

$$\begin{array}{c}
\boxed{\tau \perp \tau} \quad \frac{\alpha_1 \neq \alpha_2}{\alpha_1 \perp \alpha_2} \text{ORTH/VAR} \quad \frac{\tau_1 \perp \tau_3 \quad \tau_2 \perp \tau_4}{\tau_1 \rightarrow \tau_2 \perp \tau_3 \rightarrow \tau_4} \text{ORTH/FUN} \quad \frac{\tau_1 \perp [\alpha_1/\alpha_2]\tau_2}{\forall \alpha_1. \tau_1 \perp \forall \alpha_2. \tau_2} \text{ORTH/FORALL} \\
\\
\frac{\tau_1 \perp \tau_3 \quad \tau_2 \perp \tau_3}{\tau_1 \& \tau_2 \perp \tau_3} \text{ORTH/AND1} \quad \frac{\tau_1 \perp \tau_2 \quad \tau_1 \perp \tau_3}{\tau_1 \perp \tau_2 \& \tau_3} \text{ORTH/AND2} \quad \frac{l_1 \neq l_2}{\{l_1 : \tau_1\} \perp \{l_2 : \tau_2\}} \text{ORTH/RECLAB} \\
\\
\frac{\tau_1 \perp \tau_2}{\{l : \tau_1\} \perp \{l : \tau_2\}} \text{ORTH/REC}
\end{array}$$

Figure 1. Orthogonality between types.

$$\begin{array}{c}
\boxed{\gamma \vdash \tau} \quad \frac{\alpha \in \gamma}{\gamma \vdash \alpha} \text{wfVAR} \quad \frac{}{\gamma \vdash \top} \text{wfTOP} \quad \frac{\gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2}{\gamma \vdash \tau_1 \rightarrow \tau_2} \text{wfFUN} \quad \frac{\gamma, \alpha \vdash \tau}{\gamma \vdash \forall \alpha. \tau} \text{wfFORALL} \\
\\
\frac{\gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2 \quad \tau_1 \perp \tau_2}{\gamma \vdash \tau_1 \& \tau_2} \text{wfAND} \quad \frac{\gamma \vdash \tau}{\gamma \vdash \{l : \tau\}} \text{wfREC}
\end{array}$$

Figure 2. Well-formedness of types.

$$\text{Values } v ::= \top \mid \lambda(x:\tau).e \mid \wedge \alpha. e \mid v_1, v_2 \mid \{l = e\}$$

Figure 3. Values.

$$\begin{array}{c}
\frac{\tau_1 <: \tau}{(\tau)(v : \tau_1) \hookrightarrow v} \text{CAST/BASE} \quad \frac{(\tau)(v_1 : \tau_1) \hookrightarrow v}{(\tau)(v_1, v_2 : \tau_1 \& \tau_2) \hookrightarrow v} \text{CAST/LEFT} \quad \frac{(\tau)(v_2 : \tau_2) \hookrightarrow v}{(\tau)(v_1, v_2 : \tau_1 \& \tau_2) \hookrightarrow v} \text{CAST/RIGHT}
\end{array}$$

Figure 4. Casts.

$$\begin{array}{c}
\frac{}{v \Downarrow v} \text{DYN/VAL} \quad \frac{e_1 \Downarrow \lambda(x:\tau).e \quad e_2 \Downarrow v_2 \quad (\tau)(v_2 : \tau_2) \hookrightarrow v_3 \quad [v_3/x]e \Downarrow v}{e_1 (e_2 : \tau_2) \Downarrow v} \text{DYN/APP} \\
\\
\frac{e_1 \Downarrow \forall \alpha. e \quad [\tau/\alpha]e \Downarrow v}{e_1 \tau \Downarrow v} \text{DYN/TAPP} \quad \frac{e_1 \Downarrow v_1 \quad e_2 \Downarrow v_2}{e_1, e_2 \Downarrow v_1, v_2} \text{DYN/MERGE}
\end{array}$$

Figure 5. Dynamic semantics.

$$|\tau| = \top$$

$$\begin{aligned} |\alpha| &= \alpha \\ |\top| &= () \\ |\tau_1| \rightarrow |\tau_2| &= |\tau_1| \rightarrow |\tau_2| \\ |\forall \alpha. \tau| &= \forall \alpha. |\tau| \\ |\tau_1 \& \tau_2| &= (|\tau_1|, |\tau_2|) \\ |\{l : \tau\}| &= |\tau| \end{aligned}$$

Figure 6. Type translation.

$$\begin{aligned} & \boxed{\tau <: \tau \hookrightarrow C} \quad \frac{}{\alpha <: \alpha \hookrightarrow \lambda(x:|\alpha|).x} \text{subVAR} \quad \frac{}{\tau <: \top \hookrightarrow \lambda(x:|\tau|).()} \text{subTOP} \\ & \frac{\tau_3 <: \tau_1 \hookrightarrow C_1 \quad \tau_2 <: \tau_4 \hookrightarrow C_2}{\tau_1 \rightarrow \tau_2 <: \tau_3 \rightarrow \tau_4 \hookrightarrow \lambda(f:|\tau_1 \rightarrow \tau_2|). \lambda(x:|\tau_3|). C_2 (f (C_1 x)))} \text{subFUN} \\ & \frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2 \hookrightarrow C}{\forall \alpha_1. \tau_1 <: \forall \alpha_2. \tau_2 \hookrightarrow \lambda(f:|\forall \alpha_1. \tau_1|). \Lambda \alpha. C (f \alpha)} \text{subFORALL} \quad \frac{\tau_1 <: \tau_2 \hookrightarrow C_1 \quad \tau_1 <: \tau_3 \hookrightarrow C_2}{\tau_1 <: \tau_2 \& \tau_3 \hookrightarrow \lambda(x:|\tau_1|). (C_1 x, C_2 x)} \text{subAND} \\ & \frac{\tau_1 <: \tau_3 \hookrightarrow C}{\tau_1 \& \tau_2 <: \tau_3 \hookrightarrow \lambda(x:|\tau_1 \& \tau_2|). C (\text{proj}_1 x)} \text{subAND}_1 \quad \frac{\tau_2 <: \tau_3 \hookrightarrow C}{\tau_1 \& \tau_2 <: \tau_3 \hookrightarrow \lambda(x:|\tau_1 \& \tau_2|). C (\text{proj}_2 x)} \text{subAND}_2 \\ & \frac{\tau_1 <: \tau_2 \hookrightarrow C}{\{l : \tau_1\} <: \{l : \tau_2\} \hookrightarrow \lambda(x:|\{l : \tau_1\}|). C x} \text{subREC} \end{aligned}$$

Figure 7. Elaboration subtyping.

$$\begin{array}{c}
\boxed{\gamma \vdash e : \tau \hookrightarrow E} \quad \frac{(x, \tau) \in \gamma}{\gamma \vdash x : \tau \hookrightarrow x} \text{EVAR} \quad \frac{}{\gamma \vdash \top : \top \hookrightarrow ()} \text{ETOP} \quad \frac{\gamma, x : \tau \vdash e : \tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash \lambda(x : \tau). e : \tau \rightarrow \tau_1 \hookrightarrow \lambda(x : |\tau|). E} \text{ELAM} \\
\\
\frac{\gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_3 \hookrightarrow E_2 \quad \tau_3 <: \tau_1 \hookrightarrow C}{\gamma \vdash e_1 \ e_2 : \tau_2 \hookrightarrow E_1 \ (C \ E_2)} \text{EAPP} \quad \frac{\gamma, \alpha \vdash e : \tau \hookrightarrow E}{\gamma \vdash \Lambda \alpha. e : \forall \alpha. \tau \hookrightarrow \Lambda \alpha. E} \text{EBLAM} \\
\\
\frac{\gamma \vdash e : \forall \alpha. \tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash e \ \tau : [\tau/\alpha] \tau_1 \hookrightarrow E \ |\tau|} \text{ETAPP} \quad \frac{\gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_2 \hookrightarrow E_2 \quad \tau_1 \perp \tau_2}{\gamma \vdash e_1, e_2 : \tau_1 \ \& \ \tau_2 \hookrightarrow (E_1, E_2)} \text{EMERGE} \\
\\
\frac{\gamma \vdash e : \tau \hookrightarrow E}{\gamma \vdash \{l = e\} : \{l : \tau\} \hookrightarrow E} \text{EREC-CONSTRUCT} \quad \frac{\gamma \vdash e : \tau \hookrightarrow E \quad \tau \bullet l = \tau_1 \hookrightarrow C}{\gamma \vdash e.l : \tau_1 \hookrightarrow C \ E} \text{EREC-SELECT} \\
\\
\frac{\gamma \vdash e : \tau \hookrightarrow E \quad \tau \setminus l = \tau_1 \hookrightarrow C}{\gamma \vdash e \setminus l : \tau_1 \hookrightarrow C \ E} \text{EREC-RESTRICT} \\
\\
\boxed{\tau_1 \bullet l = \tau_2 \hookrightarrow C} \quad \overline{\{l : \tau\} \bullet l = \tau \hookrightarrow \lambda(x : |\{l : \tau\}|). x} \text{select} \\
\\
\frac{\tau_1 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_1 x)} \text{select}_1 \quad \frac{\tau_2 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_2 x)} \text{select}_2 \\
\\
\boxed{\tau_1 \setminus l = \tau_2 \hookrightarrow C} \quad \overline{\{l : \tau\} \setminus l = \top \hookrightarrow \lambda(x : |\{l : \tau\}|). ()} \text{restrict} \\
\\
\frac{\tau_1 \setminus l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \setminus l = \tau \ \& \ \tau_2 \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). (C \ (\text{proj}_1 x), \text{proj}_2 x)} \text{restrict}_1 \\
\\
\frac{\tau_2 \setminus l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \setminus l = \tau_1 \ \& \ \tau \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). (\text{proj}_1 x, C \ (\text{proj}_2 x))} \text{restrict}_2
\end{array}$$

Figure 8. Elaboration typing.