{z}$

$\breve{\alpha}_x = \alpha_x \breve{\alpha}_z$, $\frac{\breve{\alpha}_z}{\breve{\delta}_x} x = \breve{x}$, $\frac{\breve{\alpha}_z}{\breve{\delta}_x} z = \breve{z}$

$\frac{\breve{\alpha}_z}{\breve{\delta}_x} z_x = \breve{z}_x$, $\frac{\breve{\alpha}_z}{\breve{\delta}_x} x_z = \breve{x}_z$, $\breve{\delta}_z = \breve{\delta}_x \delta_z$, $\hat{\beta}_x = \hat{\alpha}_z \beta_x$

$$\frac{d\breve{x}}{d\breve{t}} = \frac{\breve{\alpha}_x + \breve{\beta}_x S}{1 + S + (\breve{z}/\breve{z}_x)^{n_{zx}}} - \breve{\delta}_x \breve{x}$$

becomes: $$\frac{\breve{\alpha}_z}{\breve{\delta}_x} \breve{\delta}_x \frac{dx}{dt} = \frac{\alpha_x \breve{\alpha}_z + \breve{\alpha}_z \beta_x S}{1 + S + \left(\frac{\breve{\alpha}_z}{\breve{\delta}_x} z \Big/ \frac{\breve{\alpha}_z}{\breve{\delta}_x} z_x\right)^{n_{zx}}} - \breve{\delta}_x \frac{\breve{\alpha}_z}{\breve{\delta}_x} x$$

$$\Rightarrow \quad \breve{\alpha}_z \frac{dx}{dt} = \frac{\breve{\alpha}_z \alpha_x + \breve{\alpha}_z \beta_x S}{1 + S + (z/z_x)^{n_{zx}}} - \breve{\alpha}_z x$$

$$\Rightarrow \quad \boxed{\frac{dx}{dt} = \frac{\alpha_x + \beta_x S}{1 + S + (z/z_x)^{n_{zx}}} - x}$$

n & s are dimensionless.

$$\frac{d\breve{z}}{d\breve{t}} = \frac{\breve{\alpha}_z}{1 + (\breve{x}/\breve{x}_z)^{n_{zx}}} - \breve{\delta}_z \breve{z}$$

becomes: $$\frac{\breve{\alpha}_z}{\breve{\delta}_x} \breve{\delta}_x \frac{dz}{dt} = \frac{\breve{\alpha}_z}{1 + \left(\frac{\breve{\alpha}_z}{\breve{\delta}_x} x \Big/ \frac{\breve{\alpha}_z}{\breve{\delta}_x} x_z\right)^{n_{zx}}} - \breve{\delta}_x \delta_z \frac{\breve{\alpha}_z}{\breve{\delta}_x} z$$

$$\Rightarrow \quad \breve{\alpha}_z \frac{dz}{dt} = \frac{\breve{\alpha}_z}{1 + (x/x_z)^{n_{zx}}} - \delta_z \breve{\alpha}_z z$$

$$\Rightarrow \quad \boxed{\frac{dz}{dt} = \frac{1}{1 + (x/x_z)^{n_{zx}}} - \delta_z z}$$

c). Given Data: $\alpha_x = 1.5$, $\beta_x = 5.0$, $z_x = 0.4$, $n_{zx} = 2.7$, $x_z = 1.5$,

$n_{xz} = 2.7$, $\delta_z = 1.0$

At S.S. $\frac{dx}{dt} = \frac{dz}{dt} = 0$

So: $\frac{\alpha_x + \beta_x S}{1 + S + (z/z_x)^{n_{zx}}} = x$; $\frac{1}{1 + (x/x_z)^{n_{zx}}} = \delta_z z$

$\Rightarrow \frac{1.5 + 5S}{1 + S + (z/0.4)^{2.7}} = x$; $\frac{1}{1 + (x/1.5)^{2.7}} = z$

The input signal is S and output expression is x

To make process easier, we can get a relationship in the form of $S = f(x)$

$\Rightarrow 1.5 + 5S = x \cdot [1 + S + (z/0.4)^{2.7}]$

Let $(z/0.4)^{2.7} = \theta$

then: $1.5 + 5S = x[1 + S + \theta]$

$\Rightarrow 1.5 + 5S = x + xS + x\theta$

$\Rightarrow S = \frac{x + x\theta - 1.5}{5 - x}$

The relationship was plotted by Matlab code (attached)

$\boxed{\text{Yes, the figure can be reproduced}}$

d). Matlab code is attached

e). The Hopf bifurcation is @ $S = 5\times10^{-1} - 6\times10^{-1}$

So I choose the value below the range: $\boxed{4\times10^{-1}}$

The figures are plotted by Julia code (attached)

and figures are attached as well.

As shown in figure, my s.s. value are:

$$[X_{ss}, Y_{ss}, Z_{ss}] = [\underbrace{0.0012,\ 0.5787,\ 0.00035}_{\text{also attached with figures} \leftarrow \text{more correct \& details see Julia.}}]$$

According to the figures they look like try to get out of phase so I guess it's incoherant when below the Hopf point

The Saddle bifurcation point occurs @ $S = 10^4 - 1.7\times10^4$

So I choose the value above the range: $\boxed{3\times10^4}$

The figures are plotted by Julia code (attached)

and figures are attached as well.

As shown in figure, my s.s. value are:

$$[X_{ss}, Y_{ss}, Z_{ss}] = [\underbrace{0.648999,\ 0.2290179,\ 0.0025}_{\text{also attached with figures} \leftarrow \text{more correct \& details see Julia.}}]$$

According to the figures, they in phase, so I guess it's coherent, because this oscillator behavior is from a stable steady state at high signal. At here, the large limit cycle is presented. Expression of cells passing through the saddle node is far from the unstable spiral center near the Hopf birfication point. So it's in phase