Implied Equity Duration:

A New Measure of Equity Security Risk

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Abstract

We derive a measure of implied equity duration as a natural extension of the traditional measure of bond duration and develop an algorithm for the empirical estimation of implied equity duration. We show that the standard empirical predictions and results for bond duration hold for our measure of implied equity duration and that implied equity duration represents an important common factor in stock returns. We also show that the book-to-market factor advocated by Fama and French (1993) acts as a noisy proxy for an underlying duration factor. Finally, we provide evidence that the long-run equity yield curve is downward sloping for durations up to 20 years. Our results suggest that existing empirical tests of asset pricing models using short holding period equity returns are misspecified.

1. Introduction

Techniques for analyzing the risk characteristics of fixed income securities have evolved within a theoretically rigorous framework based on the discounted expectations of the future cash flows of the securities. For example, constructs like duration and convexity are well established for fixed income securities and are embraced by academics and practitioners alike. The analysis of equity securities, in contrast, has evolved in a relatively ad hoc manner. Following disappointment with the performance of equilibrium pricing models such as the CAPM, academics and practitioners have adopted empirically motivated procedures for the analysis of equity risk. For example, following Fama and French (1993), a popular academic approach to modeling the risk characteristics of stock returns is through a three-factor model incorporating a market factor, a size factor and a book-to-market factor. Similarly, practitioners have embraced the notion of classifying stocks on the basis of market capitalization and the extent to which they exhibit 'style' characteristics of 'value' versus 'growth'. We bridge this gap in the analysis techniques for fixed income and equity securities by developing an implied equity duration metric that provides a rigorous theoretical and powerful empirical technique for the analysis of equity security risk.[1]

We begin by developing our estimates of implied equity duration based on Macaulay's traditional measure of bond duration. The primary obstacle in adapting the bond duration

[1] Cornell (1999) also recognizes the potential importance of duration as a measure of equity risk and provides preliminary evidence of a negative relation between dividend yields and betas.

formula to equities is in the estimation of the expected future cash distributions for equities. We develop a two-stage procedure to facilitate this task. First, using simple forecasting models based on historical financial data, we estimate the expected future cash distributions for a finite forecast horizon. Second, we assume that the remaining value implicit in the observed stock price will be distributed as a level perpetuity beyond our finite forecast horizon. We then apply the standard duration formula to compute our measure of implied equity duration. We recognize that our estimation procedure for implied equity duration represents a simple approximation based on relatively crude forecasting assumptions. Nevertheless, the resulting duration estimates have strong empirical predictive power, and our basic framework is easily adapted to incorporate more sophisticated forecasting models.

Empirical tests demonstrate the effectiveness of our measure of implied equity duration in explaining the risk characteristics of equity returns. Theory suggests that common shocks to expected returns will have a greater valuation impact on long duration equities. Consistent with the theoretical predictions, implied equity duration is strongly positively related with stock return volatility/beta. Implied equity duration also has incremental explanatory power in forecasting future volatilities/betas over past volatilities/betas. In addition, equity duration represents a strong common factor in stock returns with incremental explanatory power over the market, size and book-to-market factors advocated by Fama and French (1993). Moreover, we present evidence suggesting that the Fama-French book-to-market factor serves as a noisy proxy for our equity duration factor. Finally, we find that our equity duration factor is strongly positively correlated with aggregate equity market returns, but only weakly associated with long bond returns, suggesting that common shocks to the expected returns on equities are dominated by common shocks to the equity premium.

We also present descriptive evidence on the slope of the equity yield curve. Using average realized one-year ahead returns to proxy for expected returns, we demonstrate that the long-run equity yield curve is generally downward sloping. This result contrasts with the liquidity preference theory that is typically used to justify an upward sloping yield curve for bonds. It suggests that equity investors have a preference for locking in long-run equity returns, and so require a premium to compensate for the additional risks and transaction costs associated with 'rolling over' the cash flows from short duration equities. If this is the case, then existing empirical tests of asset pricing models using short holding period equity returns are seriously misspecified.[2] Finally, our results suggest that the well-documented 'book-to-market' effect in stock returns may reflect investors' preferences for holding long-duration equities.

The remainder of the paper is organized as follows. The next section develops our measure of implied equity duration and our empirical predictions. Section three describes our data and section four presents our results. Section five concludes.

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[2] Misspecification arises because the risk in equity returns associated with shocks to the expected return on equities is increasing in the deviation between the duration of the equity security and the intended holding period of the investor. Existing research has typically estimated risk using monthly equity returns, but if investors' intended holding periods are longer, then the use of monthly holding period returns will overstate the risk of long duration equities and understate the risk of short duration equities. We expand on this issue in section 4.4.

2. *Implied Equity Duration: Definition, Measurement and Hypotheses*

2.1 Definitions

The traditional measure of duration (*D*) for a bond is the Macaulay duration formula:

$$D = \frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{P} \tag{1}$$

where CF denotes the cash flow made at time *t*, *r* denotes the yield to maturity and *P* denotes the bond price. This measure of duration is a weighted average of the times to each of the respective cash flows on the bond, where the weights represent the relative contributions of the cash flows to the bond's value. Intuitively, duration represents the average maturity of the bond's promised cash flows.

The primary role of duration in the analysis of fixed income securities is as a measure of bond price sensitivity to changes in the yield to maturity. Differentiating the expression for the value of a bond with respect to the yield to maturity gives:

$$\frac{\partial P}{\partial r} = -P \times \frac{D}{1+r} \tag{2}$$

Intuitively, this result indicates that the relation between bond prices changes and changes in bond yields is a simple function of duration:[3]

$$\frac{\Delta P}{P} \approx -\frac{D}{1+r} \Delta r \tag{3}$$

[3] This relation is only approximate, because duration itself is a decreasing function of the yield to maturity (i.e., the convexity property).

The expression $\dfrac{D}{1+r}$ is often referred to as the 'modified duration', and provides a simple measure of the sensitivity of bond prices changes to yield changes.

Extending the duration concept to equities introduces two important new problems. The first problem is that while the flows to equity are ultimately determined by the firm's investment opportunity set, the flows to individual equity securities are also a function of the firm's financing policy. In much the same way that a bondholder can trade a bond on the open market to liquidate a bond position, management can engage in equity issuance, dividend and stock repurchase transactions that effectively liquidate the positions of certain equity holders. But just as the duration of a bond should not be affected by the trading intentions of the bondholder, the duration of an equity security should not be affected by the firm's financing policy. In other words, the relevant cash flows in computing the duration of equity are the firm-level cash flows that are determined by the firm's investment opportunity set, and so we perform our analysis at the firm-level rather than the individual security level. Thus, P denotes the market capitalization of equity (share price multiplied by shares outstanding), CF denotes the net cash distributions to all equity holders (dividends plus stock repurchases less equity issuances) and r denotes the cost of equity (expected return on equity).[4]

The second problem in extending the duration concept to equities is the substantial uncertainty regarding the amount and timing of the future net cash distributions. We address

[4] The ability of a firm to engage in market transactions to transfer expected free cash flow among stakeholders is clearly not limited to transactions between equity holders. Debt, preferred stock and warrants represent other potential claimants with whom a firm may engage in such transactions. We focus on equity securities, because they generally represent the most significant claim on the firm's free cash flows.

this problem by first partitioning the duration formula in equation (1) into two parts, a finite forecasting horizon of length T and an infinite terminal expression:

$$D = \frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}} \times \frac{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}}{P} + \frac{\sum_{t=T+1}^{\infty} t \times \frac{CF_T}{(1+r)^t}}{\sum_{t=T+1}^{\infty} \frac{CF_T}{(1+r)^t}} \times \frac{\sum_{t=T+1}^{\infty} \frac{CF_T}{(1+r)^t}}{P} \tag{4}$$

We next assume that the terminal cash flow stream consists of a level perpetuity that can be inferred by subtracting the value of the cash flows over the finite forecast period from the observed market capitalization implicit in the security price:

$$D = \frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}} \times \frac{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}}{P} + \frac{\sum_{t=T+1}^{\infty} t \times r \times (P - \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t})}{(P - \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t})} \times \frac{(P - \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t})}{P} \tag{5}$$

Intuitively, D = (cash flows from the finite period x relative weight) + (cash flows from the terminal period x relative weight). Recognizing that the duration of a level perpetuity is *(1+r)/r*, this expression simplifies to:

$$D = \frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{P} + (T + \frac{(1+r)}{r}) \times \frac{(P - \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t})}{P} \tag{6}$$

The assumption that the cash flow stream for an equity security can be partitioned into a finite forecasting period and an infinite terminal expression is standard in the equity valuation literature. The assumption that the terminal cash flows are realized as a level perpetuity is less standard. More commonly, the terminal cash flows are assumed to grow at a constant terminal rate such as the expected macroeconomic growth rate. We make the level perpetuity assumption for tractability and without loss of generality. As long as the forecasting horizon is long enough to exhaust plausible opportunities for firm-specific or industry-specific supernormal growth, the terminal growth rate will be a cross-sectional constant, and so will not be an important source of cross-sectional variation in implied equity duration.

The assumption that the terminal cash flow perpetuity can be inferred from the observed security price rests on the notion of market efficiency. For this reason, we refer to the resulting measurements of equity duration as 'implied' equity duration, because they are implied by observed security prices. In other words, our measure of equity duration is based on investors' consensus expectations, as reflected in stock prices, rather than on necessarily rational forecasts of future cash flows. We entertain violations of security market efficiency and recognize that overpriced (underpriced) securities will have relatively more (less) of their value represented in the terminal value expression, and hence will have longer (shorter) implied equity durations.

The discussion above explicitly deals with the estimation of the cash flows related to the terminal expression. Empirical implementation of the measure of duration also requires a procedure for estimating the cash flows over the finite forecast horizon. We discuss this estimation procedure next.

2.2 Empirical Implementation

The foremost remaining problem in implementing our measure of implied equity duration is in generating forecasts of cash flows over the finite forecast horizon. For the purposes of this paper, our goal is to develop a robust and parsimonious forecasting model based on widely available historical financial information and sound economic intuition. We recognize that our forecasts will generally be less accurate than those made by security analysts, who can bring significantly more information and structure to the forecasting task. However, we also recognize that existing efforts to make broad risk classifications of equity securities have been developed on an ad hoc basis based on crude financial ratios, such as size, market-to-book and price-to-earnings. Our goal is to show that using similar financial information in our more rigorously developed implied equity duration framework enhances our ability to understand and explain equity security risk characteristics.

Our forecasting procedures are based on recent evidence from the earnings-based valuation literature [e.g., Penman and Sougiannis (1998), Nissim and Penman (2000), Penman (2000)]. This literature suggests that financial accounting constructs, such as earnings and book value of equity, provide effective instruments for forecasting future cash flows. We begin by using the clean surplus relation to express net cash distributions to equity in terms of earnings and book value of equity:

$$CF_t = E_t - (BV_t - BV_{t-1}) \tag{7}$$

where E_t represents accounting earnings at the end of period t and BV_t represents the book value of equity at the end of period t. Re-arranging the right-hand side of equation (7) gives:

$$CF_t = BV_{t-1} \times \left[\frac{E_t}{BV_{t-1}} - \frac{(BV_t - BV_{t-1})}{BV_{t-1}} \right] \tag{8}$$

Equation 8 indicates that the amount of cash flows generated over a period is increasing in return on equity ($\frac{E_t}{BV_{t-1}}$) and decreasing in the rate at which the firm grows its equity base

$(\frac{(BV_t - BV_{t-1})}{BV_{t-1}})$. Thus, equation 8 establishes return on equity (ROE) and growth in equity as they key drivers of cash flow. Armed with forecasts of future ROE and growth in equity, we can compute forecasts of future cash flows. We turn next to the models used to construct the ROE and growth forecasts.

It is well established that ROE follows a slowly mean reverting process [e.g., Penman (1991)]. Moreover, both economic intuition and empirical evidence suggest that the mean to which ROE reverts should approximate the cost of equity [e.g., Nissim and Penman (2000)]. We therefore model ROE as a first-order autoregressive process with an autocorrelation coefficient based on the long-run average rate of mean reversion in ROE and a long-run mean equal to the cost of equity.

Growth in equity is primarily driven by growth in sales. Growth in equity can also arise from changes in operating asset turnover and financial leverage. The relation between growth in equity, growth in sales, turnover and leverage can be seen by first noting that book value of equity can be expressed as:

$$BV_t = \frac{S_t}{\frac{S_t}{A_t} \times \frac{A_t}{BV_t}} \tag{9}$$

where S_t denotes sales for period t, A_t denotes total assets at the end of period t, $\frac{S_t}{A_t}$ represents asset turnover for period t and $\frac{A_t}{BV_t}$ represents financial leverage at the end of period t. We can now express growth in equity as

$$\frac{BV_t - BV_{t-1}}{BV_{t-1}} = \frac{BV_t}{BV_{t-1}} - 1 = \frac{S_t}{S_{t-1}} / \left[\left(\frac{S_t}{A_t} / \frac{S_{t-1}}{A_{t-1}} \right) \times \left(\frac{A_t}{BV_t} / \frac{A_{t-1}}{BV_{t-1}} \right) \right] - 1 \tag{10}$$

Assuming that asset turnover and financial leverage remain constant over time, this expression reduces to:

$$\frac{BV_t - BV_{t-1}}{BV_{t-1}} = \frac{S_t - S_{t-1}}{S_{t-1}} \tag{11}$$

Nissim and Penman (2000) show that changes in turnover and leverage tend to be transitory. For this reason, past growth in sales provides a superior predictor of future growth in equity than does past growth in equity. Sales growth follows a mean reverting process similar to ROE, but mean reversion in sales growth tends to be more rapid [see Nissim and Penman (2000)]. Economic intuition suggests that the mean to which sales growth reverts should approximate the long-run macroeconomic growth rate.[5] We therefore model growth in equity as a first-order autoregressive process, with an autocorrelation coefficient equal to the long-run average rate of mean reversion in sales growth and a mean equal to the long-run GDP growth rate.

[5] Sales growth rates for US equities have averaged around 10% over the past 40 years [see Nissim and Penman]. This period, however, has been one of unprecedented growth for US equity markets, and the long-run macroeconomic growth rate provides a more economically reasonable ex ante estimate of long-run sales growth.

Implementation of our estimation procedure for implied equity duration requires four financial variables and 4 forecasting parameters as inputs. We summarize these inputs in table 1. The four financial variables are book value (both current and lagged one year), sales (both current and lagged one year), earnings (current) and market capitalization (current). The four forecasting parameters are the autocorrelation coefficient for ROE, the autocorrelation coefficient for sales growth, the cost of equity and the long-run GDP growth rate. We conduct our analysis using annual data and obtain the required financial variables from the annual COMPUSTAT files. Using pooled data over our sample period, we obtain average estimates of the autocorrelation coefficients for ROE and sales growth of 0.57 and 0.24 respectively. Consistent with Nissim and Penman, our estimates indicate that sales growth mean reverts more rapidly that ROE. The long-run averages for cost of equity and GDP growth rate are based on the long-run averages reported by Ibbotson (1999) of (approximately) 12% and 6% respectively. Finally, we use a finite forecast horizon of ten years, because most of the mean reversion in sales growth and ROE is complete after 10 years. We emphasize that these forecasting procedures are relatively crude. For example, certain forecasting parameters have been shown to vary systematically as a function of industry membership and other firm characteristics. However, our immediate goal is to introduce the concept of implied equity duration and demonstrate the ability of a relatively simple empirical estimation procedure to produce an effective measure of implied equity duration.

We illustrate our estimation procedure via two examples, shown in panels A and B of table 2 respectively. The two examples are for Alaska Air Group and Amazon.com at the beginning of 1999, and are designed to be illustrative of a typical low duration equity and a typical high duration equity respectively. Values for the required forecasting variables are listed at the top left of each panel and the forecasting parameters, which are assumed to be the same across firms, are listed at the top right of each panel. Forecasts of cash flows and their present values are derived for the ten-year forecast horizon. The growth rate is derived by applying the sales persistence parameter of 0.24 to revert the growth rate to its long-run mean of 6%. Similarly, ROE is derived by applying the ROE persistence parameter of 0.57 to revert ROE to its long-run mean of 12%.[6] Applying the growth rate to lagged book value generates the forecasts of future book values. Applying the forecasts of ROE to the lagged book value forecasts then generates the forecasts of earnings. Cash flow forecasts can then be backed out from earnings and book value forecasts using the clean surplus relation. The duration of the finite forecast cash flows is equal to the ratio of the time weighted present value to the present value of the forecast cash flows. The weight assigned to the finite period duration is equal to the ratio of the present value of the forecast cash flows to the market capitalization. The duration of the terminal cash flows is always equal to 19.33 [i.e., $T + (1+r)/r = 10 + 1.12/0.12=19.33$]. The weight assigned to the terminal duration is simply one minus the weight assigned to the finite period duration. Implied equity duration is then computed by taking the weighted sum of the finite and terminal durations.

The computation for Alaska Air indicates that 64% of the value implicit in the current price is expected to be realized during the finite forecast period. Alaska Air's forecast ROE exceeds its forecast growth rate in every year of the forecast period, which results in a positive cash

[6] Both Sales Growth and ROE mean revert to their long run averages in the following convex manner: $ROE_{t+1}=(ROE_t-.12)*.57+.12$ and $\text{Sales Growth}_{t+1}=(\text{Sales Growth}_t - .06)*.24 +.06$.

flow in every period. Moreover, Alaska Air's earnings-to-price ratio is relatively high, so the cash flows realized during the forecast period represent a significant fraction of the market capitalization. This results in a relatively low implied equity duration of just 10.0 years for Alaska Air. The computation for Amazon.com indicates that the cash flows realized during the forecast period amount to –21% of the value implicit in the current market capitalization. In Amazon's case, the negative current ROE and high growth rate combine to generate cash flows over the finite forecast period that are mostly negative and have a negative net present value. Implicit in this negative cash flow is the necessity of Amazon.com to access the capital markets in some capacity over the finite forecasting horizon. Not until the eighth year of the finite period does the ROE exceed the growth rate in book value, which is what is required for positive free cash flow. As a consequence of the negative weighting on the finite forecast period duration, Amazon's implied equity duration of 23.0 years exceeds the terminal period duration of 19.33 years. Thus, duration tends to be low for firms with high ROE, low growth and small market capitalizations and high for firms with low ROE, high growth and large market capitalizations.

2.3 Interpretation

Before turning to our empirical predictions, it is useful to provide some intuition for our measure of implied equity duration and to discuss its relation to some other common equity style characteristics. We can do this by considering some special cases of the implied equity duration formula in equation (6). These special cases all involve the assumption that the net cash distributions made over the finite forecasting interval take the form of a level annuity, denoted by A. The duration of a level annuity of length T is given by:

$$D_A = \frac{(1+r)}{r} - \frac{T}{(1+r)^T - 1} \tag{12}$$

and the present value of a level annuity of amount A and length T is given by:

$$PV_A \quad = \quad A \times \frac{1 - \dfrac{1}{(1+r)^T}}{r} \tag{13}$$

Substituting these two equations into equation (6) and simplifying yields:

$$D \quad = \quad T + \frac{(1+r)}{r} - \frac{A/r}{P} \times T \tag{14}$$

This expression highlights the fact that implied equity duration is decreasing in the magnitude of the net cash distributions paid over the finite forecast horizon. Differentiating (14) with respect to A gives:

$$\frac{\partial D}{\partial A} \quad = \quad \frac{-T}{r \times P} \tag{15}$$

Duration is decreasing in the magnitude of the annuity, with the rate of decrease being larger for longer forecast horizons, lower discount rates and lower stock valuations.

The first special case we consider is the case where the level annuity is zero (A=0), in which case (14) simplifies to:

$$D \quad = \quad T + \frac{(1+r)}{r} \tag{16}$$

This is simply the formula for the duration of a level perpetuity commencing in T periods. Using our assumptions of a 10-year forecast horizon and a 12% discount rate, equation (16) gives an implied duration of 19.3 years. This special case can be thought of as corresponding to a 'growth' firm that reinvests 100% of the free cash flows generated by its operating activities for a period of T years, and then distributes its accumulated value through a level dividend thereafter. In practice, most firms would distribute some of their free cash flows via dividends or stock repurchases over the finite forecasting horizon. Hence, implied equity duration will be less than 19.3 years for most firms (e.g., the Alaska Air example in table 2). However, it is also possible to have an implied equity duration that is greater than the 19.3 years implied by this special case. Such a situation occurs for 'super-growth' firms, which raise additional equity capital over the finite forecasting period in addition to reinvesting all internally generated free cash flow. In situations where the net present value of the net cash distributions over the finite forecast horizon is negative, implied equity duration will be greater than 19.3 (e.g., the Amazon example in table 2).

Equation (14) is also useful for illustrating the relation between implied equity duration and the earnings-to-price ratios and book-to-market ratios. Recall from equation (8) that the net cash distributions received over the finite forecast horizon can be expressed as:

$$CF_t = BV_{t-1} \times \left[\frac{E_t}{BV_{t-1}} - \frac{(BV_t - BV_{t-1})}{BV_{t-1}} \right]$$

If we assume that growth in equity is zero over all periods (i.e., $\frac{(BV_t - BV_{t-1})}{BV_{t-1}} = 0$) such that BV= BV_{-1} through BV_T) and perfect persistence of current ROE over the forecast period (i.e., $\frac{E_t}{BV_{t-1}} = \frac{E_0}{BV_{-1}}$), then $CF_t = E_0$. The amount of the annuity for the finite forecast horizon is now equal to earnings at the beginning of the forecast horizon, and implied equity duration becomes:

$$D \;=\; T + \frac{(1+r)}{r} - \frac{E_0}{P} \times \frac{T}{r} \tag{17}$$

In this special case, we see that there is a simple negative relation between implied equity duration and the earnings-to-price ratio. Thus, for firms where growth in equity is low and ROE is persistent, the earnings-to-price ratio can be used to provide a simple proxy for duration. Figure 2 provides estimates of implied equity duration for Alaska Air and Amazon.com based on (17) and labeled ‘earnings-to-price approximation’. The equation (17) approximations understate duration for Alaska Air and overstate duration for Amazon. These exaggerations are primarily attributable to the implicit extrapolations that this approximation imposes on the high positive ROE for Alaska Air and the high negative ROE for Amazon.com.

Next, assume that growth in equity is again zero over the forecast period (i.e., $\frac{(BV_t - BV_{t-1})}{BV_{t-1}} = 0$) but that ROE, instead of persisting, immediately mean reverts to the cost of capital in the first year of the forecast period (i.e., $\frac{E_t}{BV_{t-1}} = r$), then $CF_t = r \times BV_0$. The amount of the annuity for the finite forecast horizon is now equal to book value at the beginning of the forecast horizon multiplied by the cost of capital, and implied equity duration becomes:

$$D = T + \frac{(1+r)}{r} - \frac{BV_0}{P} \times T \tag{18}$$

In this special case, there is a simple negative relation between implied equity duration and the book-to-market ratio. Thus, for firms where growth in equity is low and ROE is quickly mean reverting, the book-to-market ratio can be used to provide a simple proxy for duration. Table 2 provides estimates of implied equity duration for Alaska Air and Amazon.com based on (18) and labeled 'book-to-market approximation'. Approximations based on equation (18) understate duration for both Alaska Air and Amazon.com. For Alaska Air, the understatement arises because the implicit assumption of no growth in equation (18) would lead to greater positive cash flows early in the forecast horizon. For Amazon, the understatement arises because the implicit assumptions of no growth and immediate mean reversion to a positive ROE would lead to more positive cash flow early in the forecast horizon.

The long-run average autocorrelation coefficients reported in table 1 indicate that innovations in ROE and growth are neither completely persistent nor completely transitory. ROE is slowly mean reverting, suggesting that both the earnings-to-price ratio and the book-to-market ratio should explain incremental variation in implied equity duration. Growth in sales also exhibits some persistence. Since growth in sales during the forecast horizon tends to reduce cash flows, firms with high sales growth should have longer equity duration. In summary, we therefore expect that our empirical estimates of implied equity duration will be decreasing in the earnings-to-price ratio, decreasing in the book-to-market ratio and increasing in sales growth.

2.4 Empirical Predictions

The primary empirical implication of duration stems from the relation between ex post holding period returns and changes in expected return. Denoting holding period returns as h and changes in expected return as Δr, equation (3) indicates that the relation between ex post holding period returns and changes in expected returns takes the following form:

$$h = \frac{\Delta P}{P} \approx -\frac{D}{1+r}\Delta r \tag{19}$$

Empirical verification of the relation in (19) is difficult, because changes in expected returns are not directly observable. Nevertheless, (19) has several useful empirical implications. First, defining volatility in terms of the standard deviation (σ), then (19) implies that:

$$\sigma(h) \approx \frac{D}{1+r}\sigma(\Delta r) \tag{20}$$

This expression indicates that the volatility of holding-period returns is a positive function of duration and provides the basis for our first empirical prediction:

P1: The volatility of equity holding period returns is increasing in equity duration.

Our first prediction relates to the total volatility of equity returns. However, asset-pricing theory suggests that non-diversifiable volatility constitutes a more relevant measure of risk. In particular, the capital asset pricing model indicates that only systematic risk (β) that is related to movements in the market portfolio should be priced. Defining h_m as the ex post holding-period return on the market portfolio, D_m as the duration of the market portfolio and r_m as the expected return on the market portfolio, we can use (19) to obtain the relation between equity β and equity duration as follows:

$$\beta(h, h_m) = \frac{\sigma(h, h_m)}{\sigma^2(h_m)} \approx \frac{D}{D_m} \times \frac{(1+r_m)}{(1+r)} \times \frac{\sigma(\Delta r, \Delta r_m)}{\sigma^2(\Delta r_m)} \tag{21}$$

Equation (21) indicates that equity duration has the effect of magnifying equity betas computed using realized holding-period returns. If we further assume that the final term in

(21) is dominated by contemporaneous correlations between r and r_m (i.e., r and r_m exhibit relatively little serial correlation), then (21) reduces to:

$$\beta(h,h_m) \approx \frac{D}{D_m} \times \frac{(1+r_m)}{(1+r)} \times \beta(r,r_m) \tag{22}$$

The final term in (22), $\beta(r,r_m)$, simply reflects the sensitivity of an equity's expected return to the expected return on the market portfolio. Since the market portfolio is simply a positive weighted average of all the individual securities in the market, $\beta(r,r_m)$ will be positive by construction for the 'typical' security. Moreover, there is a large body of empirical evidence documenting strong common expected return shocks in equities [e.g., Campbell and Shiller (1988), Campbell and Mei (1993)]. Thus, we expect $\beta(r,r_m)$ to be positive for most equity securities. Equation (22) indicates that allowing for positively correlated shocks to expected returns introduces equity duration as a source of cross-sectional variation in beta. Equation (22) forms the basis for our second prediction:

P2: Equity betas computed from realized holding-period returns are increasing in the duration of the equity relative to the duration of the market portfolio.

Tests of our second prediction build on evidence in Campbell and Mei (1993) and Cornell (1999). Campbell and Mei use a log-linear approximation to returns to estimate the proportion of the variation in beta attributable to common variation in cash flows versus common variation in expected returns. They find that the betas are largely attributed to common innovations in expected returns. Thus, their evidence implies that equation (22) should capture an empirically important source of cross-sectional variation in beta. Cornell anticipates our second prediction by recognizing that the Campbell and Mei results imply that equity duration should be an important determinant of betas. He presents preliminary tests in this respect by correlating betas with earnings-to-price ratios, dividend-to-price ratios and growth forecasts. Cornell provides mixed and indirect evidence in support of *P2*. We build on Cornell's results by constructing more direct tests of *P2*.

Our second prediction rests on the assumption that some shocks to expected returns are common across securities. However, it does not necessarily rule out the case of idiosyncratic shocks to expected returns. For example, liquidity has been proposed as an important determinant of expected returns [e.g., Amihud amd Mendelson (1983)]. Therefore, events having an impact on a firm's liquidity, such as changes in exchange listing, addition/removal from an index and the listing of derivative securities, may result in idiosyncratic shocks to expected returns. Denoting h_f and r_f as the firm-specific components of realized and expected returns respectively and substituting into (20) yields:

$$\sigma\left(h_f\right) \approx \frac{D}{1+r}\sigma\left(\Delta r_f\right) \tag{23}$$

from which we generate our third prediction:

P3: *The standard deviation of the idiosyncratic component of realized holding-period returns is increasing in equity duration.*

Our first three predictions concern associations between equity duration and common measures of equity volatility. Our remaining predictions concern the ability of equity duration to capture a common factor in stock returns. Existing academic research has focused on three significant common factors in stock returns: an overall market factor, a factor related to firm size and a factor related to the book-to-market ratio [e.g., Fama and French (1993)]. The identification of these factors has been achieved through data mining, rather than through the application of a theoretically based valuation framework. It will therefore be useful to examine the extent to which our theoretically derived duration factor stacks up against the factors identified through data mining. Of particular interest is the relation between the duration factor and the book-to-market factor, since our analysis in section 2.3 indicates that the book-to-market ratio serves as a noisy proxy for implied equity duration. The identification of a robust common factor in equity returns related to duration would also be useful to practitioners using multi-factor models to forecast equity risk. Commercially available models use as many as 15 common risk factors, but do not explicitly include a duration factor [e.g., BARRA (1999)].

We estimate mimicking factors for duration using two alternative but related procedures, in order to facilitate interpretation of our results and comparability with previous research. First, we follow the Fama and French (1993) procedure of creating a mimicking portfolio for risk characteristics. A mimicking portfolio for duration is constructed by taking the difference between the returns on stocks with high duration and the returns on stocks with low duration. We examine three specific predictions using this mimicking portfolio:

P4: A mimicking portfolio for duration captures strong common variation in stock returns.

P5: *A mimicking portfolio for duration captures similar common variation in stock returns to a mimicking portfolio for book-to-market.*

P6: *A mimicking portfolio for duration dominates a mimicking portfolio for book-to-market in explaining common returns for firms with high persistence in ROE and sales growth.*

P4 follows directly from equation (19) combined with previous evidence that there are significant common shocks to expected returns [e.g., Campbell and Mei (1993)]. Equation (19) indicates that common variation in expected returns will lead to common variation in stock prices that is magnified for high duration securities. A mimicking factor based on the difference between high and low duration portfolios will therefore pick up common variation in expected returns that is attributable to common shocks in expected returns. *P5* examines the possibility that the book-to-market factor is simply serving as a proxy for duration. If this is the case, then we would expect that the common variation in stock returns captured by the two factors is closely related. *P6* builds on *P5* by investigating whether the duration factor does a better job of capturing common variation in stock returns in firms where book-to-market provides a relatively poor proxy for implied equity duration. Recall from section 3.2 that implied equity duration is a simple function of book-to-market when ROE and sales growth are purely transitory. Thus, we expect that the mimicking factor for duration will dominate the mimicking factor for book-to-market when persistence in ROE and sales growth are high.

Our second procedure for computing a duration factor uses a straightforward regression approach and is similar to the procedure used by BARRA (1999). This procedure attempts to directly estimate the common shocks to expected returns by estimating equation (19) in cross-section:

$$h_{it} = \alpha_t + \gamma_t \frac{-D_{it}}{(1+r)} + \varepsilon_{it} \tag{24}$$

The model in (24) is estimated separately for each of the calendar months in our sample. Comparing (24) to equation (19), we see that if duration is estimated without error and shocks to expected returns are common across equities, then α_t=0 and γ_t=Δr_t. However, our estimate of duration is almost certainly measured with error and shocks to expected returns are likely to be attributable to both common and firm-specific factors. Thus, estimates of α_t will be biased upward and estimates of γ_t will be biased downward. Nevertheless, we can gauge the effectiveness of our measure of implied duration by inspecting the properties of the γ estimates of Δr. In addition, we make two specific predictions with respect to the γ estimates:

P7: *The γ estimates from equation (24) are negatively correlated with the holding period returns on the market portfolio.*

P8: *The γ estimates from equation (24) are negatively correlated with the holding period returns on long duration bonds.*

P7 follows directly from the observation that γ measures the change in the expected return on equities. Increases in the expected return on equities should lead to reductions in equity prices and lower holding period returns on equities. Thus, we should observe a negative correlation between γ and the returns on the market portfolio. P8 is more tenuous, since it requires commonality in the expected return shocks to stocks and bonds. If shocks to the risk free rate of return are a significant source of shocks to the expected returns on both stocks and bonds, then there should be a negative correlation between γ and long duration bond returns. However, if shocks to expected returns on equities are largely attributable to shocks to the equity premium, then we will still find support for *P7*, but not necessarily *P8*.

3. Data

Our sample includes all firms with available data from the NYSE, Amex and NASDAQ from 1963 through 1998. Financial statement data are obtained from the COMPUSTAT annual tapes. Earnings are measured using income before extraordinary items (annual data item #18). Market value of equity is calculated by multiplying price as of the fiscal year end (annual data item #199) with the number of shares outstanding as of the fiscal year end (data item #25). Book value of common equity (BV) represents the par value of common stock, treasury stock, additional paid in capital and retained earnings as of FYE (annual data item #60). Observations with negative book value of equity are deleted from the sample. Sales growth is calculated as the one-year discrete growth rate in annual net sales (annual data item #12).

Stock returns are drawn from the Center for Research on Securities Prices (CRSP) daily tape. We compute stock returns for portfolios of securities formed on various financial ratios computed from the COMPUSTAT financial data described above. Our timing conventions for computing portfolio returns follow those in Fama and French (1993). If we are computing returns for portfolios formed on financial data from year t-1, then we compute monthly holding period returns from July of year t through June of year t + 1. We compute excess returns for each of our stock portfolios by subtracting the one-month Treasury bill rate, measured at the beginning of the month.

We also compute three measures of equity volatility using the stock return data. We compute the standard deviation of total stock returns (σ), and we also estimate a market model regression for each firm and compute the beta (β) and residual standard deviation (σ_f) for each stock. The market model regressions are estimated using weekly holding period returns over a two-year period. For each observation, we compute volatility using both historical and forward data. The historical estimates employ data from the two-year period ending at the end of the fiscal year from which we obtain our financial data. The forward estimates use data from the two-year period beginning at the end of the fiscal year from which we obtain our financial data.

To be included in our final sample, a firm must have non-missing values for all the required variables from COMPUSTAT and must have at least some of the required return data available on CRSP. This sample consists of 126,870 firm-year observations. Of these observations, data was available to compute at least one of the volatility metrics for 102,684 observations. We also winsorize the one-percent tails of each of the financial ratios computed using the COMPUSTAT data to eliminate the influence of extreme outliers.

Finally, we obtain data on monthly percent long-term government bond returns from Ibbotson Associates. We construct our excess long-bond return series by subtracting the one-month Treasury bill rate, measured at the beginning of the month.

4. *Results*

4.1 DESCRIPTIVE STATISTICS

We begin by reporting descriptive statistics for our estimates of implied equity duration. We also present comparable statistics for several related financial measures. Section 2.3 demonstrates that duration is related to the book-to-market ratio (negative expected relation), the earnings-to-price ratio (negative expected relation), sales growth (positive expected relation) and market capitalization (positive expected relation). Moreover, each of these measures has been proposed as a potential proxy for risk in prior research. We therefore include descriptive statistics on these measures.

Panel A of table 3 reports univariate statistics on duration. Duration has a mean of 15.1 years and a standard deviation of 4.1 years. The lower quartile value is 13.3 and the upper quartile value is 17.4. Thus, for most firms duration is quite close to 19.3 years, the value of duration in the special case where no cash distributions are made in the finite forecast horizon. Most firms therefore distribute relatively little of the value represented in their stock price during the 10-year finite forecast period. However, the minimum value of duration is –16.8 years, indicating that there are exceptions. A negative value for duration requires that the present value of the cash flows over the finite forecast horizon exceeds the market value of the stock. One explanation for such a situation is that the stock is underpriced. An alternative explanation is that our forecasting model has incorrectly forecast that past profitability will continue into the future. At the other extreme, the maximum value of duration is 32.0 years. The fact that duration exceeds 19.3 years suggests that the present value of the cash flows over the finite forecast horizon is negative. For duration to be so much greater than 19.3 years, the negative present value of the finite forecast period cash flows must be large relative to the market capitalization.

Panel B of table 3 reports the correlations between our estimate of implied equity duration and the other financial measures. The correlations are generally strong and are consistently of the expected signs. Implied equity duration is strongly negatively correlated with book-to-market (Pearson=-0.67; Spearman=-0.73) and earnings-to-price (Pearson=-0.79; Spearman=-0.76) and positively correlated with sales growth (Pearson=0.20; Spearman=0.19). Thus, high duration stocks are 'glamour' stocks with low book-to-market and earnings-to-price ratios and high sales growth, while low duration stocks are 'value' stocks with high book-to-market and earnings-to-price ratios and low sales growth. It is also noteworthy that the correlations between book-to-market and earnings-to-price (Pearson=0.57; Spearman=0.58) are lower than the respective correlations of each of these variables with duration. In other words, duration does an excellent job of synthesizing common variation in book-to-market and earnings-to-price. Book-to-market, earnings-to-price and sales growth have all been proposed as empirical proxies for unidentified common risk factors in stock returns. The correlations in

Table 3 are consistent with implied equity duration representing the underlying common factor represented by each of these variables.

4.2 VOLATILITY RESULTS

The first three predictions outlined in section 2.4 concern the relation between implied equity duration and stock return volatility. This section presents the results of tests of these predictions. We begin in table 4 by providing evidence on the association between our estimates of implied equity duration and historical stock return volatility. Table 5 then provides evidence on the ability of duration to forecast future stock return volatility.

Panel A of Table 4 presents correlations between our estimates of implied equity duration and estimates of the standard deviation of monthly stock returns. The standard deviation of monthly stock returns is estimated using weekly stock returns from the two years leading up to the year from which we obtain the financial data to construct our duration estimates. We also report correlations for the related financial measures identified in section 4.1. Consistent with our first prediction, *P1*, we see that implied equity duration has a strong positive correlation with stock return volatility (Pearson=0.19, Spearman=0.23). We also see that book-to-market, earnings-to-price, sales growth and market capitalization all have significant correlations with stock return volatility. However, in the case of book-to-market, earnings-to-price and sales growth, the correlations are much weaker than they are for implied duration. Moreover, the sign of the correlations for these variables are the same as the sign of their correlations with implied equity duration. The results for these variables are therefore consistent with them simply serving as noisy proxies for duration. For market capitalization, however, the correlations with stock return volatility are negative and the Spearman correlation, is stronger than the corresponding return for implied duration. The strong negative correlations for market capitalization cannot be explained by a duration proxy story, and are probably attributable to the greater cash flow volatility of smaller, less diversified firms.

Panels B and C of table 4 look at the correlations between implied equity duration and the systematic and firm-specific components of volatility respectively. The volatility decomposition is achieved through market model regressions using weekly stock returns over the two years leading up to the duration measurement year. Panel B measures systematic volatility using the beta coefficients (β) from the market model regressions. Consistent with *P2*, there is a strong positive correlation between relative duration and beta (Pearson=0.12; Spearman=0.19). We again find that the correlations for book-to-market, earnings-to-price and sales growth are somewhat weaker, and are of the same sign as their respective correlations with duration. The results for these variables are again consistent with them simply serving as noisy proxies for duration. In contrast, the sign of the correlations on market capitalization switches from negative to positive from panel A to panel B. Small firms have higher total volatility, while large firms have greater systematic volatility. This result is again consistent with the higher return volatility of small firms arising from greater volatility in their underlying cash flows.

Finally, Panel C reports the correlations for the firm-specific component of stock return volatility (σ_f), which is measured using the standard deviation of the residuals from the market model regressions described above. Consistent with *P3*, there is a strong positive correlation between implied duration and σ_f (Pearson=0.18; Spearman=0.22). We again find that the correlations for book-to-market, earnings-to-price and sales growth are somewhat weaker, and are of the same sign as their respective correlations with duration. The results for these variables are again consistent with them simply serving as noisy proxies for duration. Finally, the correlations for market capitalization are large and negative, confirming the idea that the higher return volatility of small firms arises from greater firm-specific cash flow volatility due to lack of diversification.

Table 5 investigates the ability of implied equity duration to forecast future stock return volatility. We use the same measures of stock return volatility as table 4, but the measures are now estimated using weekly stock returns in the two years following the computation of implied equity duration. Instead of reporting correlations, we report regressions of our volatility metrics on implied equity duration. This approach allows us to include lagged values of the volatility metrics as competing explanatory variables. For our estimates of implied equity duration to be useful from a forecasting perspective, they must have incremental explanatory power over lagged values of the volatility metrics. Panel A of table 5 provides evidence of the hypothesized positive relation between implied equity duration and future stock return volatility. Panels B and C confirm that the positive relation extends to both the systematic and firm-specific components of return volatility. Finally, we find that the implied equity duration still loads with a significant positive coefficient when we include lagged values of the respective volatility metrics in the regressions. Thus, implied equity duration is incrementally useful in forecasting future stock return volatility and its components.

In summary, section 4.2 provides three key findings concerning the relation between implied equity duration and stock return volatility. First, we find strong evidence of the hypothesized positive relation between implied equity duration and stock return volatility. Second, we show that association of variables like book-to-market and earnings-to-price with stock return volatility appears to derive, at least in part, from their ability to serve as proxies for implied equity duration. Finally, we show that implied equity duration is incrementally useful over past stock return volatility in forecasting future stock return volatility.

4.3 COMMON FACTOR RESULTS

Our next set of tests investigates whether duration represents a significant common factor in stock returns. We begin by following the Fama and French (1993) methodology of constructing a mimicking portfolio to capture common variation in stocks associated with the duration factor. Fama and French identify three distinct common factors in returns: an overall market factor, a size factor and a book-to-market factor. Our earlier results in section 4.1 suggest that the book-to-market factor is quite likely to represent a noisy proxy for the duration factor. Our tests therefore focus on examining the effects of replacing the mimicking portfolio for book-to-market with a mimicking portfolio for duration.

In constructing mimicking portfolios for size and book-to-market, we use the exact procedures described in Fama and French (1993). Six portfolios are formed from sorts on size and book-to-market. Two size groupings (S and B) are formed around the NYSE median and three book-to-market groupings (L, M and H) are formed around the NYSE 30th and 70th percentiles. Six value-weighted portfolios are constructed from the intersection of these groupings (S/L, S/M, S/H, B/L, B/M, B/H). The mimicking factor for size (SMB) is constructed by taking the difference, each month, between the simple average of the returns on the three small-stock portfolios and the three big-stock portfolios. Similarly, the mimicking factor for book-to-market (HML) is the difference, each month, between the simple average of the returns on the two high-book-to-market portfolios and the two low-book-to-market portfolios. Following Fama and French, we then evaluate whether the mimicking portfolios capture strong common variation in returns. This is accomplished by forming 25 value-weighted portfolios based on 5 size and 5 book-to-market groupings and estimating time series regressions of the portfolio returns on the mimicking portfolios.

The results in table 6A are from regressions on the 25 portfolio returns on the mimicking portfolios for size and book-to-market. These results are a replication of those in Fama and French's table 5, and differ only in that we use an extra 84 months of data. Consistent with the results of Fama and French, we find that SMB and HML typically capture substantial time-series variation in stock returns, with 17 of the 25 R^2 values above 0.2 and 5 above 0.25. Moreover the slopes on SMB are all monotonically decreasing in size and the slopes on HML are all monotonically increasing in HML. Thus, we confirm Fama and French's result that size and book to market represent important common factors in equity returns.

Table 6B replicates the results in table 6A, but replaces book-to-market with implied equity duration. Thus, both the mimicking portfolio (HDMLD) and the 25 dependent variable portfolios are constructed by replacing the implied equity duration variable for the book-to-market variable. Both the theory and evidence provided earlier in this paper indicate that book-to-market is strongly negatively correlated with equity duration. Thus, the low (high) book-to-market portfolios serve as proxies for high (low) duration portfolios. In order to facilitate a direct comparison between tables 6A and 6B, we list the duration portfolios going from left to right from high duration to low duration. The results in table 6B indicate that the R^2 values are higher than the corresponding values in 6A for 14 of the 25 portfolios. These portfolios are bolded in table 6B. Note that there is a distinct pattern in the bolded R^2 values, with the duration factor dominating in the large, high duration securities and the book-to-market factor dominating in the small, low duration securities. We will provide an explanation for this result below. The mean (median) R^2 value over the 25 portfolios is 0.338 (0.3630) for the duration factor versus 0.322 (0.3370) for the book-to-market factor. Thus, duration has a small edge over book-to-market in its ability to explain common variation in stock returns. Focusing on the slope coefficients, we see that the spread between the slopes on the high and low duration portfolios exceeds the spread between the low and high book-to-market portfolios for all five size groupings. Overall, implied equity duration has a small edge over book-to-market in explaining common variation in stock returns.

The pattern in the relative R^2 values between tables 6A and 6B is distinct, with the duration factor dominating in the large, high duration securities and the book-to-market factor dominating in the small, low duration securities. There is a straightforward explanation for this pattern. Recall that we estimate implied equity duration based on the unconditional average levels of autocorrelation in growth and ROE. In reality, the autocorrelation coefficients vary as a function of firm characteristics. In section 2.3, we demonstrated that implied equity duration is a direct function of the book-to-market ratio when persistence in growth and ROE is zero. A natural explanation for the results in table 6, therefore, is that growth and ROE are less persistent for small, low duration securities. Table 7 provides results that are consistent with this explanation. Table 7 reports the autocorrelation coefficients for growth and ROE estimated separately for each of the 25 portfolios. Panel A reports results for the portfolios formed on size and book-to-market, while panel B reports results for the portfolios formed on size and duration. There are very clear patterns in the autocorrelation coefficients. Both ROE and growth are more persistent in large, high duration (low book-to-market) equities. Thus, book-to-market provides a poor proxy for duration in these stocks. Conversely, ROE and growth are less persistent in small, low duration (high book-to-market) equities. Thus, book-to-market provides a good proxy for duration in these stocks. Moreover, since our measure of implied equity duration is based on the unconditional average levels of persistence, it should provide a relatively poor proxy for duration for small, low duration (high book-to-market) stocks. This is exactly what is borne out by the pattern of R^2 values in tables 6A and 6B.

Tables 8A and 8B, repeat the analysis in tables 6A and 6B incorporating the market factor as an additional source of common variation, and are analogous to table 6 in Fama and French. The addition of the market factor increases the R^2 values considerably. Nevertheless, the R^2 values continue to have a small edge in the regressions using the duration factor over the book-to-market factor. The results in table 8B indicate that the R^2 values are higher than the corresponding values in 8A for 18 of the 25 portfolios. These portfolios are bolded in table 8B. The mean (median) R^2 value over the 25 portfolios is 0.911 (0.917) for the duration factor versus 0.905 (0.916) for the book-to-market factor. It is also noteworthy that we no longer see the distinct pattern in relative R^2 values that we saw in table 6. The addition of the market factor appears to have picked up some of the common variation missed by book-to-market in the small, low duration portfolios. Focusing on the slope coefficients, we again see that the spread between the slopes on the high and low duration portfolios exceeds the spread between the low and high book-to-market portfolios for all five size groupings. Thus, the superiority of duration over book-to-market is robust to the inclusion of the market factor.

In summary, the results in tables 6 through 8 confirm predictions *P4* through *P6*. Duration represents a strong common factor in stock returns and accounts for more overall variation in returns than the book-to-market factor. Moreover, the dominance of the duration factor over the book-to-market factor is concentrated in portfolios where book-to-market provides a relatively poor measure of equity duration. Thus, it appears that the ability of book-to-market to capture common variation in returns arises because it serves as a noisy proxy for equity duration. We turn next to our final set of tests, in which we use our measure of implied equity duration to directly estimate common shocks to expected equity returns.

Our final set of tests directly estimate the change in the expected return on equities and investigate the properties of the resulting estimates. Recall from section 2.4 that the relation between holding period returns on equities and expected returns can be expressed as:

$$h = \frac{\Delta P}{P} \approx -\frac{D}{1+r}\Delta r$$

If there are common shocks to the expected return on equities (Δr), then we can estimate these shocks by estimating cross-sectional regressions of the form:

$$h_{it} = \alpha_t + \gamma_t \frac{-D_{it}}{(1+r)} + \varepsilon_{it}$$

with the γ_t providing an estimate of the change in expected return (Δr) for period t. Empirical estimation of this regression is subject to several specification problems. First, the relation is only approximate and not valid for large values of Δr (the convexity property). This should not create a serious problem, since our estimation uses monthly data, and monthly changes in expected return are unlikely to be large enough to create serious violations of the linearity assumption. The second problem is an errors-in-variables problem arising from the use of empirical estimates for duration (D) and expected returns (r). This problem will cause the intercept in the regression to be positive and the slope to be biased toward zero, thus understating the magnitude of the estimated changes in expected returns. We therefore expect our estimates of the change in expected return on equities to understate the true values. Nevertheless, our empirical estimates will provide a lower bound on the magnitude of these shocks and further evidence on the ability of duration to capture a common factor in expected returns.

We begin in panel A of table 9 and figure 1A by looking at the distributional properties of our estimates of change in expected return on equities (Δr). Δr ranges from a low of –0.82% to a high of 1.51%. The low of –0.82% occurred in October of 1969, a month in which the market rose by over 5%. The high of 1.51% occurred in June of 1970, a month when the market fell by over 11%. During the best month for the market in our sample period (October 1974), the market rose by over 16% and Δr was less than –0.5%. Conversely, during the worst month for the market in our sample period (October 1987), the market fell by over 22% and Δr exceeded 0.5%. Thus, our analysis suggests that our lower bound estimates of Δr exhibit substantial temporal variation. Moreover, significant shocks to expected returns are associated with significant shocks to holding period returns of the opposite sign, consistent with the predictions of basic valuation theory. The distribution of Δr is right-skewed (skewness=0.54) and highly leptokurtic (kurtosis=5.04). It is well known that monthly market returns are left skewed and leptokurtic. Our results suggest that these properties in returns can be attributed, at least in part, to related properties in the distribution of shocks to expected returns.

Finally, panel B of table 9 reports the correlations between Δr and other common factors in returns. Recall that we predict a negative correlation between Δr and both the market return and the excess long bond return. For the market return, both the Pearson and Spearman correlations are strongly negative (-0.45 and –0.45 respectively). Visual confirmation of the negative correlation between our estimates of Δr and the market return are provided in

Figures 1B (monthly realizations) and 1C (12-month moving averages). For the long bond return, however, the correlations are negative but statistically insignificant. This latter result is somewhat puzzling. One explanation for the result is that shocks to the risk-free component of expected equity returns are extremely small relative to shocks to the equity premium. However, the relatively strong correlation between the market return and the long-bond return is difficult to reconcile with this explanation. Alternatively, shocks to the risk-free rate may be correlated with shocks to short-term cash flows that are greater for short duration equities and hence confound the reported correlations. Panel B of table 9 also reports correlations with the common factors from our Fama-French tests. As would be expected, Δr is highly negatively correlated with our mimicking portfolio for duration (Pearson=-0.73 and Spearman=-0.72) and the mimicking portfolio for book-to-market (Pearson=0.57, Spearman=0.57). However, of these three duration-like factors (Δr, HML, HDMLD), Δr has the strongest correlation with the market return. This result is consistent with Δr representing our most efficient estimate of the common factor in returns related to duration. Such a result is comforting, because the Δr estimates are derived by directly estimating the underlying theoretical relation between holding period returns and duration rather than from the ad hoc Fama-French portfolio grouping procedures.

4.4 DESCRIPTIVE EVIDENCE ON THE EQUITY YIELD CURVE

We complete our empirical analysis by reporting descriptive evidence on the equity yield curve. A yield curve plots the yield to maturity on a class of securities as a function of the number of years to maturity, and is a common feature of fixed income security analysis. At a given point in time, the shape of the yield curve is influenced by expectations about future short rates. Averaged over longer periods of time, the yield curve can provide information about the preferred investment horizons of investors. This information should arise because a risk averse investor will pay a premium to avoid the risk associated with the uncertainty in realized returns that results from investing in a security with a maturity that differs from the investor's desired horizon. The usually observed upward slope of the yield curve for treasury bonds, especially for short maturities, is the empirical basis for the liquidity preference theory, which holds that short-term investors dominate the market for treasuries. There is, however, no corresponding evidence concerning the preferred investment horizons of equity investors.

The construction of an equity yield curve is more complicated than the construction of a Treasury bond yield curve for two reasons. First, in constructing a yield curve, one would ideally like to compute the 'pure' yield curve, using securities that make a single 'bullet' payment at a specific maturity date. Under such circumstances, there is no ambiguity concerning the maturity of the security. In computing the yield curve for treasuries, the existence of coupon payments slightly complicates this process. This problem is further complicated for equities, since the payouts from an equity security are typically realized gradually over long periods of time. We therefore use implied equity duration to measure the 'average' maturity of an equity security.

The second problem in constructing an equity yield curve is that without a fixed schedule of future payoffs, it is difficult to compute the implied yield on an equity security. In this respect, recall that our empirical technique for estimating the terminal cash flows on an equity security involved assuming a constant yield and solving for the implied terminal cash flows.

Thus, it would be circular to impute the yield from our assumed cash flows. Instead, we rely on rational expectations and assume that anticipated and realized yields will converge when averaged over long periods of time. Accordingly, we estimate anticipated yields on securities of varying durations by computing their average realized annual returns following the measurement of implied equity duration. We compute realized returns over the 12-month period beginning 4 months after the end of the fiscal year from which we obtain the financial inputs for our duration computation. This ensures that the information used to compute duration would have been available to market participants. The resulting sample consists of 110,072 annual return observations. We then partition these observations into the following six categories implied equity duration categories: <1 year (4,027 observations), 1-5years (3,904 observations), 6-10 years (12,875 observations), 11-15 years (42,201 observations), 6-20 years (39,808 observations) and >20 years (7,157 observations).

Figure 2 reports the average realized future annual returns for each duration category. There is a distinctive downward sloping yield curve over the first 20 years, with realized future returns declining monotonically from 33.8% for the lowest duration category to 9.7% for the 16-20 year category. It appears that investors have a preference for long duration equities, and so require a substantial premium for holding short duration equities. This is in contrast to treasury bonds, where we typically see an upward sloping yield curve, indicating a preference for short duration bonds. At first glance, these results seem somewhat difficult to reconcile, but they are consistent with the popular investment maxim 'stocks for the long run'. It appears that investors require a significant equity premium to hold short duration equities, but are prepared to hold long-duration equities for a substantially lower premium. Perhaps the additional cash flow risks and transaction costs associated with equities are viewed to be unattractive for short-term investments, but less of a concern for long-run investments. Beyond 20 years, the equity yield curve begins to slope up again, suggesting that the desired investment horizon for equity investors is in the 15-20 year range, and an additional premium is required for even longer duration equities.

Our evidence suggesting that equities are priced by investors with multi-year investment horizons has important implications for existing empirical tests of asset pricing theories. Existing research typically uses short holding period returns (weekly, monthly or annual return data). Most of the systematic volatility in stock returns over such short holding periods is attributable to expected return shocks [Campbell and Mei (1993)]. However, our evidence suggests that it is premature to claim that this volatility represents a source of risk that should be priced by investors. For investors with long investment horizons, this short-term price volatility is irrelevant. Indeed, the long duration equities, which have the most short-term price volatility, represent the least risky securities for investors with long investment horizons. Investing in long duration equities allows these investors to lock in the current expected return, and hence immunize themselves from future expected return shocks. Investing in short duration equities would expose investors to the reinvestment risk associated with rolling over their investments at uncertain future rates of return.

We note in closing that the results for the equity yield curve have a close relation to the debate over the 'book-to-market' effect in stock returns. Recall that implied equity duration and book-to-market are negatively correlated. Thus, given the book-to-market effect, it should come as no surprise that high duration firms display lower realized future returns. What we offer is a new potential explanation for these results. Investors in equities have a preference for locking in long holding period returns, and so require a premium for holding short duration equities. However, following Lakonishok, Shleifer and Vishny (1994), another explanation is that investors bid the prices of 'glamour' securities to irrationally high levels, resulting in low book-to-market ratios, high implied equity durations and lower expected future returns. While distinguishing between these competing explanations is difficult, our explanation is supported by our theory and evidence that duration captures an important common factor in stock returns.

5. Conclusions

In this paper, we develop a measure of equity duration and provide a simple algorithm for the empirical estimation of equity duration. We show that the standard empirical predictions and results for bond duration hold for our measure of equity duration and that equity duration represents an important common factor in stock returns. We also show that the book-to-market ratio can be used to construct a simple but effective proxy for equity duration, and that the Fama and French (1993) book-to-market factor can be interpreted as a duration factor. Finally, we show how equity duration can be used to impute the common shock to the expected return on equities.

We acknowledge that the forecasting model we use to estimate equity duration is crude. Improvements in the forecasting model should lead to improved estimates of equity duration and more refined measures of equity risk. We also emphasize that our empirical predictions in no way rely on equilibrium asset pricing theory or related assumptions, such as investor risk aversion. We simply assume that investors require a positive expected return to invest in equities, and that this expected return is subject to common shocks. However, our finding that the equity yield curve is downward sloping suggests that investors prefer to hold equities for the long-run. Investors in short duration equities demand a premium to compensate for the additional risk associated with reinvesting the cash flows received from short duration equities at uncertain future rates of return. This finding also suggests that existing empirical tests of asset pricing theories using short holding period equity returns may be seriously misspecified. For even though long duration equities have greater short-term price volatility, they minimize the risk associated with expected return shocks for investors with long investment horizons.

Finally, our measure of implied equity duration provides a natural and intuitive ranking of stocks' style characteristics on the value/growth dimension that is popular among practitioners. Currently, index providers such as Standard and Poor's, Dow Jones and Russell compete to provide the 'best' indices of value and growth stocks.[7] Yet their growth and value classifications are based on ad hoc reasoning and data-motivated statistical procedures. By combining information about expected growth, expected profitability and current stock price into a single and rigorously developed measure, implied equity duration provides an attractive alternative to the ad hoc measures of value and growth proposed by practitioners.

[7] Standard and Poor's use the BARRA classification of value versus growth, which is based on book-to-market. Dow Jones and Russell use more complex measures that combine more than one indicator of value and growth. A comparison of the alternative approaches is provided at http://208.198.167.32/dj_style/index.html.

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TABLE 1
Summary of Financial Variables and Forecasting Parameters Used in the Estimation of Implied Equity Duration

Panel A: Financial Variables

Financial Variable	Compustat Definition
Book Value of Equity (BV)	Data Item 60
Earnings (E)	Data Item 18 = Income before extraordinary items
Sales (S)	Data Item 12
Market Capitalization	Data Item 199 x Data Item 25

Panel B. Forecasting Parameters

Forecasting Parameter	Value
Autocorrelation Coefficient for Return on Equity	0.57
Cost of Equity Capital	0.12
Autocorrelation Coefficient for Growth in Sales/Book Value	0.24
Long-Run Growth Rate in Sales/Book Value	0.06

The autocorrelation coefficients are based on pooled autoregressions for Return on Equity and Sales Growth using a sample of 139,404, observations over Compustat years 1950 to 1999. The Cost of Equity Capital and Long-Run Growth Rates are based on their long-run historical averages.

TABLE 2 Panel A
Examples Illustrating the Computation of Implied Equity Duration for Alaska Air Group and Amazon.com for 1999

Calculation of Implied Equity Duration for Alaska Air in 1999

Input data ($millions, except percentages)		**Forecasting Parameters**	
Price (P_0)	685.90	*Autocorr. Coeff. for ROE*	57%
Lagged Book Value (B_{-1})	789.50	*Cost of equity capital (r)*	12%
Book Value (B_0)	930.70	*Autocorr. Coeff. for Growth*	24%
Growth rate ($(S_0-S_{-1})/S_{-1}$)	9.70%	*Long-Run Growth Rate*	6%
Earnings (E_0)	134.20		

Forecast Model

Time Period (t)	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Growth Rate	9.70%	6.89%	6.21%	6.05%	6.01%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%
ROE_t (E_t/B_{t-1})	17.00%	14.85%	13.62%	12.93%	12.53%	12.30%	12.17%	12.10%	12.06%	12.03%	12.02%
BV_t	930.70	994.81	1,056.62	1,120.55	1,187.92	1,259.23	1,334.80	1,414.89	1,499.78	1,589.77	1,685.15
$E_t=B_{t-1}*ROE_t$	134.20	138.20	135.53	136.57	140.38	146.12	153.27	161.48	170.57	180.45	191.06
$CF_t=B_{t-1}+E_t-BV_t$		74.09	73.72	72.64	73.01	74.81	77.70	81.39	85.68	90.46	95.67
$PV(CF_t)$		66.15	58.77	51.70	46.40	42.45	39.37	36.82	34.60	32.62	30.80
$t*PV(CF_t)$		66.15	117.54	155.10	185.59	212.25	236.20	257.72	276.84	293.60	308.04

$\Sigma(PV(CF_t))$	439.69	*Terminal PV*	246.21
$\Sigma(t*PV(CF_t))$	2,109.04		
10 Year Duration	*4.80*	*Terminal Duration*	*19.33*
10 Year Weight	*0.64*	*Terminal Weight*	*0.36*

Implied Equity Duration ***10.01*** **years**

Earnings-to-Price Approximation **3.03** **Years**

Book-to-Market Approximation **5.76** **Years**

TABLE 2 Panel B
Examples Illustrating the Computation of Implied Equity Duration for Alaska Air Group and Amazon.com for 1999

Calculation of Implied Equity Duration for Amazon.com in 1999

Input data ($millions, except percentages)		**Forecasting Parameters**	
Price (P_0)	8,905.00	*Autocorr. Coeff. for ROE*	57%
Lagged Book Value (B_{-1})	138.75	*Cost of equity capital (r)*	12%
Book Value (B_0)	266.28	*Autocorr. Coeff. for Growth*	24%
Growth rate ($(S_0-S_{-1})/S_{-1}$)	168.90%	*Long-Run Growth Rate*	6%
Earnings (E_0)	-719.97		

Forecast Model **Time Period (t)**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Growth Rate	168.90%	45.10%	15.38%	8.25%	6.54%	6.13%	6.03%	6.01%	6.00%	6.00%	6.00%
ROE_t (E_t/B_{t-1})	-518.90%	-290.61%	-160.49%	-86.32%	-44.04%	-19.94%	-6.21%	1.62%	6.08%	8.63%	10.08%
BV_t	266.28	386.36	445.80	482.58	514.15	545.66	578.57	613.33	650.14	689.15	730.50
$E_t=B_{t-1}*ROE_t$	(719.97)	(773.84)	(620.07)	(384.80)	(212.54)	(102.54)	(33.87)	9.38	37.32	56.09	69.45
$CF_t=B_{t-1}+E_t-BV_t$		(893.92)	(679.50)	(421.59)	(244.10)	(134.06)	(66.78)	(25.38)	0.51	17.08	28.10
$PV(CF_t)$		(798.14)	(541.69)	(300.08)	(155.13)	(76.07)	(33.83)	(11.48)	0.20	6.16	9.05
$t*PV(CF_t)$		(798.14)	1,083.39)	(900.24)	(620.52)	(380.33)	(203.01)	(80.35)	1.63	55.44	90.48

$\Sigma(PV(CF_t))$	-1,901.00	*Terminal PV*	10,806
$\Sigma(t*PV(CF_t))$	-3,918.42		
10 Year Duration	*2.06*	*Terminal Duration*	*19.33*
10 Year Weight	*(0.21)*	*Terminal Weight*	*1.21*

Implied Equity Duration ***23.02* years**

Earnings-to-Price Approximation **26.07 years**
Book-to-Market Approximation **19.03 years**

TABLE 3
Descriptive Statistics for Estimates of Implied Equity Duration (Duration) and Other Related Equity Security Characteristics

Panel A: Univariate Statistics

	Obs	Mean	Std. Dev.	Min	Low	Median	Upper	Max
Duration	126870	15.13	4.09	-16.75	13.30	15.63	17.36	31.97
Book-to-Market	126870	0.86	0.73	0.02	0.38	0.67	1.11	7.58
Earnings-to-Price	102083	0.09	0.07	0.00	0.05	0.08	0.12	0.66
Sales Growth	126870	0.17	0.30	-0.69	0.01	0.12	0.26	1.00
Market Cap.	126870	749.35	3192.00	0.65	16.52	65.41	308.89	64261.30

Panel B: Correlations (Pearson above the diagonal, Spearman below the diagonal)

	Duration	Book-to-Market	Earnings-to-Price	Sales Growth	Market Cap.
Duration	-	-0.67	-0.79	0.20	0.08
Book-to-Market	-0.73	-	0.57	-0.22	-0.13
Earnings-to-Price	-0.76	0.58	-	-0.07	-0.11
Sales Growth	0.19	-0.27	-0.07	-	-0.01
Market Cap.	0.16	-0.37	-0.21	0.10	-

See table 2 for the calculation of duration for fiscal year t. Book-to-Market is calculated as book value of equity divided by the market value of equity measured at the end of fiscal-year t. Earnings-to-Price is earnings divided by the market value of equity measured at the end of fiscal-year t. Sales Growth is calculated as $(Sales_t – Sales_{t-1}) / Sales_{t-1}$, where t is the current fiscal year. Market Capitalization (Market Cap.) is the market value of equity measured at the end of fiscal-year t.

TABLE 4
Correlation Between Equity Volatility and Implied Equity Duration, Book-to-Market, Earnings-to-Price, Sales Growth and Size.

Panel A: Volatility is the Standard Deviation of Stock Returns [σ]

	Duration	Book-to-market	Earnings-to-Price	Sales Growth	Market Cap.
Pearson Corr of σ with	0.19	-0.03	-0.04	0.08	-0.16
Spearman Corr of σ with	0.23	-0.09	-0.12	0.04	-0.49
Observations	102,684	102,684	83,155	102,684	102,684

Panel B: Volatility is the Stock Return Beta [β]

	Relative Duration	Book-to-market	Earnings-to-Price	Sales Growth	Market Cap.
Pearson Corr of β with	0.12	-0.10	-0.06	0.07	0.06
Spearman Corr of β with	0.19	-0.15	-0.09	0.08	0.18
Observations	102,684	102,684	83,155	102,684	102,684

Panel C: Volatility is the Standard Deviation of Firm-Specific Stock Returns [σ_f]

	Duration	Book-to-market	Earnings-to-Price	Sales Growth	Market Cap.
Pearson Corr of σ_f with	0.18	-0.02	-0.03	0.07	-0.16
Spearman Corr of σ_f with	0.22	-0.07	-0.12	0.03	-0.54
Observations	102,684	102,684	83,155	102,684	102,684

Relative Duration for firm i in year t is calculated as $Duration_{it}/(Market\ Duration_t)$. Market Duration is the value-weighted average of all firms with a measure of duration in fiscal year t. See table 2 for the calculation of duration for firm i in fiscal year t. Book-to-Market is calculated as book value of equity divided by the market value of equity measured at the end of fiscal-year t. Earnings-to-Price is earnings divided by the market value of equity measured at the end of fiscal-year t. Sales Growth is calculated as $(Sales_t - Sales_{t-1}) / Sales_{t-1}$, where t is the current fiscal year. Market Capitalization (Market Cap.) is the market value of equity measured at the end of fiscal-year t.
β for firm i for fiscal year t is estimated via a market model regression. The regression is run using weekly returns for a period of two years ending at the end of the fiscal year from which we obtain the data to compute each of the financial ratios. The standard deviation of stock returns [σ] is the standard deviation of the weekly returns calculated over the same two-year period. The standard deviation of firm-specific stock returns [σ_f] is the standard deviation of the residuals from the market model regression.
All correlations are significant at the 0.0001 level.

TABLE 5
Forecasting Ability of Implied Equity Duration with Respect to Equity Security Volatility
Model 1: Volatility(t+1) = $\alpha + \delta$ Duration(t)
Model 2: Volatility(t+1) = $\alpha + \delta$ Duration(t) + χVolatility(t)

Panel A: Volatility is Standard Deviation of Stock Returns [σ]

	Intercept	Duration	Volatility(t)	Adj. R^2
Model 1				
Coefficient	0.039	0.002		0.04
Standard Error	0.000	0.000		
t-statistic	95.39	60.15		
Model 2				
Coefficient	0.009	0.001	0.662	0.46
Standard Error	0.000	0.000	0.003	
t-statistic	26.91	34.08	236.85	

Panel B: Volatility is Stock Return Beta [β]

	Intercept	Relative Duration	Volatility(t)	Adj. R^2
Model 1				
Coefficient	0.580	0.3177		0.02
Standard Error	0.008	0.008		
t-statistic	71.76	40.35		
Model 2				
Coefficient	0.329	0.197	0.39	0.19
Standard Error	0.008	0.008	0.00	
t-statistic	41.21	25.85	120.80	

Panel C: Volatility is Standard Deviation of Firm-Specific Stock Returns [σ_S]

	Intercept	Duration	Volatility(t)	Adj. R^2
Model 1				
Coefficient	0.036	0.002		0.04
Standard Error	0.000	0.000		
t-statistic	80.18	54.91		
Model 2				
Coefficient	0.009	0.001	0.649	0.41
Standard Error	0.000	0.000	0.003	
t-statistic	23.12	30.82	215.28	

The number of observations in the Model 1 regressions is 83,785 and in Model 2 regression is 71,491.
Relative Duration for firm i in year t is calculated as $Duration_{it}/(Market\ Duration_t)$. Market Duration is the value-weighted average of all firms with a measure of duration in fiscal year t. See table 2 for the calculation of duration for firm i in fiscal year t.
β for firm i for fiscal year t is estimated via a market model regression. The regression is run using weekly returns for a period of two years starting following the year from which we obtain the data to compute each of the financial ratios. The standard deviation of stock returns [σ] is the standard deviation of the weekly returns calculated over the same two-year period. The standard deviation of firm-specific stock returns [σ_f] is the standard deviation of the residuals from the market model regression.
All correlations are significant at the 0.0001 level.

TABLE 6A

Regressions of excess stock returns (in percent) on the mimicking returns for the size (*SMB*) and book-to-market equity (*HML*) factors: July 1964 to June 1998.[a]

R(t)-RF(t)=a+sSMB(t)+hHML(t)+e(t)

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity
Book-to-market equity (*BE/ME*) quintiles

Size quintile	Low	2	3	4	High	Low	2	3	4	High
			s					*T-values (s)*		
Small	1.939	1.756	1.615	1.541	1.524	22.42	21.60	22.67	22.07	22.77
2	1.708	1.448	1.224	1.148	1.322	18.70	18.12	16.21	16.20	17.24
3	1.397	1.138	0.972	0.920	1.113	16.48	14.85	13.80	12.81	14.01
4	0.892	0.772	0.697	0.695	0.843	11.02	10.04	9.23	9.04	9.54
Big	0.237	0.264	0.251	0.310	0.514	3.21	3.72	3.46	4.29	5.86
			h					*T-values*		
Small	-0.370	-0.226	-0.087	0.065	0.274	-4.81	-3.14	-1.37	1.06	4.60
2	-0.643	-0.329	-0.099	0.043	0.239	-7.92	-4.64	-1.48	0.69	3.52
3	-0.640	-0.319	-0.067	0.065	0.219	-8.51	-4.68	-1.07	1.03	3.11
4	-0.678	-0.335	-0.085	0.044	0.168	-9.44	-4.90	-1.26	0.65	2.14
Big	-0.610	-0.325	-0.154	0.089	0.188	-9.29	-5.15	-2.38	1.39	2.41
			Adj R^2							
Small	0.55	0.52	0.54	0.53	0.55					
2	0.49	0.45	0.38	0.37	0.41					
3	0.45	0.36	0.30	0.27	0.31					
4	0.33	0.22	0.16	0.15	0.17					
Big	0.18	0.08	0.03	0.03	0.08					

[a] *SMB* (small minus big), the return on the mimicking portfolio for the common size factor in stock returns, is the difference each month between the simple average of the percent returns on the three small-stock portfolios (*S/L*, *S/M*, and *S/H*) and the simple average of the returns on the three big-stock portfolios (*B/L*, *B/M*, and *B/H*). *HML* (high minus low), the return on the mimicking portfolio for the common book-to-market equity factor in returns, is the difference each month between the simple average of the returns on the two high-*BE/ME* portfolios (*S/H* and *B/H*) and the average of the returns on the two low-*BE/ME* portfolios (*S/L* and *B/L*).

The 25 size-*BE/ME* stock portfolios are formed as follows. Each year t from 1964 to 1999 NYSE quintile breakpoints for size (*ME*, stock price times shares outstanding), measured at the end of June, are used to allocate NYSE, Amex, and NASDAQ stocks to five size quintiles. Similarly, NYSE quintile breakpoints for *BE/ME* are used to allocate NYSE, Amex, and NASDAQ stocks to five book-to-market equity quintiles. In *BE/ME*, *BE* is book common equity for the fiscal year ending in calendar year $t-1$, and *ME* is for the end of December of $t-1$. The 25 size-*BE/ME* portfolios are formed as the intersections of the five size and the five *BE/ME* groups. Value-weighted percent monthly returns on the portfolios are calculated from July of year t to June of $t+1$.

TABLE 6B

Regressions of excess stock returns (in percent) on the mimicking returns for the size (*SMB*) and Duration (*HDMLD*) factors: July 1964 to June 1998.[a]

R(t) - RF(t) = a + sSMB(t) + dHDMLD(t)+e(t)

Dependent variable: Excess returns on 25 stock portfolios formed on size and Duration quintiles

Size quintile	High	2	3	4	Low	High	2	3	4	Low
			s					*T-values (s)*		
Small	1.882	1.671	1.473	1.446	1.460	22.04	20.93	20.57	20.51	20.69
2	1.501	1.296	1.159	1.097	1.250	16.99	16.31	16.17	15.02	16.24
3	1.217	1.038	0.863	0.847	1.083	14.53	13.48	12.03	11.84	13.47
4	0.839	0.670	0.625	0.565	0.800	10.36	8.66	8.34	7.34	8.76
Big	0.140	0.190	0.166	0.233	0.430	1.99	2.65	2.31	3.14	5.06
			d					*T-values (d)*		
Small	0.725	0.437	0.228	0.060	-0.103	7.75	4.99	2.90	0.77	-1.33
2	1.095	0.579	0.261	0.028	-0.119	11.31	6.65	3.32	0.35	-1.41
3	1.029	0.505	0.225	0.012	-0.092	11.21	5.98	2.86	0.15	-1.04
4	1.073	0.553	0.262	0.048	-0.023	12.09	6.52	3.19	0.57	-0.23
Big	1.091	0.597	0.310	0.034	-0.113	14.15	7.60	3.95	0.42	-1.21
			Adj R^2							
Small	**0.64**	**0.55**	0.53	0.51	0.51					
2	**0.54**	**0.46**	**0.42**	0.36	0.39					
3	**0.49**	**0.38**	**0.31**	0.26	0.30					
4	**0.42**	**0.25**	**0.18**	0.12	0.15					
Big	**0.34**	**0.15**	**0.05**	0.02	0.05					

[a]*SMB* (small minus big), the return on the mimicking portfolio for the common size factor in stock returns, is the difference each month between the simple average of the

percent returns on the three small-stock portfolios (*S/L*, *S/M*, and *S/H*) and the simple average of the returns on the three big-stock portfolios (*B/L*, *B/M*, and *B/H*). *HML* (high minus low), the return on the mimicking portfolio for the common book-to-market equity factor in returns, is the difference each month between the simple average of the returns on the two high-*BE/ME* portfolios (*S/H* and *B/H*) and the average of the returns on the two low-*BE/ME* portfolios (*S/L* and *B/L*). *HDMLD* (high minus low), the return on the mimicking portfolio for the duration factor in returns, is the difference each month between the simple average of the returns on the two high-*duration* portfolios (*S/HD* and *B/HD*) and the average of the returns on the two low-*duration* portfolios (*S/LD* and *B/LD*)

The 25 size-*Duration* stock portfolios are formed as follows. Each year *t* from 1964 to 1999 NYSE quintile breakpoints for size (*ME*, stock price times shares outstanding), measured at the end of June, are used to allocate NYSE, Amex, and NASDAQ stocks to five size quintiles. Similarly, NYSE quintile breakpoints for *Duration* are used to allocate NYSE, Amex, and NASDAQ stocks to five duration quintiles. The 25 size-*Duration* portfolios are formed as the intersections of the five size and the five *Duration* groups. Value-weighted percent monthly returns on the portfolios are calculated from July of year *t* to June of *t*+1.

TABLE 7
Autocorrelation Coefficients for ROE and Sales Growth stratified by Size and BM Quintiles and by Size and Duration Quintiles.

$$ROE(t) = a + b[ROE(t-1)]+\varepsilon(t)$$

$$\text{Sales Growth}(t) = a + b[\text{Sales Growth }(t-1)]+\upsilon(t)$$

Panel A: 25 stock portfolios formed on *size* and *book-to-market* quintiles

Size quintile	Low	2	3	4	High	Low	2	3	4	High
			ROE					*Sales Growth*		
Small	0.51	0.40	0.40	0.36	0.34	0.13	0.11	0.10	0.12	0.10
2	0.62	0.42	0.30	0.28	0.29	0.12	0.20	0.15	0.14	0.14
3	0.62	0.40	0.30	0.29	0.30	0.19	0.25	0.26	0.23	0.23
4	0.59	0.34	0.34	0.35	0.35	0.34	0.31	0.27	0.30	0.28
Big	0.52	0.41	0.40	0.39	0.31	0.52	0.33	0.25	0.21	0.27

Panel B: 25 stock portfolios formed on *size* and *duration* quintiles

Size quintile	High	2	3	4	Low	High	2	3	4	Low
			ROE					*Sales Growth*		
Small	0.34	0.16	0.10	0.13	0.04	0.12	0.12	0.11	0.16	0.11
2	0.47	0.17	0.24	0.25	0.21	0.13	0.21	0.15	0.18	0.15
3	0.54	0.23	0.27	0.31	0.31	0.18	0.31	0.28	0.25	0.21
4	0.65	0.31	0.38	0.36	0.30	0.28	0.38	0.28	0.27	0.25
Big	0.63	0.56	0.43	0.46	0.40	0.53	0.41	0.27	0.22	0.18

The 25 size-*book-to-market* stock portfolios are formed as follows. Each year *t* from 1964 to 1999 firms are ranked based on size into five quintiles. Similarly, firms are ranked on book-to-market value of equity into five book-to-market quintiles. The 25 size-*book-to-market* portfolios are formed as the intersections of the five size and the five *book-to-market* groups. The 25 size-*Duration* stock portfolios are formed as follows. Each year *t* from 1964 to 1999 firms are ranked based on size into five quintiles. Similarly, firms are ranked on *Duration* in to five duration quintiles. The 25 size-*Duration* portfolios are formed as the intersections of the five size and the five *Duration* groups. Return on equity (ROE) and Sales Growth are calculated at the end of the fiscal year prior to the ranking procedure.

TABLE 8A

Regressions of excess stock returns (in percent) on the market factor and mimicking returns for the size (*SMB*) and book-to-market equity (*HML*) factors: July 1964 to June 1998.[a]

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t)+e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity
Book-to-market equity (*BE/ME*) quintiles

Size quintile	Low	2	3	4	High	Low	2	3	4	High
			b					*T-values (b)*		
Small	1.096	1.038	0.964	0.941	0.909	37.19	38.45	53.92	52.26	55.35
2	1.196	1.088	1.022	0.967	1.028	43.59	56.44	54.13	57.75	50.43
3	1.141	1.044	0.970	0.979	1.058	51.67	56.88	62.92	57.12	48.13
4	1.097	1.057	1.041	1.040	1.145	54.54	61.75	63.61	53.88	41.33
Big	1.003	0.969	0.988	0.974	1.108	54.83	57.30	56.20	51.94	36.62
			s					*T-values(s)*		
Small	1.437	1.280	1.173	1.110	1.107	32.53	31.65	43.76	41.10	44.98
2	1.160	0.949	0.755	0.705	0.851	28.19	32.86	26.68	28.06	27.82
3	0.874	0.660	0.528	0.471	0.628	26.41	23.97	22.82	18.34	19.06
4	0.389	0.288	0.220	0.218	0.318	12.90	11.21	8.98	7.53	7.66
Big	-0.223	-0.180	-0.201	-0.136	0.006	-8.11	-7.09	-7.64	-4.84	0.13
			h					*T-values (h)*		
Small	-0.051	0.076	0.194	0.340	0.539	-1.33	2.15	8.31	14.50	25.21
2	-0.295	-0.013	0.198	0.326	0.539	-8.26	-0.53	8.04	14.93	20.32
3	-0.309	-0.015	0.215	0.351	0.528	-10.74	-0.62	10.73	15.74	18.45
4	-0.360	-0.027	0.218	0.348	0.502	-13.73	-1.23	10.24	13.83	13.91
Big	-0.319	-0.043	0.134	0.373	0.511	-13.38	-1.97	5.83	15.30	12.97
			Adj R^2							
Small	0.89	0.89	0.94	0.93	0.94					
2	0.90	0.93	0.92	0.92	0.91					
3	0.92	0.92	0.93	0.91	0.89					
4	0.91	0.92	0.92	0.89	0.83					
Big	0.89	0.89	0.88	0.86	0.77					

[a]*SMB* (small minus big), the return on the mimicking portfolio for the common size factor in stock returns, is the difference each month between the simple average of the percent returns

on the three small-stock portfolios (*S/L*, *S/M*, and *S/H*) and the simple average of the returns on the three big-stock portfolios (*B/L*, *B/M*, and *B/H*). *HML* (high minus low), the return on the mimicking portfolio for the common book-to-market equity factor in returns, is the difference each month between the simple average of the returns on the two high-*BE/ME* portfolios (*S/H* and *B/H*) and the average of the returns on the two low-*BE/ME* portfolios (*S/L* and *B/L*).

The 25 size-*BE/ME* stock portfolios are formed as follows. Each year *t* from 1964 to 1999 NYSE quintile breakpoints for size (*ME*, stock price times shares outstanding), measured at the end of June, are used to allocate NYSE, Amex, and NASDAQ stocks to five size quintiles. Similarly, NYSE quintile breakpoints for *BE/ME* are used to allocate NYSE, Amex, and NASDAQ stocks to five book-to-market equity quintiles. In *BE/ME*, *BE* is book common equity for the fiscal year ending in calendar year $t-1$, and *ME* is for the end of December of $t-1$. The 25 size-*BE/ME* portfolios are formed as the intersections of the five size and the five *BE/ME* groups. Value-weighted percent monthly returns on the portfolios are calculated from July of year *t* to June of $t+1$.

TABLE 8B

Regressions of excess stock returns (in percent) on the market factor and mimicking returns for the size (*SMB*) and duration (*HDMLD*) factors: July 1964 to June 1998.[a]

R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + dHDMLD(t)+e(t)

	Dependent variable: Excess returns on 25 stock portfolios formed on size and Duration quintiles									
Size quintile	High	2	3	4	Low	High	2	3	4	Low
			b					*T-values (b)*		
Small	1.090	1.008	0.960	0.951	0.942	38.14	36.30	50.44	52.94	48.84
2	1.152	1.078	0.963	0.984	1.038	42.29	55.24	51.19	52.06	52.94
3	1.127	1.035	0.959	0.979	1.080	51.88	51.65	49.72	60.21	51.26
4	1.099	1.048	1.017	1.050	1.202	55.54	54.40	55.42	58.18	44.51
Big	0.954	0.982	0.976	1.009	1.108	55.44	60.94	57.09	55.70	42.38
			s					*T-values (s)*		
Small	1.455	1.276	1.097	1.073	1.091	35.14	31.72	39.78	41.24	39.06
2	1.050	0.874	0.781	0.712	0.843	26.63	30.92	28.69	26.01	29.68
3	0.775	0.632	0.487	0.463	0.660	24.64	21.77	17.42	19.68	21.62
4	0.408	0.260	0.226	0.154	0.329	14.25	9.30	8.52	5.88	8.41
Big	-0.234	-0.195	-0.217	-0.162	-0.004	-9.38	-8.34	-8.75	-6.18	-0.10
			d					*T-values (d)*		
Small	0.143	-0.102	-0.285	-0.448	-0.606	3.08	-2.26	-9.25	-15.41	-19.42
2	0.480	0.003	-0.254	-0.497	-0.674	10.88	0.09	-8.33	-16.26	-21.23
3	0.427	-0.048	-0.287	-0.511	-0.669	12.13	-1.48	-9.20	-19.42	-19.63
4	0.486	-0.007	-0.281	-0.513	-0.665	15.16	-0.21	-9.45	-17.56	-15.22
Big	0.581	0.072	-0.211	-0.505	-0.705	20.85	2.77	-7.63	-17.22	-16.66
			Adj R^2							
Small	**0.91**	**0.89**	0.93	**0.93**	0.93					
2	**0.91**	**0.93**	**0.92**	0.91	**0.92**					
3	**0.93**	0.91	0.90	**0.92**	**0.90**					
4	**0.93**	0.91	0.90	**0.90**	**0.85**					
Big	**0.92**	**0.91**	**0.89**	**0.88**	**0.82**					

[a]*SMB* (small minus big), the return on the mimicking portfolio for the common size factor in stock returns, is the difference each month between the simple average of the percent returns on the three small-stock portfolios (*S/L*, *S/M*, and *S/H*) and the simple average of the returns on the three big-stock portfolios (*B/L*, *B/M*, and *B/H*). *HML* (high minus low), the return on the mimicking portfolio for the common book-to-market equity factor in returns, is the difference each month between the simple average of the returns on the two high-*BE/ME* portfolios (*S/H* and *B/H*) and the average of the returns on the two low-*BE/ME* portfolios (*S/L* and *B/L*). *HDMLD* (high minus low), the return on the mimicking portfolio for the duration factor in returns, is the difference each month between the simple average of the returns on the two high-*duration* portfolios (*S/HD* and *B/HD*) and the average of the returns on the two low-*duration* portfolios (*S/LD* and *B/LD*)

The 25 size-*Duration* stock portfolios are formed as follows. Each year *t* from 1964 to 1999 NYSE quintile breakpoints for size (*ME*, stock price times shares outstanding), measured at the end of June, are used to allocate NYSE, Amex, and NASDAQ stocks to five size quintiles. Similarly, NYSE quintile breakpoints for *Duration* are used to allocate NYSE, Amex, and NASDAQ stocks to duration quintiles. The 25 size-*Duration* portfolios are formed as the intersections of the five size and the five *Duration* groups. Value-weighted percent monthly returns on the portfolios are calculated from July of year *t* to June of *t*+1.

TABLE 9
Descriptive statistics (in percent) and correlations between the estimated change in expected return (Δr) and other common factors in stock returns.

Panel A: Univariate Statistics

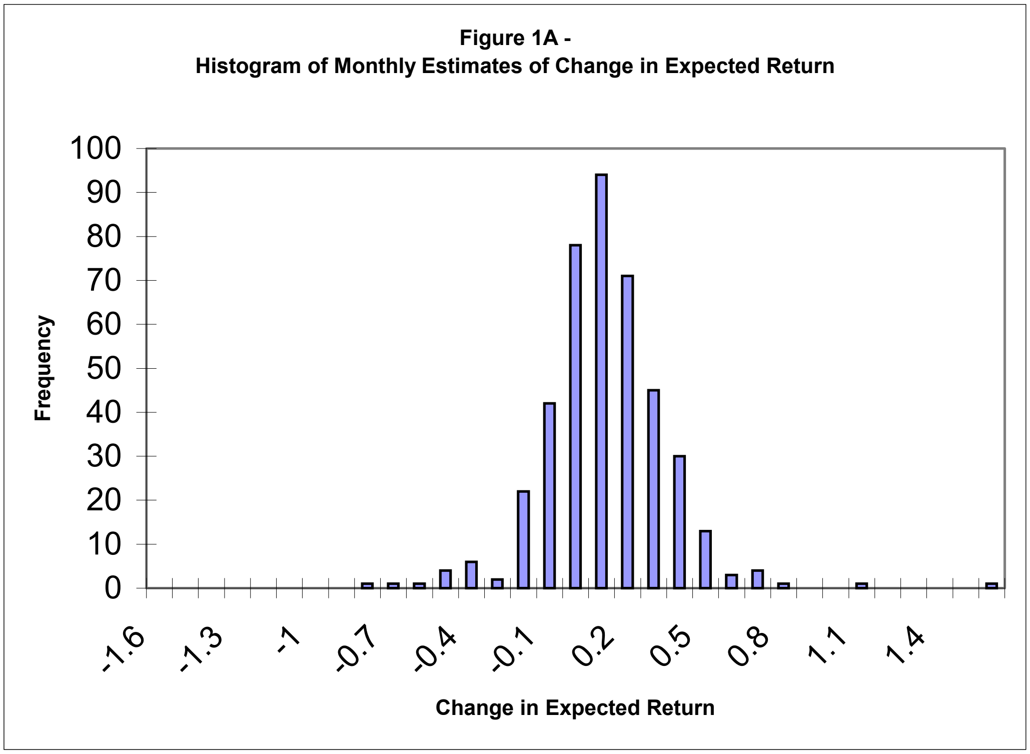
	Obs	Mean	Std. Dev.	Min	Low	Median	Upper	Max
Market [RM-RF]	420	.53	4.30	-22.82	-1.95	.72	3.28	16.00
Excess Long Bond (TERM)	420	.11	3.02	-8.69	-1.63	-.02	1.82	12.02
Change in expected return (Δr)	420	.05	.23	-.82	-.06	.05	.18	1.51
Size (SMB)	420	.27	2.81	-10.01	-1.42	.09	1.98	9.06
Book-to-market (HML)	420	.41	3.17	-14.22	-2.39	-.58	1.34	16.50
Duration (HDMLD)	420	-.50	2.61	-8.69	-2.01	-.56	1.01	9.96

Panel B: Correlations (Pearson above the diagonal, Spearman below the diagonal)

	Market	Term	Δr	SMB	HML	HDMLD
Market	-	0.33 (0.0001)	-0.45 (0.0001)	0.31 (0.0001)	-0.24 (0.0001)	0.35 (0.0001)
Term	0.36 (0.0001)	-	-0.08 (0.1155)	-0.12 (0.0119)	-0.02 (0.6531)	0.01 (0.7888)
Δr	-0.45 (0.0001)	-0.06 (0.2000)	-	-0.28 (0.0001)	0.57 (0.0001)	-0.73 (0.0001)
SMB	0.25 (0.0001)	-0.10 (0.0177)	-0.25 (0.0001)	-	-0.05 (0.0001)	0.17 (0.0001)
HML	-0.26 (0.0001)	-0.06 (0.2019)	0.57 (0.0001)	-0.12 (0.0137)	-	-0.77 (0.0004)
HDMLD	0.33 (0.0001)	0.01 (0.9428)	-0.72 (0.0001)	0.18 (0.0001)	-0.76 (0.0001)	-

Correlation between factor returns, where the factors are the market return, long-term bond return, change in expected return, size, book-to-market, and duration. Pearson Correlation Coefficients in the upper right diagonals and Spearman Correlation Coefficients in the lower left diagonal (p-values in parentheses): July 1964 to December 1999, 420 months. The excess long-bond return (TERM) is computed as the difference between the long-run government bond return and the one-month Treasury bill return. The change in the expected return (Δr) is the estimate of γ from cross sectional regressions of the form: $h_{it} = \alpha_t + \gamma_t \frac{-D_{it}}{(1+r)} + \varepsilon_{it}$. *SMB* (small minus big), the return on the mimicking portfolio for the common size factor in stock returns, is the difference each month between the simple average of the percent returns on the three small-stock portfolios (*S/L*, *S/M*, and *S/H*) and the simple average of the returns on the three big-stock portfolios (*B/L*, *B/M*, and *B/H*). *HML* (high minus low), the return on the mimicking portfolio for the common book-to-market equity factor in returns, is the difference each month between the simple average of the returns on the two high-*BE/ME* portfolios (*S/H* and *B/H*) and the average of the returns on the two low-*BE/ME* portfolios (*S/L* and *B/L*). *HDMLD* (high minus low), the return on the mimicking portfolio for the duration factor in returns, is the difference each month between the simple average of the returns on the two high-*duration* portfolios (*S/HD* and *B/HD*) and the average of the returns on the two low-*duration* portfolios (*S/LD* and *B/LD*).

Figures 1A – 1C
Graphical Illustration of Monthly Estimates of the Change in Expected Return (Δr)



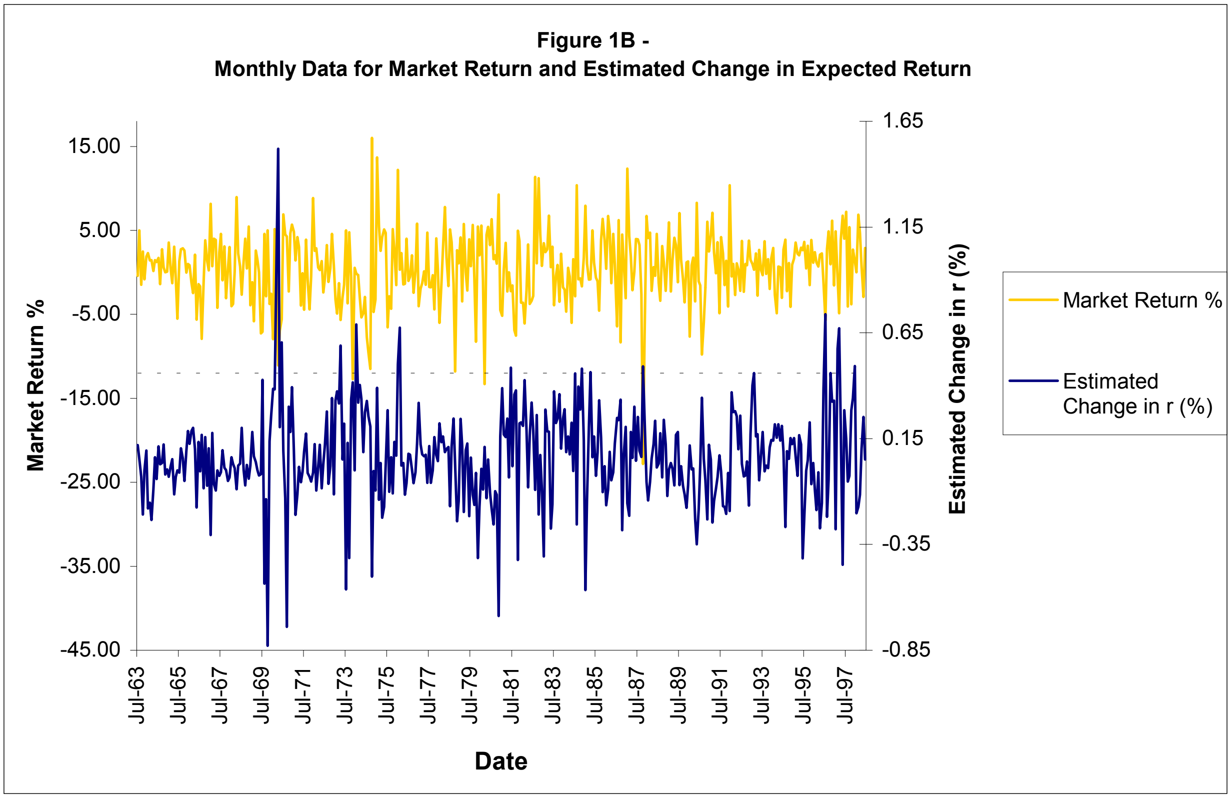


Figure 1B -
Monthly Data for Market Return and Estimated Change in Expected Return
15.00
5.00
-5.00
-15.00
-25.00
-35.00
-45.00
Market Return %
1.65
1.15
0.65
0.15
-0.35
-0.85
Estimated Change in r (%)
Jul-63
Jul-65
Jul-67
Jul-69
Jul-71
Jul-73
Jul-75
Jul-77
Jul-79
Jul-81
Jul-83
Jul-85
Jul-87
Jul-89
Jul-91
Jul-93
Jul-95
Jul-97
Date
Market Return %
Estimated Change in r (%)

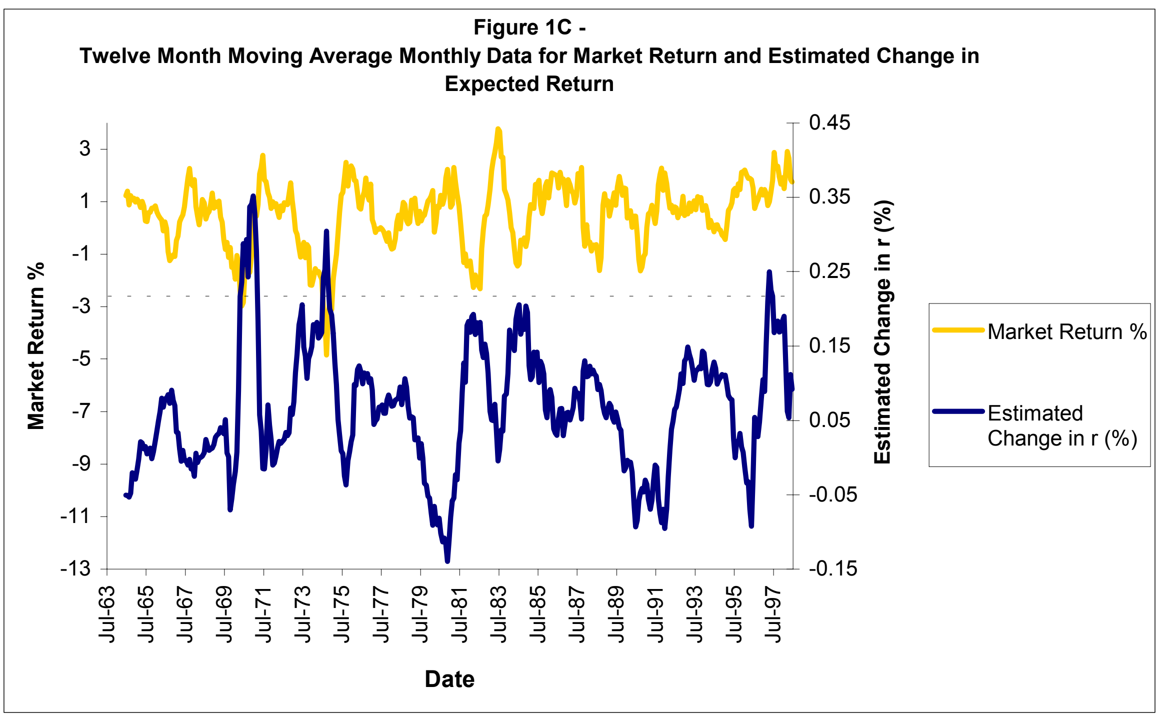


Figure 1C -
Twelve Month Moving Average Monthly Data for Market Return and Estimated Change in Expected Return
3
1
-1
-3
-5
-7
-9
-11
-13
Market Return %
0.45
0.35
0.25
0.15
0.05
-0.05
-0.15
Estimated Change in r (%)
Jul-63
Jul-65
Jul-67
Jul-69
Jul-71
Jul-73
Jul-75
Jul-77
Jul-79
Jul-81
Jul-83
Jul-85
Jul-87
Jul-89
Jul-91
Jul-93
Jul-95
Jul-97
Date
Market Return %
Estimated Change in r (%)

Figure 2

Equity Yield Curve:
Observations are Sorted into Categories Based on Implied Equity Duration. Average Annual Buy-Hold Returns are Computed Starting 4 Months after the Fiscal Year-End for Each Duration Category and Plotted to Give the Equity Yield Curve.