1 Introduction

The programms contained in 'dstpy3' package can be used to calculate the direct scattering transform for the nonlinear Schrödinger equation. For extensive description, see the papers

- G. Boffetta and A. R. Osborne: Computation of the Direct Scattering Transform for the Nonlineare Schroedinger Equation, Journal of Comp. Phys. **102**, p 252-264, (1992)
  `http://personalpages.to.infn.it/~boffetta/Papers/bo92.pdf`
- Mansoor I. Yousefi, Frank R. Kschischang:
  - Information Transmission using the Nonlinear Fourier Transform, Part I `https://arxiv.org/abs/1202.3653`
  - Information Transmission using the Nonlinear Fourier Transform, Part II `https://arxiv.org/abs/1204.0830`
  - Information Transmission using the Nonlinear Fourier Transform, Part III `https://arxiv.org/abs/1302.2875`

2 What the algorithms *should* do

For a given field, the direct scattering transform shall be calculated. The problem can be formulated as eigenvalue problem

$$v_t = \begin{pmatrix} -j\lambda & q(x) \\ -q^*(x) & j\lambda \end{pmatrix} v$$

with the eigenvalue λ. Assuming the normalized time and field vectors $q(x)$ and x are present in discretized form. For an eigenvalue candidate ζ, v can be calculated by integration:

$$v[k+1] = v[k] + \mathrm{d}x \cdot P[k]v[k] \tag{1}$$

with

$$P[k] = \begin{pmatrix} -j\zeta & q[k] \\ -q^*[k] & j\zeta \end{pmatrix} \tag{2}$$

and the starting value

$$v[0] = \begin{pmatrix} \exp(j\zeta\, x_{\max}) \\ 0 \end{pmatrix} \quad . \tag{3}$$

The scattering coefficients $a(\zeta)$ and $b(\zeta)$ then can be calculated from v:

$$a(\zeta) = v_1(x_{\max}) \exp(j\zeta \cdot x_{\max}) \tag{4}$$
$$b(\zeta) = v_2(x_{\max}) \exp(-j\zeta \cdot x_{\max}) \tag{5}$$

The candidate ζ can is an eigenvalue, when $a(\zeta) = 0$ (or very small, when performing the integration numerically).

For root finding, it is desirable to have the derivative of a. To do this, one needs to calculate

$$\frac{\mathrm{d}a}{\mathrm{d}\zeta} = -jx_{\mathrm{max}} \exp(-j\zeta \cdot x_{\mathrm{max}}) \cdot v_1'(\zeta) \quad . \tag{6}$$

The derivative of v with respect to ζ is v', which can be integrated with

$$v'[k+1] = v'[k] + \mathrm{d}x \left(P'[k]v[k] + P[k]v'[k]\right) \tag{7}$$

and the initial value

$$v'[0] = \begin{pmatrix} jx_{\mathrm{max}} \exp(j\zeta \cdot x_{\mathrm{max}}) \\ 0 \end{pmatrix} \quad . \tag{8}$$

From this, one gets

$$a'(\zeta) = \left(v_1'(x_{\mathrm{max}}) + jx_{\mathrm{max}} \cdot v_1(x_{\mathrm{max}})\right) \exp(j\zeta \cdot x_{\mathrm{max}}) \quad . \tag{9}$$

To simultaneously integrate v and v', one can solve

$$\begin{pmatrix} v[k+1] \\ v'[k+1] \end{pmatrix} = \begin{pmatrix} v[k] \\ v'[k] \end{pmatrix} + \mathrm{d}x \cdot \begin{pmatrix} P[k] & 0 \\ P'[k] & P[k] \end{pmatrix} \begin{pmatrix} v[k] \\ v'[k] \end{pmatrix} \quad . \tag{10}$$

To find some eigenvalue with $a(\lambda) = 0$ from a starting value ζ, one can follow the root-finding procedure:

$$\zeta_{k+1} = \zeta_k - \frac{a(\zeta_k)}{a'(\zeta_k)} \tag{11}$$