Simulations of state-of-the-art fermionic neural network wave functions with diffusion Monte Carlo

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Recently developed neural network-based ab-initio solutions (Pfau et. al arxiv:1909.02487v2) for finding ground states of fermionic systems can generate state-of-the-art results on a broad class of systems. In this work, we improve the results for this Ansatz with Diffusion Monte Carlo. Additionally, we introduce several modifications to the network (Fermi Net) and optimization method (Kronecker Factored Approximate Curvature) that reduce the number of required resources while maintaining or improving the modelling performance. In terms of the model, we remove redundant computations and alter the way data is handled in the permutation equivariant function. The Diffusion Monte Carlo results exceed or match state-of-the-art performance for all systems investigated: atomic systems Be-Ne, and the carbon cation C^+ .

Outline

The problem

- What are we trying to do?
- Why would we want to do that?

Related work

- What has been done before?
- What is related now?

The model

- What did we use to model the system?

The algorithms

 What algorithms were used to solve the problem?

Some results

What results did we get?

Next steps

Where is this going?

The problem

Quantum particles/systems

- Superposition (waves)
- **Entanglement** (non-classical correlations)
- Exponential amount of information to model

$$\mathbf{R}_1$$
 \mathbf{r}_2 \mathbf{R}_2 \mathbf{R}_2 \mathbf{R}_3 \mathbf{R}_4 \mathbf{r}_4

$$\hat{H}\psi(X) = E\psi(X)$$

Related work

Solving the QMB with ANNs Carleo & Troyer, Science 10, 2017

- Spin system wave functions
- Boltzmann machine
- First demonstration

SchNet

Schütt et al, J. Chem. Phys. 148, 2019

- Atomic/Molecular properties predictions
- Graph NN

Fermi Net

Pfau et al, Nature Chemistry 12, 2020

- Atomic/molecular wave functions
- Multi-electron orbitals
- Graph NN
- Expensive, but very accurate

PauliNet

Hermann et al, Nature Chemistry 12, 2020

- Atomic/molecular wave functions
- Slater-Jastrow-backflow wave function
- Graph NN
- Fast, but less accurate

Related work

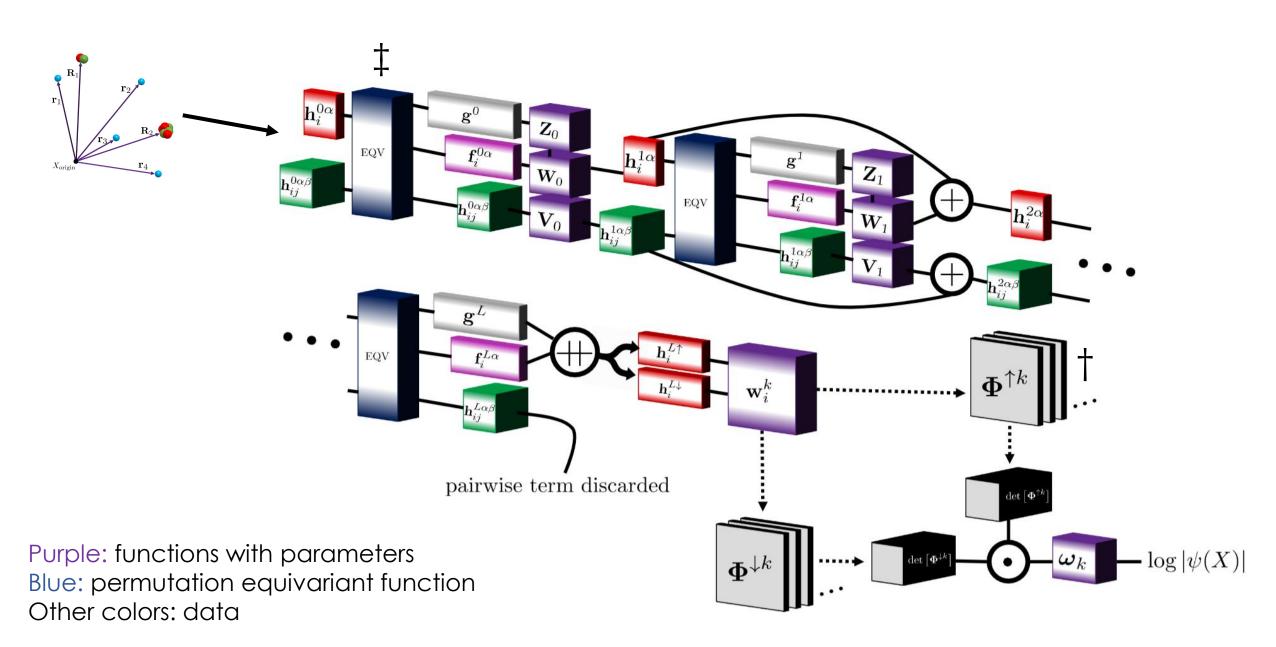
Contributions of this work

- Improvements in efficiency to Fermi Net
- First (known to us) application of DMC to neural network Ansatz
- Matching or exceeding state-of-the-art results on atomic systems Be-Ne and ion C⁺

Other differences

- Changes to the optimisation (KFAC)

The model



The model

$$\downarrow \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \longrightarrow f(x) \longrightarrow \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} \longrightarrow g(z) \longrightarrow \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} \longrightarrow \det \begin{vmatrix} h_0(y_0) & h_0(y_1) & h_0(y_2) \\ h_1(y_0) & h_1(y_1) & h_1(y_2) \\ h_2(y_0) & h_2(y_1) & h_2(y_2) \end{bmatrix} \longrightarrow \psi(x)$$

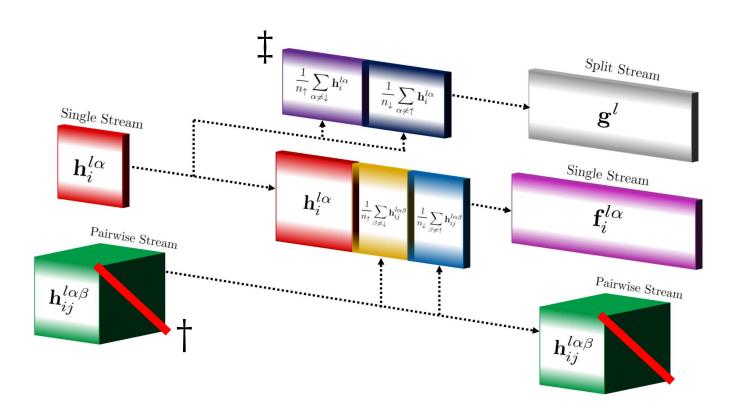
$$\begin{bmatrix} x_1 \\ x_0 \\ x_2 \end{bmatrix} \longrightarrow f(x) \longrightarrow \begin{bmatrix} z_1 \\ z_0 \\ z_2 \end{bmatrix} \longrightarrow g(z) \longrightarrow \begin{bmatrix} y_1 \\ y_0 \\ y_2 \end{bmatrix} \longrightarrow \det \begin{vmatrix} h_0(y_1) & h_0(y_0) & h_0(y_2) \\ h_1(y_1) & h_1(y_0) & h_1(y_2) \\ h_2(y_1) & h_2(y_0) & h_2(y_2) \end{bmatrix} \longrightarrow -\psi(x)$$

Swapping inputs results in change of sign of the wave function = antisymmetric = fermionic system

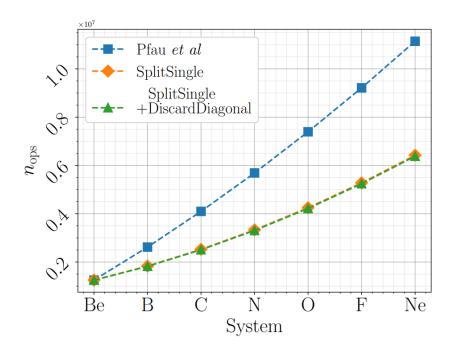
- Pfaffian wave function https://pubmed.ncbi.nlm.nih.gov/16711968/
- Vandermonde wave function https://ieeexplore.ieee.org/document/9297114/

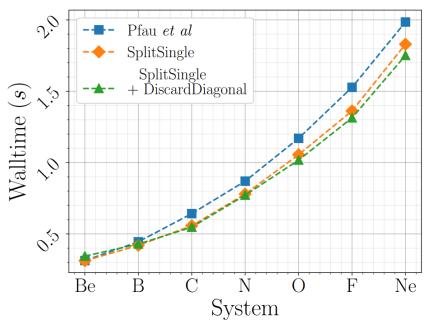
- ...

The model



- \dagger Remove redundant diagonal elements i=j
- ‡ Split data that is the same in all indices





The algorithms

Variational Monte Carlo <u>VMC</u>

Iterative improvement of the wave function Ansatz by changing the parameters performed under the Born-Oppenheimer approximation (fixed nuclei)

Kronecker Factored
Approximate Curvature

KFAC

Approximate second order optimization related to stochastic reconfiguration

Diffusion Monte Carlo **DMC**

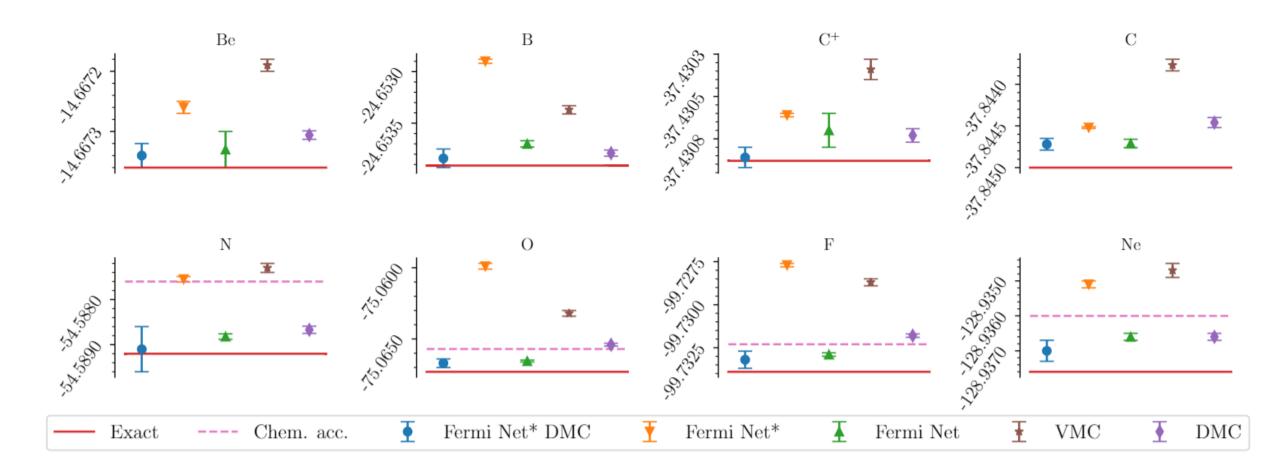
Projector method iteratively improving the wave function by reweighting the walkers performed under the fixed-node approximation

Approximate stochastic reconfiguration

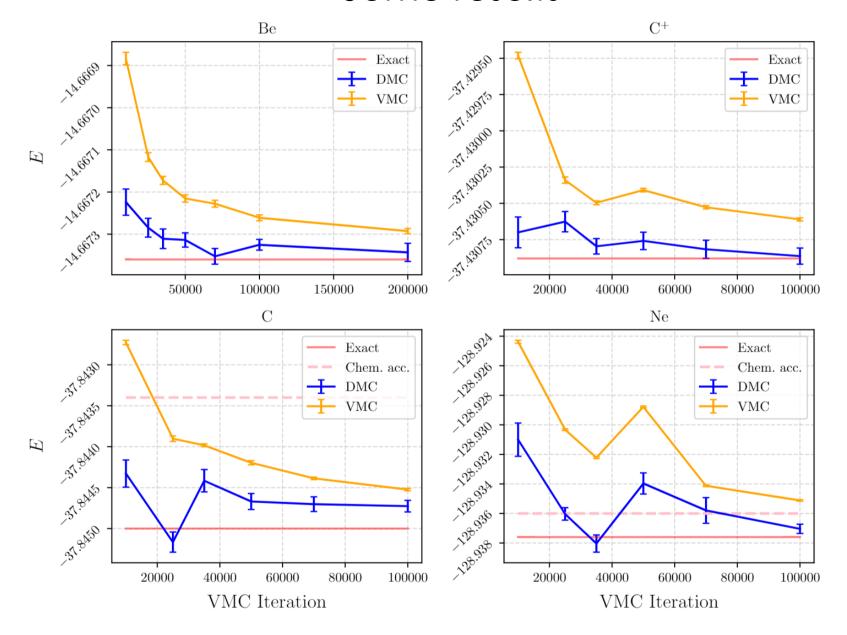
Some results

Atom	Fermi Net* + DMC	Fermi Net*	Fermi Net [7]	VMC [31]	DMC [31]	HF	Exact [32]
	(This work)	(This work)					
$\overline{\ }$ Be	-14.66734(2)	-14.66726(1)	-14.66733(3)	-14.66719(1)	-14.667306(7)	-14.35188	-14.66736
$_{\mathrm{B}}$	-24.65384(9)	-24.65290(2)	-24.65370(3)	-24.65337(4)	-24.65379(3)	-24.14899	-24.65391
C^{+}	-37.43086(6)	-37.43061(1)	$-37.4307(1)^{\dagger}$	-37.43034(6)	-37.43073(4)	-36.87037	-37.43088
\mathbf{C}	-37.84472(7)	-37.84452(1)	-37.84471(5)	-37.84377(7)	-37.84446(6)	-37.08959	-37.8450
N	-54.5891(5)	-54.58755(6)	-54.58882(6)	-54.5873(1)	-54.58867(8)	-53.5545	-54.5892
O	-75.0667(3)	-75.0599(2)	-75.06655(7)	-75.0632(2)	-75.0654(1)	-73.6618	-75.0673
\mathbf{F}	-99.7332(5)	-99.7277(1)	-99.7329(1)	-99.7287(2)	-99.7318(1)	-97.9865	-99.7339
Ne	-128.9370(3)	-128.9351(1)	-128.9366(1)	-128.9347(2)	-128.9366(1)	-126.6045	-128.9376

Some results



Some results



Next steps

Improved Ansatz design

- Can other changes be made to Fermi Net?
- Other wave functions?

Improved VMC

- CG SR?
- KFAC features?
- Mixed precision training?

Other systems

- Solids?
- Where else is VMC used?

Other VMC methods

- Pseudopotentials?

Scaling

- Most efficient way to model large systems?
- What are the technical challenges of scaling?

Final Comments

Long-term vision

- Infrastructure (GPUs and deep learning frameworks) will continue to improve
- Quantum chemistry neural network methods will adopt other standard quantum chemistry methods
- Neural network methods will supplement or replace standard quantum chemistry methods

Takeaways

- We improved the Fermi Net by changing how data was handled and extended with diffusion Monte Carlo
- There is a long-term case for investing in neural network method for modelling wave functions