Homework 5

In [20]:
```
import FPI_systems_script as fp
import numpy as np
import newton_method_nd_script as newton
import matplotlib.pyplot as plt
```

Problem 1

a)

In [23]:
```
x=1
y=1
f = lambda x,y: 3*x**2-y**2
g = lambda x,y: 3*x*y**2-x**3-1
tol = 1e-6
xn=np.array([0])
yn=np.array([0])
i = 0
nmax = 100
print(x-xn[0])
while i<nmax:
    i+=1
    xn= np.append(xn,[x-(1/6)*f(x,y)-(1/18)*g(x,y)])
    yn = np.append(yn,[ y-(1/6)*g(x,y)])
    x=xn[i]
    y=yn[i]
    print("|x,y=",x,",",y,"|n=",i,"|")
```

```
1
|x,y= 0.6111111111111112 , 0.8333333333333334 |n= 1 |
|x,y= 0.537627648224356 , 0.825845907636031 |n= 2 |
|x,y= 0.5098526471793883 , 0.8350754051244156 |n= 3 |
|x,y= 0.4997639081553767 , 0.8460583046842721 |n= 4 |
|x,y= 0.4970514812695153 , 0.8546596424061794 |n= 5 |
|x,y= 0.49712847475471206 , 0.8602593332311694 |n= 6 |
|x,y= 0.49796584037338293 , 0.8634534504994335 |n= 7 |
|x,y= 0.49877852266327083 , 0.8650705340475626 |n= 8 |
|x,y= 0.49935234523423094 , 0.8657885093270078 |n= 9 |
|x,y= 0.49969573668160794 , 0.8660529592641877 |n= 10 |
|x,y= 0.49987716481688294 , 0.8661171234315466 |n= 11 |
|x,y= 0.49996205493101037 , 0.8661081278813082 |n= 12 |
|x,y= 0.49999613062601456 , 0.8660817949315464 |n= 13 |
|x,y= 0.5000065274435789 , 0.8660583435897949 |n= 14 |
|x,y= 0.5000074742301264 , 0.8660424479279674 |n= 15 |
|x,y= 0.5000055743369228 , 0.8660331988708097 |n= 16 |
|x,y= 0.5000034477437609 , 0.8660284298701143 |n= 17 |
|x,y= 0.5000018733305437 , 0.8660262575922866 |n= 18 |
|x,y= 0.5000009037890777 , 0.8660254195493174 |n= 19 |
|x,y= 0.5000003788539021 , 0.8660251867758471 |n= 20 |
|x,y= 0.5000001265332682 , 0.8660251860299434 |n= 21 |
|x,y= 0.5000000212920462 , 0.8660252486871047 |n= 22 |
|x,y= 0.49999998648531685 , 0.8660253105232057 |n= 23 |
|x,y= 0.49999998090778286 , 0.8660253542851718 |n= 24 |
|x,y= 0.4999999849003055 , 0.866025380492036 |n= 25 |
|x,y= 0.4999999903464918 , 0.8660253943528655 |n= 26 |
|x,y= 0.49999999461637457 , 0.8660254008502335 |n= 27 |
|x,y= 0.49999999733330663 , 0.866025403466688 |n= 28 |
|x,y= 0.49999999884301444 , 0.8660254042709514 |n= 29 |
|x,y= 0.49999999958814473 , 0.8660254043495316 |n= 30 |
|x,y= 0.49999999990995775 , 0.866025404207803 |n= 31 |
|x,y= 0.5000000000235898 , 0.8660254040469915 |n= 32 |
|x,y= 0.5000000000477254 , 0.8660254039274053 |n= 33 |
|x,y= 0.5000000000405211 , 0.8660254038535676 |n= 34 |
|x,y= 0.5000000000268617 , 0.8660254038135036 |n= 35 |
|x,y= 0.5000000000153876 , 0.8660254037942027 |n= 36 |
|x,y= 0.5000000000078209 , 0.8660254037861278 |n= 37 |
|x,y= 0.5000000000035025 , 0.8660254037834412 |n= 38 |
|x,y= 0.5000000000013154 , 0.8660254037829974 |n= 39 |
|x,y= 0.50000000000034 , 0.8660254037832926 |n= 40 |
|x,y= 0.49999999999997624 , 0.8660254037837039 |n= 41 |
|x,y= 0.49999999999988404 , 0.8660254037840279 |n= 42 |
|x,y= 0.4999999999998924 , 0.8660254037842348 |n= 43 |
|x,y= 0.4999999999999258 , 0.86602540378435 |n= 44 |
|x,y= 0.4999999999999563 , 0.866025403784407 |n= 45 |
|x,y= 0.49999999999997724 , 0.8660254037844316 |n= 46 |
|x,y= 0.49999999999998945 , 0.8660254037844404 |n= 47 |
|x,y= 0.49999999999999584 , 0.8660254037844423 |n= 48 |
|x,y= 0.4999999999999988 , 0.8660254037844417 |n= 49 |
|x,y= 0.4999999999999999 , 0.8660254037844407 |n= 50 |
|x,y= 0.5000000000000003 , 0.8660254037844399 |n= 51 |
|x,y= 0.5000000000000003 , 0.8660254037844393 |n= 52 |
|x,y= 0.5000000000000002 , 0.8660254037844389 |n= 53 |
|x,y= 0.5000000000000002 , 0.8660254037844388 |n= 54 |
|x,y= 0.5000000000000002 , 0.8660254037844387 |n= 55 |
```

```
|x,y= 0.5000000000000001 , 0.8660254037844386 |n= 56 |
|x,y= 0.5 , 0.8660254037844386 |n= 57 |
|x,y= 0.5 , 0.8660254037844386 |n= 58 |
|x,y= 0.5 , 0.8660254037844386 |n= 59 |
|x,y= 0.5 , 0.8660254037844386 |n= 60 |
|x,y= 0.5 , 0.8660254037844386 |n= 61 |
|x,y= 0.5 , 0.8660254037844386 |n= 62 |
|x,y= 0.5 , 0.8660254037844386 |n= 63 |
|x,y= 0.5 , 0.8660254037844386 |n= 64 |
|x,y= 0.5 , 0.8660254037844386 |n= 65 |
|x,y= 0.5 , 0.8660254037844386 |n= 66 |
|x,y= 0.5 , 0.8660254037844386 |n= 67 |
|x,y= 0.5 , 0.8660254037844386 |n= 68 |
|x,y= 0.5 , 0.8660254037844386 |n= 69 |
|x,y= 0.5 , 0.8660254037844386 |n= 70 |
|x,y= 0.5 , 0.8660254037844386 |n= 71 |
|x,y= 0.5 , 0.8660254037844386 |n= 72 |
|x,y= 0.5 , 0.8660254037844386 |n= 73 |
|x,y= 0.5 , 0.8660254037844386 |n= 74 |
|x,y= 0.5 , 0.8660254037844386 |n= 75 |
|x,y= 0.5 , 0.8660254037844386 |n= 76 |
|x,y= 0.5 , 0.8660254037844386 |n= 77 |
|x,y= 0.5 , 0.8660254037844386 |n= 78 |
|x,y= 0.5 , 0.8660254037844386 |n= 79 |
|x,y= 0.5 , 0.8660254037844386 |n= 80 |
|x,y= 0.5 , 0.8660254037844386 |n= 81 |
|x,y= 0.5 , 0.8660254037844386 |n= 82 |
|x,y= 0.5 , 0.8660254037844386 |n= 83 |
|x,y= 0.5 , 0.8660254037844386 |n= 84 |
|x,y= 0.5 , 0.8660254037844386 |n= 85 |
|x,y= 0.5 , 0.8660254037844386 |n= 86 |
|x,y= 0.5 , 0.8660254037844386 |n= 87 |
|x,y= 0.5 , 0.8660254037844386 |n= 88 |
|x,y= 0.5 , 0.8660254037844386 |n= 89 |
|x,y= 0.5 , 0.8660254037844386 |n= 90 |
|x,y= 0.5 , 0.8660254037844386 |n= 91 |
|x,y= 0.5 , 0.8660254037844386 |n= 92 |
|x,y= 0.5 , 0.8660254037844386 |n= 93 |
|x,y= 0.5 , 0.8660254037844386 |n= 94 |
|x,y= 0.5 , 0.8660254037844386 |n= 95 |
|x,y= 0.5 , 0.8660254037844386 |n= 96 |
|x,y= 0.5 , 0.8660254037844386 |n= 97 |
|x,y= 0.5 , 0.8660254037844386 |n= 98 |
|x,y= 0.5 , 0.8660254037844386 |n= 99 |
|x,y= 0.5 , 0.8660254037844386 |n= 100 |
```

This looks like it converges linearly as we get 1 digit every 3 iterations roughly

b)

These entries represent the inverse of the jacobian of $F(X) = (f(x), g(x))$

c)

In [10]:
```
dfdx = lambda x:6*x
dfdy = lambda y:-2*y
dgdx = lambda x,y: 3*y**2-3*x**2
dgdy = lambda x,y: 6*x*y

x0 = np.array([1,1])
tol=1e-14
nmax=100

def F(x):
    return np.array([3*x[0]**2-x[1]**2,3*x[0]*x[1]**2-x[0]**3-1])

def JF(x):
        return np.array([[6*x[0],-2*x[1]],[3*x[1]**2-3*x[0]**2,6*x[0]*x[1]]])
```

In [11]:
```
r,rn,nf,nj = newton.newton_method_nd(F,JF,x0,tol,nmax)
```

In [16]:
```
print("r=",r)
print("iterations = ",nf)
```

```
r= [0.5       0.8660254]
iterations =  7
```

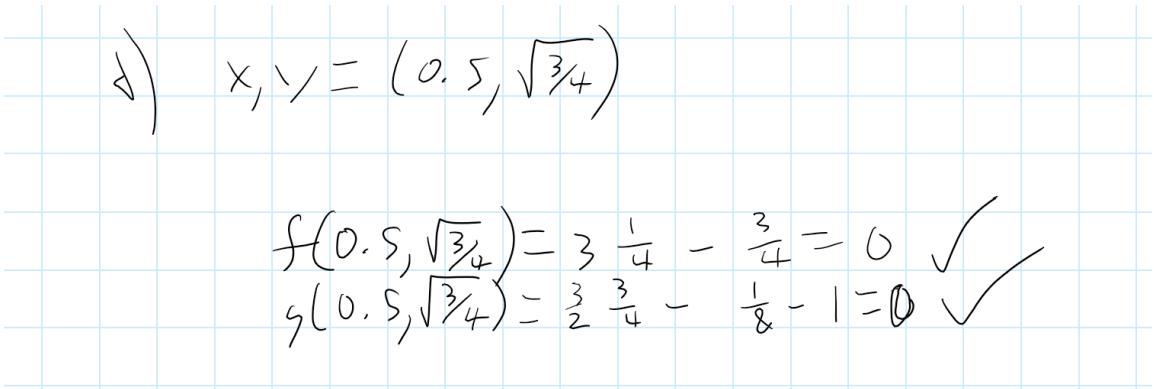
In [17]:
```
for a in rn:
    print("|",a,"|")
```

```
| [1. 1.] |
| [0.61111111 0.83333333] |
| [0.50365908 0.85249442] |
| [0.49996412 0.86604564] |
| [0.5       0.8660254] |
| [0.5       0.8660254] |
| [0.5       0.8660254] |
```

This converges much faster than the fixed point method.

d)

The correct numerical result is $x = 0.5, y = \sqrt{3/4}$



Problem 2

Here we need to satisfy $|g_i(X_0)| < 1$ for every i.

(2) want $\left|\frac{\delta g_i(\vec{x})}{\delta x_j}\right| \le \frac{K}{n}$ $n=2$, $K \le 1$ so:

$$\left|\frac{\delta g_i(\vec{x})}{\delta x_j}\right| \le \frac{1}{2}$$

$$\left|\frac{\delta g_0(\vec{x})}{\delta x_0}\right| = \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \cdot 2(x+y)\left(1+(x+y)^2\right)^{-1/2} \le 1/2$$

$\left|\frac{\delta g_0}{\delta x_0}\right|$: $\left|(x+y)\left(1+(x+y)^2\right)^{-1/2}\right| \le \frac{1}{\sqrt{2}}$

$\left|\frac{\delta g_0}{\delta x_1}\right|$: $\left|(x+y)\left(1+(x+y)^2\right)^{-1/2}\right| \le 1/\sqrt{2}$

$\left|\frac{\delta g_1}{\delta x_0}\right|$: $\left|(x-y)\left(1+(x-y)^2\right)^{-1/2}\right| \le \frac{1}{\sqrt{2}}$

$\left|\frac{\delta g_1}{\delta x_1}\right|$: $\left|-(x-y)\left(1+(x-y)^2\right)^{-1/2}\right| \le 1/\sqrt{2}$

$$\left|(x+y)\left(1+(x+y)^2\right)^{-1/2}\right| \le 1/\sqrt{2}$$
$$\left|(x-y)\left(1+(x-y)^2\right)^{-1/2}\right| \le 1/\sqrt{2}$$

$$|x+y| \le \sqrt{1+(x+y)^2}\,\frac{1}{\sqrt{2}}$$
$$|x-y| \le \sqrt{1+(x-y)^2}\,\frac{1}{\sqrt{2}}$$

$$(x+y)^2 \le \left(1+(x+y)^2\right)\frac{1}{2}$$
$$(x-y)^2 \le \left(1+(x-y)^2\right)\frac{1}{2}$$

$\frac{(x+y)^2}{2} \le 1/2 \rightarrow x^2+2xy+y^2 \le 1$ (1)

$\frac{(x-y)^2}{2} \le 1/2$ $\quad x^2-2xy+y^2 \le 1$ (2)

Triangle ineq.:

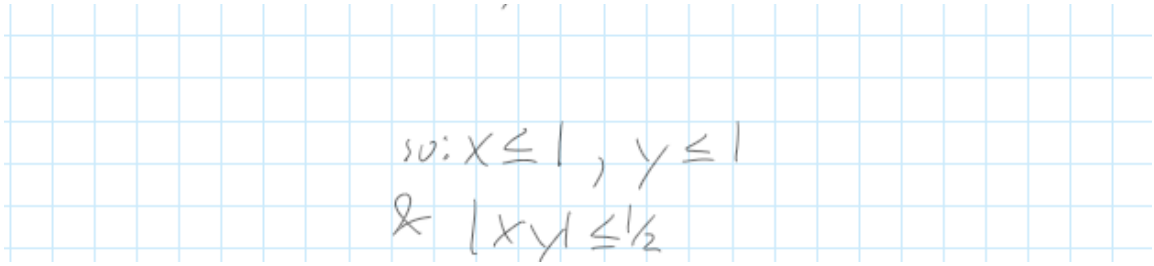
$$|(1)+(2)| \le |(1)|+|(2)| \le 2$$

drop "1" as x, y are squared

$$2x^2+2y^2 \le 2$$
$$x^2+y^2 \le 1$$

&: $|(1)-(2)| \le |(1)|+|(2)| \le 2$

$$|4xy| \le 2$$
$$|xy| \le 1/2$$



Problem 3

a)

(3)

Normal line: $l_n = \frac{x - x_n}{\frac{\delta f}{\delta x}(x_n, y_n)} - \frac{y - y_n}{\frac{\delta f}{\delta y}(x_n, y_n)} = 0$

a) & $f(x,y) = 0$

so define $F(\vec{x}) = \begin{bmatrix} l_n(\vec{x}) \\ f(\vec{x}) \end{bmatrix} = 0$

& apply 2D newton's to above.

so: $J_F(\vec{x}) = \begin{bmatrix} \frac{\delta l}{\delta x} & \frac{\delta l}{\delta y} \\ \frac{\delta f}{\delta x} & \frac{\delta f}{\delta y} \end{bmatrix} = \begin{bmatrix} \frac{1}{\frac{\delta f}{\delta x}(x_n, y_n)} & \frac{-1}{\frac{\delta f}{\delta y}(x_n, y_n)} \\ \frac{\delta f}{\delta x} & \frac{\delta f}{\delta y} \end{bmatrix}$

lecture 13 ⇒ so $\vec{x}_{k+1} = \vec{x}_k - J_F^{-1}(\vec{x}_k) F(\vec{x}_k)$ ⇐ replace k w/ n

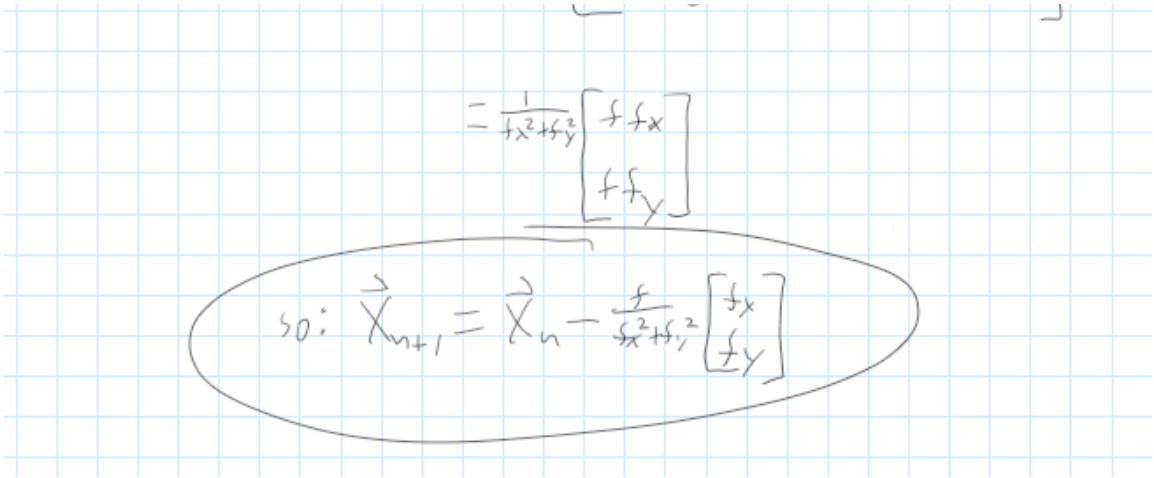
$$J_F^{-1}(\vec{x}_n) = \frac{1}{f_x^2 + f_y^2} \begin{bmatrix} f_x f_y^2 & f_x \\ -f_x^2 f_y & f_y \end{bmatrix}$$

$$\det(J_F) = \frac{\delta f}{\delta y}\frac{\delta f}{\delta x}^{-1} + \frac{\delta f}{\delta x}\frac{\delta f}{\delta y}^{-1} = f_y f_x / (f_y^2 + f_x^2)$$

$$J_F^{-1}(\vec{x}_n) F(\vec{x}_n) = \frac{1}{\det(J_F)} \begin{bmatrix} f_x f_y^2 (l_n) + f_x (f_n) \\ -f_x^2 f_y (l_n) + f_y (f_n) \end{bmatrix}$$

$$= \frac{1}{f_x^2 + f_y^2} \begin{bmatrix} \cancel{(x - x_n) f_y^2} \cancel{- f f_y (x - x_n)} + f f_x \\ \cancel{- f_x f_y (x - x_n)} + \cancel{(y - y_n) f_x^2} + f f_y \end{bmatrix}$$

$$= \frac{1}{f_x^2 + f_y^2} \begin{bmatrix} 0 + f f_x \\ 0 + f f_y \end{bmatrix}$$



b)

Here we need to plug our function into the newton's method function.

In [18]:
```
f = lambda x,y,z: x**2+4*y**2+4*z**2-16
fx = lambda x: 2*x
fy = lambda y: 8*y
fz = lambda z: 8*z
```

In [49]:
```
n=0
x0=y0=z0=1
xn3 = np.array([[x0,y0,z0]])
"""print(xn3)"""
while n<nmax:
    d = -f(x0,y0,z0)/(fx(x0)**2+fy(y0)**2+fz(z0)**2)
    x0=x0+d*fx(x0)
    y0=y0+d*fy(y0)
    z0=z0+d*fz(z0)
    xn3 = np.vstack((xn3,[[x0,y0,z0]]))
    print("|x|y|z|")
    print("|",x0,"|",y0,"|",z0,"|")
    n+=1
"""print(xn3[:,0])"""
t = range(n+1)
plt.plot(t,(1.093642317388195-xn3[:,0])/1.093642317388195 )
plt.xlim(0,10)
plt.xlabel("n")
plt.ylabel("Relative error of x")
```

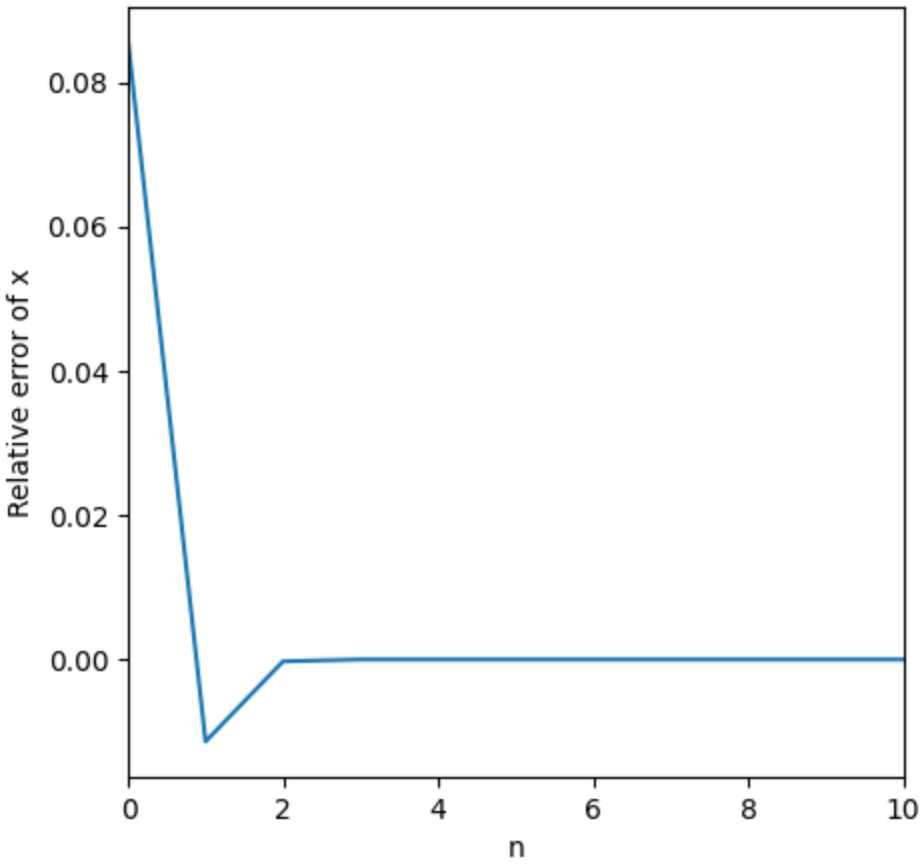
```
|x|y|z|
| 1.106060606060606 | 1.4242424242424243 | 1.4242424242424243 |
|x|y|z|
| 1.0939261558696716 | 1.3617416944918586 | 1.3617416944918586 |
|x|y|z|
| 1.0936424626088905 | 1.3603291057551175 | 1.3603291057551175 |
|x|y|z|
| 1.093642317388233 | 1.3603283832232336 | 1.3603283832232336 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230444 | 1.3603283832230444 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
```

```
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
```

```
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
```

```
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
|x|y|z|
| 1.093642317388195 | 1.3603283832230446 | 1.3603283832230446 |
```

Out[49]:
```
Text(0, 0.5, 'Relative error of x')
```



This plot is indicative of all 3 variables x, y, z, which all converge after ~4-5 iterations. We also can see from the table that 2-3 digits are "locked" after each step, and that the accuracy increases by ~10^{-2} each iteration.

In [ ]: