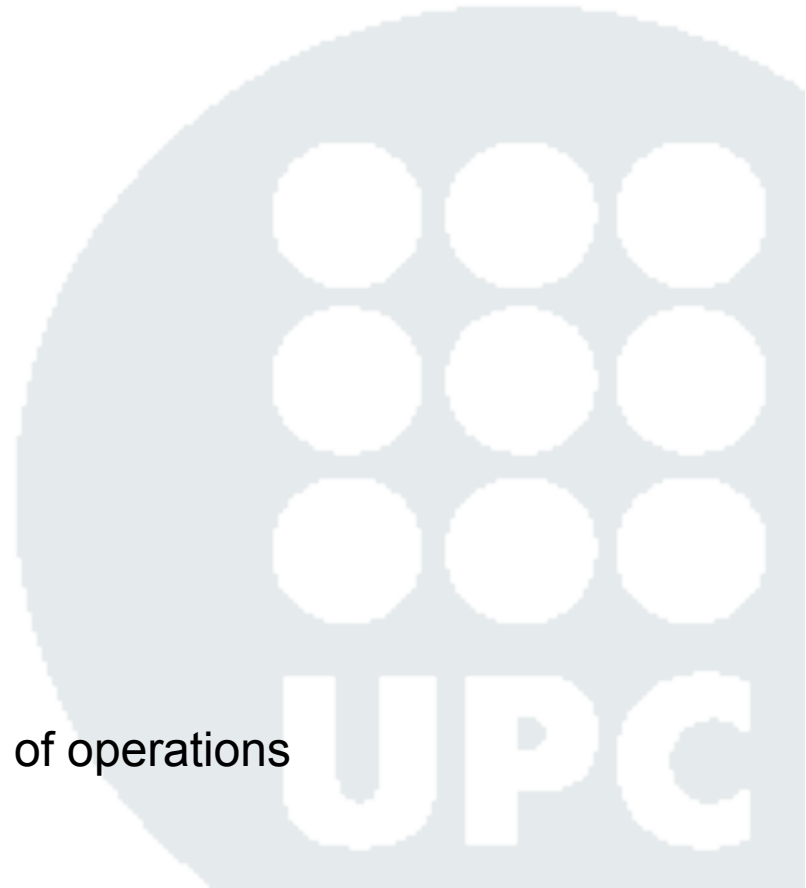


## 3.1 Languages: relational algebra

- Introduction
- Relational algebra operations
  - Union
  - Renaming
  - Intersection
  - Difference
  - Cartesian product
  - Selection
  - Projection
  - Combination (join)
- Relational algebra sequence of operations



## Introduction

- The **data manipulation languages** (DML) can be classified as:

- Languages based on relational algebra
- Languages based on relational calculus (e.g.: SQL)

But most of them use elements from both lines  
(SQL also incorporates elements from algebra).

- **Relational calculus:** Has its basis in the predicate calculations

**Declarative What?**

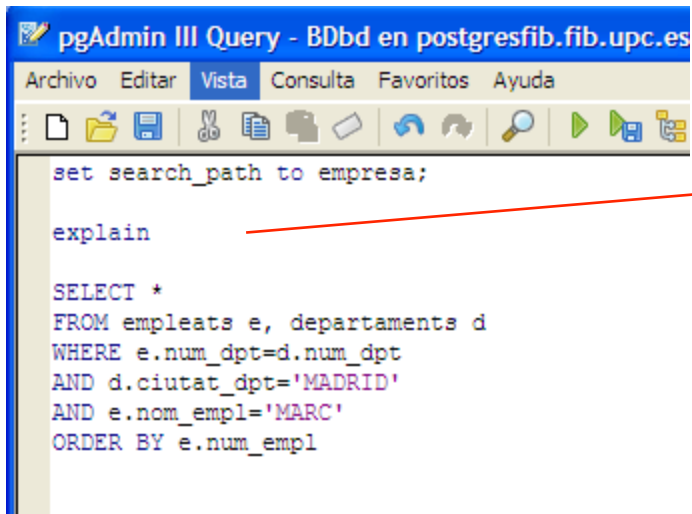
- **Relational algebra:** Has its basis in the set theory  
(remember that the relations are sets)

**Procedimental How?**

- **Interest** of relational algebra:

- Helps to understand which querying functionalities must provide a relational language
- The current standard version of SQL incorporates relational algebra operations
- The DBMS process and optimize the queries based on relational algebra (remember that algebra is procedimental and, for example, SQL is declarative)

# SQL: WHAT?



```

pgAdmin III Query - BDdb en postgresfib.fib.upc.es
Archivo  Editar  Vista  Consulta  Favoritos  Ayuda

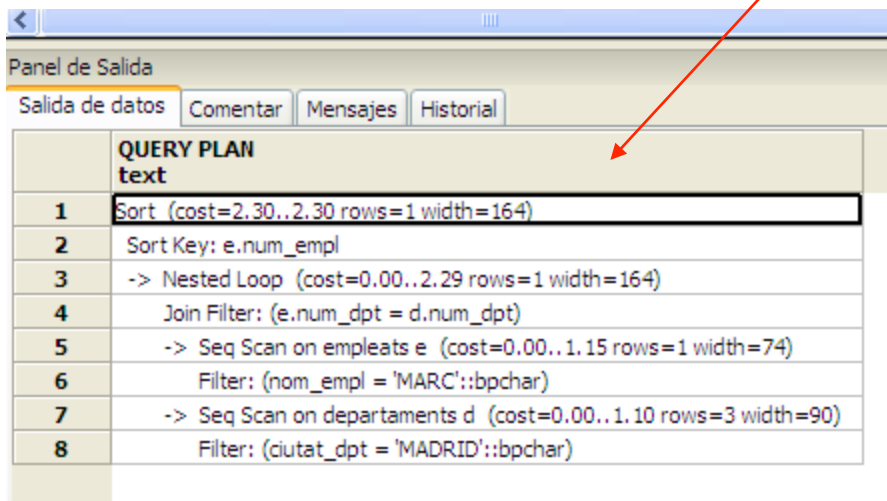
set search_path to empresa;

explain

SELECT *
FROM empleats e, departaments d
WHERE e.num_dpt=d.num_dpt
AND d.ciutat_dpt='MADRID'
AND e.nom_empl='MARC'
ORDER BY e.num_empl
  
```

With this instruction 'EXPLAIN' we tell Postgres to explain us HOW it will execute this sentence

# ALGEBRA: HOW?



	QUERY PLAN text
1	Sort (cost=2.30..2.30 rows=1 width=164)
2	Sort Key: e.num_empl
3	-> Nested Loop (cost=0.00..2.29 rows=1 width=164)
4	Join Filter: (e.num_dpt = d.num_dpt)
5	-> Seq Scan on empleats e (cost=0.00..1.15 rows=1 width=74)
6	Filter: (nom_empl = 'MARC')::bpchar
7	-> Seq Scan on departaments d (cost=0.00..1.10 rows=3 width=90)
8	Filter: (ciutat_dpt = 'MADRID')::bpchar

The result contains, among other things (cost, result width, algorithms), the expression in relational algebra that represents the best possible sequence of execution to resolve the previous query.

- 4. Sort of result from 3
- 3. Join between results from 1 and 2
- 2. Selection over employees
- 1. Selection over departments

## Operations of the relational algebra

- **1st classif.:** Set operations

- Union
- Intersection
- Difference
- Cartesian product

Operations specifically relational

- Selection
- Projection
- Combination (join)
- Renaming

- **2nd classif.:** Primitive operations

- Union
- Difference
- Cartesian product
- Selection
- Projection
- Renaming

Non-primitive operations

- Intersection
- Combination (join)

- **3rd classif.:** Binary operations

- Union
- Intersection
- Difference
- Cartesian product
- Combination (join)

Unary operations

- Selection
- Projection
- Renaming

- **Relational closure:** Both the operands and the result of a relational algebra operation are relations

Ex:  $T = R \cup S$

## Example

MODULE-CN(module, avg-surface)

B6	10
B2	20

OFFICE(module-dp, num-dp, superfície)

B6	25	10
B6	27	10
B2	25	15
B2	30	25

{module-dp} is a foreign key  
referencing MODULE-CN

ADM-PERSONNEL(num-per, name, surname, module, num)

100	Joan	Soler	B6	25
150	Clara	Bellsolà	B6	25

{module, num} is a foreign key  
referencing OFFICE

LAB-PERSONNEL(num-per, name, surname, module, num)

150	Clara	Bellsolà	B6	25
110	Núria	Nogué	B2	25
200	Jordi	Moles	B6	27
230	Pere	Roig	NULL	NULL

{module, num} is a foreign  
key referencing OFFICE

## Union

ADM-PERSONNEL( <u>num-per</u> , name, surname, module, num)					
100	Joan	Soler	B6	25	
150	Clara	Bellsolà	B6	25	

LAB-PERSONNEL( <u>num-per</u> , name, surname, module, num)					
150	Clara	Bellsolà	B6	25	
110	Núria	Nogué	B2	25	
200	Jordi	Moles	B6	27	
230	Pere	Roig	NULL	NULL	

$$R = \text{ADM-PERSONNEL} \cup \text{LAB-PERSONNEL}$$

R( <u>num-per</u> , name, surname, module, num)					
100	Joan	Soler	B6	25	
150	Clara	Bellsolà	B6	25	
110	Núria	Nogué	B2	25	
200	Jordi	Moles	B6	27	
230	Pere	Roig	NULL	NULL	

There are not repeated tuples!!!

- The schema attributes of the resulting relation  $T \cup S$  are the same attributes of the T schema or S schema.
- The **body** of the resulting relation  $T \cup S$  is the set of tuples belonging to the body of T, belonging to the body of S, or belonging to both of them
- To make the union between two relations, T and S must be **compatible relations**.
- In the case where the attributes from T and S have different names, we need to rename the attributes from one of the relations so that the relations will be compatible.

## Compatible relationships

- Some relational algebra operations, such as the union, only make sense when applied to compatible relations (with “similar” tuples)

- Example: we can make the union

$\text{ADM-PERSONNEL} \cup \text{LAB PERSONNEL}$

because the tuples from both relations are similar. However, there is no point in making the union

$\text{ADM-PERSONNEL} \cup \text{OFFICES}$

- **Two relations T and S are compatible if:**

- They have schemas with an identic attribute set, and the domains for each pair of attributes are the same in T and in S.

Example:

ADM-PERSONNEL and LAB-PERSONNEL are clearly compatible:

ADM-PERSONNEL(num-per, name, surname, module, num)

LAB-PERSONNEL(num-per, name, surname, module, num)



## Renaming

DOC-PERSONNEL(num-per, name, surname, module-dp, num-dp)					
400	Jaume	Cases	Omega	119	
500	Pau	Pou	B6	123	



**R = DOC-PERSONNEL {module-dp -> module, num-dp -> num}**

R(num-per, name, surname, module, num)					
400	Jaume	Cases	Omega	119	
500	Pau	Pou	B6	123	

- The **schema attributes** of the resulting relation is the same as the initial relation except the attributes names that have been renamed.
- The **body** of the resulting relation does not change.



## Intersection

ADM-PERSONNEL( <u>num-per</u> , name, surname, module, num)					
100	Joan	Soler	B6	25	
150	Clara	Bellsolà	B6	25	

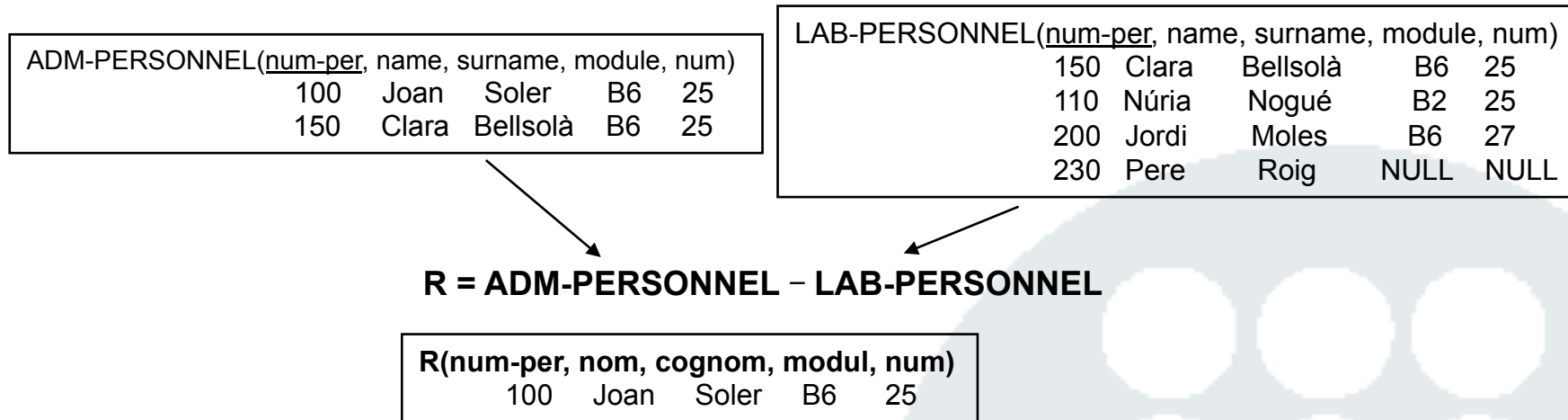
LAB-PERSONNEL( <u>num-per</u> , name, surname, module, num)					
150	Clara	Bellsolà	B6	25	
110	Núria	Nogué	B2	25	
200	Jordi	Moles	B6	27	
230	Pere	Roig	NULL	NULL	

$$R = \text{ADM-PERSONNEL} \cap \text{LAB-PERSONNEL}$$

R( <u>num-per</u> , name, surname, module, num)					
150	Clara	Bellsolà	B6	25	

- The schema attributes of the result relationship  $T \cap S$  are the same attributes of the T schema or the S schema.
- The **body** of the relations resulting of  $T \cap S$  is the set of tuples belonging to the body of both relations
- To make the intersections between two relations, T and S must be **compatible relationships**
- In the case where the attributes from T and S do have different names, we need to rename the attributes from one of the relationships so that the relations will be compatible

## Difference



- The **schema attributes** of the result relation  $T - S$  are the same attributes of the  $T$  schema or the  $S$  schema.
- The **body** of the relation resulting of  $T - S$  is the set of tuples belonging to the body of  $T$  but not to the body of  $S$ .
- To make the difference between two relationships,  $T$  and  $S$  must be **compatible relations**.
- In the case where  $T$  and  $S$  have different attribute names, we need to rename the attributes from one of the relationships so that the relations will be compatible

## Cartesian product

MODULE-CN(module, avg-surface)	
B6	10
B2	20

OFFICE(modul-dp, num-dp, surface)		
B6	25	10
B6	27	10
B2	25	15
B2	30	25

**R = MODULE-CN × OFFICE**

R(module, avg-surface, module-dp, num-dp, surface)				
B6	10	B6	25	10
B6	10	B6	27	10
B6	10	B2	25	15
B6	10	B2	30	25
B2	20	B6	25	10
B2	20	B6	27	10
B2	20	B2	25	15
B2	20	B2	30	25

- The **schema attributes** of the resulting relation  $T \times S$  are all the attributes from  $T$  plus all the attributes from  $S$ .
- If  $T$  and  $S$  have attributes with the same name, previously we will need to rename one of the two relations to avoid the ambiguity.
- The **body** of the resulting relation from  $T \times S$  is the set of all the tuples of the form  $\langle v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_m \rangle$  where  $\langle v_1, v_2, \dots, v_n \rangle$  belongs to the body of  $T$  and  $\langle w_1, w_2, \dots, w_m \rangle$  belongs to the body of  $S$ .
- To make the cartesian product of two relations  $T$  and  $S$ ,  $T$  and  $S$  must not be **compatible relations**.

## Selection

OFFICE(module-dp, num-dp, surface)		
B6	25	10
B6	27	10
B2	25	15
B2	30	25



**R = OFFICE(module-dp= 'B2' AND surface > 16)**

R(module-de, num-de, surface)		
B2	30	25

- **T(C)** denotes the selection of T with the condition C, being C the selection condition
- The condition C is composed by one or more comparisons of the form:

$$A_i \theta V_j \text{ or } A_i \theta A_j$$

where  $A_i$  and  $A_j$  are attributes of the relation T,  $V_j$  is a constant value, and  $\theta$  is a comparison operator ( $=$ ,  $<>$ ,  $<$ ,  $<=$ ,  $>$ ,  $>=$ ). The comparisons have to be interrelated by one of the logical operators AND ( $\wedge$ ), OR ( $\vee$ ).

- The **schema attributes** of the resultint relation T(C), are the same attributes of the T schema.
- The **body** of T(C) is the set of tuples that belong to the body of T that satisfy the condition C.

## Projection

ADM-PERSONNEL(per-num, name, surname, module, num)					
100	Joan	Soler	B6	25	
150	Clara	Bellsolà	B6	25	



**R = ADM-PERSONNEL[module, num]**

R(module, num)	
B6	25

In relational algebra, **THERE ARE NOT REPEATED TUPLES**, because the result of the operations are sets.

- **$T[A_i, A_j, \dots, A_k]$**  denotes the projection of  $T$  over  $\{A_i, A_j, \dots, A_k\}$ , being  $\{A_i, A_j, \dots, A_k\}$  a subset of the attributes of the relation  $T$  schema.
- The **schema attributes** of the resulting relationship  $T[A_i, A_j, \dots, A_k]$ , are the attributes  $\{A_i, A_j, \dots, A_k\}$
- The **body** of the resulting relationship  $T[A_i, A_j, \dots, A_k]$  is the subset of all the tuples with the form  $\langle t.A_i, t.A_j, \dots, t.A_k \rangle$  where  $t$  is a tuple that belongs to the body of  $T$  and where  $t.A_p$  denotes the value for the attribute  $A_p$  from the tuple  $t$ .

## Combination (join)

MODULE-CN(module, avg-surface)		
B6	10	
B2	20	

OFFICE(module-dp, num-dp, surface)		
B6	25	10
B6	27	10
B2	25	15
B2	30	25

**R = MODULE-CN[module=module-dp]OFFICE**

R(module, avg-surface, module-dp, num-dp, surface)				
B6	10	B6	25	10
B6	10	B6	27	10
B2	20	B2	25	15
B2	20	B2	30	25

**R = MODULE-CN[module=module-dp, avg-surface<=surface]OFFICE**

R(module, avg-surface, module-dp, num-dp, surface)				
B6	10	B6	25	10
B6	10	B6	27	10
B2	20	B2	30	25

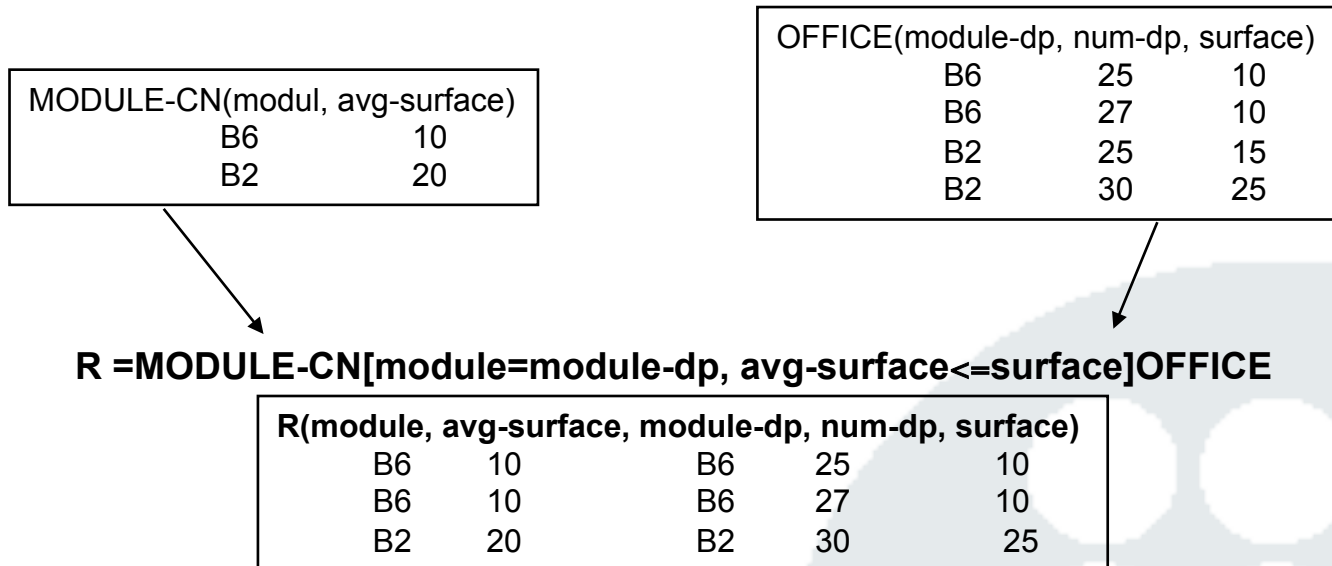
- **T[B]S** denotes the combination of T and S with the condition B
- The **condition B** of a combination **T [B] S** is composed by one or more comparisons with the form:

$$A_i \theta A_j$$

where  $A_i$  is an attribute of the relation T,  $A_j$  is an attribute of the relation S,  $\theta$  is a comparison operator ( $=, \neq, <, \leq, >, \geq$ ) and it matches that  $A_i$  and  $A_j$  have the same domain.

- The different conditions in a combination or join are separated by commas.

## Combination (join)



- The **schema attributes** of the relation resulting of  $T[B]S$  are all the attributes of  $T$  and all the attributes of  $S$ .
- If  $T$  and  $S$  have some identical attribute name, previously we will need to rename one of the two relations to avoid the ambiguity.
- The **body** of the resulting relation  $T[B]S$  is the set of tuples belonging to the extension of the cartesian product  $T \times S$  that match all the comparisons that form the combination condition  $B$ .

## Combination (join): Types of “joins”

- “**θ-join**”: The “join” is also called “θ-join”
- “**Equi-join**”: Particular case of “join” where all the comparisons of the condition have the operator ‘=’.

ADM-PERSONNEL(per-num, name, surname, module, num)					
100	Joan	Soler	B6	25	
150	Clara	Bellsolà	B6	25	

OFFICE(module-dp, num-dp, surface)		
B6	25	10
B6	27	10
B2	25	15
B2	30	25

**R = ADM-PERSONNEL[module=module-dp, num=num-dp]OFFICE**

R(num-per, name, surname, module, num, module-dp, num-dp, surface)								
100	Joan	Soler	B6	25	B6	25	10	
150	Clara	Bellsolà	B6	25	B6	25	10	

- “**Natural join**”: Variant of the “equi-join” without all the unnecessary attributes. We denote it using \*. The difference with the “equi-join” is that in the natural join schema, the second attribute of each comparison does not appear anymore in the resulting schema.

ADM-PERSONNEL(per-num, name, surname, module, num)					
100	Joan	Soler	B6	25	
150	Clara	Bellsolà	B6	25	

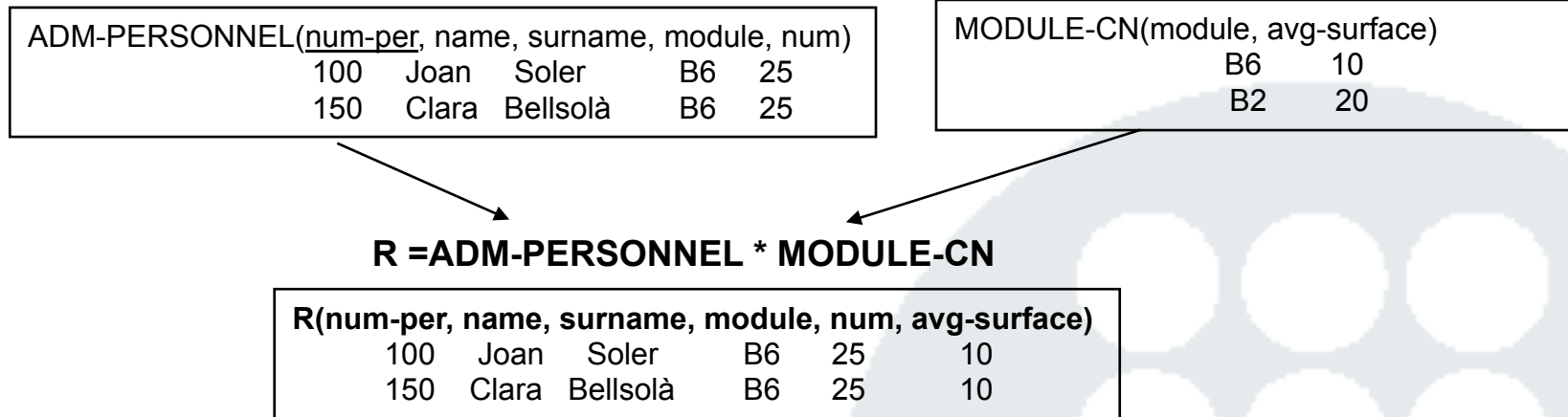
OFFICE(module-dp, num-dp, surface)		
B6	25	10
B6	27	10
B2	25	15
B2	30	25

**R = ADM-PERSONNEL[module\*module-dp, num\*num-dp]OFFICE**

R(num-per, name, surname, module, num, surface)						
100	Joan	Soler	B6	25	10	
150	Clara	Bellsolà	B6	25	10	



## Combination (join): implicit “Natural join”



- The **implicit “natural join”**: Variant of the “natural-join” where we do not specify the condition of the combination and therefore we assume (by default) that the combination condition corresponds to the one of a “natural join”, where all the pairs of attributes with the same name on both relations are matched .
- **T \* S** denotes the implicit “natural join” of T and S.

## Sequences of operations of relational algebra

**Example:** Get the offices (module and number) from the modules that have an average surface bigger than 15.

MODULE-CN(module, avg-surface)	
B6	10
B2	20

OFFICE(module-dp, num-dp, surface)		
B6	25	10
B6	27	10
B2	25	15
B2	30	25

**A = MODULE-CN(avg-surface > 15)**

**B = A{module -> module-dp}**

**C = OFFICE \* B**

**R = C[module-dp, num-dp]**

R( modul-dp, num-dp)	
B2	25
B2	30

- The **queries** of a relational DB may be expressed in terms of relation algebra **sequences of operations**.
- The sequences of operations allow us to define a relation that contains exactly what we want to query.