# Chapter 5
# Network Layer: Control Plane

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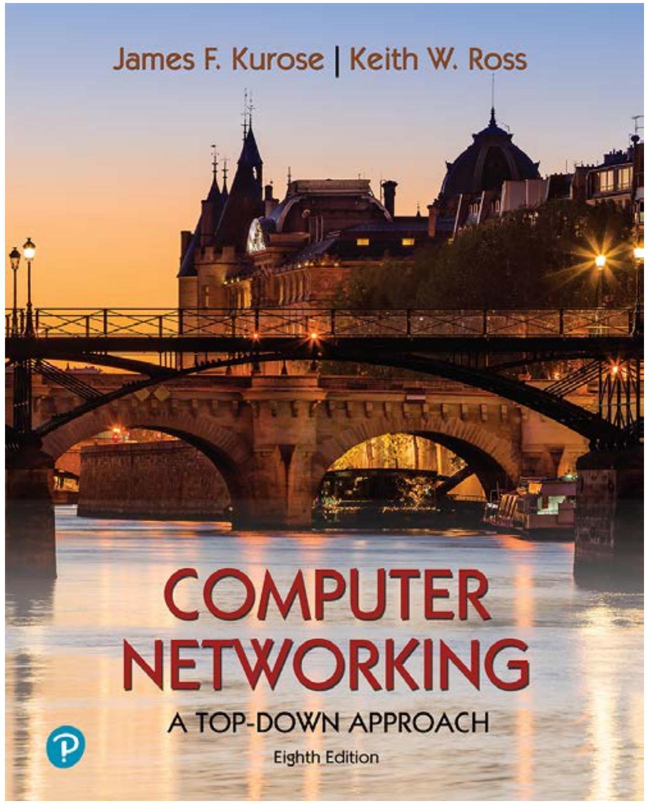
Thanks and enjoy! JFK/KWR

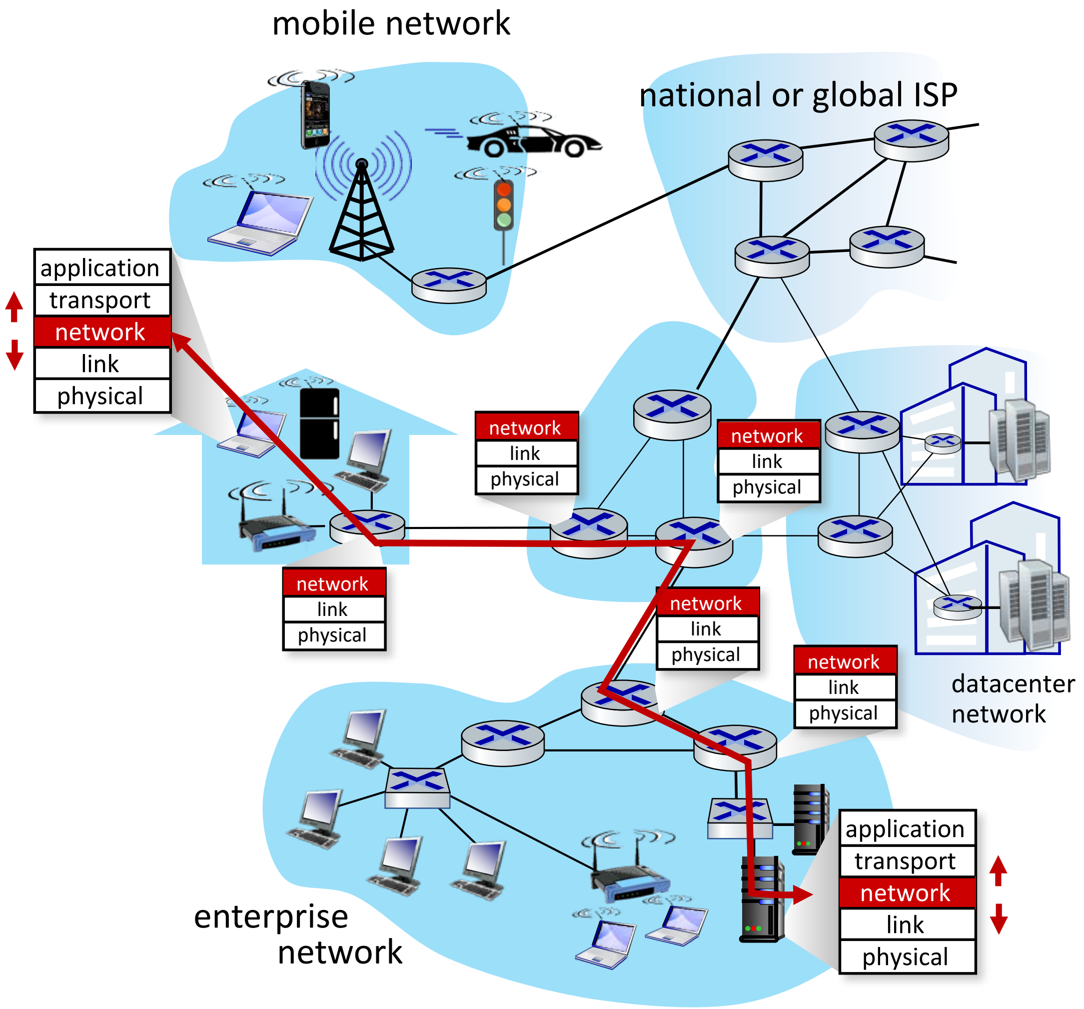





*Computer Networking: A Top-Down Approach*
8th edition
Jim Kurose, Keith Ross
Pearson, 2020

# Network-layer services and protocols

- transport segment from sending to receiving host
  - sender: encapsulates segments into datagrams, passes to link layer
  - receiver: delivers segments to transport layer protocol
- network layer protocols in *every Internet device*: hosts, routers
- routers:
  - examines header fields in all IP datagrams passing through it
  - moves datagrams from input ports to output ports to transfer datagrams along end-end path



# Two key network-layer functions

network-layer functions:

- *forwarding:* move packets from a router's input link to appropriate router output link
- *routing:* determine route taken by packets from source to destination
  - *routing algorithms*

analogy: taking a trip

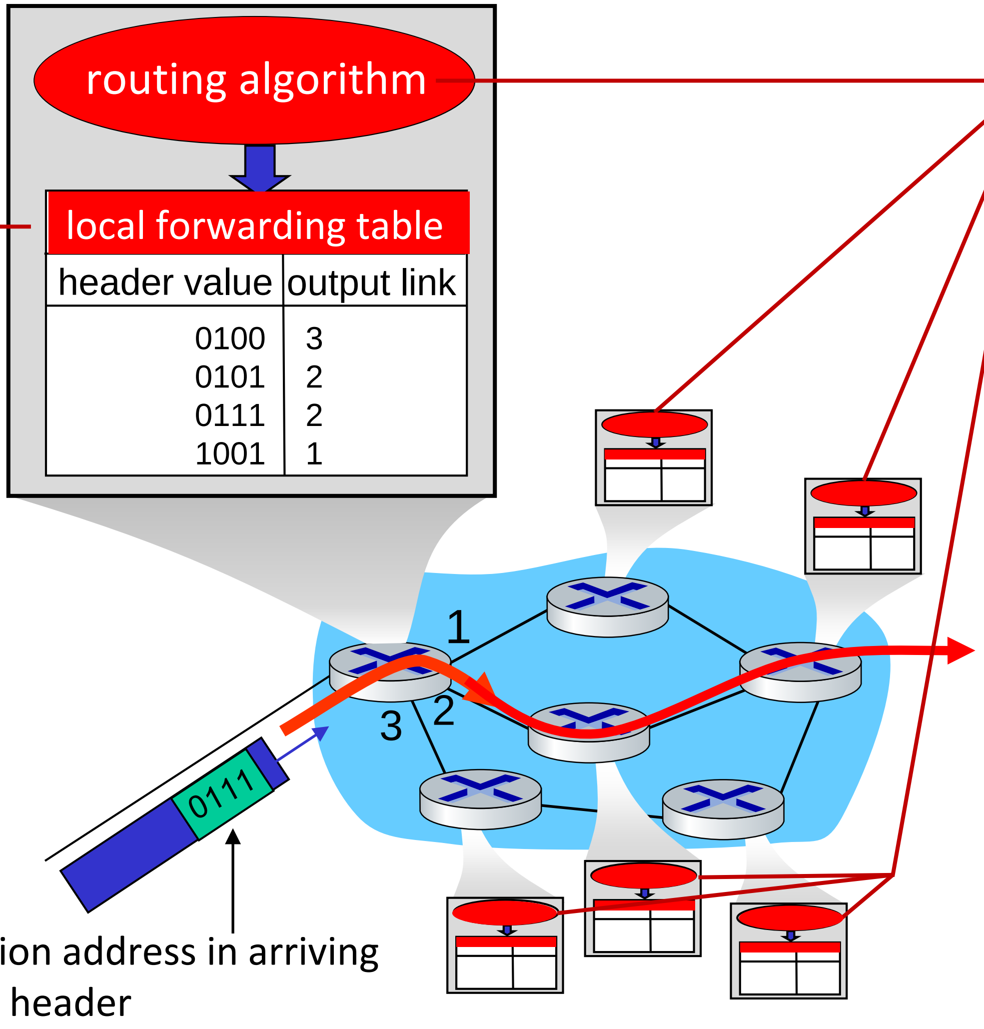
- *forwarding:* process of getting through single interchange
- *routing:* process of planning trip from source to destination

forwarding

routing

# Two key network-core functions

*Forwarding:*
- aka "switching"
- *local* action: move arriving packets from router's input link to appropriate router output link

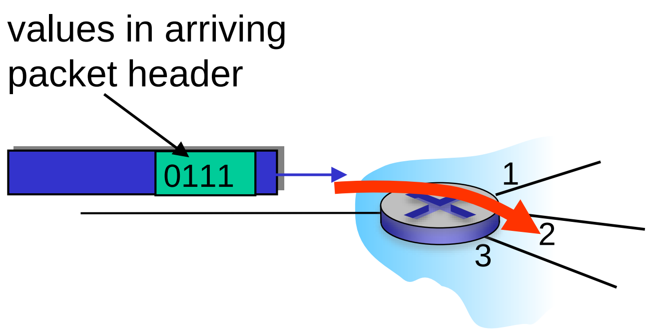



*Routing:*
- *global* action: determine source-destination paths taken by packets
- routing algorithms

# Network layer: data plane, control plane

## Data plane:

- *local*, per-router function
- determines how datagram arriving on router input port is forwarded to router output port




## Control plane

- *network-wide* logic
- determines how datagram is routed among routers along end-end path from source host to destination host

# Network-layer functions

- forwarding: move packets from router's input to appropriate router output

*data plane*

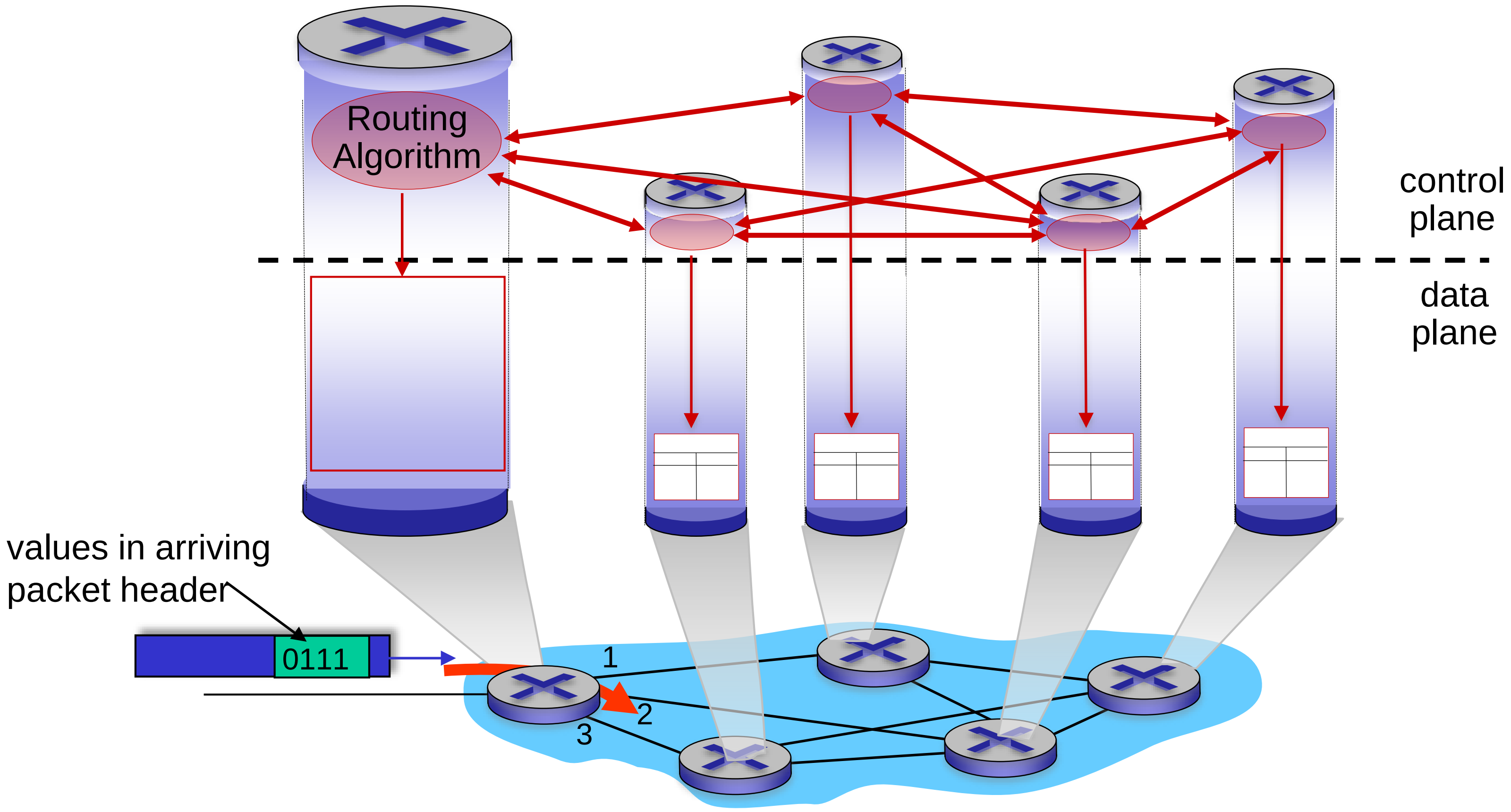
- routing: determine route taken by packets from source to destination

*control plane*

- two control-plane approaches:
  - *traditional routing algorithms:* implemented in routers
  - *software-defined networking (SDN)*: implemented in (remote) servers

# Per-router control plane

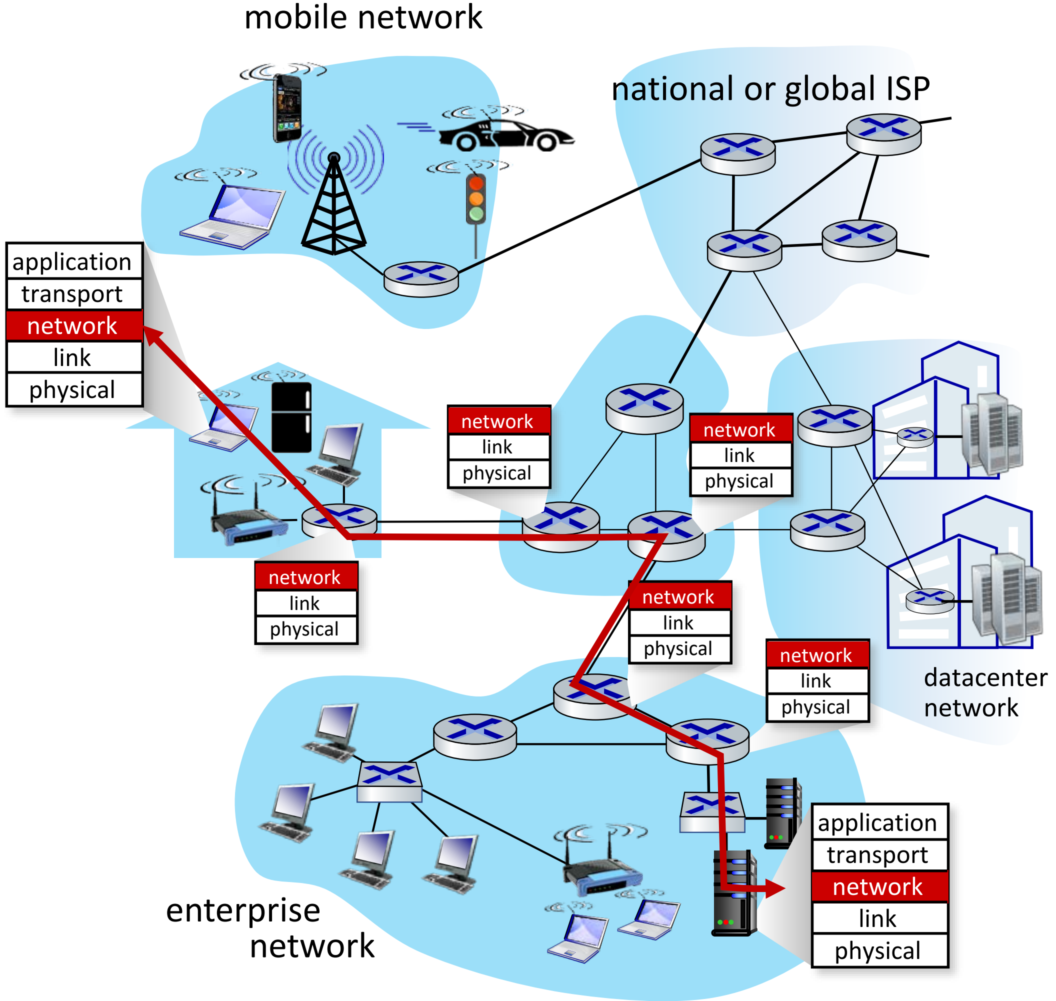
Individual routing algorithm components *in each and every router* interact in the control plane



# Routing protocols

Routing protocol goal: determine "good" paths (equivalently, routes), from sending hosts to receiving host, through network of routers

- path: sequence of routers packets traverse from given initial source host to final destination host
- "good": least "cost", "fastest", "least congested"
- routing: a "top-10" networking challenge!



# Network-layer service model

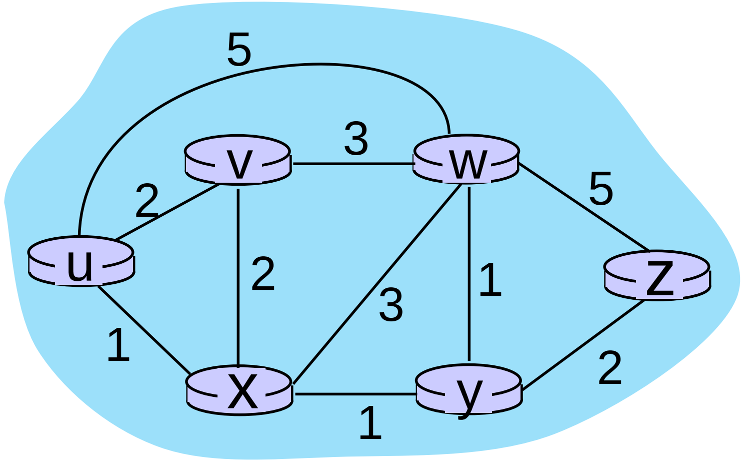
| Network Architecture | Service Model | Quality of Service (QoS) Guarantees ? | | | |
|---|---|---|---|---|---|
| | | Bandwidth | Loss | Order | Timing |
| Internet | best effort | none | no | no | no |

**Internet "best effort" service model**

*No* guarantees on:

i. successful datagram delivery to destination
ii. timing or order of delivery
iii. bandwidth available to end-end flow

# Graph abstraction: link costs




$c_{a,b}$: cost of *direct* link connecting *a* and *b*

*e.g.*, $c_{w,z} = 5$, $c_{u,z} = \infty$

cost defined by network operator: could always be 1, or inversely related to bandwidth, or inversely related to congestion

graph: $G = (N,E)$

$N$: set of routers = { $u, v, w, x, y, z$ }

$E$: set of links ={ $(u,v), (u,x), (v,x), (v,w), (x,w), (x,y), (w,y), (w,z), (y,z)$ }

# Routing algorithm classification

**global:** all routers have *complete* topology, link cost info
- "link state" algorithms

*How fast do routes change?*

**static:** routes change slowly over time

**dynamic:** routes change more quickly
- periodic updates or in response to link cost changes

**decentralized:** iterative process of computation, exchange of info with neighbors
- routers initially only know link costs to attached neighbors
- "distance vector" algorithms

*global or decentralized information?*

# Dijkstra's link-state routing algorithm

- centralized: network topology, link costs known to *all* nodes
  - accomplished via "link state broadcast"
  - all nodes have same info
- computes least cost paths from one node ("source") to all other nodes
  - gives *forwarding table* for that node
- iterative: after *k* iterations, know least cost path to *k* destinations

## notation

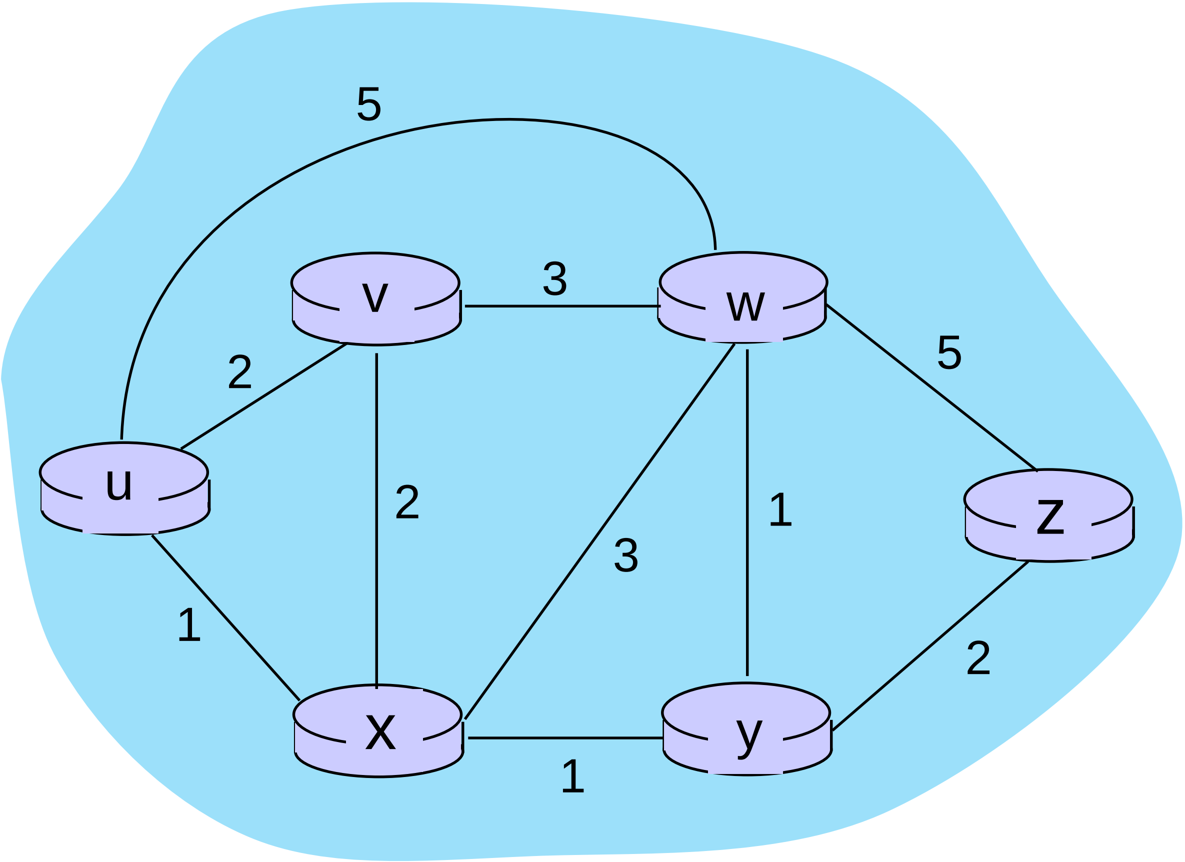
- $c_{x,y}$: direct link cost from node *x* to *y*; = ∞ if not direct neighbors
- *D(v):* *current* estimate of cost of least-cost-path from source to destination *v*
- *p(v):* predecessor node along path from source to *v*
- *N':* set of nodes whose least-cost-path *definitively* known

# Dijkstra's link-state routing algorithm

*Initialization:*
  $N' = \{u\}$ /* compute least cost path from u to all other nodes */
  for all nodes $v$
    if $v$ adjacent to $u$ /* $u$ initially knows direct-path-cost only to direct neighbors */
      then $D(v) = c_{u,v}$ /* but may not be *minimum* cost! */
    else $D(v) = \infty$

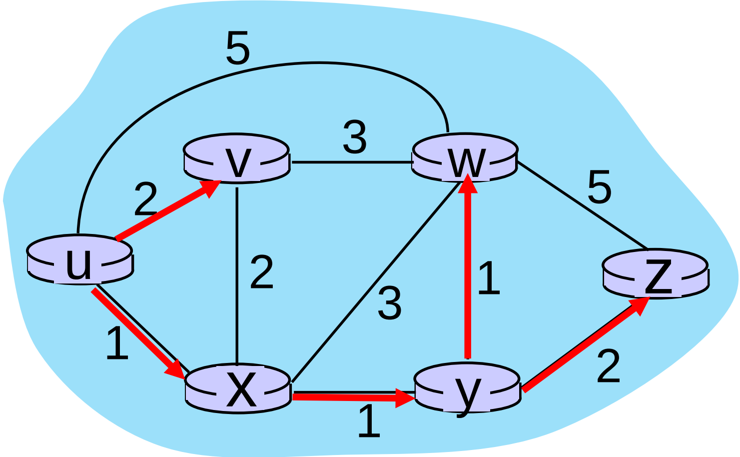
*Loop*
  find $w$ not in $N'$ such that $D(w)$ is a minimum
  add $w$ to $N'$
  update $D(v)$ for all $v$ adjacent to $w$ and not in $N'$ :
    $D(v) = \min(D(v),\ D(w) + c_{w,v})$
  /* new least-path-cost to $v$ is either old least-cost-path to $v$ or known
  least-cost-path to $w$ plus direct-cost from $w$ to $v$ */
*until all nodes in $N'$*

# Routing Algorithm: An Example



# Dijkstra's algorithm: an example

| Step | N' | D(v),p(v) | D(w),p(w) | D(x),p(x) | D(y),p(y) | D(z),p(z) |
|---|---|---|---|---|---|---|
| 0 | u | 2,u | 5,u | 1,u | ∞ | ∞ |
| 1 | ux | 2,u | 4,x | | 2,x | ∞ |
| 2 | uxy | 2,u | 3,y | | | 4,y |
| 3 | uxyv | | 3,y | | | 4,y |
| 4 | uxyvw | | | | | 4,y |
| 5 | uxyvwz | | | | | |

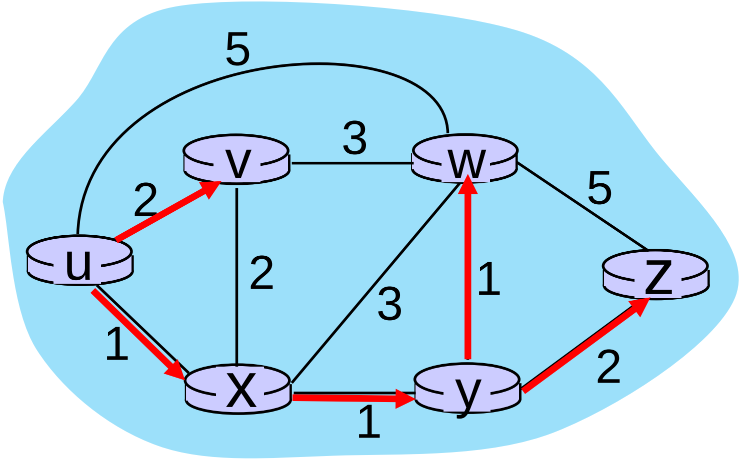



Initialization (step 0): For all a: if *a* adjacent to then $D(a) = c_{u,a}$

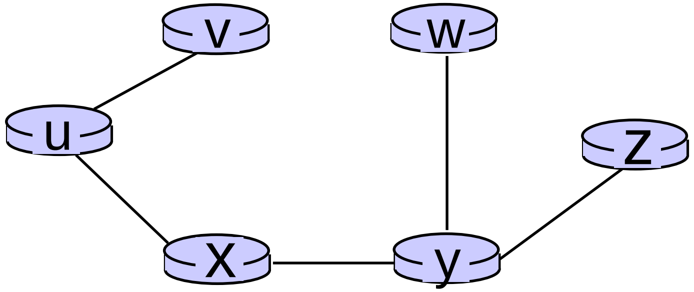
find *a* not in *N'* such that *D(a)* is a minimum
add *a* to *N'*
update *D(b)* for all *b* adjacent to *a* and not in *N'* :

**$D(b) = \min(D(b), D(a) + c_{a,b})$**

# Dijkstra's algorithm: an example




resulting least-cost-path tree from u:




resulting forwarding table in u:

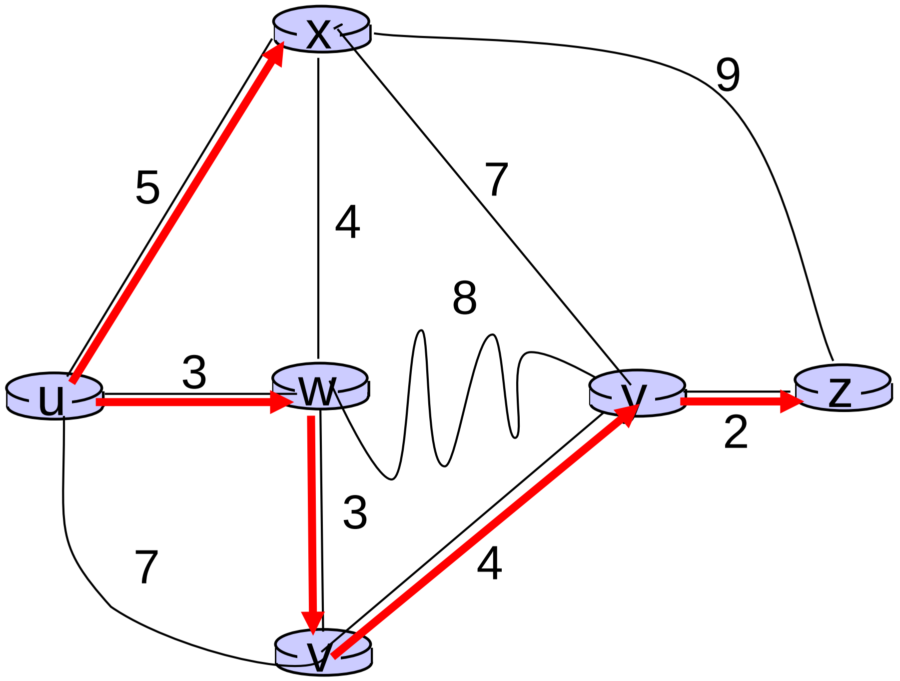
| destination | outgoing link |
|---|---|
| v | (u,v) |
| x | (u,x) |
| y | (u,x) |
| w | (u,x) |
| x | (u,x) |

route from *u* to *v* directly

route from u to all other destinations via *x*

# Dijkstra's algorithm: another example

| Step | N' | v<br>D(v), p(v) | w<br>D(w), p(w) | x<br>D(x), p(x) | y<br>D(y), p(y) | z<br>D(z), p(z) |
|---|---|---|---|---|---|---|
| 0 | u | 7,u | 3,u | 5,u | ∞ | ∞ |
| 1 | uw | 6,w | | 5,u | 11,w | ∞ |
| 2 | uwx | 6,w | | | 11,w | 14,x |
| 3 | uwxv | | | | 10,v | 14,x |
| 4 | uwxvy | | | | | 12,y |
| 5 | uwxvyz | | | | | |




## notes:

- construct least-cost-path tree by tracing predecessor nodes
- ties can exist (can be broken arbitrarily)

# Distance vector algorithm

Based on *Bellman-Ford* (BF) equation (dynamic programming):

Bellman-Ford equation

Let $D_x(y)$: cost of least-cost path from $x$ to $y$.
Then:

$$D_x(y) = \min_v \{ c_{x,v} + D_v(y) \}$$

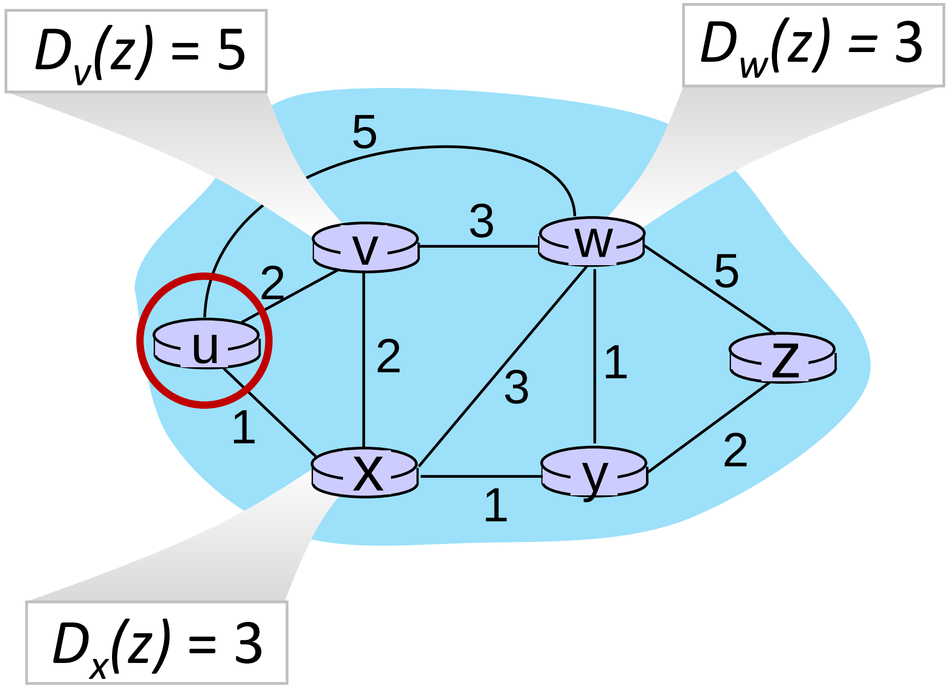
*v*'s estimated least-cost-path cost to *y*

*min* taken over all neighbors *v* of *x*

direct cost of link from *x* to *v*

# Bellman-Ford Example

Suppose that *u*'s neighboring nodes, *x,v,w,* know that for destination *z*:




Bellman-Ford equation says:

$$D_u(z) = \min \{ c_{u,v} + D_v(z), \; c_{u,x} + D_x(z), \; c_{u,w} + D_w(z) \}$$

$$= \min \{2 + 5, \; 1 + 3, \; 5 + 3\} = 4$$

*node achieving minimum (x) is next hop on estimated least-cost path to destination (z)*

# Distance vector algorithm

key idea:

- from time-to-time, each node sends its own distance vector estimate to neighbors
- when *x* receives new DV estimate from any neighbor, it updates its own DV using B-F equation:

$$D_x(y) \leftarrow \min_v\{c_{x,v} + D_v(y)\} \quad \text{for each node } y \in N$$

- under minor, natural conditions, the estimate $D_x(y)$ *converge to the actual least cost* $d_x(y)$

# Distance vector algorithm:

each node:

*wait* for (change in local link cost or msg from neighbor)

↓

*recompute* DV estimates using DV received from neighbor

↓

if DV to any destination has changed, *notify* neighbors

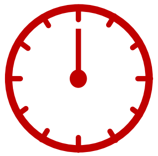
iterative, asynchronous: each local iteration caused by:

- local link cost change
- DV update message from neighbor

distributed, self-stopping: each node notifies neighbors *only* when its DV changes

- neighbors then notify their neighbors – *only if necessary*
- no notification received, no actions taken!

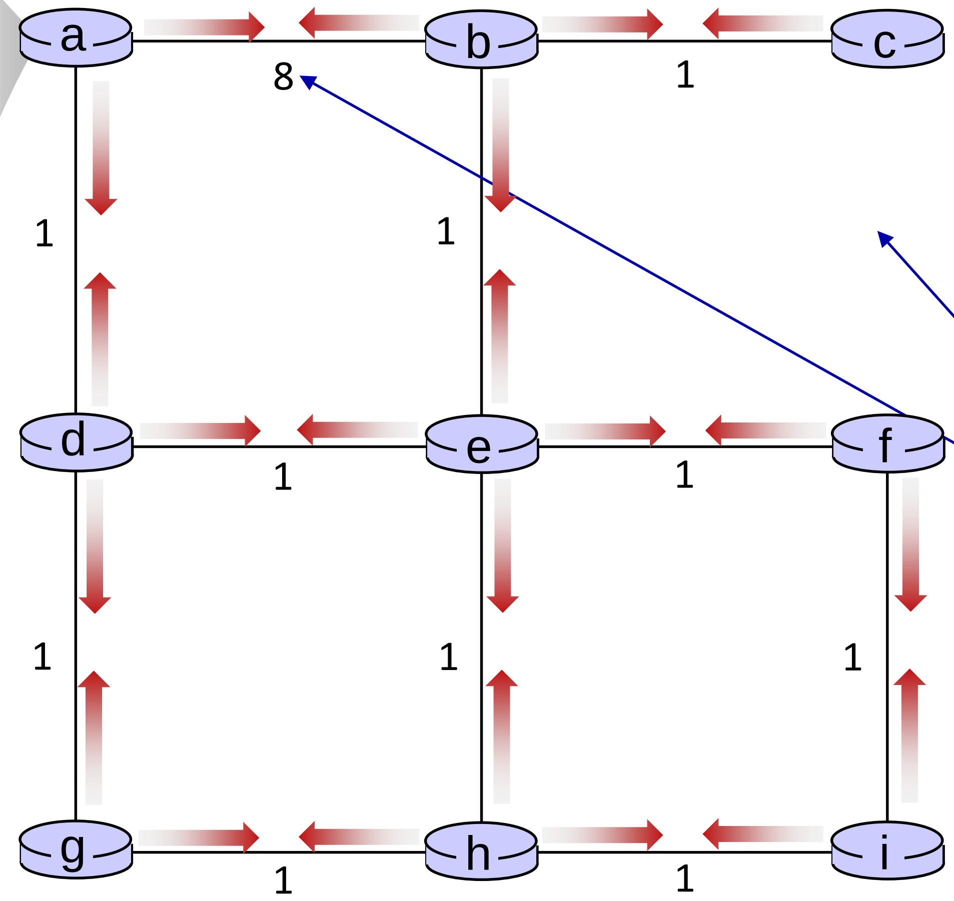
# Distance vector: example



t=0

- All nodes have distance estimates to nearest neighbors (only)
- All nodes send their local distance vector to their neighbors

| DV in a: |
|---|
| $D_a(a)=0$ |
| $D_a(b) = 8$ |
| $D_a(c) = \infty$ |
| $D_a(d) = 1$ |
| $D_a(e) = \infty$ |
| $D_a(f) = \infty$ |
| $D_a(g) = \infty$ |
| $D_a(h) = \infty$ |
| $D_a(i) = \infty$ |



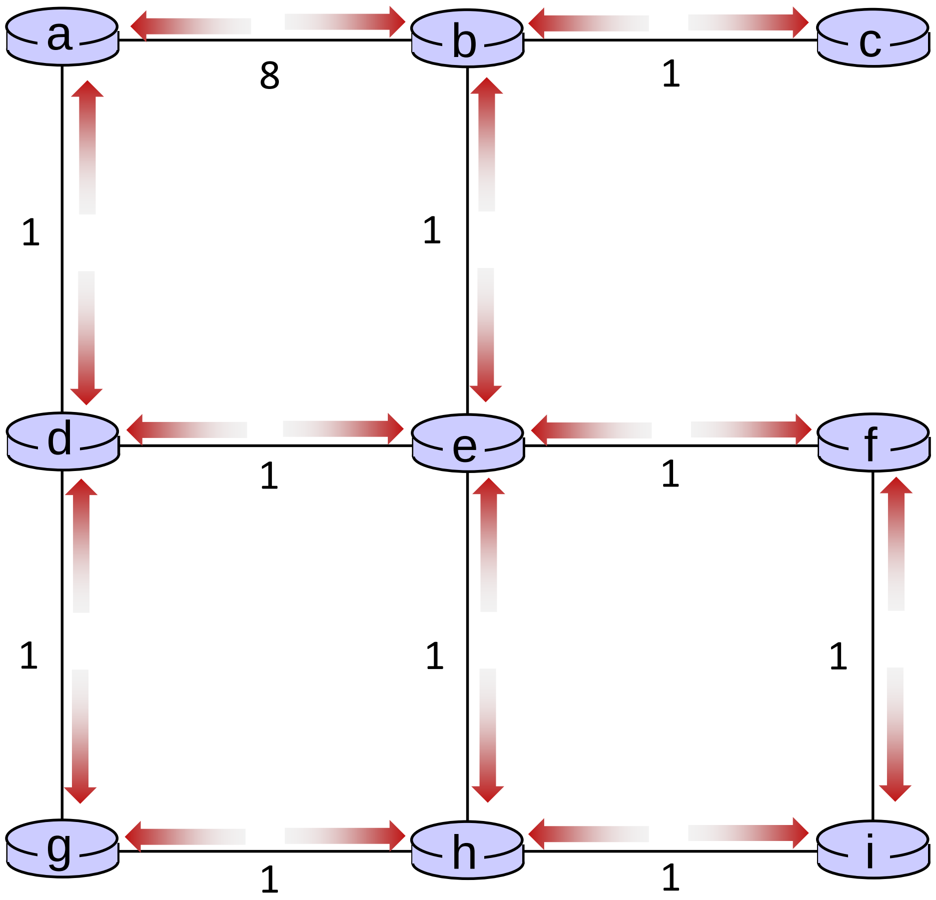

A few asymmetries:
- missing link
- larger cost

# Distance vector example: iteration

t=1

All nodes:

- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors

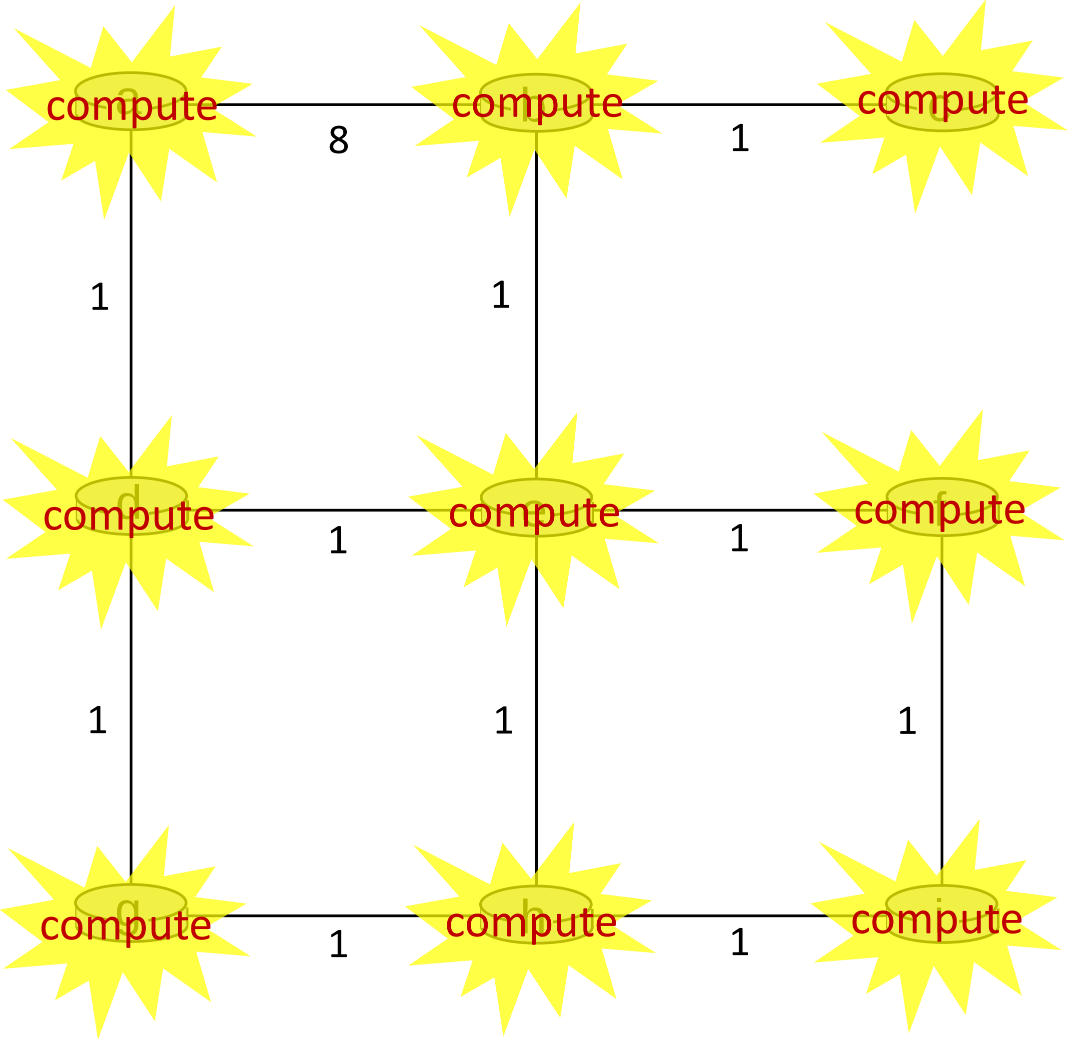


# Distance vector example: iteration

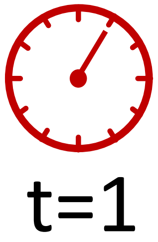
t=1

All nodes:

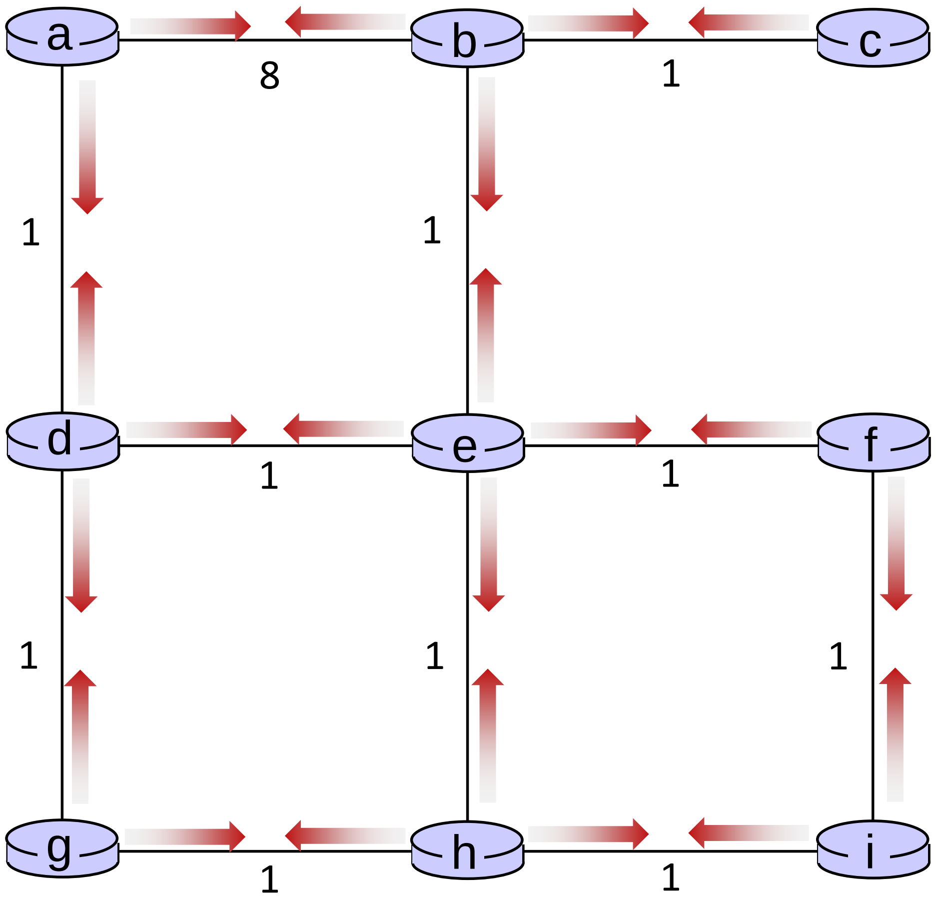
- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors



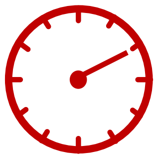
# Distance vector example: iteration




All nodes:
- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors

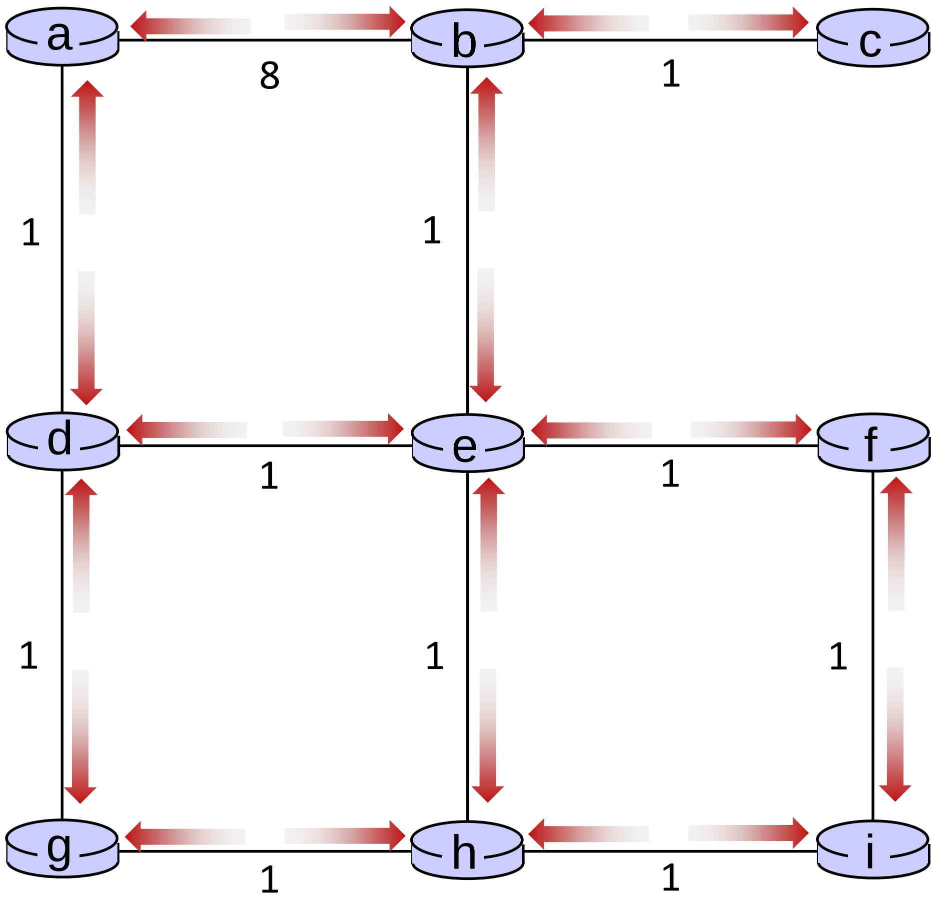


# Distance vector example: iteration

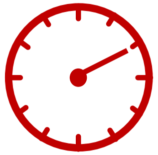


t=2

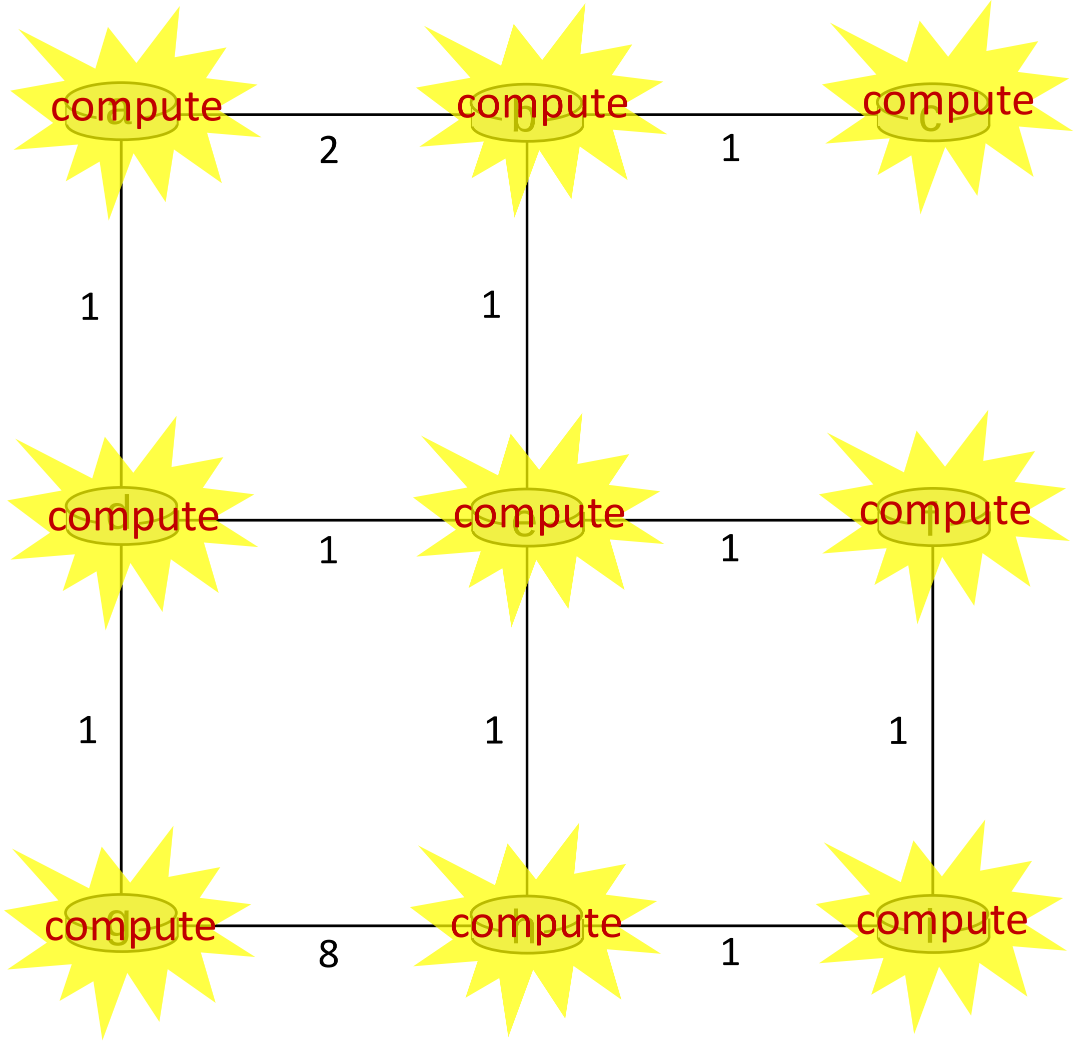
All nodes:
- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors



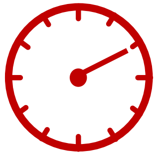
# Distance vector example: iteration




All nodes:
- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors



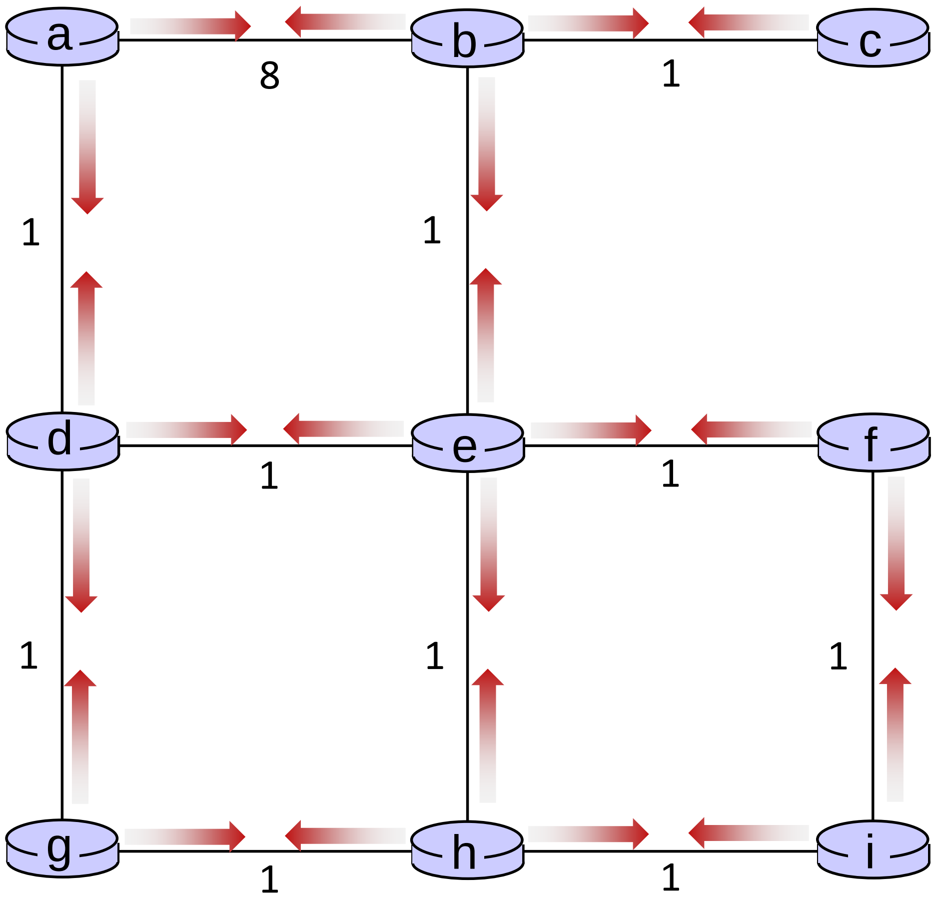
# Distance vector example: iteration



t=2

All nodes:

- receive distance vectors from neighbors
- compute their new local distance vector
- send their new local distance vector to neighbors

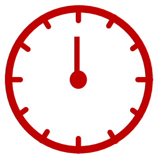


# Distance vector example: iteration

…. and so on

Let’s next take a look at the iterative *computations* at nodes

# Distance vector example:



t=1

- b receives DVs from a, c, e

**DV in a:**

$D_a(a)=0$
$D_a(b) = 8$
$D_a(c) = \infty$
$D_a(d) = 1$
$D_a(e) = \infty$
$D_a(f) = \infty$
$D_a(g) = \infty$
$D_a(h) = \infty$
$D_a(i) = \infty$

**DV in b:**

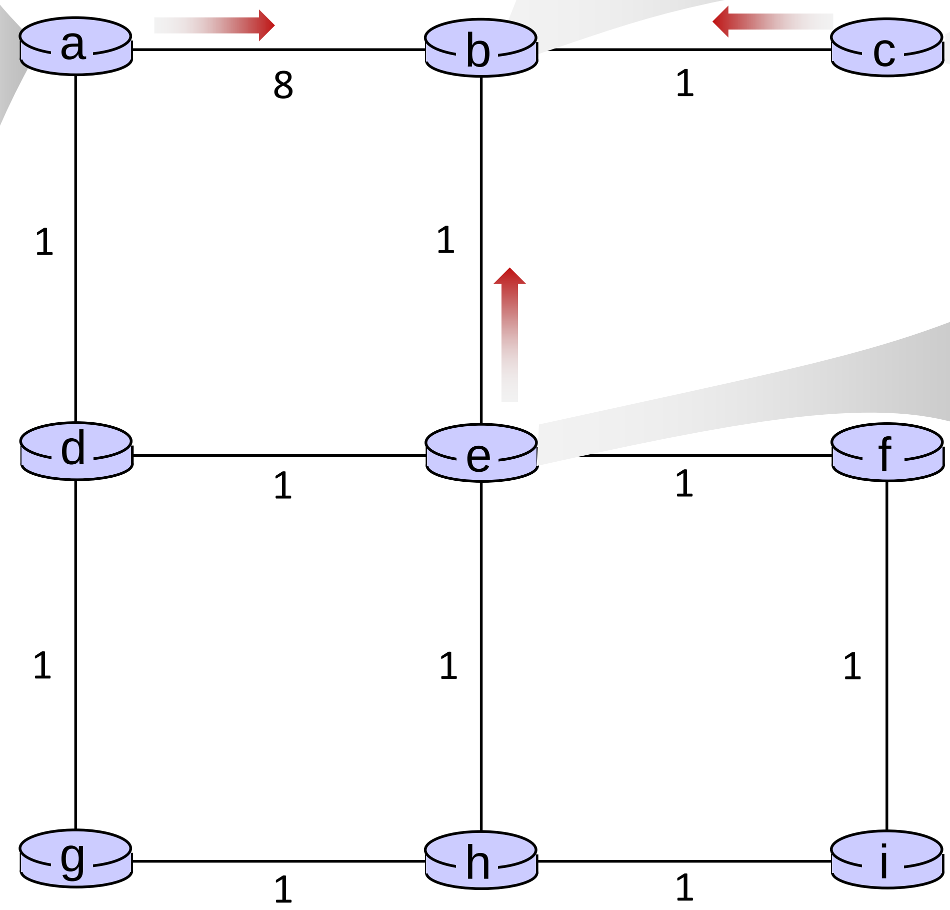
$D_b(a) = 8$ $D_b(f) = \infty$
$D_b(c) = 1$ $D_b(g) = \infty$
$D_b(d) = \infty$ $D_b(h) = \infty$
$D_b(e) = 1$ $D_b(i) = \infty$

**DV in c:**

$D_c(a) = \infty$
$D_c(b) = 1$
$D_c(c) = 0$
$D_c(d) = \infty$
$D_c(e) = \infty$
$D_c(f) = \infty$
$D_c(g) = \infty$
$D_c(h) = \infty$
$D_c(i) = \infty$

**DV in e:**

$D_e(a) = \infty$
$D_e(b) = 1$
$D_e(c) = \infty$
$D_e(d) = 1$
$D_e(e) = 0$
$D_e(f) = 1$
$D_e(g) = \infty$
$D_e(h) = 1$
$D_e(i) = \infty$



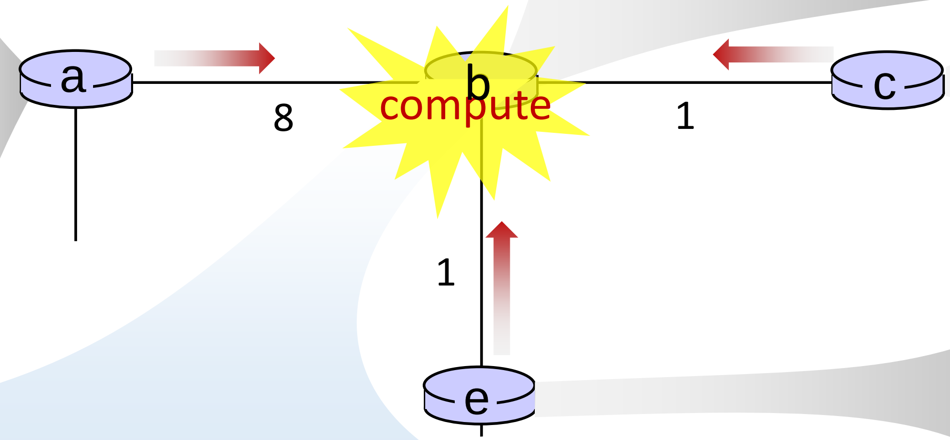
# Distance vector example:

t=1

- b receives DVs from a, c, e, computes:

**DV in a:**

$D_a(a)=0$
$D_a(b) = 8$
$D_a(c) = \infty$
$D_a(d) = 1$
$D_a(e) = \infty$
$D_a(f) = \infty$
$D_a(g) = \infty$
$D_a(h) = \infty$
$D_a(i) = \infty$




**DV in b:**

| | |
|---|---|
| $D_b(a) = 8$ | $D_b(f) = \infty$ |
| $D_b(c) = 1$ | $D_b(g) = \infty$ |
| $D_b(d) = \infty$ | $D_b(h) = \infty$ |
| $D_b(e) = 1$ | $D_b(i) = \infty$ |

**DV in c:**

$D_c(a) = \infty$
$D_c(b) = 1$
$D_c(c) = 0$
$D_c(d) = \infty$
$D_c(e) = \infty$
$D_c(f) = \infty$
$D_c(g) = \infty$
$D_c(h) = \infty$
$D_c(i) = \infty$

**DV in e:**

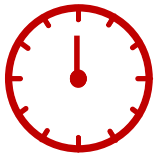
$D_e(a) = \infty$
$D_e(b) = 1$
$D_e(c) = \infty$
$D_e(d) = 1$
$D_e(e) = 0$
$D_e(f) = 1$
$D_e(g) = \infty$
$D_e(h) = 1$
$D_e(i) = \infty$

$$D_b(a) = \min\{c_{b,a}+D_a(a),\ c_{b,c}+D_c(a),\ c_{b,e}+D_e(a)\} = \min\{8,\infty,\infty\} = 8$$
$$D_b(c) = \min\{c_{b,a}+D_a(c),\ c_{b,c}+D_c(c),\ c_{b,e}+D_e(c)\} = \min\{\infty,1,\infty\} = 1$$
$$D_b(d) = \min\{c_{b,a}+D_a(d),\ c_{b,c}+D_c(d),\ c_{b,e}+D_e(d)\} = \min\{9,2,\infty\} = 2$$
$$D_b(e) = \min\{c_{b,a}+D_a(e),\ c_{b,c}+D_c(e),\ c_{b,e}+D_e(e)\} = \min\{\infty,\infty,1\} = 1$$
$$D_b(f) = \min\{c_{b,a}+D_a(f),\ c_{b,c}+D_c(f),\ c_{b,e}+D_e(f)\} = \min\{\infty,\infty,2\} = 2$$
$$D_b(g) = \min\{c_{b,a}+D_a(g),\ c_{b,c}+D_c(g),\ c_{b,e}+D_e(g)\} = \min\{\infty, \infty, \infty\} = \infty$$
$$D_b(h) = \min\{c_{b,a}+D_a(h),\ c_{b,c}+D_c(h),\ c_{b,e}+D_e(h)\} = \min\{\infty, \infty, 2\} = 2$$
$$D_b(i) = \min\{c_{b,a}+D_a(i),\ c_{b,c}+D_c(i),\ c_{b,e}+D_e(i)\} = \min\{\infty, \infty, \infty\} = \infty$$

**DV in b:**

| | |
|---|---|
| $D_b(a) = 8$ | $D_b(f) =2$ |
| $D_b(c) = 1$ | $D_b(g) = \infty$ |
| $D_b(d) = 2$ | $D_b(h) = 2$ |
| $D_b(e) = 1$ | $D_b(i) = \infty$ |

# Distance vector example:



t=1

- c receives DVs from b

**DV in a:**

$D_a(a)=0$
$D_a(b) = 8$
$D_a(c) = \infty$
$D_a(d) = 1$
$D_a(e) = \infty$
$D_a(f) = \infty$
$D_a(g) = \infty$
$D_a(h) = \infty$
$D_a(i) = \infty$

**DV in b:**

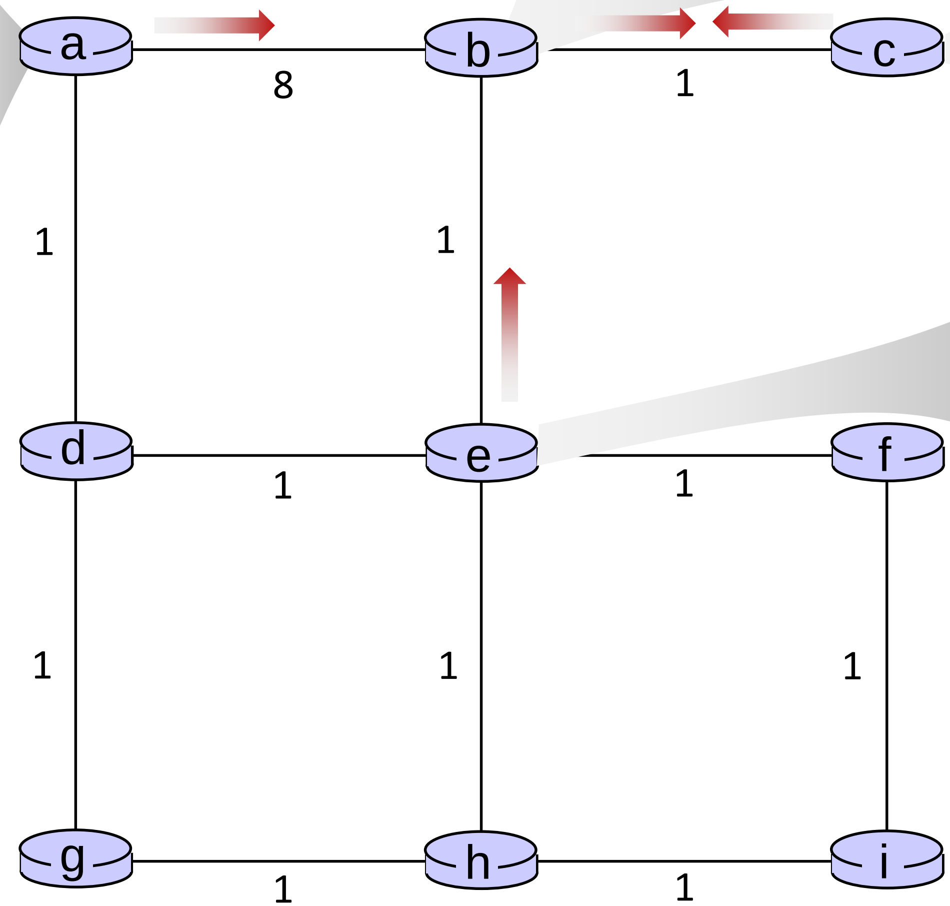
| | |
|---|---|
| $D_b(a) = 8$ | $D_b(f) = \infty$ |
| $D_b(c) = 1$ | $D_b(g) = \infty$ |
| $D_b(d) = \infty$ | $D_b(h) = \infty$ |
| $D_b(e) = 1$ | $D_b(i) = \infty$ |

**DV in c:**

$D_c(a) = \infty$
$D_c(b) = 1$
$D_c(c) = 0$
$D_c(d) = \infty$
$D_c(e) = \infty$
$D_c(f) = \infty$
$D_c(g) = \infty$
$D_c(h) = \infty$
$D_c(i) = \infty$

**DV in e:**

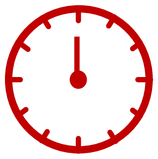
$D_e(a) = \infty$
$D_e(b) = 1$
$D_e(c) = \infty$
$D_e(d) = 1$
$D_e(e) = 0$
$D_e(f) = 1$
$D_e(g) = \infty$
$D_e(h) = 1$
$D_e(i) = \infty$



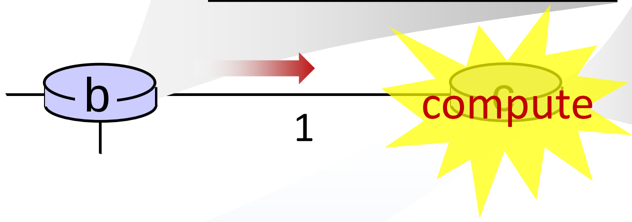
# Distance vector example:

| DV in b: | |
|---|---|
| $D_b(a) = 8$ | $D_b(f) = \infty$ |
| $D_b(c) = 1$ | $D_b(g) = \infty$ |
| $D_b(d) = \infty$ | $D_b(h) = \infty$ |
| $D_b(e) = 1$ | $D_b(i) = \infty$ |

| DV in c: |
|---|
| $D_c(a) = \infty$ |
| $D_c(b) = 1$ |
| $D_c(c) = 0$ |
| $D_c(d) = \infty$ |
| $D_c(e) = \infty$ |
| $D_c(f) = \infty$ |
| $D_c(g) = \infty$ |
| $D_c(h) = \infty$ |
| $D_c(i) = \infty$ |



t=1




- c receives DVs from b computes:

$D_c(a) = \min\{c_{c,b}+D_b(a)\} = 1 + 8 = 9$

$D_c(b) = \min\{c_{c,b}+D_b(b)\} = 1 + 0 = 1$

$D_c(d) = \min\{c_{c,b}+D_b(d)\} = 1+ \infty = \infty$

$D_c(e) = \min\{c_{c,b}+D_b(e)\} = 1 + 1 = 2$

$D_c(f) = \min\{c_{c,b}+D_b(f)\} = 1+ \infty = \infty$

$D_c(g) = \min\{c_{c,b}+D_b(g)\} = 1+ \infty = \infty$

$D_c(h) = \min\{c_{bc,b}+D_b(h)\} = 1+ \infty = \infty$

$D_c(i) = \min\{c_{c,b}+D_b(i)\} = 1+ \infty = \infty$

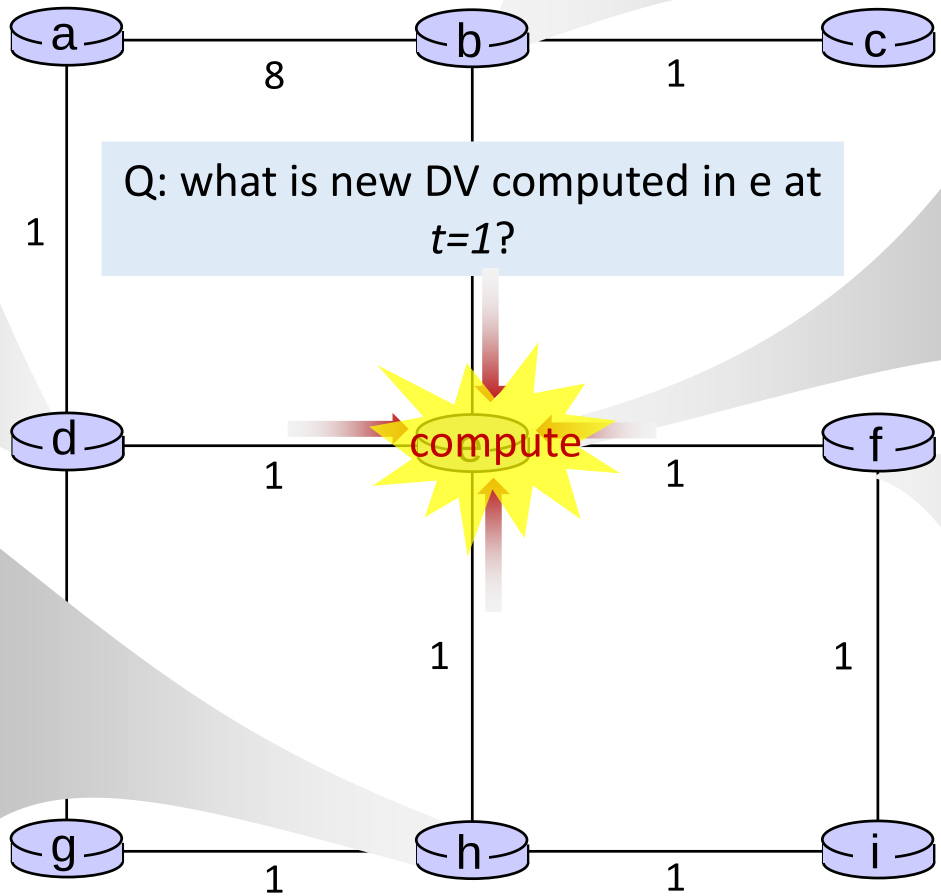
| DV in c: |
|---|
| $D_c(a) = 9$ |
| $D_c(b) = 1$ |
| $D_c(c) = 0$ |
| $D_c(d) = 2$ |
| $D_c(e) = \infty$ |
| $D_c(f) = \infty$ |
| $D_c(g) = \infty$ |
| $D_c(h) = \infty$ |
| $D_c(i) = \infty$ |

* Check out the online interactive exercises for more examples: http://gaia.cs.umass.edu/kurose_ross/interactive/

# Distance vector example:

t=1

- e receives DVs from b, d, f, h




**DV in b:**

| | |
|---|---|
| $D_b(a) = 8$ | $D_b(f) = \infty$ |
| $D_b(c) = 1$ | $D_b(g) = \infty$ |
| $D_b(d) = \infty$ | $D_b(h) = \infty$ |
| $D_b(e) = 1$ | $D_b(i) = \infty$ |

**DV in d:**

$D_c(a) = 1$
$D_c(b) = \infty$
$D_c(c) = \infty$
$D_c(d) = 0$
$D_c(e) = 1$
$D_c(f) = \infty$
$D_c(g) = 1$
$D_c(h) = \infty$
$D_c(i) = \infty$

**DV in e:**

$D_e(a) = \infty$
$D_e(b) = 1$
$D_e(c) = \infty$
$D_e(d) = 1$
$D_e(e) = 0$
$D_e(f) = 1$
$D_e(g) = \infty$
$D_e(h) = 1$
$D_e(i) = \infty$

**DV in h:**

$D_c(a) = \infty$
$D_c(b) = \infty$
$D_c(c) = \infty$
$D_c(d) = \infty$
$D_c(e) = 1$
$D_c(f) = \infty$
$D_c(g) = 1$
$D_c(h) = 0$
$D_c(i) = 1$

**DV in f:**

$D_c(a) = \infty$
$D_c(b) = \infty$
$D_c(c) = \infty$
$D_c(d) = \infty$
$D_c(e) = 1$
$D_c(f) = 0$
$D_c(g) = \infty$
$D_c(h) = \infty$
$D_c(i) = 1$

# Distance vector: state information diffusion

Iterative communication, computation steps diffuses information through network:

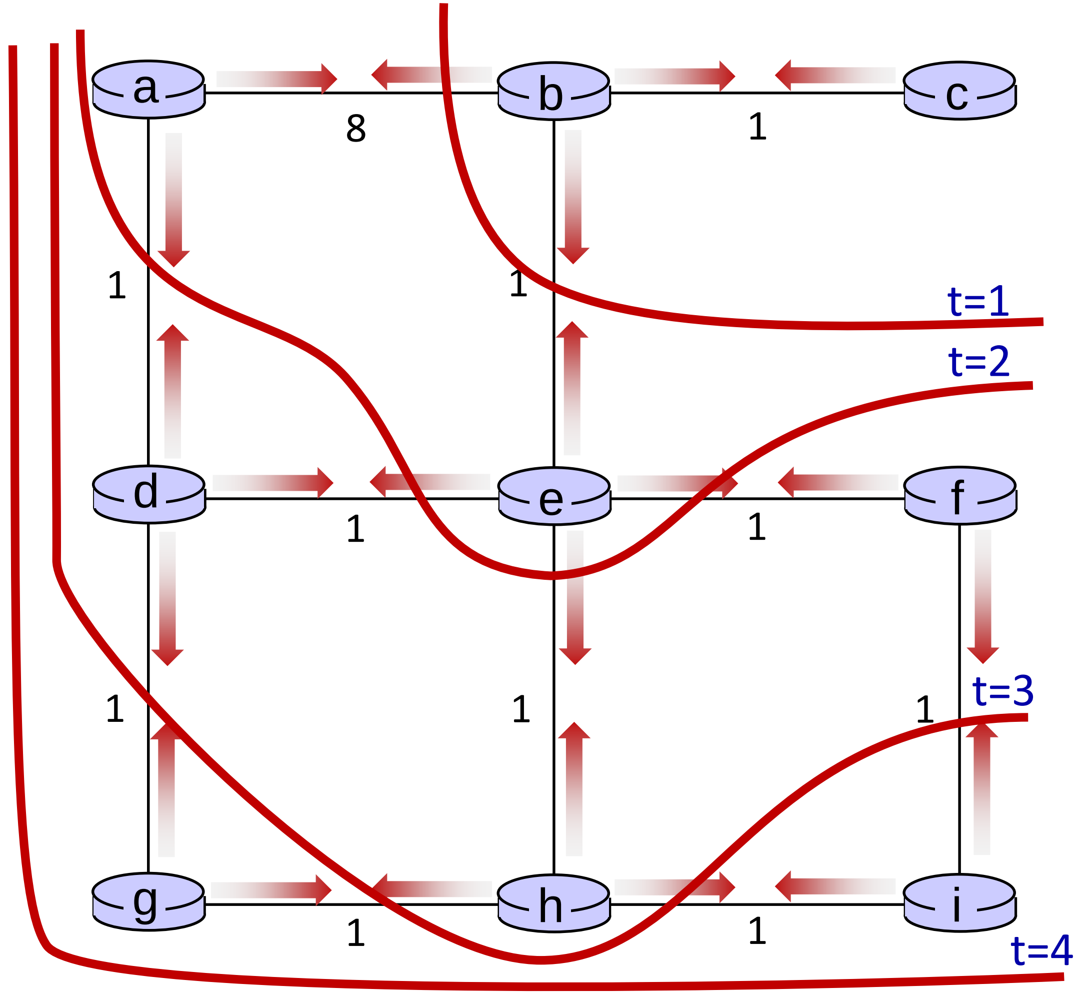
t=0 c's state at t=0 is at c only

t=1 c's state at t=0 has propagated to b, and may influence distance vector computations up to **1** hop away, i.e., at b

t=2 c's state at t=0 may now influence distance vector computations up to **2** hops away, i.e., at b and now at a, e as well

t=3 c's state at t=0 may influence distance vector computations up to **3** hops away, i.e., at b,a,e and now at c,f,h as well

t=4 c's state at t=0 may influence distance vector computations up to **4** hops away, i.e., at b,a,e, c, f, h and now at g,i as well



# Comparison of LS and DV algorithms

**message complexity**

LS: $n$ routers, $O(n^2)$ messages sent

DV: exchange between neighbors; convergence time varies

**speed of convergence**

LS: $O(n^2)$ algorithm, $O(n^2)$ messages
- may have oscillations

DV: convergence time varies
- may have routing loops
- count-to-infinity problem

**robustness:** what happens if router malfunctions, or is compromised?

LS:
- router can advertise incorrect *link* cost
- each router computes only its *own* table

DV:
- DV router can advertise incorrect *path* cost ("I have a *really* low cost path to everywhere"): black-holing
- each router's table used by others: error propagate thru network

# Distance vector: another example

$D_x()$

| from \ cost to | x | y | z |
|---|---|---|---|
| x | 0 | 2 | 7 |
| y | ∞ | ∞ | ∞ |
| z | ∞ | ∞ | ∞ |

$D_y()$

| from \ cost to | x | y | z |
|---|---|---|---|
| x | ∞ | ∞ | ∞ |
| y | 2 | 0 | 1 |
| z | ∞ | ∞ | ∞ |

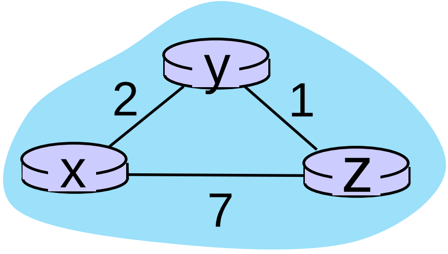
$D_z()$

| from \ cost to | x | y | z |
|---|---|---|---|
| x | ∞ | ∞ | ∞ |
| y | ∞ | ∞ | ∞ |
| z | 7 | 1 | 0 |

| from \ cost to | x | y | z |
|---|---|---|---|
| x | 0 | 2 | 3 |
| y | 2 | 0 | 1 |
| z | 7 | 1 | 0 |

$D_x(z) = \min\{c_{x,y} + D_y(z), c_{x,z} + D_z(z)\}$
$= \min\{2+1 , 7+0\} = 3$

$D_x(y) = \min\{c_{x,y} + D_y(y), c_{x,z} + D_z(y)\}$
$= \min\{2+0 , 7+1\} = 2$




*time*

# Distance vector: another example

$D_x()$

| from | cost to x | y | z |
|---|---|---|---|
| x | 0 | 2 | 7 |
| y | ∞ | ∞ | ∞ |
| z | ∞ | ∞ | ∞ |

$D_y()$

| from | cost to x | y | z |
|---|---|---|---|
| x | ∞ | ∞ | ∞ |
| y | 2 | 0 | 1 |
| z | ∞ | ∞ | ∞ |

$D_z()$

| from | cost to x | y | z |
|---|---|---|---|
| x | ∞ | ∞ | ∞ |
| y | ∞ | ∞ | ∞ |
| z | 7 | 1 | 0 |

| from | cost to x | y | z |
|---|---|---|---|
| x | 0 | 2 | 3 |
| y | 2 | 0 | 1 |
| z | 7 | 1 | 0 |

| from | cost to x | y | z |
|---|---|---|---|
| x | 0 | 2 | 7 |
| y | 2 | 0 | 1 |
| z | 7 | 1 | 0 |

| from | cost to x | y | z |
|---|---|---|---|
| x | 0 | 2 | 7 |
| y | 2 | 0 | 1 |
| z | 3 | 1 | 0 |

| from | cost to x | y | z |
|---|---|---|---|
| x | 0 | 2 | 3 |
| y | 2 | 0 | 1 |
| z | 3 | 1 | 0 |

| from | cost to x | y | z |
|---|---|---|---|
| x | 0 | 2 | 3 |
| y | 2 | 0 | 1 |
| z | 3 | 1 | 0 |

| from | cost to x | y | z |
|---|---|---|---|
| x | 0 | 2 | 3 |
| y | 2 | 0 | 1 |
| z | 3 | 1 | 0 |

*time*