Probability And Mathmatical Statistics
Homework 8

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2.

记同时工作的终端数为 X，则 $X \sim B(120,0.05)$,$EX=6$,$DX=npq=5.7$.
由拉普拉斯中心极限定理，X 近似服从 $N(6,5.7)$.

$$
\begin{aligned}
P(X \geq 10) &= 1-P(X<10) \\
&= 1-P(\frac{X-6}{\sqrt{5.7}} \leq \frac{4}{\sqrt{5.7}}) \\
&= 1-\Phi(1.68) \\
&= 0.04750
\end{aligned}
$$

4

(1)
记每个舍入误差为 X_i,$X=\sum X_i$ 则 $EX_i=0$,$DX_i=\frac{1}{12}$.
由独立同分布情形的中心极限定理，X 近似服从 $N(0,125)$.

$$
\begin{aligned}
&P(|X| \leq 15) \\
&= P(-15 \leq X \leq 15) \\
&= P(-\frac{3}{\sqrt{5}} \leq \frac{X}{\sqrt{125}} \leq \frac{3}{\sqrt{5}}) \\
&= \Phi(\frac{3}{\sqrt{5}})-\Phi(-\frac{3}{\sqrt{5}}) \\
&= 2\Phi(\frac{3}{\sqrt{5}})-1 \\
&= 0.8198 \\
&P(|X| \geq 15)=1-P(|X| \leq 15)=0.1802
\end{aligned}
$$

(2)
设最多可有 n 个数相加使得误差总和的绝对值小于 10 的概率不小于 0.96. 令 $X=\sum_{i=1}^n X_i$.$EX=0$,$DX=\frac{n}{12}$. 由中心极限定理，$X \sim^{\text{近似}} N(0,\frac{n}{12})$.

$$
\begin{aligned}
&P(|X| \leq 10) \\
&= P(-10 \leq X \leq 10) \\
&= P(-\frac{10}{\sqrt{\frac{n}{12}}} \leq \frac{X}{\sqrt{\frac{n}{12}}} \leq \frac{10}{\sqrt{\frac{n}{12}}}) \quad = 2\Phi(\frac{10}{\sqrt{\frac{n}{12}}})-1 > 0.96
\end{aligned}
$$

即 $\Phi(\frac{10}{\sqrt{\frac{n}{12}}}) > 0.98$. 查表得

$$
\begin{aligned}
&\frac{10}{\sqrt{\frac{n}{12}}} \geq 2.06 \\
&n \leq 282.78 \\
&n \leq 282
\end{aligned}
$$

所以最多 282 个数相加使得误差总和的绝对值小于 10 的概率不小于 0.96.

5.

$$EX_k=\int_0^1 6x^2(1-x)=\frac{1}{2}$$

$$EX_k^2=\int_0^1 6x^3(1-x)=\frac{3}{10}$$

$$DX_k=EX_k^2-(EX_k)^2=\frac{3}{10}-\frac{1}{4}=\frac{1}{20}$$

$$\because \frac{1}{n^2}D(\sum X_k)=\frac{1}{20n} \to 0(n \to \infty)$$

$\therefore$ 由马尔科夫大数定理, 序列 $\{X_n\}$ 服从大数定理.

$$\therefore \frac{1}{n}\sum_{k=1}^n X_k \xrightarrow{P} \frac{1}{n}\sum_{k=1}^n EX_k=\frac{1}{2}$$

7.

$$
\begin{aligned}
E(lnX_i) &= \int_0^1 (lnx) * 1dx \\
&= xlnx|_0^1 - \int_0^1 xdlnx \\
&= -1
\end{aligned}
$$

因为 $E(lnX_i)$ 有限,从而 $\{lnX_i\}$ 服从辛钦大数定律。

$$
\begin{aligned}
& lnZ_n = \frac{1}{n}\sum_{i=1}^{n} lnX_i \xrightarrow{P} -1 \\
& \because f(x) = e^x \text{连续} \\
& \therefore Z_n \xrightarrow{P} \frac{1}{e}
\end{aligned}
$$

补充

已知 $g(.,.)$ 在 (a,b) 处连续，即有

$\forall\epsilon > 0, \exists\delta > 0, |x-a| < \delta, |y-b| < \delta,$ 有 $|g(x,y)-g(a,b)| < \epsilon$

已知 $X_n \xrightarrow{P} a$,$Y_n \xrightarrow{P} b$, 所以有

$$
\begin{aligned}
& \forall\epsilon > 0 \\
& \lim_{n\to\infty} P(|X_n - a| < \epsilon) = 1 \\
& \lim_{n\to\infty} P(|Y_n - b| < \epsilon) = 1 \\
& \therefore \lim_{n\to\infty} P(|X_n - a| < \epsilon, |Y_n - b| < \epsilon) = 1 \qquad (*)
\end{aligned}
$$

下面证明 $g(X_n, Y_n) \xrightarrow{P} g(a,b)$. 取 (*) 式中 $\epsilon = \delta$, 有

$$
P(|X_n - a| < \delta, |Y_n - b| < \delta) = 1
$$

$$
\begin{aligned}
& \forall\epsilon > 0 \\
& 1 = \lim_{n\to\infty} P(|x - a| < \delta, |y - b| < \delta) \\
& \leq \lim_{n\to\infty} P(|g(X_n, Y_n) - g(a,b)| < \epsilon) \text{由连续的定义} \\
& = 1
\end{aligned}
$$

$$
\therefore \forall\epsilon > 0 \lim_{n\to\infty} P(|g(X_n, Y_n) - g(a,b)| < \epsilon) = 1
$$

$$
\therefore g(X_n, Y_n) \xrightarrow{P} g(a,b)
$$