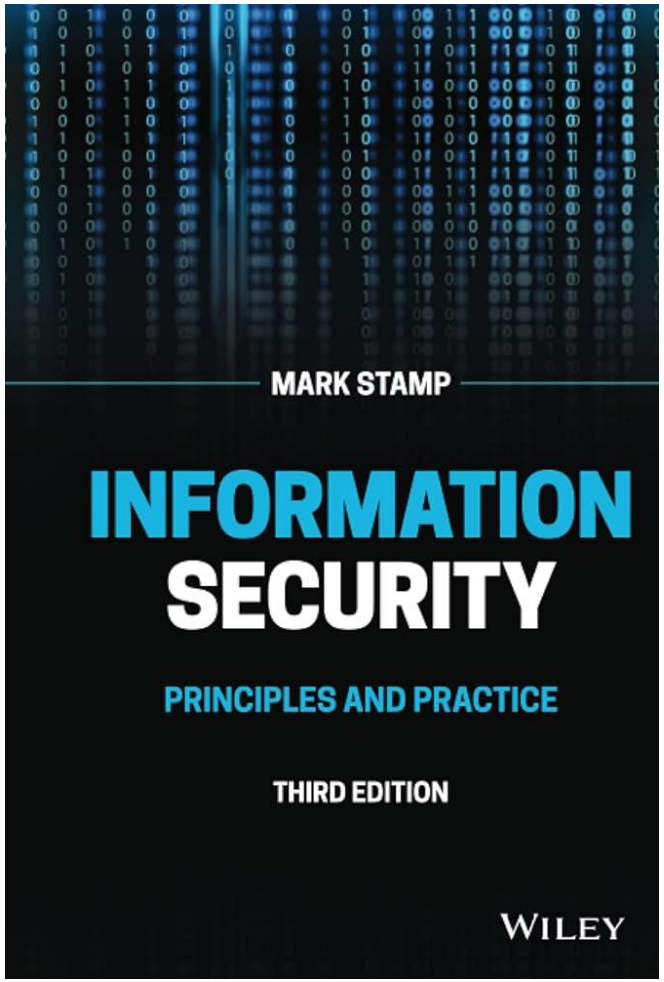
بسم الله الرحمن الرحیم

مبانی رایانش امن

جلسه ۸

مجتبی خلیلی
دانشکده برق و کامپیوتر
دانشگاه صنعتی اصفهان


MARK STAMP
INFORMATION
SECURITY
PRINCIPLES AND PRACTICE
THIRD EDITION
WILEY

- فصل سوم کتاب

Diffie-Hellman

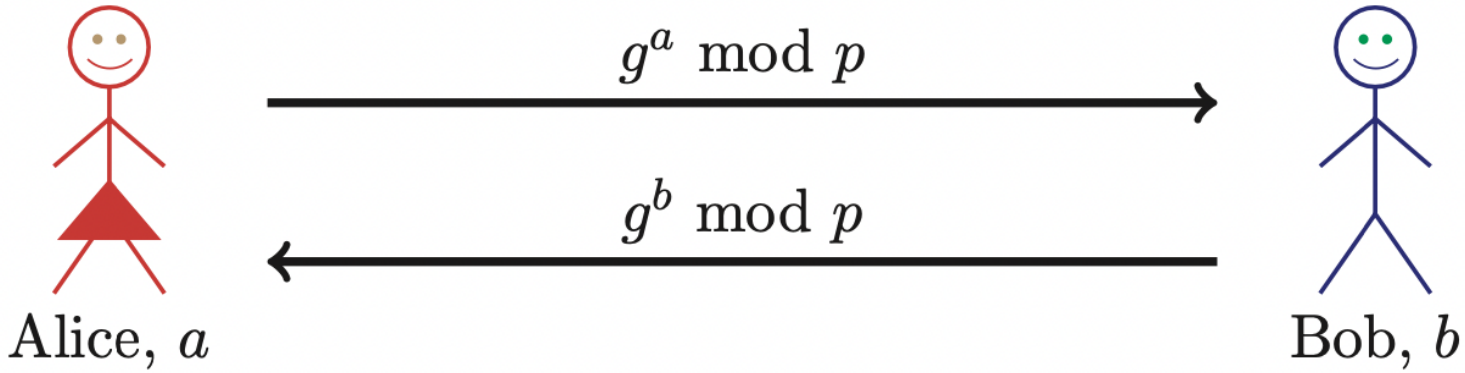
- هدف: توافق روی یک کلید (متقارن) با وجود دشمن روی خط ارتباطی

Diffie-Hellman Key Exchange

- A "key exchange" algorithm
  - Used to establish a shared symmetric key
  - ***Not*** for encrypting or signing
- Let p be prime, let g be a **generator**
  - For any $x \in \{1,2,\ldots,p-1\}$ there is n s.t. $x = g^n \bmod p$
- Based on **discrete log** problem
  - **Given:** g, p, and $g^k \bmod p$
  - **Find:** exponent k

Diffie-Hellman

- **Public:** g and p
- **Private:** Alice's exponent a, Bob's exponent b



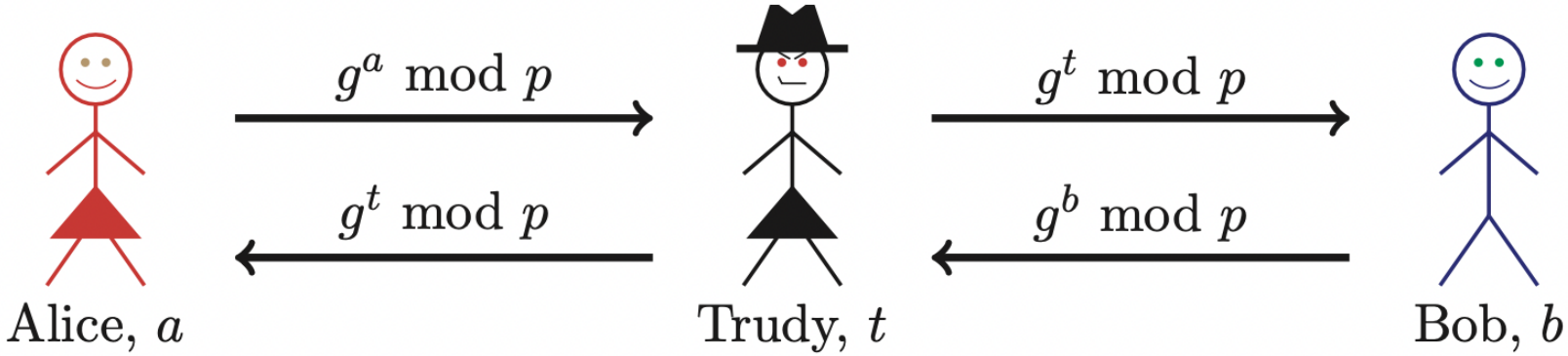
- Alice computes $(g^b)^a = g^{ba} = g^{ab} \bmod p$
- Bob computes $(g^a)^b = g^{ab} \bmod p$
- They can use $K = g^{ab} \bmod p$ as symmetric key

Diffie-Hellman

- Suppose Bob and Alice use Diffie-Hellman to determine symmetric key $K = g^{ab} \bmod p$
- Trudy can see $g^a \bmod p$ and $g^b \bmod p$
  - But… $g^a g^b \bmod p = g^{a+b} \bmod p \neq g^{ab} \bmod p$
- If Trudy can find a or b, she gets K
- If Trudy can solve **discrete log** problem, she can find a or b

Diffie-Hellman

- Subject to man-in-the-middle (MiM) attack



- Trudy shares secret $g^{at} \bmod p$ with Alice
- Trudy shares secret $g^{bt} \bmod p$ with Bob
- Alice and Bob don't know Trudy is MiM

Public Key Cryptography

- Two keys, one to encrypt, another to decrypt
  - Alice uses Bob's **public key** to encrypt
  - Only Bob's **private key** decrypts the message
- Based on "trap door, one way function"
  - "One way" means easy to compute in one direction, but hard to compute in other direction
  - Example: Given p and q, product N = pq easy to compute, but hard to find p and q from N
  - "Trap door" is used when creating key pairs

Public Key Cryptography

- Encryption
  - Suppose we **encrypt** M with Bob's public key
  - Bob's private key can **decrypt** C to recover M
- Digital Signature
  - Bob **signs** by "encrypting" with his private key
  - Anyone can **verify** signature by "decrypting" with Bob's public key
  - But only Bob could have signed
  - Like a handwritten signature, but much better…

RSA

- Invented by Clifford Cocks (GCHQ) and **R**ivest, **S**hamir, and **A**dleman (MIT)
  - RSA is the *gold standard* in public key crypto
- Let p and q be two large prime numbers
- Let N = pq be the **modulus**
- Choose e relatively prime to (p–1)(q–1)
- Find d such that ed = 1 mod (p–1)(q–1)
- **Public key** is (N,e)
- **Private key** is d

RSA

- Message M is treated as a number
- To encrypt M we compute

  $C = M^e \bmod N$
- To decrypt ciphertext C, we compute

  $M = C^d \bmod N$
- Recall that e and N are public
- If Trudy can factor N = pq, she can use e to easily find d since $ed = 1 \bmod (p-1)(q-1)$
- So, **factoring the modulus breaks RSA**
  - Is factoring the only way to break RSA?

Does RSA Really Work?

- Given $C = M^e \bmod N$ we want to show that
  $M = C^d \bmod N = M^{ed} \bmod N$
- We'll need **Euler's Theorem:**
  If x is relatively prime to n then $x^{\varphi(n)} = 1 \bmod n$
- Facts:
  1) $ed = 1 \bmod (p-1)(q-1)$
  2) By definition of "mod", $ed = k(p-1)(q-1) + 1$
  3) $\varphi(N) = (p-1)(q-1)$
- Then $ed - 1 = k(p-1)(q-1) = k\varphi(N)$
- So, $C^d = M^{ed} = M^{(ed-1)+1} = M \cdot M^{ed-1} = M \cdot M^{k\varphi(N)}$
  $= M \cdot (M^{\varphi(N)})^k \bmod N = M \cdot 1^k \bmod N =$ **M mod N**

RSA Keypair

- Generate RSA key pair…
  - Select "large" primes $p = 11$, $q = 3$
  - Then $N = pq = 33$ and $(p - 1)(q - 1) = 20$
  - Choose $e = 3$ (relatively prime to 20)
  - Find d such that $ed = 1 \bmod 20$
    - We find that $d = 7$ works
- **Public key:** $(N, e) = (33, 3)$
- **Private key:** $d = 7$

Textbook RSA Example

- **Public key:** (N, e) = (33, 3)
- **Private key:** d = 7
- Suppose message to encrypt is M = 8
- Ciphertext C is computed as

  $C = M^e \bmod N = 8^3 = 512 = 17 \bmod 33$

- Decrypt C to recover the message M by

  $M = C^d \bmod N = 17^7 = 410{,}338{,}673$
  $= 12{,}434{,}505 * 33 + 8 = 8 \bmod 33$

- Why is this "textbook" RSA?

More Efficient RSA (1)

- Modular exponentiation example
  - $5^{20} = 95367431640625 = 25 \bmod 35$
- A better way: **repeated squaring**
  - 20 = 10100 base 2
  - (1, 10, 101, 1010, 10100) = (1, 2, 5, 10, 20)
  - Note that $2 = 1 \cdot 2$, $5 = 2 \cdot 2 + 1$, $10 = 2 \cdot 5$, $20 = 2 \cdot 10$
  - $5^1 = 5 \bmod 35$
  - $5^2 = (5^1)^2 = 5^2 = 25 \bmod 35$
  - $5^5 = (5^2)^2 \cdot 5^1 = 25^2 \cdot 5 = 3125 = 10 \bmod 35$
  - $5^{10} = (5^5)^2 = 10^2 = 100 = 30 \bmod 35$
  - $5^{20} = (5^{10})^2 = 30^2 = 900 = 25 \bmod 35$
- No huge numbers and it's efficient!