

Computational Intelligence

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Outline

- Neural Networks in Practice
 - Regularization
 - Why Regularization Reduces Overfitting?
 - Regularization Techniques
 - L2 Regularization
 - Dropout

Neural Networks in Practice: Regularization

Regularization

What is it?

Technique that constrains our optimization problem to discourage complex models

Regularization

What is it?

Technique that constrains our optimization problem to discourage complex models

Why do we need it?

Improve generalization of our model on unseen data

 m : training data
 n_x : سامانه داده های مورد تمرین

Regularization in Logistic regression

$$\min_{w,b} J(w, b)$$

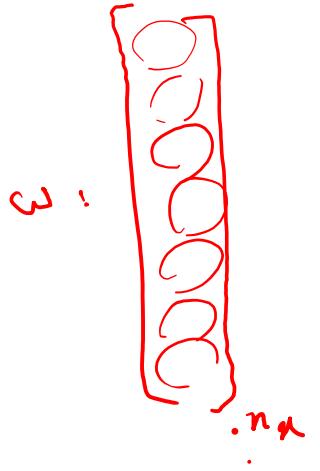
$$w \in \mathbb{R}^n \quad b \in \mathbb{R}$$

$$J(w, b) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) + \boxed{\frac{\lambda}{2m} \|w\|_2^2} + \frac{\lambda}{2m} b^2$$

$$L_2 \text{ regularization: } \|w\|_2^2 = \sum_{j=1}^{n_x} w_j^2 = w^T w$$

λ = Regularization Parameter

$$L_1 \text{ reg... : } \frac{\lambda}{2m} \sum_{j=1}^{n_x} |w_j| = \frac{\lambda}{2m} \|w\|_1$$



Regularization in Neural network

$$J(\omega^{[1]}, b^{[1]}, \dots, \omega^{[L]}, b^{[L]}) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m} \sum_{l=1}^L \|\omega^{[l]}\|_F^2$$

$$\|\omega^{[l]}\|_F^2 = \sum_{i=1}^n \sum_{j=1}^{n^{[l-1]}} (\omega_{ij}^{[l]})^2$$

$$\|\cdot\|_2^2 \quad \|\cdot\|_F^2$$

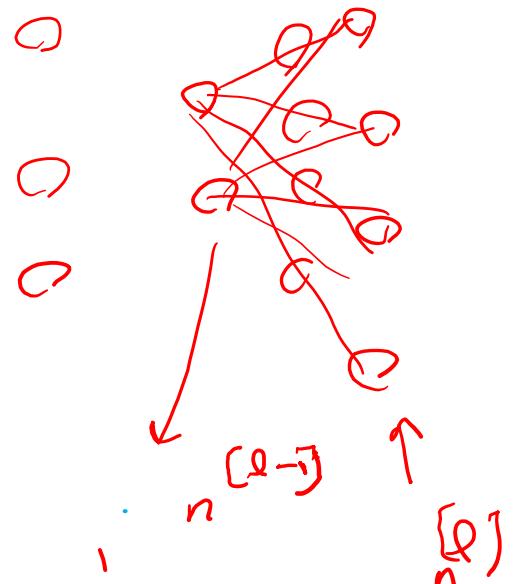
"Frobenius norm"

$$\begin{aligned} d\omega^{[l]} &= \underbrace{(\text{from backprop})}_{\omega^{[l]}} + \underbrace{\frac{\lambda}{m} \omega^{[l]}}_{\frac{\partial J}{\partial \omega^{[l]}}} \\ \omega^{[l]} &= \omega^{[l]} - \alpha d\omega^{[l]} \end{aligned}$$

$$\omega^{[l]} = \underbrace{\omega^{[l]}}_{\omega^{[l]} - \alpha \left[\text{(from back)} + \frac{\lambda}{m} \omega^{[l]} \right]}$$

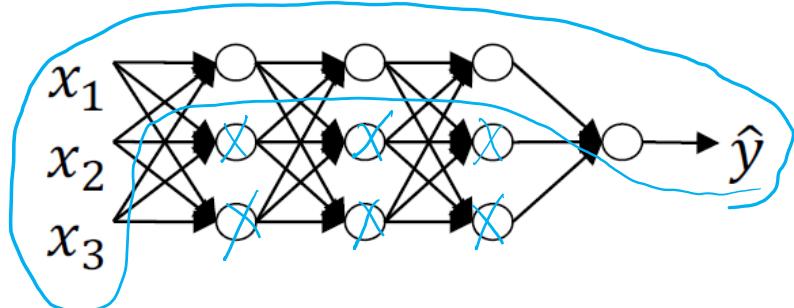
weight decay

$$= \left(1 - \frac{\alpha \lambda}{m}\right) \omega^{[l]} - \alpha \text{ (from back...)}$$



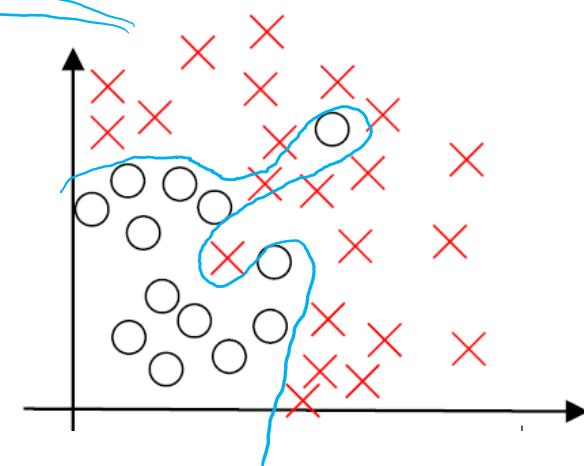
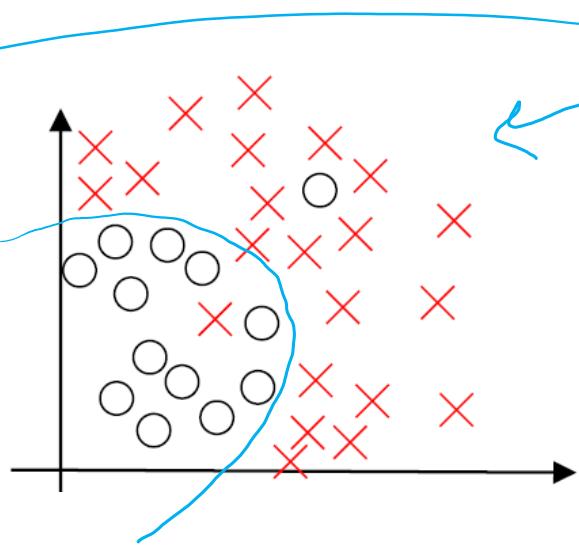
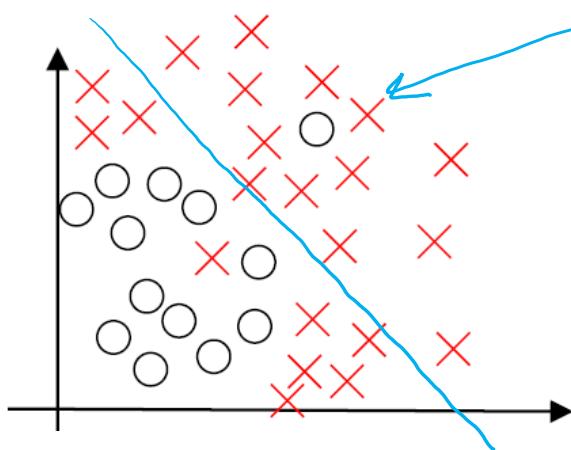
Regularization: Why Regularization Reduces Overfitting?

How does regularization prevent overfitting?

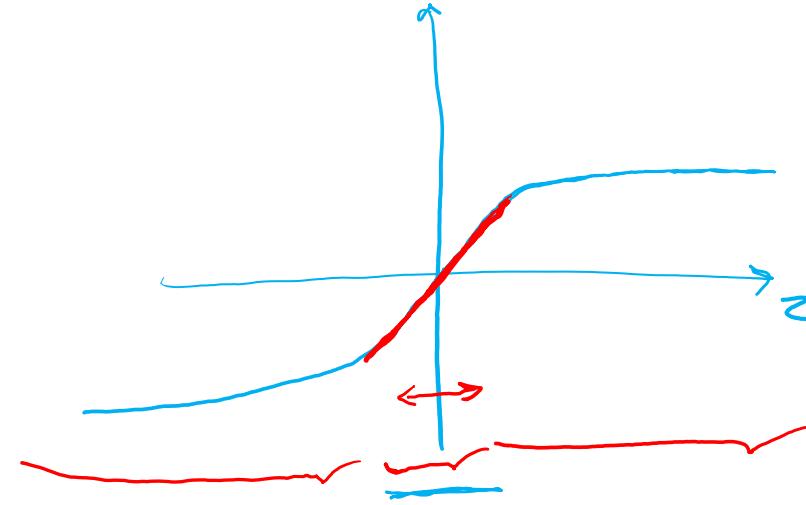


$$J(\omega^{[0]}, b^{[0]}, \dots, \omega^{[L]}, b^{[L]}) = \frac{1}{m} \sum_{i=1}^m h(\hat{y}^{(i)}, y^{(i)}) + \frac{\lambda}{2m} \sum_{l=1}^L \|\omega^{(l)}\|_F^2$$

Below the equation, there is a condition: $\|\omega^{(L)}\| \approx 0$. To the right of this condition, there is a diagram showing a vector $\omega^{(L)}$ with a length very close to zero.



How does regularization prevent overfitting?

 $\lambda \uparrow$ $w^{[l]} \downarrow$

$$\underline{z}^{[l]} = \underline{w}^{[l]} \underline{a}^{[l-1]} + b^{[l]}$$

 $\lambda \text{ large } \Rightarrow \text{linear}$ 

Regularization Techniques:

II: Dropout

I :

L_2 Regularization

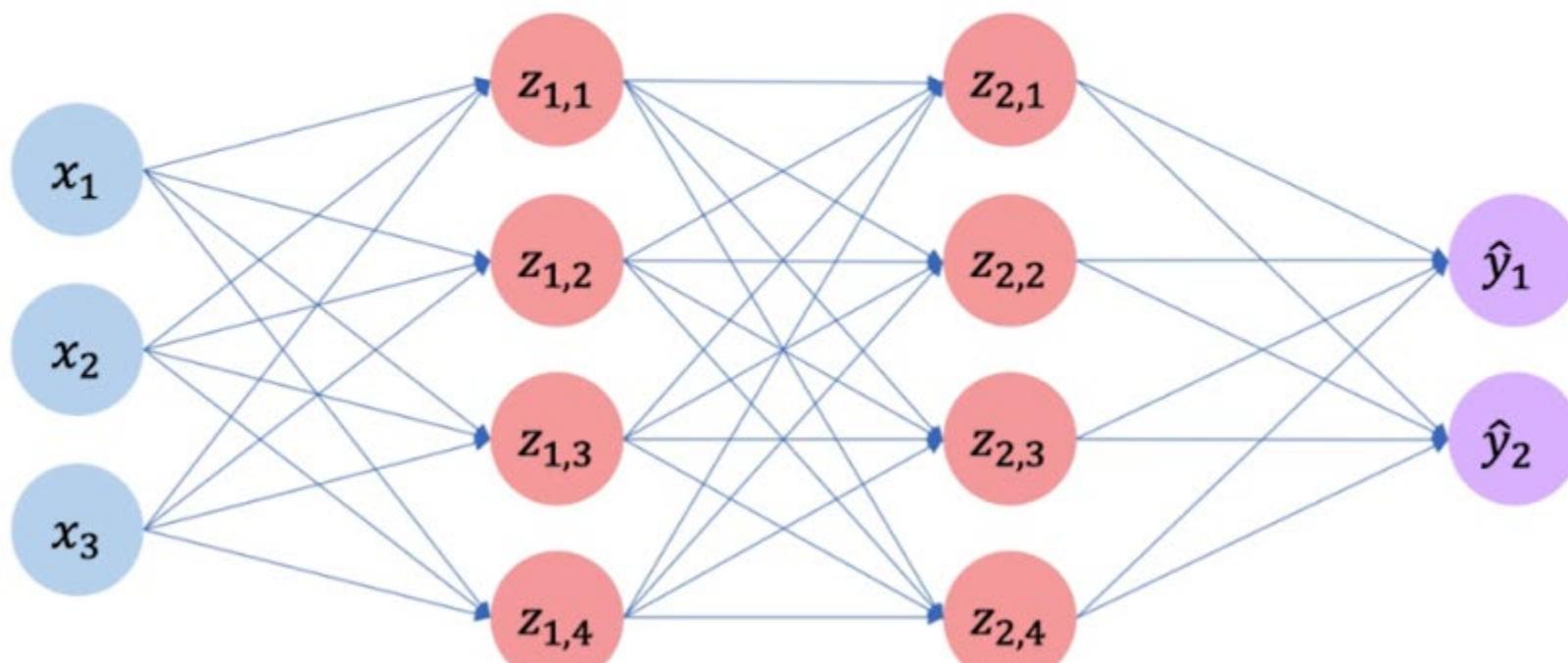
$$\frac{\lambda}{2m} \sum_{i=1}^q \|\omega^{(l)}\|_F^2$$

II :

Dropout

Regularization II: Dropout

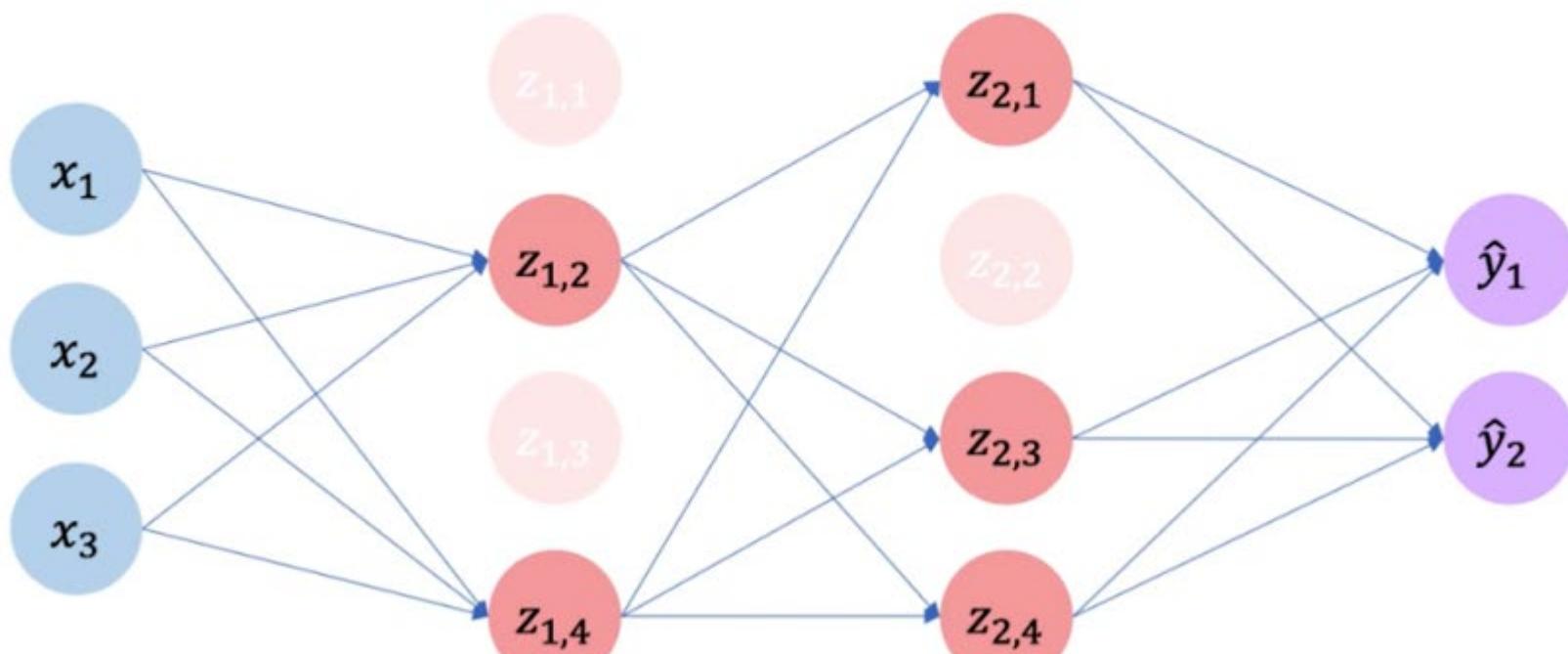
- During training, randomly set some activations to 0



Regularization II: Dropout

- During training, randomly set some activations to 0
 - Typically 'drop' 50% of activations in layer
 - Forces network to not rely on any 1 node

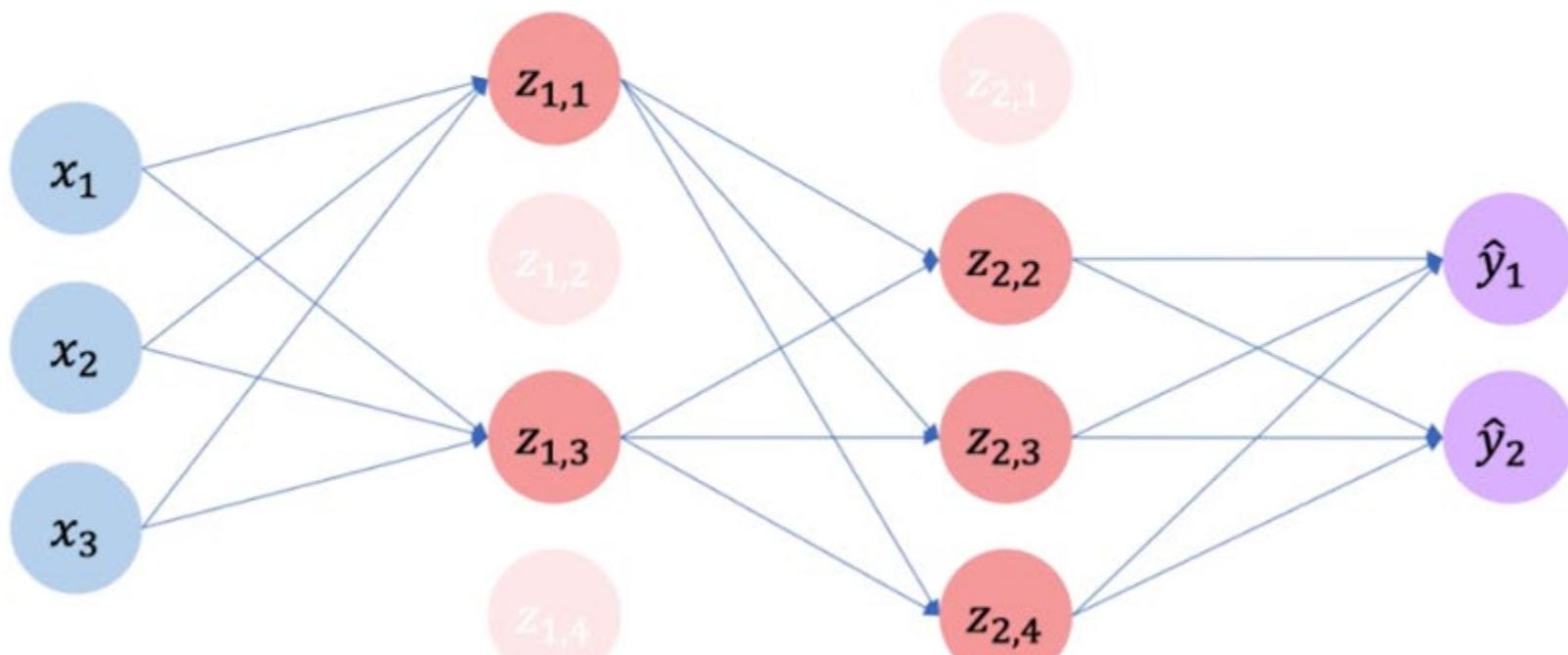
 `tf.keras.layers.Dropout(p=0.5)`



Regularization II: Dropout

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 `tf.keras.layers.Dropout(p=0.5)`



Implementing dropout (“Inverted dropout”)

Illustrate with layer $L=3$

$$\text{Keep-Prob} = 0.8 \Rightarrow$$

0.2

$d_3 = \text{np.random.rand}(a_3.\text{shape}[0], a_3.\text{shape}[1]) < \text{Keep-Prob}$

$a_3 = \text{np.multiply}(a_3, d_3)$

$a_3 /= \text{Keep-Prob}$

50 unit \rightarrow 10 unit shut off

$$Z^{[4]} = w^{[4]} a^{[3]} + b^{[4]}$$

↑ reduced 20%
 $/= 0.8$

Making predictions at test time

$$a^{(0)} = X$$

No drop out

$$z^{(1)} = w^{(1)} a^{(0)} + b^{(1)}$$

$$a^{(1)} = g^{(1)}(z^{(1)})$$

$$z^{(2)} = w^{(2)} a^{(1)} + b^{(2)}$$

$$a^{(2)} = g^{(2)}(z^{(2)})$$

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↓
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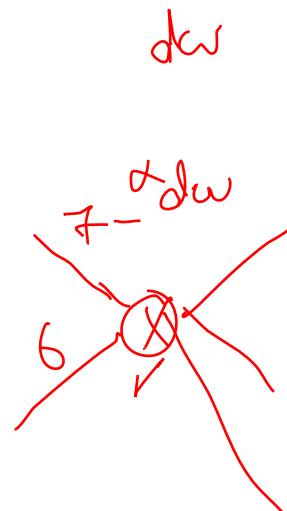
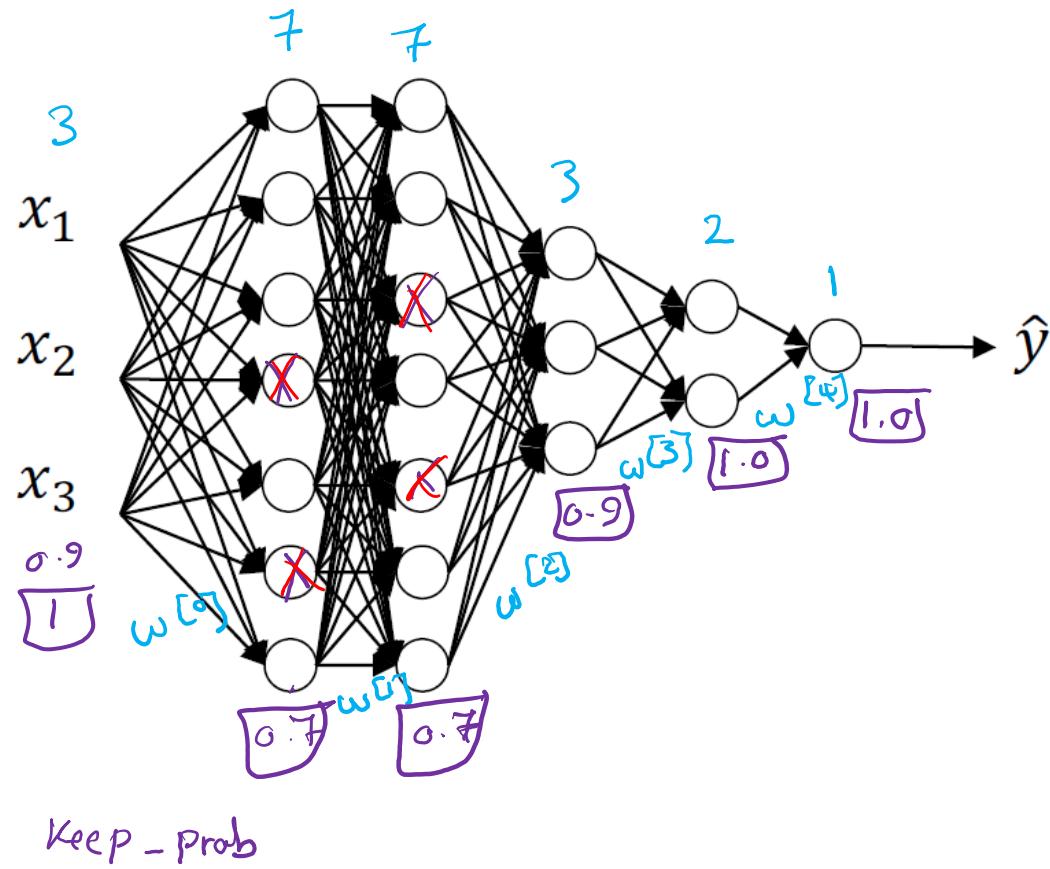
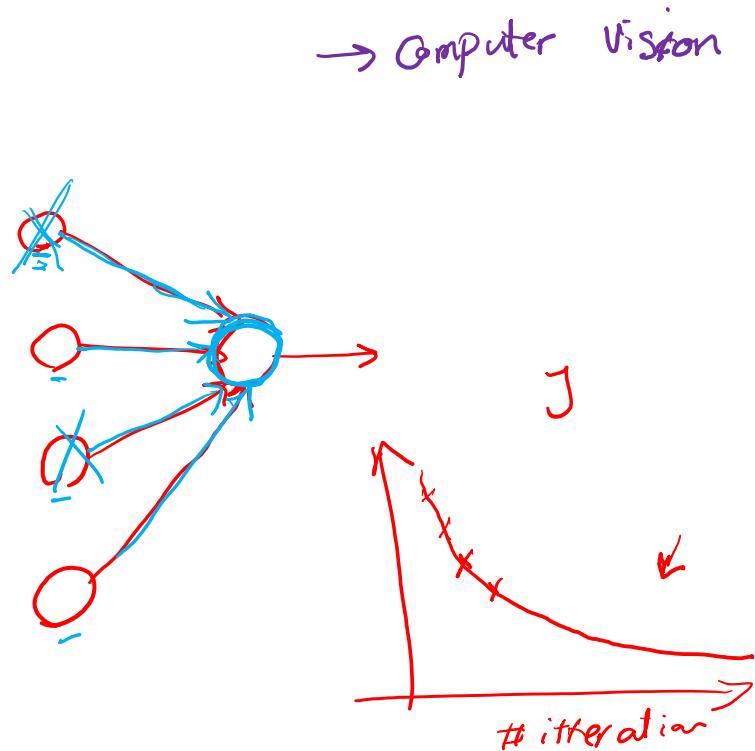
/ = keep-prab

Understanding Dropout

Why does drop-out work?

Intuition: Can't rely on any one feature, so have to spread out weights

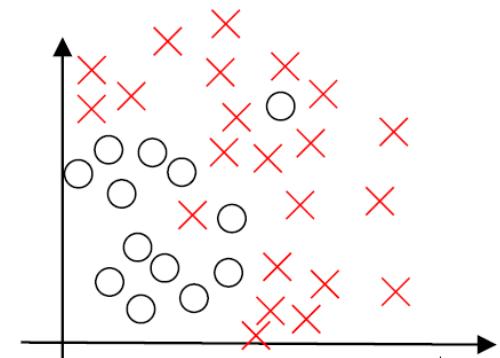
Shrink weight $\rightarrow L_2$



Core Foundation Review

Regularization

Technique that constrains our optimization problem to discourage complex models



How does regularization prevent overfitting

Regularization techniques: L2, Dropout

