# Higher-rank Polymorphism: Type Inference and Extensions

by

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# Declaration

I declare that this thesis represents my own work, except where due acknowledgment is made, and that it has not been previously included in a thesis, dissertation or report submitted to this University or to any other institution for a degree, diploma or other qualifications.

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# Acknowledgments

# Contents

Declaration i

Acknowledgments iii

List of Figures vii

List of Tables ix

I Prologue 1

1 Introduction 3
- 1.1 Contributions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3
- 1.2 Organization . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4

2 Background 7

II Technical Contributions 9

3 Higher Rank Type Inference With The Application Mode 11

4 Higher Rank Gradual Types 13

5 Type Promotion 15

III Related and Future Work 17

6 Related Work 19

7 Future Work 21

IV Epilogue 23

8 Conclusion 25

Bibliography 27

# List of Figures

# List of Tables

# Part I

# Prologue

# 1 Introduction

mention that in this thesis when we say "higher-rank polymorphism" we mean "predicative implicit higher-rank polymorphism".

## 1.1 Contributions

In summary the contributions of this thesis are:

- Chapter 3 proposes a new design for type inference higher-rank polymorphism.
  - We design a variant of bi-directional type checking where the inference mode is combined with a new, so-called, application mode. The application mode naturally propagates type information from arguments to the functions.
  - With the application mode, we give a new design for type inference of higher-ranked type, which generalizes the HM type system, supports a polymorphic let as syntactic sugar, and infers higher rank types. We present a syntax-directed specification, an elaboration semantics to System F, and an algorithmic type system with completeness and soundness proofs.
- Chapter 4 extends higher-rank polymorphism with gradual types.
  - We define a framework for consistent subtyping with
    - a new definition of consistent subtyping that subsumes and generalizes that of Siek and Taha [2007] and can deal with polymorphism and top types;
    - and a syntax-directed version of consistent subtyping that is sound and complete with respect to our definition of consistent subtyping, but still guesses instantiations.
  - Based on consistent subtyping, we present he calculus GPC. We prove that our calculus satisfies the static aspects of the refined criteria for gradual typing [Siek et al. 2015], and is type-safe by a type-directed translation to $\lambda B$ [Ahmed et al. 2009].

  - We present a sound and complete bidirectional algorithm for implementing the declarative system based on the design principle of Garcia and Cimini [2015].

- Chapter 5 presents a variant of unification for higher-rank polymorphism
  - We propose a process named promotion, which promotes a type so that it is well-typed in the prefix context of a type variable.
  - We apply promotion to higher-rank polymorphism and redesign the type variable instantiation.
  - We further explore the design of promotion by an application to the kind inference problem for datatypes.
    - ⋆ We formalize Haskell98' s datatype declarations, providing both a declarative specification and syntax-driven algorithm for kind inference. We prove that the algorithm is sound and observe how Haskell98' s technique of defaulting unconstrained kinds to $\star$ leads to incompleteness. We believe that ours is the first formalization of this aspect of Haskell98.
    - ⋆ We then present a type and kind language that is unified and dependently typed, modeling the challenging features for kind inference in modern Haskell. We include both a declarative specification and a syntax-driven algorithm. The algorithm is proved sound, and we observe where and why completeness fails. In the design of our algorithm, we must choose between completeness and termination; we favor termination but conjecture that an alternative design would regain completeness. Unlike other dependently typed languages, we retain the ability to infer top-level kinds instead of relying on compulsory annotations.

Many metatheory in the paper comes with Coq proofs, including type safety, coherence, algorithmic soundness and completeness, etc.[1]

## 1.2 Organization

This thesis is largely based on the publications by the author [Xie et al. 2018, 2019a,b; Xie and Oliveira 2017, 2018], as indicated below.

Chapter 3: Ningning Xie and Bruno C. d. S. Oliveira. 2018. "Let Arguments Go First". In European Symposium on Programming (ESOP).

[1]For convenience, whenever possible, definitions, lemmas and theorems have hyperlinks (click ☞) to their Coq counterparts.

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Ningning Xie, Richard Eisenberg and Bruno C. d. S. Oliveira. 2020. "Kind Inference for Datatypes". In Symposium on Principles of Programming Languages (POPL).

# 2 Background

# Part II

# Technical Contributions

# 3 Higher Rank Type Inference With The Application Mode

# 4 Higher Rank Gradual Types

# 5 Type Promotion

# Part III

# Related and Future Work

# 6 Related Work

# 7 Future Work

# Part IV

# Epilogue

# 8 Conclusion

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