Semantic Argument (proof rules)

- negation:

$$\frac{I \models \neg\varphi}{I \not\models \varphi} \qquad \frac{I \not\models \neg\varphi}{I \models \varphi}$$

- conjunction:

$$\frac{I \models \varphi \wedge \psi}{\begin{array}{l} I \models \varphi \\ I \models \psi \end{array}} \qquad \frac{I \not\models \varphi \wedge \psi}{I \not\models \varphi \quad | \quad I \not\models \psi}$$

('|' forks computation in two branches that both need to be proved)

- disjunction:

$$\frac{I \models \varphi \vee \psi}{I \models \varphi \quad | \quad I \models \psi} \qquad \frac{I \not\models \varphi \vee \psi}{\begin{array}{l} I \not\models \varphi \\ I \not\models \psi \end{array}}$$

Semantic Argument (proof rules)

- implication:

$$\frac{I \models \varphi \to \psi}{I \not\models \varphi \quad | \quad I \models \psi} \qquad \frac{I \not\models \varphi \to \psi}{\begin{array}{c} I \models \varphi \\ I \not\models \psi \end{array}}$$

- iff:

$$\frac{I \models \varphi \leftrightarrow \psi}{I \models \varphi \wedge \psi \quad | \quad I \not\models \varphi \vee \psi} \qquad \frac{I \not\models \varphi \leftrightarrow \psi}{I \models \varphi \wedge \neg\psi \quad | \quad I \models \neg\varphi \wedge \psi}$$

- contradiction:

$$\frac{I \models \varphi \qquad I \not\models \varphi}{I \models \bot}$$

Semantic Argument (modus ponens)

Modus ponens (MP) is the following useful rule:

$$\frac{I \models F \qquad I \models F \to G}{I \models G}$$

MP is sometimes also called implication elimination.