$$\boxed{\tau \perp \tau}$$ $$\frac{\alpha_1 \neq \alpha_2}{\alpha_1 \perp \alpha_2} \;\text{O\scriptsize RTH}::\text{V\scriptsize AR} \qquad \frac{\tau_1 \perp \tau_3 \quad \tau_2 \perp \tau_4}{\tau_1 \to \tau_2 \perp \tau_3 \to \tau_4} \;\text{O\scriptsize RTH}::\text{F\scriptsize UN} \qquad \frac{\tau_1 \perp [\alpha_1/\alpha_2]\tau_2}{\forall \alpha_1.\tau_1 \perp \forall \alpha_2.\tau_2} \;\text{O\scriptsize RTH}::\text{F\scriptsize ORALL}$$

$$\frac{\tau_1 \perp \tau_3 \quad \tau_2 \perp \tau_3}{\tau_1 \,\&\, \tau_2 \perp \tau_3} \;\text{O\scriptsize RTH}::\text{A\scriptsize ND}1 \qquad \frac{\tau_1 \perp \tau_2 \quad \tau_1 \perp \tau_3}{\tau_1 \perp \tau_2 \,\&\, \tau_3} \;\text{O\scriptsize RTH}::\text{A\scriptsize ND}2 \qquad \frac{l_1 \neq l_2}{\{l_1 : \tau_1\} \perp \{l_2 : \tau_2\}} \;\text{O\scriptsize RTH}::\text{R\scriptsize EC}\text{L\scriptsize AB}$$

$$\frac{\tau_1 \perp \tau_2}{\{l : \tau_1\} \perp \{l : \tau_2\}} \;\text{O\scriptsize RTH}::\text{R\scriptsize EC}$$

Figure 1. Orthogonality between types.

$$\boxed{\gamma \vdash \tau}$$ $$\frac{\alpha \in \gamma}{\gamma \vdash \alpha} \;\textit{wf}\text{V\scriptsize AR} \qquad \frac{}{\gamma \vdash \top} \;\textit{wf}\text{T\scriptsize OP} \qquad \frac{\gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2}{\gamma \vdash \tau_1 \to \tau_2} \;\textit{wf}\text{F\scriptsize UN} \qquad \frac{\gamma, \alpha \vdash \tau}{\gamma \vdash \forall \alpha.\tau} \;\textit{wf}\text{F\scriptsize ORALL}$$

$$\frac{\gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2 \quad \tau_1 \perp \tau_2}{\gamma \vdash \tau_1 \,\&\, \tau_2} \;\textit{wf}\text{A\scriptsize ND} \qquad \frac{\gamma \vdash \tau}{\gamma \vdash \{l : \tau\}} \;\textit{wf}\text{R\scriptsize EC}$$

Figure 2. Well-formedness of types.

$$\text{Values} \quad v \quad ::= \quad \top \mid \lambda(x{:}\tau).\,e \mid \Lambda\alpha.e \mid v_1,, v_2 \mid \{l = e\}$$

Figure 3. Values.

$$\frac{}{(\tau)(v : \tau) \hookrightarrow (v : \tau)} \;\text{C\scriptsize AST}::\text{B\scriptsize ASE} \qquad \frac{(\tau)(v_1 : \tau_1) \hookrightarrow (v : \tau)}{(\tau)(v_1,, v_2 : \tau_1 \,\&\, \tau_2) \hookrightarrow (v : \tau)} \;\text{C\scriptsize AST}::\text{L\scriptsize EFT}$$

$$\frac{(\tau)(v_2 : \tau_2) \hookrightarrow (v : \tau)}{(\tau)(v_1,, v_2 : \tau_1 \,\&\, \tau_2) \hookrightarrow (v : \tau)} \;\text{C\scriptsize AST}::\text{R\scriptsize IGHT}$$

Figure 4. Casts.

$$\frac{}{v \Downarrow v} \;\text{D\scriptsize YN}::\text{V\scriptsize AL} \qquad \frac{e_1 \Downarrow \lambda(x{:}\tau).\,e \quad e_2 \Downarrow v_2 \quad (\tau)(v_2 : \tau_2) \hookrightarrow v_3 \quad [v_3/x]e \Downarrow v}{e_1\,(e_2 : \tau_2) \Downarrow v} \;\text{D\scriptsize YN}::\text{A\scriptsize PP}$$

$$\frac{e_1 \Downarrow \forall \alpha.\,e \quad [\tau/\alpha]e \Downarrow v}{e_1\,\tau \Downarrow v} \;\text{D\scriptsize YN}::\text{T\scriptsize APP} \qquad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1,, e_2 \Downarrow v_1,, v_2} \;\text{D\scriptsize YN}::\text{M\scriptsize ERGE}$$

Figure 5. Dynamic semantics.

$$\boxed{|\tau| = \mathsf{T}}$$

$$|\alpha| = \alpha$$
$$|\top| = ()$$
$$|\tau_1| \to |\tau_2| = |\tau_1| \to |\tau_2|$$
$$|\forall \alpha.\, \tau| = \forall \alpha.\, |\tau|$$
$$|\tau_1 \mathbin{\&} \tau_2| = (|\tau_1|, |\tau_2|)$$
$$|\{l : \tau\}| = |\tau|$$

Figure 6. Type translation.

$$\boxed{\tau <: \tau \hookrightarrow C}$$

$$\dfrac{}{\alpha <: \alpha \hookrightarrow \lambda(x{:}|\alpha|).\, x} \ \text{sub}\textsc{var} \qquad \dfrac{}{\tau <: \top \hookrightarrow \lambda(x{:}|\tau|).\, ()} \ \text{sub}\textsc{top}$$

$$\dfrac{\tau_3 <: \tau_1 \hookrightarrow C_1 \qquad \tau_2 <: \tau_4 \hookrightarrow C_2}{\tau_1 \to \tau_2 <: \tau_3 \to \tau_4 \hookrightarrow \lambda(f{:}|\tau_1 \to \tau_2|).\, \lambda(x{:}|\tau_3|).\, C_2 \ (f \ (C_1 \ x))} \ \text{sub}\textsc{fun}$$

$$\dfrac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2 \hookrightarrow C}{\forall \alpha_1.\, \tau_1 <: \forall \alpha_2.\, \tau_2 \hookrightarrow \lambda(f{:}|\forall \alpha_1.\, \tau_1|).\, \Lambda \alpha.\, C \ (f \ \alpha)} \ \text{sub}\textsc{forall} \qquad \dfrac{\tau_1 <: \tau_2 \hookrightarrow C_1 \qquad \tau_1 <: \tau_3 \hookrightarrow C_2}{\tau_1 <: \tau_2 \mathbin{\&} \tau_3 \hookrightarrow \lambda(x{:}|\tau_1|).\, (C_1 \ x, C_2 \ x)} \ \text{sub}\textsc{and}$$

$$\dfrac{\tau_1 <: \tau_3 \hookrightarrow C}{\tau_1 \mathbin{\&} \tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1 \mathbin{\&} \tau_2|).\, C \ (\text{proj}_1 x)} \ \text{sub}\textsc{and}_1 \qquad \dfrac{\tau_2 <: \tau_3 \hookrightarrow C}{\tau_1 \mathbin{\&} \tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1 \mathbin{\&} \tau_2|).\, C \ (\text{proj}_2 x)} \ \text{sub}\textsc{and}_2$$

$$\dfrac{\tau_1 <: \tau_2 \hookrightarrow C}{\{l : \tau_1\} <: \{l : \tau_2\} \hookrightarrow \lambda(x{:}|\{l : \tau_1\}|).\, C \ x} \ \text{sub}\textsc{rec}$$

Figure 7. Elaboration subtyping.

$$\boxed{\gamma \vdash e : \tau \hookrightarrow E}$$

$$\frac{(x,\tau) \in \gamma}{\gamma \vdash x : \tau \hookrightarrow x} \ \text{EVAR}$$

$$\frac{}{\gamma \vdash \top : \top \hookrightarrow ()} \ \text{ETOP}$$

$$\frac{\gamma, x{:}\tau \vdash e : \tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash \lambda(x{:}\tau).\,e : \tau \to \tau_1 \hookrightarrow \lambda(x{:}|\tau|).\,E} \ \text{ELAM}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \to \tau_2 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_3 \hookrightarrow E_2 \quad \tau_3 <: \tau_1 \hookrightarrow C}{\gamma \vdash e_1 \ e_2 : \tau_2 \hookrightarrow E_1 \ (C \ E_2)} \ \text{EAPP}$$

$$\frac{\gamma, \alpha \vdash e : \tau \hookrightarrow E}{\gamma \vdash \Lambda\alpha.e : \forall\alpha.\,\tau \hookrightarrow \Lambda\alpha.E} \ \text{EBLAM}$$

$$\frac{\gamma \vdash e : \forall\alpha.\,\tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash e \ \tau : [\tau/\alpha]\tau_1 \hookrightarrow E \ |\tau|} \ \text{ETAPP}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_2 \hookrightarrow E_2 \quad \tau_1 \perp \tau_2}{\gamma \vdash e_1,, e_2 : \tau_1 \ \& \ \tau_2 \hookrightarrow (E_1, E_2)} \ \text{EMERGE}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E}{\gamma \vdash \{l = e\} : \{l : \tau\} \hookrightarrow E} \ \text{EREC-CONSTRUCT}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E \quad \tau \bullet l = \tau_1 \hookrightarrow C}{\gamma \vdash e.l : \tau_1 \hookrightarrow C \ E} \ \text{EREC-SELECT}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E \quad \tau \setminus l = \tau_1 \hookrightarrow C}{\gamma \vdash e \setminus l : \tau_1 \hookrightarrow C \ E} \ \text{EREC-RESTRICT}$$

$$\boxed{\tau_1 \bullet l = \tau_2 \hookrightarrow C}$$

$$\frac{}{\{l : \tau\} \bullet l = \tau \hookrightarrow \lambda(x{:}|\{l : \tau\}|).\,x} \ \text{select}$$

$$\frac{\tau_1 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1 \ \& \ \tau_2|).\,C \ (\text{proj}_1 x)} \ \text{select}_1$$

$$\frac{\tau_2 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1 \ \& \ \tau_2|).\,C \ (\text{proj}_2 x)} \ \text{select}_2$$

$$\boxed{\tau_1 \setminus l = \tau_2 \hookrightarrow C}$$

$$\frac{}{\{l : \tau\} \setminus l = \top \hookrightarrow \lambda(x{:}|\{l : \tau\}|).\,()} \ \text{restrict}$$

$$\frac{\tau_1 \setminus l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \setminus l = \tau \ \& \ \tau_2 \hookrightarrow \lambda(x{:}|\tau_1 \ \& \ \tau_2|).\,(C \ (\text{proj}_1 x), \text{proj}_2 x)} \ \text{restrict}_1$$

$$\frac{\tau_2 \setminus l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \setminus l = \tau_1 \ \& \ \tau \hookrightarrow \lambda(x{:}|\tau_1 \ \& \ \tau_2|).\,(\text{proj}_1 x, C \ (\text{proj}_2 x))} \ \text{restrict}_2$$

Figure 8. Elaboration typing.