Types	A, B, C	$::=$	α	Type variable
		$\mid$	$\top$	Top type
		$\mid$	$A \to B$	Function type
		$\mid$	$\forall \alpha.\, A$	Universal quantification
		$\mid$	$A \cap B$	Intersection type
		$\mid$	$(A * B) \Rightarrow C$	Disjoint constraint
Expressions	e	$::=$	x	Variable
		$\mid$	$\top$	Top
		$\mid$	$\lambda(x{:}A).\, e$	Lambda
		$\mid$	$e_1\ e_2$	Application
		$\mid$	$\Lambda \alpha.\, e$	Big lambda
		$\mid$	$e\ A$	Type application
		$\mid$	$e_1 ,, e_2$	Merge
		$\mid$	$\mathsf{assume}(A * B).\, e$	Disjointness assumption
		$\mid$	$e\ \_$	Disjointness check
Contexts	Γ	$::=$	ϵ	
		$\mid$	Γ, α	
		$\mid$	$\Gamma, x{:}A$	
		$\mid$	$\Gamma, \alpha_1 * \alpha_2$	

Figure 1. Syntax.

Definition 1. (Disjointness) Two sets S and T are *disjoint* if there does not exist an element x, such that $x \in S$ and $x \in T$.

Definition 2. (Disjointness) Two types A and B are *disjoint* if there does not exist an expression e, which is not a merge, such that $\epsilon \vdash e : A'$, $\epsilon \vdash e : B'$, $A' <: A$, and $B' <: B$.

Definition 3. (Disjointness) Two types A and B are *disjoint* if their least common supertype is $\top$.

$$\boxed{\Gamma \vdash e : A \hookrightarrow E}$$

$$\frac{x{:}A \in \Gamma}{\Gamma \vdash x : A \hookrightarrow x}\ \textsc{Ty/Var} \qquad \frac{}{\Gamma \vdash \top : \top \hookrightarrow ()}\ \textsc{Ty/Top}$$

$$\frac{\Gamma, x{:}A \vdash e : B \hookrightarrow E \qquad \Gamma \vdash A}{\Gamma \vdash \lambda(x{:}A).\, e : A \to B \hookrightarrow \lambda(x{:}|A|).\, E}\ \textsc{Ty/Lam}$$

$$\frac{\Gamma \vdash e_1 : A_1 \to A_2 \hookrightarrow E_1 \quad \Gamma \vdash e_2 : A_3 \hookrightarrow E_2 \qquad A_3 <: A_1 \hookrightarrow C}{\Gamma \vdash e_1\ e_2 : A_2 \hookrightarrow E_1\ (C\ E_2)}\ \textsc{Ty/App}$$

$$\frac{\Gamma, \alpha \vdash e : A \hookrightarrow E}{\Gamma \vdash \Lambda \alpha.\, e : \forall \alpha.\, A \hookrightarrow \Lambda \alpha.\, E}\ \textsc{Ty/BLam}$$

$$\frac{\Gamma \vdash e : \forall \alpha.\, B \hookrightarrow E \qquad \Gamma \vdash A}{\Gamma \vdash e\ A : [A/\alpha]B \hookrightarrow E\ |A|}\ \textsc{Ty/TApp}$$

$$\frac{\Gamma \vdash e_1 : A \hookrightarrow E_1 \quad \Gamma \vdash e_2 : B \hookrightarrow E_2 \qquad \Gamma \vdash A * B}{\Gamma \vdash e_1 ,, e_2 : A \cap B \hookrightarrow (E_1, E_2)}\ \textsc{Ty/Merge}$$

$$\frac{\Gamma, A_1 * A_2 \vdash e : B \hookrightarrow E}{\Gamma \vdash \mathsf{assume}(A_1 * A_1).\, e : B \hookrightarrow E}\ \textsc{Ty/DisjointAssume}$$

$$\frac{\Gamma \vdash e : (A_1 * A_2) \Rightarrow B \hookrightarrow E \qquad \Gamma \vdash A_1 * A_2}{\Gamma \vdash e\ \_ : B \hookrightarrow E}\ \textsc{Ty/DisjointCheck}$$

Figure 2. Typing.