Semidefinite Programming for NLOS Error Mitigation in TDOA Localization

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***Abstract*— Non-line-of-sight (NLOS) error mitigation for the time-of-arrival (TOA) localization has been extensively studied, but these methods cannot be directly applied for the time-difference-of-arrival (TDOA) systems. Recent work has applied convex optimization for NLOS error mitigation in TDOA systems. Issues remain unsolved with this technique include the convex hull problem, reference-anchor selection problem, and difficulties dealing with a wide range of NLOS-caused ranging errors. This letter proposes a new technique to transform a TDOA model into a TOA model and develops a semidefinite programming method with new constraints for effective NLOS error mitigation in TDOA systems. Major advantages of this method include: 1) it resolves the issues such as convex hull and reference-anchor selection issues that existing schemes are facing; 2) it does not require any *a priori* information about NLOS links or NLOS error statistics; and 3) it achieves a better performance than existing convex optimization schemes, which is verified in both simulation and real experiments.**

***Index Terms*— Time-difference-of-arrival (TDOA) positioning, non-line-of-sight (NLOS), semidefinite programming (SDP).**

I. Introduction

TIME-OF-ARRIVAL (TOA) [1] and time-difference-of-arrival (TDOA) [2] techniques are widely used in indoor positioning systems. A common issue that reduces the accuracy of these systems is non-line-of-sight (NLOS) propagation. NLOS error mitigation and NLOS link identification for TOA localization have been investigated extensively, assuming that *a priori* information of NLOS links and/or NLOS error statistics are available [3]–[7], or other forms of system/channel resources are available [8]–[12]. Convex optimization has recently been applied for NLOS error mitigation in TOA systems without requiring such information in [13]–[15], and in [16], where an unknown source transmission time is assumed.

Convex optimization is also applied in TDOA systems for NLOS error mitigation [17]–[22], but there are many unsolved issues. For TOA systems in NLOS conditions, since the NLOS bias is positive and is typically much larger than measurement noise, an additional constraint restricting the target to be inside a circle with the range as the radius and the anchor as the center can be applied in the optimization process. In TDOA localization, however, NLOS-caused ranging errors could be positive or negative, making the problem more complex.

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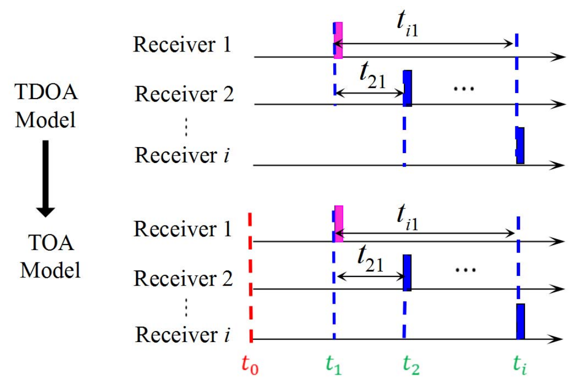

Fig. 1. Transformation of a TDOA model into a TOA model.

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Also, convex optimization for TDOA systems might encounter the convex hull problem; that is, the estimated positions lie in the convex hull formed by the sensor nodes [20]. In addition, since a reference must be selected first to obtain the observations, it is possible that an NLOS link might be chosen as the reference, which will further degrade the performance. To avoid choosing an NLOS link as the reference, different methods have been developed: in [18] a method that tries each anchor as the reference to estimate the location and then uses a linear combination of these estimated locations as the final result is developed; in [20] an LOS link is identified first, which is then used as the reference.

These methods result in very complex estimators, and still do not effectively solve all the problems. The methods developed for NLOS mitigation in TOA localization cannot be applied directly to TDOA systems due to their architectural differences and some of the issues discussed above [17].

To address the issues of NLOS mitigation in TDOA systems that existing schemes are facing, a new method that transforms a TDOA architecture into a TOA architecture is proposed in this letter, together with a semidefinite programming (SDP) method with new constraints for this method. Also, the proposed technique does not require *a priori* information about NLOS links or NLOS error statistics, like existing convex optimization schemes. And it achieves a much better performance than existing convex optimization schemes, which will be validated in both simulation and real experiments.

II. Proposed Method

Consider a TDOA system with M time-synchronized anchors, assumed to be receivers, where the first anchor is used as the reference, without loss of generality. As shown in Fig. 1, the distance-difference between the source node, which is assumed to be a transmitter, to the ith receiver and the reference receiver is expressed as

$$\begin{aligned} r_{i1} &= ct_{i1} = r_i - r_1 \\ &= (d_i + b_i - d_1 - b_1) + (n_i - n_1), \quad i = 1, \cdots, M, \qquad (1) \end{aligned}$$

where r_i and d_i are the measured and true distances from the source node to the ith anchor, respectively, t_{i1} is the signal propagation time difference between the ith and first (reference) receiver, b_i represents the NLOS-caused positive bias, and n_i is the range-measurement noise, which is commonly modeled as a zero-mean Gaussian variable with variance σ^2 [17]. Note that both LOS and NLOS conditions are considered in this letter; if the ith link is an LOS link, then $b_i = 0$.

The basic idea being developed here is to localize the source by using an emulated TOA model but with the known information from the TDOA model, such as t_{i1} [24]. In TOA systems, all anchors must be time-synchronized with the target; in TDOA systems, target-anchor synchronization is not necessary, but all anchors must be time-synchronized. In the TDOA model in Fig. 1, if the first receiver is designated as the reference, then t_{i1} will be the observed timing difference between the ith and the reference receivers, which is the only available information represented as

$$t_{i1} = t_i - t_1,\ i = 1, \cdots, M, \tag{2}$$

where t_i and t_1 are time instants when the signal arrives at the first and the ith receiver, respectively. If t_0 represents the time instant when the signal leaves the transmitter, then the resulting TOA model can be described as

$$r_i = d_i + b_i + n_i \tag{3a}$$
$$(t_i - t_0)c = d_i + b_i + n_i,\ i = 1, \cdots, M. \tag{3b}$$

Note that t_i, t_1, and t_0 in (2) and (3b) are unknown in TDOA systems. However, if t_1, the time instant for the reference receiver, is set at some value, for example, 0.1 seconds, then t_i could be calculated as $t_{i1} - t_1$, and t_0 will be some value less than or equal to 0.1 seconds.

Squaring both sides of (3b) followed by some algebraic manipulations yields

$$\begin{aligned} c^2t_i^2 + c^2t_0^2 - 2c^2t_it_0 - d_i^2 - b_i^2 - 2b_id_i &= 2n_i(b_i + d_i) + n_i^2 \\ &= \epsilon_i. \end{aligned} \tag{4}$$

Let

$$q_i = b_i^2 + 2b_id_i. \tag{5}$$

Eq. (4) simplifies to

$$\epsilon_i = c^2t_i^2 + c^2t_0^2 - 2c^2t_it_0 - d_i^2 - q_i. \tag{6}$$

Since the measurement noise n_i is generally much smaller than $b_i + d_i$ in practice, the n_i^2 term in ϵ_i can be neglected; even in LOS cases where $b_i = 0$, this approximation is valid as long as n_i is small relative to the actual range.

The nonlinear least-squares estimator of the unknown parameters $\boldsymbol{\theta}, \boldsymbol{q}, \boldsymbol{d}, t_0$ is expressed as

$$\begin{aligned} &\underset{\boldsymbol{d},\boldsymbol{b},\boldsymbol{\theta},\boldsymbol{q},\boldsymbol{t_0}}{\operatorname{argmin}} \sum_{i=1}^{M} w_i \left(c^2t_i^2 + c^2t_0^2 - 2c^2t_it_0 - d_i^2 - q_i\right)^2, \\ &\text{s.t.}\ \ q_i = b_i^2 + 2b_id_i \end{aligned} \tag{7}$$

where w_i is a positive weight and $\boldsymbol{\theta}$ is the location of the target to be estimated, which could be 2-dimensional (2D) or 3D with coordinates (x, y) and (x, y, z), respectively. Eq. (7) is non-linear and non-convex. Introduce two new variables:

$$s = t_0^2 \tag{8a}$$
$$h_i = d_i^2 = ||\boldsymbol{V}_i - \boldsymbol{\theta}||_2^2, \tag{8b}$$

where $\boldsymbol{V}_i$ is the position of the ith receiver, which could be 2D or 3D like $\boldsymbol{\theta}$. Eq. (8b) can be written in vector-matrix form by using the Schur complement as [14]

$$h_i = \begin{pmatrix} \boldsymbol{V}_i \\ -1 \end{pmatrix}^T \begin{pmatrix} \boldsymbol{I}_2 & \boldsymbol{\theta} \\ \boldsymbol{\theta} & z \end{pmatrix} \begin{pmatrix} \boldsymbol{V}_i \\ -1 \end{pmatrix}; \tag{9a}$$
$$\begin{pmatrix} \boldsymbol{I}_2 & \boldsymbol{\theta} \\ \boldsymbol{\theta} & z \end{pmatrix} \succcurlyeq \boldsymbol{0}, \tag{9b}$$

where z is a new variable added.

Since b_i and d_i are positive, q_i in (5) satisfies

$$q_i \geq 0. \tag{10}$$

Eq. (7) is transformed into an SDP problem as

$$\begin{aligned} &\underset{\boldsymbol{h},\boldsymbol{s},\boldsymbol{c},\boldsymbol{\theta},z,t_0}{\operatorname{argmin}} \sum_{i=1}^{M} w_i \left(c^2t_i^2 + c^2s - 2c^2t_it_0 - h_i - q_i\right)^2 \\ &\qquad\qquad + \sum_{i=1}^{M} \rho(q_i^2 + s^2) \\ &\text{s.t.}\quad \text{Eqs. (9) and (10)} \end{aligned} \tag{11}$$

where ρ is a penalization factor, which is required when the problem is ill-posed [14]. Although (11) may be solved mathematically, a good performance is not guaranteed.

Since t_0 is a key variable for the model transformation, a few constraints on t_0 are developed to improve its estimation accuracy. First, a geometric constraint:

$$(t_i - t_0)c + (t_j - t_0)c \geq ||(\boldsymbol{V}_i - \boldsymbol{V}_j)||_2, \quad i \neq j. \tag{12}$$

Since the ith and jth receivers as well as the transmitter can form a triangle, (12) holds because the sum of two sides of a triangle will not be smaller than the third side. Second, since t_i is the arrival time of the signal at the ith receiver,

$$t_i \geq t_0. \tag{13}$$

Since the biases of NLOS links in (3b) are positive and typically much larger than the measurement noise n_i,

$$r_i = (t_i - t_0)c \geq d_i \quad \text{or}\ r_i^2 = (t_i - t_0)^2c^2 \geq d_i^2. \tag{14}$$

Further with (1):

$$r_i^2 = (t_i - t_0)^2c^2 \tag{15a}$$
$$= (t_i^2 - t_it_0 + t_0^2 - t_it_0)c^2 \tag{15b}$$
$$\leq (t_i^2 - t_it_0)c^2. \tag{15c}$$

Eq. (15b) can be relaxed to be linear as expressed in (15c) since $t_i \geq t_0$ and the time instants are positive, that is, $t_0 \geq 0, t_i \geq 0$. Therefore, $t_0^2 - t_it_0 \leq 0$, and (15c) follows.

Eqs. (14) and (15) show that

$$(t_i^2 - t_it_0)c^2 \geq r_i^2 \geq d_i^2 = h_i. \tag{16}$$

It is possible that (16) may not be feasible if $b_i = 0$ and the noise is negative (i.e., $n_i < 0$), since in such a case, $r_i < d_i$, as shown in (3a). We resort to the soft-minimum method [23]

TABLE I

ESTIMATORS CONSIDERED IN THE PERFORMANCE COMPARISON

Estimator	Description
PROPOSED	The proposed estimator.
SDR-ROBUST	The robust semidefinite relaxation method in [17].
SDP	The SDP estimator in [20].
CRLB	CRLB for TDOA in NLOS conditions [24].

to resolve this problem. In this method, a variable $u_i > 0$ is introduced in (16) to make it valid:

$$(t_i^2 - t_i t_0)c^2 + u_i \geq d_i^2 = h_i, \tag{17a}$$

$$u_i \geq 0. \tag{17b}$$

A problem with this approach is that a large u_i will loosen this constraint. To ensure a strict constraint, the following item is added to the objective function

$$\mu \sum_{i=1}^{M} u_i^2, \tag{18}$$

where μ (>0) is to be determined. With the method in [23], the constraint in (17) tends to choose a proper value of u_i to ensure feasibility of the constraint while (18) in the objective function optimizes the value of u_i to make the constraint tight.

The SDP problem is summarized as

$$\begin{aligned} \underset{\boldsymbol{h},\boldsymbol{q},\boldsymbol{\theta},\boldsymbol{u},z,t_0,s}{\operatorname{argmin}} \quad & \sum_{i=1}^{M} w_i \left(c^2 t_i^2 + c^2 s - 2c^2 t_i t_0 - h_i - q_i\right)^2 \\ & + \sum_{i=1}^{M} \rho (q_i^2 + s^2) + \mu \sum_{i=1}^{M} u_i^2 \\ \text{s.t.} \quad & \text{Eqs. (9), (10), (12), (13), (17).} \end{aligned} \tag{19}$$

This technique completely resolves the problems that existing schemes face for NLOS mitigation in TDOA systems such as the convex hull problem and difficulty selecting a proper reference anchor, since after the transformation, we are effectively dealing with a TOA system. Additionally, it does not require any prior information about the NLOS links or NLOS error statistics.

III. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Setup and Results

The performances of the methods listed in Table I are compared via simulation and experimental results. In the simulation, the sources are uniformly distributed in a $50m \times 50m$ space. And the eight anchors are placed at $(\pm 20, \pm 20)m$, $(\pm 20, 0)m$ and $(0, \pm 20)m$. As adopted in [16], the NLOS bias is assumed to be uniformly distributed over $[0m, 10m]$, and is unknown to all estimators. The mean-squared error (MSE) of the estimators is simulated under different noise conditions. The optimal weights for each link should be set according to the link biases. Since in practice the biases are unknown, as in [23], the weighting elements w_i are all set to 1. The penalization factor is set as $\rho = 0.01$, and μ is set as $\mu = 1$.

Note that t_1 should be given a proper value (e.g., $t_1 = 0.1$ or at least greater than 0) that guarantees the validity of (16). Also in (19), c^2, and timing information such as t_0 and t_i is directly involved in the calculation. Since c^2 is huge relative to the difference between t_i and t_0, computation might cause a loss of precision since a tiny change in t_0 will cause large error in the final estimation. To avoid this potential loss of precision, t_i is multiplied by 10^8 while c is scaled by 10^{-8}. This normalization avoids the loss of computation precision but does not affect the final result.

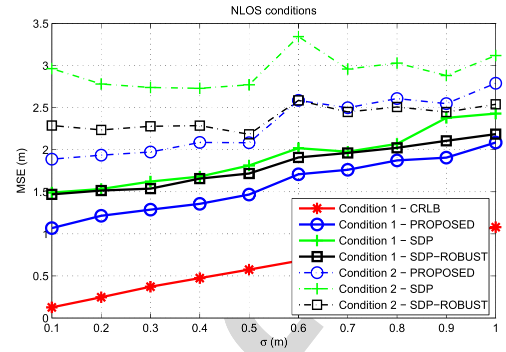

Fig. 2. MSEs for different values of σ and NLOS conditions.

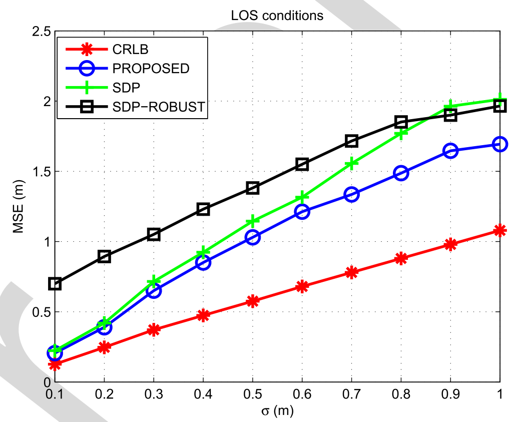

Fig. 3. MSEs for different values of σ and LOS conditions.

Simulation results are shown in Fig. 2, where

- Condition 1: $1 \sim 5$ of the 8 links are NLOS links;
- Condition 2: $6 \sim 8$ of the 8 links are NLOS links, which represents an unrealistically severe NLOS situation.

The Cramer-Rao lower bound (CRLB) for TDOA systems in NLOS conditions [24] is used as the performance benchmark. It is observed from Fig. 2 that under Condition 1, the proposed estimator significantly outperforms other convex optimization estimators. In Condition 2, the unrealistically severe NLOS situation, the the proposed estimator still outperforms other estimators when $\sigma \leq 0.6m$; after this point, SDR-ROBUST starts to perform slightly better than the proposed method.

Since the proposed algorithm is developed to mitigate NLOS errors, it should still work well in LOS conditions. Comparison in LOS conditions is shown in Fig. 3. The MSE of the proposed estimator grows faster with σ than that of the CRLB. However, it still outperforms other NLOS mitigation estimators in LOS conditions.

Also, the proposed scheme does not have the convex hull problem [20]. In the simulation, about 30% of chosen target locations are outside (5 meters away) of the hull formed by the anchors, and unlike with existing methods, the estimated positions with the proposed schemes do not fall in the hull.

TABLE II
AVERAGE ERROR AND STD FOR 2D TARGET POSITION ESTIMATES

	Proposed	SDP	SDP-ROBUST
Average error (cm)	110.4	160.1	136
Error std (cm)	131.2	171	140.1

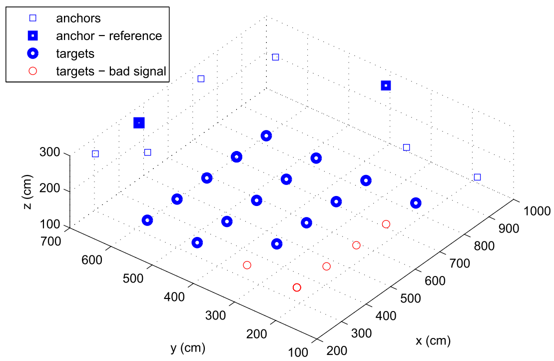

Fig. 4. Experiment setup.

B. Experimental Setup and Results

A TDOA localization system with an 80 MHz bandwidth WiFi signal (IEEE 802.11ac) generated from an iPhone 6 as the target is set up in a laboratory to test the proposed algorithm. The setup is shown in Fig. 4. In this experiment, two independently working clusters of receivers are used. Each cluster has four synchronized receivers, with one as the reference receiver.

NLOS propagation for each target position is created by randomly blocking 0 to 8 links of the signal between the target and the receivers. For each position, 500 sets of data are acquired. The average error and its standard deviation (std) of the two dimensional (2D) location estimates are shown in Table II. Here error is defined as the distance between the actual position and the estimated position. Because the receiver gain is very low and due to limitation of available hardware, for some of the positions marked in red in Fig. 4, the received signals are too weak to generate valid results for any algorithm. Thus, these positions are excluded in the comparison. Also, since SDP-ROBUST requires prior information of the maximum NLOS bias. In this experiment the maximum bias is set to be 3 meters, which maximize the performance compared to other settings. Two dimensional localization is considered in this experiment. The results also show that the proposed scheme has a superior performance than existing schemes.

IV. CONCLUSION

After analyzing the issues of NLOS mitigation in TDOA systems with existing methods, we first propose a scheme to transform a TDOA system into an effective TOA system to resolve these issues. Then, we formulate an SDP problem with new constraints to mitigate NLOS effects without requiring any prior information about the NLOS links or NLOS error statistics, a major advantage of the proposed scheme over some existing techniques. The performance of the proposed scheme is compared with those of existing schemes in simulations as well as in experiments in a realistic environment. The comparison results show that the proposed scheme has a significant performance gain in nearly every scenario evaluated.

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