Repayment calculate

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1. Yearly Rate to Monthly Rate

P: principle . R^y = yearly Rate R^m = monthly Rate

$\because P \times (1+R^y) = P(1+R^m)^{12}$ | OR $R^m = R^y/12$?

$\therefore R^m = (1+R^y)^{1/12} - 1$

2. Compound Interest Repayment Calculate

$r = R^m$ n = term months P: principle

PV = present value M: montly Repayment.

(How to get the formula for M calculate)

We first analyse it month by month.

month	we will talk about the interest I_k of every month and remain P_k and after repay P_k'
1	$I_1 = P * r$ $P_1 = P * (1+r)$ repay m. then end of month 1 $P_1' = P_1 - m = P(1+r) - m$
2	$I_2 = P_1' * r$ $P_2 = P_1' * (1+r)$ $P_2' = P_2 - m = P_1'(1+r) - m$ $= [P(1+r) - m](1+r) - m$ $= P(1+r)^2 - m(1+r) - m$ $= P(1+r)^2 - m(1+r+1) = P(r+1)^2 - m[1+(1+r)]$
3	$I_3 = P_2' * r$ $P_3 = P_2'(1+r)$ $P_3' = P_2'(1+r) - m$ $= [P(1+r)^2 - m[1+(1+r)]](1+r) - m$ $= P(1+r)^3 - m[1+(1+r)](r+1) - m = P(r+1)^3 - m[(1+r)^2 + (1+r) + 1]$
4	$P_4' = P_3'(1+r) - m = P(1+r)^4 - m[(r+1)^2 + (r+1) + 1][r+1] - m$ $= P(r+1)^4 - m[(r+1)^3 + (r+1)^2 + (r+1) + 1]$
⋮ k	$P_k' = P(r+1)^k - m\boxed{[(r+1)^{k-1} + (r+1)^{k-2} + \cdots (r+1) + 1]}$
⋮	$P_n' = P(r+1)^n - m\underbrace{[(r+1)^{n-1} + (r+1)^{n-2} + \cdots (r+1)^1 + 1]}_{n\text{. item}}$ $\dfrac{1[1-(r+1)^n]}{1-(r+1)}$
n	$P_n' = P(r+1)^n - m\dfrac{(r+1)^n - 1}{r} = 0$

Important, we need to construct geometric progression to calculate sum.

sum of geometric progression $S_n = \dfrac{a_1(1-q^n)}{1-q}$

$a_1 = 1$
$q = r+1$

① $S_n = a_1 + a_2 \cdots a_n$, q: common ratio

② $qS_n = a_1q + a_2q \cdots a_nq$

$\because a_1q = a_2 \quad a_2q = a_3 \cdots$

$\therefore qS_n = \boxed{a_2 + a_3 \cdots + a_n} + \boxed{a_nq}$

& $\because a_n = a_1q^{n-1}$

$\therefore a_nq = a_1q^n$

$\therefore$ ③ $qS_n = \boxed{(S_n - a_1)} + \boxed{a_1q^n}$

$\therefore S_n = \dfrac{a_1q^n - a_1}{q-1}$

$= \dfrac{a_1(1-q^n)}{1-q}$

End of period · term $P_n' = 0$, so we need to deduce P_n' by p, r, m

$P_n' = f(p, r, m) = 0$.

$$\therefore m = \frac{P(r+1)^n \cdot r}{(r+1)^n - 1}$$

$$= \frac{pr}{1 - \dfrac{1}{(r+1)^n}}$$

$$= \frac{pr}{1 - (r+1)^{-n}}$$

This formula is usually used in programming