Semantic Argument for FOL

To decide validity of FOL formulae, we extend the semantic argument method from PL using the following proof rules:

- universal quantification 1: $\dfrac{I \models \forall x\, \varphi}{I \triangleleft \{x \mapsto t\} \models \varphi}$ for *any term* t

- existential quantification 1: $\dfrac{I \not\models \exists x\, \varphi}{I \triangleleft \{x \mapsto t\} \not\models \varphi}$ for *any term* t

  In practice, we often choose t that was already introduced earlier (in order to obtain a contradiction); t can contain an eigenvariable.

- universal quantification 2: $\dfrac{I \not\models \forall x\, \varphi}{I \triangleleft \{x \mapsto v\} \not\models \varphi}$ for a *fresh eigenvariable* v

- existential quantification 2: $\dfrac{I \models \exists x\, \varphi}{I \triangleleft \{x \mapsto v\} \models \varphi}$ for a *fresh eigenvariable* v

  The value v cannot have been used in the proof before.

The *eigenvariables* are not interpreted; they are *symbolic names*/*metavariables*.

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- contradiction:

$$\frac{\begin{array}{c} J : I \triangleleft \cdots \models p(s_1, \ldots, s_n) \\ K : I \triangleleft \cdots \not\models p(t_1, \ldots, t_n) \end{array}}{I \models \bot} \qquad \text{for } 1 \leq i \leq n : \alpha_J[s_i] = \alpha_K[t_i]$$

- rules for $(=)$ will be introduced in the next lecture (about theories)