تکلیف اول

مجتبی ملائی
۴۰۱۳۱۳۸۳

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۱. اگر توابع فعال ساز را حذف کنیم، فرمول نهایی به شکل زیر خواهد بود:

$$\hat{y} = \sum_{j=1}^{4}(\sum_{i=1}^{3} x_i w_{ij}^{(1)} + b_j^{(1)}) w_i^{(2)} + b_j^{(2)}$$

که میتوان آن را به صورت زیر نوشت:

$$\hat{y} = \sum_{j=1}^{4}(\sum_{i=1}^{3} x_i w_{ij}^{(1)} w_j^{(2)} + b_j^{(1)} w_j^{(2)}) + b_j^{(2)}$$

$$\hat{y} = \sum_{i=1}^{3}(x_i(\sum_{j=1}^{4} w_{ij}^{(1)} w_j^{(2)}) + \sum_{j=1}^{4}(b_j^{(1)} w_j^{(2)})) + \sum_{j=1}^{4} b_j^{(2)}$$

حالا اگر قرار دهیم:

$$w_i' = \sum_{j=1}^{4} w_{ij}^{(1)} w_j^{(2)}, b' = \sum_{j=1}^{4} b_j^{(1)} w_j^{(2)} + b_j^{(2)}$$

خواهیم داشت:

$$\hat{y} = \sum_{i=1}^{3} x_i w_i' + b'$$

که در واقع همان ساختار یک شبکه عصبی بدون لایه پنهان است.

۲. **آ)** $\frac{\partial \mathcal{L}}{\partial \hat{y}} = -\frac{y}{\hat{y}} + \frac{(1-y)}{1-\hat{y}}$

ب) $\frac{\partial \hat{y}}{\partial z^{(2)}} = \sigma(z^{(2)})(1 - \sigma(z^{(2)})) = \hat{y}(1-\hat{y})$

ج) $\frac{\partial a_j^{(1)}}{\partial z_j^{(1)}} = 1 - \tanh^2(z_j^{(1)}) = 1 - (a_j^{(1)})^2$

د) $\frac{\partial J}{\partial z^{(2)}} = \frac{\partial J}{\partial \mathcal{L}} \frac{\partial \mathcal{L}}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z^{(2)}} = \frac{1}{m}(-\frac{y}{\hat{y}} + \frac{(1-y)}{1-\hat{y}})\hat{y}(1-\hat{y}) = \frac{1}{m}(\hat{y} - y)$

ه) $\frac{\partial \mathcal{L}}{\partial w_{ij}^{(1)}} = \frac{\partial \mathcal{L}}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial a_j^{(1)}} \frac{\partial a_j^{(1)}}{\partial z_j^{(1)}} \frac{\partial z_j^{(1)}}{\partial w_{ij}^{(1)}} = (\hat{y} - y)(w_j^{(2)})(1 - (a_j^{(1)})^2)(x_i^{(1)})$

و) $\frac{\partial \mathcal{L}}{\partial b_j^{(1)}} = \frac{\partial \mathcal{L}}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial a_j^{(1)}} \frac{\partial a_j^{(1)}}{\partial z_j^{(1)}} \frac{\partial z_j^{(1)}}{\partial b_j^{(1)}} = (\hat{y} - y)(w_j^{(2)})(1 - (a_j^{(1)})^2)$

ز) $\frac{\partial \mathcal{L}}{\partial w_j^{(2)}} = \frac{\partial \mathcal{L}}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial w_j^{(2)}} = (\hat{y} - y) a_j^{(1)}$

ح) $\frac{\partial \mathcal{L}}{\partial b^{(2)}} = \frac{\partial \mathcal{L}}{\partial z^{(2)}} \frac{\partial z^{(2)}}{\partial b^{(2)}} = \hat{y} - y$

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forward propagation:

$$z_j^{(1)} = [-0.1425, 0.1036, 0.7293, -0.1154]$$

$$a_j^{(1)} = [-0.14154322, 0.10323094, 0.62263691, -0.11489045]$$

$$z^{(2)} = 0.31613438$$

$$\hat{y} = 0.57838188$$

$$\mathcal{L} = 0.54752093, J = 0.54752093$$

back-propagation:

$$\frac{\partial \mathcal{L}}{\partial w_j^{(2)}} = \begin{bmatrix} -0.05967719 \\ 0.04352403 \\ 0.262515 \\ -0.04843989 \end{bmatrix}$$

$$\frac{\partial \mathcal{L}}{\partial b^{(2)}} = 0.42161812$$

$$\frac{\partial \mathcal{L}}{\partial w_{ij}^{(1)}} = \begin{bmatrix} 0.02375734 & -0.03357857 & 0.00890675 & 0.00478461 \\ 0.04648176 & -0.0656972 & 0.01742624 & 0.00936119 \\ 0.06920618 & -0.09781583 & 0.02594574 & 0.01393777 \end{bmatrix}$$

$$\frac{\partial \mathcal{L}}{\partial b_j^{(1)}} = \begin{bmatrix} 0.1032928 \\ -0.14599378 \\ 0.03872499 \\ 0.02080264 \end{bmatrix}$$