(1) Did you receive any help whatsoever from anyone in solving this assignment? Yes / No. If you answered 'yes', give full details:

Yes. Xiaoyu Bai explained to me how to analyze d-separation for a Bayes network in problem 4. We also discussed some ambiguity in problem 6.

(2) Did you give any help whatsoever to anyone in solving this assignment? Yes / No. If you answered 'yes', give full details:

No.

1.1 Yes, because $P(X, Y) = P(X) * P(Y)$:
$P(X=1, Y=1) = 1/8$, $P(X=1) * P(Y=1) = (3/8 + 1/8) * (1/8 + 1/8) = 1/8$
$P(X=1, Y=0) = 3/8$, $P(X=1) * P(Y=0) = (3/8 + 1/8) * (3/8 + 3/8) = 3/8$
$P(X=0, Y=1) = 1/8$, $P(X=0) * P(Y=1) = (3/8 + 1/8) * (1/8 + 1/8) = 1/8$
$P(X=0, Y=0) = 3/8$, $P(X=0) * P(Y=0) = (3/8 + 1/8) * (3/8 + 3/8) = 3/8$

1.2 No, because there exists case when $P(X|Y, Z) \neq P(X|Z)$, since $P(X=0|Y=0, Z=1) = 0$, while $P(X=0|Z=1) = 1/4$

1.3 No, because there exists case when $P(X|Y, W) \neq P(X|W)$, since $P(X=0|Y=0, W=0) = 3/7$, while $P(X=0|W=0) = 4/7$

1.4 Yes, because $P(X|Y, C) = P(X|Y)P(C) = P(X)P(C)$, and $P(X|C)=P(X)P(C)$, they are equal

2.1 By adding pseudo counts, $P(w_7=1|L=T) = 0.25$, $P(w_7=1|L=F) = 0.5$, $P(w_{10}=1|L=T) = 0.25$, $P(w_{10}=1|L=F) = 0.5$

$P(L_5=T|E_5) = P(w_7=1|L=T) P(w_{10}=1|L=T) P(L=T)/(P(w_7=1|L=T) P(w_{10}=1|L=T) P(L=T) + P(w_7=1|L=F) P(w_{10}=1|L=F) P(L=F)) = 0.5 * 0.25 * 0.25 / (0.5 * 0.25 * 0.25 + 0.5 * 0.5 * 0.5) = 1/5$

$P(L_5=F|E_5) = 4/5$

2.2 By adding pseudo counts, $P(w_2=1|L=T) = 0.75$, $P(w_2=1|L=F) = 0.5$, $P(w_3=1|L=T) = 0.75$, $P(w_3=1|L=F) = 0.25$, $P(w_{11}=1|L=T) = 0.5$, $P(w_{11}=1|L=F) = 0.25$

$P(L_6=T|E_6) = P(w_2=1|L=T) P(w_3=1|L=T) P(w_{11}=1|L=T) P(L=T)/(P(w_2=1|L=T) P(w_3=1|L=T) P(w_{11}=1|L=T) P(L=T) + P(w_2=1|L=F) P(w_3=1|L=F) P(w_{11}=1|L=F) P(L=F)) = 9/10$

$P(L_5=T|E_5) = 1/10$

2.3 By performing the M step, the resulted parameters are

$P(w_3=1|L=T) = 0.7647$, $P(w_3=1|L=F) = 0.2245$, $P(w_{10}=1|L=T) = 0.2353$, $P(w_{10}=1|L=F) = 0.5714$, $P(L=T)= 0.5167$, $P(L=F)= 0.4833$

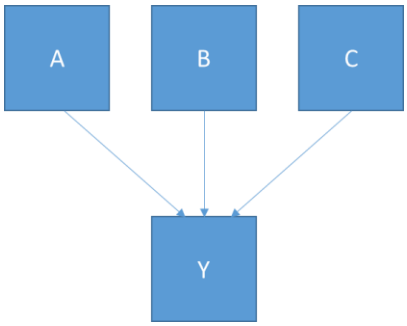
So the classification will be

$P(L=T|E_{final}) = P(w_3=1|L=T) P(w_{10}=1|L=T)P(L=T)/(P(w_3=1|L=T) P(w_{10}=1|L=T)P(L=T) + P(w_3=1|L=F) P(w_{10}=1|L=F)P(L=F)) = 0.6$

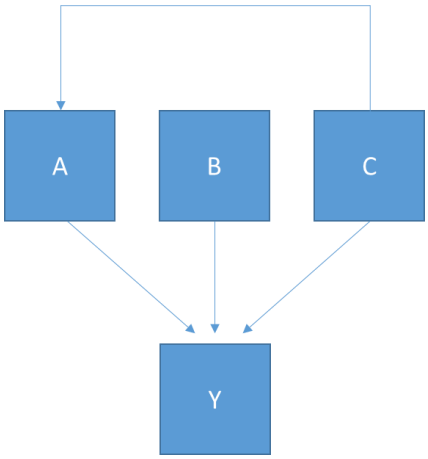
$P(L=F|E_{final}) = 0.4$

It will be classified as true.

3.1 $P(Y,A,B,C) = P(Y|A,B,C)P(A)P(B)P(C)$



3.2 $P(Y,A,B,C) = P(Y|A,B,C)P(A|C)P(C)P(B)$



3.3 {B, C, D, E, F, G, H}

3.4 $P(Y, B, C, D, E, F, G, H) = P(B)P(C)P(E|B,C)P(Y|E)P(H|Y,E)P(G|Y)P(F|Y,D)P(D|A)$

4.1 {A, B, C, E, F, H, I}

4.2 {B, D, G, H, I}

4.3 {A, B, C, D, E, G, H, I}

4.4 {}

4.5 {}

4.6 {I}

5.1 $P(A=F,B=F,C=F,D=F) = P(A=F)P(B=F|A=F)P(C=F|B=F)P(D=F|B=F,C=F) = 0.8*0.6*0.8*0.1 = 0.0384$

5.2 $P(B=T|A=T,C=F) = P(A=T,B=T,C=F)/P(A=T,C=F)$

Let $X = P(A=T,B=T,C=F) = P(A=T,B=T,C=F, D=T) + P(A=T,B=T,C=F, D=F) = P(A=T)P(B=T|A=T)P(C=F|B=T)(P(D=T|B=T,C=F) + P(D=F|B=T,C=F)) = 0.1*0.5*0.2*(0.98+0.02) = 0.01$

Let $Y = P(A=T,B=F,C=F) = P(A=T)P(B=F|A=T)P(C=F|B=F)(P(D=T|B=F,C=F) + P(D=F|B=F,C=F)) = 0.8*0.5*0.2*(0.9+0.1) = 0.08$

So $P(A=T,C=F) = X + Y$, and $P(B=T|A=T,C=F) = X/(X+Y) = 1/9$

So $P(B=F|A=T,C=F) = 8/9$

5.3 Based on $P(A,B,C,D) = P(A)P(B|A)P(C|B)P(D|B,C)$

$P(D=T|A=F) = P(D=T,A=F)/P(A=F) = (P(A=F,B=T,C=T,D=T) + P(A=F,B=T,C=F,D=T) + P(A=F,B=F,C=T,D=T) + P(A=F,B=F,C=F,D=T)) /P(A=F) = (0.8*0.4*0.9*0.99 + 0.8*0.4*0.1*0.98 + 0.8*0.6*0.2*0.95 + 0.8*0.6*0.8*0.9) / 0.8 = 0.9416$

$P(D=F|A=F) = 0.0584$

5.4 Based on $P(A,B,C,D) = P(A)P(B|A)P(C|B)P(D|B,C)$

$P(D=T|B=F) = P(D=T,B=F)/P(B=F) = (P(A=T,B=F,C=T,D=T) + P(A=T,B=F,C=F,D=T) + P(A=F,B=F,C=T,D=T) + P(A=F,B=F,C=F,D=T)) / (P(B=F|A=T) + P(B=F|A=F)) = (0.2*0.5*0.2*0.95 + 0.2*0.5*0.8*0.9 + 0.8*0.6*0.2*0.95 + 0.8*0.6*0.8*0.9) / (0.5*0.2 + 0.6*0.8) = 0.91$

$P(D=F|B=F) = 0.09$

5.5 D is conditionally independent on A given A, B, C. So $P(D=T|A=T,B=T,C=F) = P(D=T|B=T,C=F) = 0.99$

6. For the first E step

1. $P(C=1|A=0,B=1,D=0) = P(A=0,B=1,D=0,C=1)/P(A=0,B=1,D=0) = P(A=0,B=1,D=0,C=1) / (P(A=0,B=1,D=0,C=1) + P(A=0,B=1,D=0,C=0)) = 1/2$

2. $P(C=1|A=0,B=1,D=0) = 1/2$

For the first M step

1. $P(C=1|B=1) = P(C=1,B=1)/P(B=1)$, where $P(C=1, B=1) = (4+1/2)/17$, $P(B=1) = (8+1)/17$, so the result will be $(4+1/2)/9 = 1/2$

$P(C=0|B=1) = 1/2$

2. $P(C=1|B=0) = P(C=1,B=0)/P(B=0) = (4/17)/(8/17) = ½$

$P(C=0|B=0) = 1/2$

3. $P(A=0,B=1,C=0,D=1) = 1/17$

For the second E step

1. $P(C=1|A=0,B=1,D=0) = P(A=0,B=1,D=0,C=1)/P(A=0,B=1,D=0) = (1+1/2)/3 = 1/2$