

First-class Polymorphism: Type Inference and Extensions

by

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A thesis submitted in partial fulfillment of the requirements for
the degree of Doctor of Philosophy
at The University of Hong Kong

February 2021

Abstract of thesis entitled
“First-class Polymorphism: Type Inference and Extensions”

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for the degree of Doctor of Philosophy
at The University of Hong Kong
in February 2021

DECLARATION

I declare that this thesis represents my own work, except where due acknowledgment is made, and that it has not been previously included in a thesis, dissertation or report submitted to this University or to any other institution for a degree, diploma or other qualifications.

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February 2021

ACKNOWLEDGMENTS

CONTENTS

DECLARATION	I
ACKNOWLEDGMENTS	III
LIST OF FIGURES	VII
LIST OF TABLES	IX
I PROLOGUE	1
1 INTRODUCTION	3
1.1 Contributions	3
1.2 Organization	4
2 BACKGROUND	7
II TECHNICAL CONTRIBUTIONS	9
3 HIGHER RANK TYPE INFERENCE WITH THE APPLICATION MODE	11
4 HIGHER RANK GRADUAL TYPES	13
5 TYPE PROMOTION	15
III RELATED AND FUTURE WORK	17
6 RELATED WORK	19
7 FUTURE WORK	21
	 v

Contents

IV EPILOGUE	23
8 CONCLUSION	25
BIBLIOGRAPHY	27

LIST OF FIGURES

LIST OF TABLES

PART I

PROLOGUE

1 INTRODUCTION

1.1 CONTRIBUTIONS

In summary the contributions of this thesis are:

- Chapter 3 proposes a new design for type inference predicative implicit higher-rank polymorphism.
 - We design a variant of bi-directional type checking where the inference mode is combined with a new, so-called, application mode. The application mode naturally propagates type information from arguments to the functions.
 - With the application mode, we give a new design for type inference of higher-ranked type, which generalizes the HM type system, supports a polymorphic let as syntactic sugar, and infers higher rank types. We present a syntax-directed specification, an elaboration semantics to System F, and an algorithmic type system with completeness and soundness proofs.
- Chapter 4 extends predicative implicit higher-rank polymorphism with gradual types.
 - We define a framework for consistent subtyping with
 - * a new definition of consistent subtyping that subsumes and generalizes that of Siek and Taha [2007] and can deal with polymorphism and top types;
 - * and a syntax-directed version of consistent subtyping that is sound and complete with respect to our definition of consistent subtyping, but still guesses instantiations.
 - Based on consistent subtyping, we present the calculus GPC. We prove that our calculus satisfies the static aspects of the refined criteria for gradual typing [Siek et al. 2015], and is type-safe by a type-directed translation to λB [Ahmed et al. 2009].
 - We present a sound and complete bidirectional algorithm for implementing the declarative system based on the design principle of Garcia and Cimini [2015].

1 Introduction

- Chapter 5 presents a variant of unification for implicit higher-rank polymorphism
 - We propose a process named promotion, which promotes a type so that it is well-typed in the prefix context of a type variable.
 - We apply promotion to predicative implicit higher-rank polymorphism and redesign the type variable instantiation.
 - We further explore the design of promotion by an application to the kind inference problem for datatypes.
 - ★ We formalize Haskell98’s datatype declarations, providing both a declarative specification and syntax-driven algorithm for kind inference. We prove that the algorithm is sound and observe how Haskell98’s technique of defaulting unconstrained kinds to \star leads to incompleteness. We believe that ours is the first formalization of this aspect of Haskell98.
 - ★ We then present a type and kind language that is unified and dependently typed, modeling the challenging features for kind inference in modern Haskell. We include both a declarative specification and a syntax-driven algorithm. The algorithm is proved sound, and we observe where and why completeness fails. In the design of our algorithm, we must choose between completeness and termination; we favor termination but conjecture that an alternative design would regain completeness. Unlike other dependently typed languages, we retain the ability to infer top-level kinds instead of relying on compulsory annotations.


Many metatheory in the paper comes with Coq proofs, including type safety, coherence, algorithmic soundness and completeness, etc.¹

1.2 ORGANIZATION

This thesis is largely based on the publications by the author [Xie et al. 2018, 2019a,b; Xie and Oliveira 2017, 2018], as indicated below.

Chapter 3: Ningning Xie and Bruno C. d. S. Oliveira. 2018. “Let Arguments Go First”. In European Symposium on Programming (ESOP).

Chapter 4: Ningning Xie, Xuan Bi, and Bruno C. d. S. Oliveira. 2018. “Consistent Subtyping for All”. In European Symposium on Programming (ESOP).

¹For convenience, whenever possible, definitions, lemmas and theorems have hyperlinks (click ) to their Coq counterparts.

Ningning Xie, Xuan Bi, Bruno C. d. S. Oliveira, and Tom Schrijvers. 2019. “Consistent Subtyping for All”. In ACM Transactions on Programming Languages and Systems (TOPLAS).

Chapter 5: Ningning Xie and Bruno C. d. S. Oliveira. 2017. “Towards Unification for Dependent Types” (Extended abstract), In Draft Proceedings of Trends in Functional Programming (TFP).

Ningning Xie, Richard Eisenberg and Bruno C. d. S. Oliveira. 2020. “Kind Inference for Datatypes”. In Symposium on Principles of Programming Languages (POPL).

2 BACKGROUND

PART II

TECHNICAL CONTRIBUTIONS

3 HIGHER RANK TYPE INFERENCE WITH THE APPLICATION MODE

4 HIGHER RANK GRADUAL TYPES

5 TYPE PROMOTION

PART III

RELATED AND FUTURE WORK

6 RELATED WORK

7 FUTURE WORK

PART IV

EPILOGUE

8 CONCLUSION

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