Simulation and Timeline

July 15, 2021

1 Robust TFBoost: Simulation

1.1 Data generation

- We generated data sets $D = \{(x_i, y_i), i = 1, ..., N\}$, consisting of a predictor $x_i \in \mathcal{L}_2$ and a scalar response y_i that follow the model:

$$y_i = r(x_i) + \rho\epsilon_i, \tag{1}$$

where the errors ϵ_i are i.i.d, r is the regression function, and $\rho > 0$ is a constant that controls the signal-to-noise ratio (SNR):

$$\text{SNR} = \frac{\text{Var}(r(X))}{\text{Var}(\rho\epsilon)}.$$

- To sample the functional predictors x_i, we considered the model:

$$x_i(t) = \mu(t) + \sum_{p=1}^{4} \sqrt{\lambda_j}\xi_{ij}\phi_j(t), \tag{2}$$

where $\mu(t) = 2\text{sin}(t\pi)\text{exp}(1-t)$, $\lambda_1 = 0.8, \lambda_2 = 0.3, \lambda_3 = 0.2$, and $\lambda_4 = 0.1$, $\xi_{ij} \sim N(0, 1)$, and ϕ_j are the first four eigenfunctions of the "Mattern" covariance function $\gamma(s, t)$ with parameters $\rho = 3, \sigma = 1, \nu = 1/3$:

$$\gamma(s,t) = C\left(\frac{\sqrt{2\nu}|s-t|}{\rho}\right),\ C(u) = \frac{\sigma^2 2^{1-\nu}}{\Gamma(\nu)}u^\nu K_\nu(u),$$

where $\Gamma(.)$ is the Gamma function and K_ν is the modified Bessel function of the second kind. For each subject i, we evaluate x_i on a dense and regular grid $t_1, ..., t_{100}$ equally spaced in $\mathcal{I} = [0, 1]$.

- We considered five regression functions:
  - $r_1(X) = \int_{\mathcal{I}} \left(\sin\left(\frac{3}{2}\pi t\right) + \sin\left(\frac{1}{2}\pi t\right)\right) X(t)dt$,

- $r_2(X) = (\xi_1 + \xi_2)^{1/3}$, where $\xi_1 = \int_{\mathcal{I}}(X(t) - \mu(t))\psi_1(t)dt$ and $\xi_2 = \int_{\mathcal{I}}(X(t) - \mu(t))\psi_2(t)dt$ are projections onto the first two FPCs (ψ_1 and ψ_2) of X with mean $\mu(t) = E(X(t))$,
- $r_3(X) = 5\exp\left(-\frac{1}{2}\left|\int_{\mathcal{I}} x(t)\log(|x(t)|)dt\right|\right)$,
- $r_4(X) = 5\text{sigmoid}\left(\int_{\mathcal{I}} X(t)^2\sin(2\pi t)dt\right)$, where $\text{sigmoid}(u) = 1/(1 + \exp(-u))$, and
- $r_5(X) = 5\left(\sqrt{\left|\int_{\mathcal{I}_1}\cos(2\pi t^2)X(t)dt\right|} + \sqrt{\left|\int_{\mathcal{I}_2}\sin(X(t))dt\right|}\right)$, where $\mathcal{I}_1 = [0, 0.5]$ and $\mathcal{I}_2 = (0.5, 1]$.

- For clean data (C_0), we generated ϵ_i in (1) from $N(0, 1)$ and selected ρ that corresponds to SNR = 5.

  For contaminated data, we sampled 10% training samples as outliers and let the set of their indices be I_o. The outliers belong to one of the five types introduced below. For $j \in I_o$,

  - C_1: *Shape outliers*

    In (1), $\epsilon_j \sim N(10, 0.25)$
    In (2), $\xi_{j,2} \sim N(10, 0.25)$ and the other parameters stay the same.

  - C_2: *Magnitude outliers*

    $x_j = 2\tilde{x}_j, y_j = 4\tilde{y}_j$, where $(\tilde{x}_j, \tilde{y}_j)$ were generated as clean data.

  - C_3: *Point-type measurement error outliers*

    Randomly sample 10 points form $t_1, ..., t_{100}$ and denote them as $t_{j,o_1}, ..., t_{j,o_{10}}$. For $k = 1, ..., 10$,

    $$x_j(t_{j,o_k}) = \tilde{x}_j(t_{j,o_k}) + \eta_{j,o_k},$$

    where $\eta_{j,o_k} \sim 0.5N(10, 0.25) + 0.5N(-10, 0.25)$, $y_j = \tilde{y}_j$, and $(\tilde{x}_j, \tilde{y}_j)$ were generated as clean data.

  - C_4: *Interval-type measurement error outliers*

    Randomly sample one interval from intervals $[t_1, ..., t_{10}]$, ...,$[t_{91}, ..., t_{100}]$, and denote the interval as $t_{j,o}, ..., t_{j,o+9}$
    For $k = 0, ..., 9$,

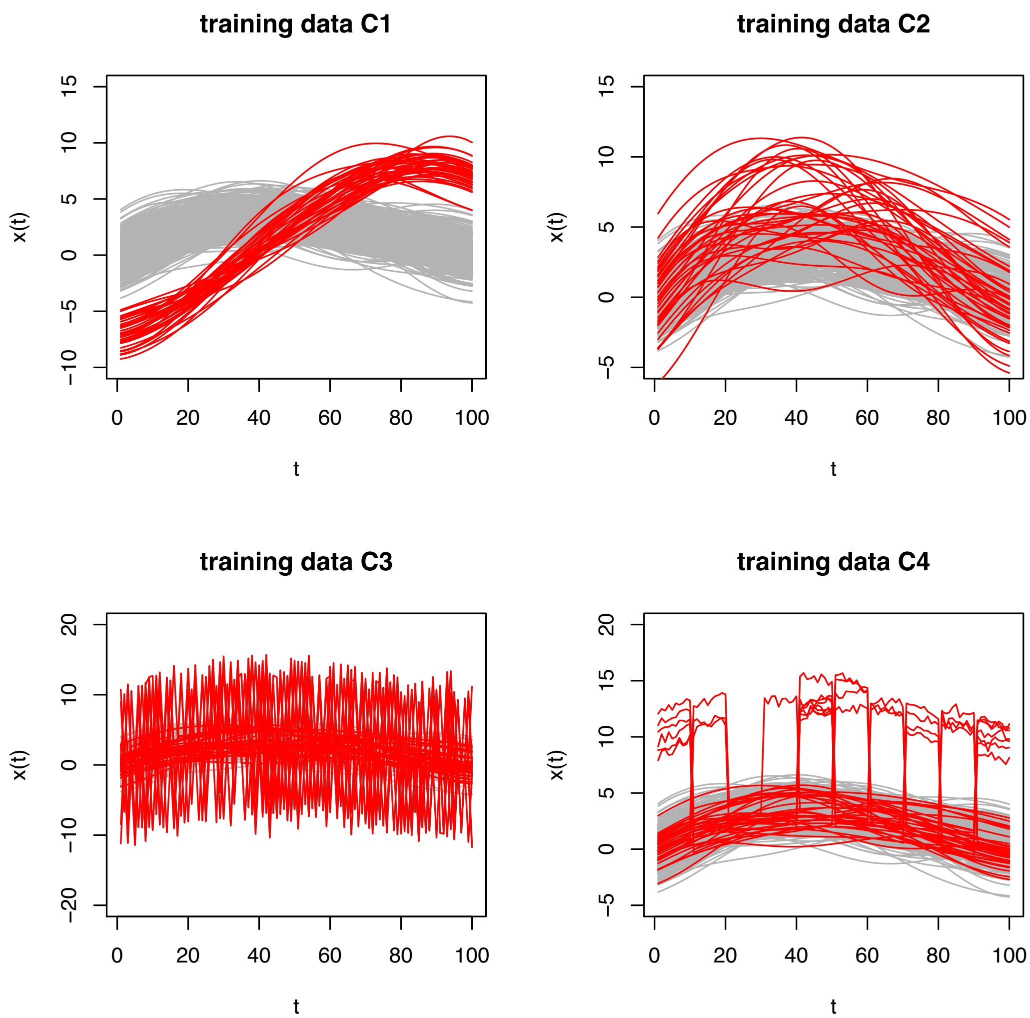
    $$x_j(t_{j,o+k}) = \tilde{x}_j(t_{j,o+k}) + \eta_{j,o+k},$$

    where $\eta_{j,o+k} \sim N(10, 0.25)$, $y_j = \tilde{y}_j$, and $(\tilde{x}_j, \tilde{y}_j)$ were generated as clean data.

  - C_5: *Pure vertical outliers*

    $$\epsilon_j \sim N(10, 0.25)$$

1.2 Visualize the outliers



1.3 Model comparison

For each setting, we used 100 independently generated datasets and compared the performance of the following methods:

- `TFBoost(L2)`: tree-based functional boosting with L2 loss
- `TFBoost(LAD)`: tree-based functional boosting with LAD loss
- `TFBoost(RR)`: tree-based functional boosting modified to follow the framework of RRBoost

- `FPPR`: functional projection pursuit regression (Ferraty et al., 2013),
- `FGAM`: functional generalized additive models (McLean et al., 2014),
- `MFLM`: Sieve M-estimator for a semi-functional linear model Huang et al. (2015)
- `RFSIR`: robust functional sliced inverse regression (Wang et al., 2017)
- `RFPLM`: robust estimation for semi-functional linear regression models (Boente et al., 2020)

1.4 Results

	C_0	C_1	C_2	C_3	C_4	C_5
TFBoost(L2)	0.144 (0.006)	0.151 (0.009)	0.206 (0.030)	0.144 (0.006)	0.146 (0.006)	1.690 (0.271)
TFBoost(LAD)	0.151 (0.010)	0.202 (0.027)	0.205 (0.030)	0.150 (0.008)	0.151 (0.008)	**0.158** (0.012)
TFBoost(RR)	0.162 (0.017)	0.193 (0.136)	0.215 (0.110)	0.158 (0.014)	0.157 (0.012)	0.159 (0.015)
FPPR	0.137 (0.007)	0.202 (0.076)	0.164 (0.050)	0.137 (0.007)	0.149 (0.013)	1.845 (0.517)
FGAM	**0.130** (0.005)	**0.143** (0.008)	**0.153** (0.016)	**0.130** (0.005)	**0.133** (0.005)	1.205 (0.080)
RFPLM	**0.130** (0.006)	**0.130** (0.006)	**0.130** (0.006)	**0.130** (0.006)	**0.131** (0.006)	**0.130** (0.006)
MFLM	**0.129** (0.006)	0.761 (0.054)	0.269 (0.033)	**0.130** (0.006)	0.138 (0.006)	0.166 (0.014)
RFSIR	0.137 (0.008)	0.145 (0.014)	0.157 (0.025)	0.138 (0.007)	0.142 (0.006)	1.727 (0.587)

Table 1: Summary statistics of test errors for data generated from r_1; displayed in the form of mean (sd).

	C_0	C_1	C_2	C_3	C_4	C_5
TFBoost(L2)	**0.181** (0.008)	**0.193** (0.010)	0.223 (0.023)	**0.183** (0.009)	**0.184** (0.010)	1.789 (0.347)
TFBoost(LAD)	0.186 (0.010)	0.257 (0.056)	0.208 (0.020)	0.188 (0.011)	**0.188** (0.011)	**0.195** (0.011)
TFBoost(RR)	0.196 (0.014)	0.225 (0.063)	**0.198** (0.019)	0.202 (0.021)	0.198 (0.023)	**0.203** (0.020)
FPPR	**0.181** (0.009)	0.347 (0.133)	0.288 (0.058)	**0.183** (0.011)	0.196 (0.024)	1.886 (0.545)
FGAM	0.226 (0.012)	0.243 (0.015)	0.276 (0.027)	0.233 (0.013)	0.233 (0.012)	1.343 (0.094)
RFPLM	0.286 (0.014)	0.286 (0.014)	0.290 (0.016)	0.287 (0.014)	0.288 (0.017)	0.286 (0.014)
MFLM	0.285 (0.014)	2.032 (0.099)	0.325 (0.028)	0.389 (0.023)	0.626 (0.032)	0.344 (0.024)
RFSIR	0.183 (0.009)	**0.218** (0.061)	**0.202** (0.015)	0.185 (0.010)	0.193 (0.013)	1.551 (0.569)

Table 2: Summary statistics of test errors for data generated from r_2; displayed in the form of mean (sd).

1.5 Timeline

- 2021/07:

	C_0	C_1	C_2	C_3	C_4	C_5
TFBoost(L2)	**0.305** (0.016)	**0.314** (0.020)	0.518 (0.090)	**0.306** (0.015)	**0.308** (0.016)	1.968 (0.360)
TFBoost(LAD)	0.319 (0.019)	0.382 (0.036)	0.383 (0.049)	0.317 (0.018)	0.318 (0.016)	**0.326** (0.021)
TFBoost(RR)	0.333 (0.032)	0.370 (0.059)	**0.337** (0.041)	0.337 (0.040)	0.328 (0.028)	**0.335** (0.027)
FPPR	**0.303** (0.018)	0.446 (0.105)	0.606 (0.360)	0.313 (0.022)	0.318 (0.022)	1.845 (0.453)
FGAM	0.319 (0.017)	0.331 (0.017)	0.442 (0.061)	0.321 (0.016)	0.319 (0.017)	1.445 (0.112)
RFPLM	0.380 (0.018)	0.379 (0.019)	0.381 (0.018)	0.379 (0.019)	0.382 (0.019)	0.379 (0.019)
MFLM	0.377 (0.018)	1.365 (0.080)	0.485 (0.046)	0.886 (0.063)	2.165 (0.129)	0.445 (0.028)
RFSIR	0.310 (0.019)	**0.310** (0.019)	**0.337** (0.030)	**0.311** (0.020)	**0.311** (0.017)	1.825 (0.677)

Table 3: Summary statistics of test errors for data generated from r_3; displayed in the form of mean (sd).

	C_0	C_1	C_2	C_3	C_4	C_5
TFBoost(L2)	**0.321** (0.015)	**0.333** (0.015)	0.681 (0.322)	**0.324** (0.014)	**0.328** (0.014)	2.037 (0.267)
TFBoost(LAD)	**0.338** (0.017)	0.404 (0.026)	0.552 (0.161)	**0.340** (0.014)	**0.345** (0.017)	**0.361** (0.026)
TFBoost(RR)	0.347 (0.036)	0.490 (0.273)	0.591 (0.667)	0.365 (0.036)	0.374 (0.048)	**0.360** (0.040)
FPPR	0.362 (0.029)	0.417 (0.045)	0.538 (0.271)	0.384 (0.040)	0.415 (0.043)	1.960 (0.386)
FGAM	0.408 (0.019)	0.417 (0.018)	**0.491** (0.054)	0.415 (0.017)	0.411 (0.019)	1.659 (0.151)
RFPLM	0.544 (0.032)	0.544 (0.031)	0.544 (0.032)	0.555 (0.040)	0.561 (0.039)	0.543 (0.032)
MFLM	0.538 (0.030)	0.544 (0.032)	0.826 (0.104)	0.645 (0.037)	0.827 (0.048)	0.630 (0.048)
RFSIR	0.341 (0.020)	**0.363** (0.021)	**0.421** (0.154)	0.348 (0.018)	0.354 (0.024)	2.415 (0.512)

Table 4: Summary statistics of test errors for data generated from r_4; displayed in the form of mean (sd).

	C_0	C_1	C_2	C_3	C_4	C_5
TFBoost(L2)	**0.583** (0.032)	**0.628** (0.045)	1.725 (0.678)	**0.593** (0.033)	**0.590** (0.036)	2.322 (0.278)
TFBoost(LAD)	0.622 (0.034)	0.694 (0.055)	1.292 (0.315)	**0.633** (0.039)	**0.634** (0.030)	**0.677** (0.066)
TFBoost(RR)	0.694 (0.092)	0.869 (0.307)	1.280 (1.596)	0.703 (0.083)	0.723 (0.084)	**0.686** (0.077)
FPPR	**0.608** (0.057)	0.718 (0.175)	0.967 (0.911)	0.638 (0.049)	0.673 (0.073)	2.364 (0.482)
FGAM	0.610 (0.041)	**0.670** (0.070)	**0.776** (0.100)	0.643 (0.046)	0.641 (0.048)	1.909 (0.127)
RFPLM	0.891 (0.045)	1.045 (0.078)	0.890 (0.045)	0.889 (0.046)	0.895 (0.047)	0.888 (0.045)
MFLM	0.881 (0.039)	1.125 (0.067)	1.698 (0.189)	1.421 (0.080)	2.527 (0.118)	0.998 (0.052)
RFSIR	0.677 (0.052)	0.690 (0.063)	**0.821** (0.183)	0.672 (0.052)	0.650 (0.053)	2.379 (0.518)

Table 5: Summary statistics of test errors for data generated from r_5; displayed in the form of mean (sd).

- TFBoost: revise paper (submit?)
  - Robust TFBoost: simulation
  - thesis: draft the background chapter

- 2021/08:
  - TFBoost: submit paper and package
  - thesis: draft the background, RRBoost and TFBoost chapters
  - Robust TFBoost: simulation and real example
  - record JSM presentation

- 2021/09:
  - thesis: draft Robust TFBoost chapter
  - Sparse TFBoost: simulation

- 2021/09:
  - thesis: draft robust TFBoost, Sparse TFBoost chapters
  - Sparse TFBoost: simulation and real example

- 2021/10:
  - thesis: draft Sparse TFBoost chapter, conclusion and future work

- 2021/11:
  - thesis: first draft complete, start revising

- 2021/12 (end of year):
  - thesis: second draft

- Before 2022/04:
  - thesis defence

References

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