$$\boxed{\tau \perp \tau} \qquad \frac{\alpha_1 \neq \alpha_2}{\alpha_1 \perp \alpha_2}\ \textsf{orthog}\text{var} \qquad \frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_4}{\tau_1 \to \tau_2 \perp \tau_3 \to \tau_4}\ \textsf{orthog}\text{fun} \qquad \frac{\tau_1 \perp [\alpha_1/\alpha_2]\tau_2}{\forall \alpha_1 .\, \tau_1 \perp \forall \alpha_2 .\, \tau_2}\ \textsf{orthog}\text{forall}$$

$$\frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_3}{\tau_1 \,\&\, \tau_2 \perp \tau_3}\ \textsf{orthog}\text{and}_1 \qquad \frac{\tau_1 \perp \tau_2 \qquad \tau_1 \perp \tau_3}{\tau_1 \perp \tau_2 \,\&\, \tau_3}\ \textsf{orthog}\text{and}_2 \qquad \frac{l_1 \neq l_2}{\{l_1 : \tau_1\} \perp \{l_2 : \tau_2\}}\ \textsf{orthog}\text{reclab}$$

$$\frac{\tau_1 \perp \tau_2}{\{l : \tau_1\} \perp \{l : \tau_2\}}\ \textsf{orthog}\text{rec}$$

Figure 1. Orthogonality between types.

$$\boxed{\gamma \vdash \tau} \qquad \frac{\alpha \in \gamma}{\gamma \vdash \alpha}\ \textit{wf}\text{var} \qquad \frac{}{\gamma \vdash \top}\ \textit{wf}\text{top} \qquad \frac{\gamma \vdash \tau_1 \qquad \Gamma \vdash \tau_2}{\gamma \vdash \tau_1 \to \tau_2}\ \textit{wf}\text{fun} \qquad \frac{\gamma, \alpha \vdash \tau}{\gamma \vdash \forall \alpha .\, \tau}\ \textit{wf}\text{forall}$$

$$\frac{\gamma \vdash \tau_1 \qquad \Gamma \vdash \tau_2 \qquad \tau_1 \perp \tau_2}{\gamma \vdash \tau_1 \,\&\, \tau_2}\ \textit{wf}\text{and} \qquad \frac{\gamma \vdash \tau}{\gamma \vdash \{l : \tau\}}\ \textit{wf}\text{rec}$$

Figure 2. Well-formedness of types.

$$\boxed{|\tau| = T}$$

$$\begin{aligned}
|\alpha| &= \alpha \\
|\top| &= () \\
|\tau_1| \to |\tau_2| &= |\tau_1| \to |\tau_2| \\
|\forall \alpha .\, \tau| &= \forall \alpha .\, |\tau| \\
|\tau_1 \,\&\, \tau_2| &= (|\tau_1|, |\tau_2|) \\
|\{l : \tau\}| &= |\tau|
\end{aligned}$$

Figure 3. Type translation.

$$\boxed{\tau <: \tau \hookrightarrow C} \qquad \frac{}{\alpha <: \alpha \hookrightarrow \lambda(x{:}|\alpha|).\, x}\ \textsf{sub}\text{var} \qquad \frac{}{\tau <: \top \hookrightarrow \lambda(x{:}|\tau|).\, ()}\ \textsf{sub}\text{top}$$

$$\frac{\tau_3 <: \tau_1 \hookrightarrow C_1 \qquad \tau_2 <: \tau_4 \hookrightarrow C_2}{\tau_1 \to \tau_2 <: \tau_3 \to \tau_4 \hookrightarrow \lambda(f{:}|\tau_1 \to \tau_2|).\, \lambda(x{:}|\tau_3|).\, C_2\ (f\ (C_1\ x))}\ \textsf{sub}\text{fun}$$

$$\frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2 \hookrightarrow C}{\forall \alpha_1 .\, \tau_1 <: \forall \alpha_2 .\, \tau_2 \hookrightarrow \lambda(f{:}|\forall \alpha_1 .\, \tau_1|).\, \Lambda \alpha .\, C\ (f\ \alpha)}\ \textsf{sub}\text{forall} \qquad \frac{\tau_1 <: \tau_2 \hookrightarrow C_1 \qquad \tau_1 <: \tau_3 \hookrightarrow C_2}{\tau_1 <: \tau_2 \,\&\, \tau_3 \hookrightarrow \lambda(x{:}|\tau_1|).\, (C_1\ x, C_2\ x)}\ \textsf{sub}\text{and}$$

$$\frac{\tau_1 <: \tau_3 \hookrightarrow C}{\tau_1 \,\&\, \tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathsf{proj}_1 x)}\ \textsf{sub}\text{and}_1 \qquad \frac{\tau_2 <: \tau_3 \hookrightarrow C}{\tau_1 \,\&\, \tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathsf{proj}_2 x)}\ \textsf{sub}\text{and}_2$$

$$\frac{\tau_1 <: \tau_2 \hookrightarrow C}{\{l : \tau_1\} <: \{l : \tau_2\} \hookrightarrow \lambda(x{:}|\{l : \tau_1\}|).\, C\ x}\ \textsf{sub}\text{rec}$$

Figure 4. Elaboration subtyping.

$$\boxed{\gamma \vdash e : \tau \hookrightarrow E}$$

$$\frac{(x,\tau) \in \gamma}{\gamma \vdash x : \tau \hookrightarrow x}\ \text{Evar} \qquad \frac{}{\gamma \vdash \top : \top \hookrightarrow ()}\ \text{Etop} \qquad \frac{\gamma, x{:}\tau \vdash e : \tau_1 \hookrightarrow E \qquad \gamma \vdash \tau}{\gamma \vdash \lambda(x{:}\tau).\, e : \tau \to \tau_1 \hookrightarrow \lambda(x{:}|\tau|).\, E}\ \text{Elam}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \to \tau_2 \hookrightarrow E_1 \qquad \gamma \vdash e_2 : \tau_3 \hookrightarrow E_2 \qquad \tau_3 <: \tau_1 \hookrightarrow C}{\gamma \vdash e_1\ e_2 : \tau_2 \hookrightarrow E_1\ (C\ E_2)}\ \text{Eapp} \qquad \frac{\gamma, \alpha \vdash e : \tau \hookrightarrow E}{\gamma \vdash \Lambda\alpha.\, e : \forall\alpha.\,\tau \hookrightarrow \Lambda\alpha.\, E}\ \text{Eblam}$$

$$\frac{\gamma \vdash e : \forall\alpha.\,\tau_1 \hookrightarrow E \qquad \gamma \vdash \tau}{\gamma \vdash e\ \tau : [\tau/\alpha]\tau_1 \hookrightarrow E\ |\tau|}\ \text{Etapp} \qquad \frac{\gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \qquad \gamma \vdash e_2 : \tau_2 \hookrightarrow E_2 \qquad \tau_1 \perp \tau_2}{\gamma \vdash e_1 ,, e_2 : \tau_1 \,\&\, \tau_2 \hookrightarrow (E_1, E_2)}\ \text{Emerge}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E}{\gamma \vdash \{l = e\} : \{l{:}\tau\} \hookrightarrow E}\ \text{Erec-construct} \qquad \frac{\gamma \vdash e : \tau \hookrightarrow E \qquad \tau \bullet l = \tau_1 \hookrightarrow C}{\gamma \vdash e.l : \tau_1 \hookrightarrow C\ E}\ \text{Erec-select}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E \qquad \tau \setminus l = \tau_1 \hookrightarrow C}{\gamma \vdash e \setminus l : \tau_1 \hookrightarrow C\ E}\ \text{Erec-restrict}$$

$$\boxed{\tau_1 \bullet l = \tau_2 \hookrightarrow C}$$

$$\frac{}{\{l{:}\tau\} \bullet l = \tau \hookrightarrow \lambda(x{:}|\{l{:}\tau\}|).\, x}\ \mathtt{select} \qquad \frac{\tau_1 \bullet l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathtt{proj}_1 x)}\ \mathtt{select}_1$$

$$\frac{\tau_2 \bullet l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathtt{proj}_2 x)}\ \mathtt{select}_2$$

$$\boxed{\tau_1 \setminus l = \tau_2 \hookrightarrow C}$$

$$\frac{}{\{l{:}\tau\} \setminus l = \top \hookrightarrow \lambda(x{:}|\{l{:}\tau\}|).\, ()}\ \mathtt{restrict}$$

$$\frac{\tau_1 \setminus l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \setminus l = \tau \,\&\, \tau_2 \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, (C\ (\mathtt{proj}_1 x), \mathtt{proj}_2 x)}\ \mathtt{restrict}_1$$

$$\frac{\tau_2 \setminus l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \setminus l = \tau_1 \,\&\, \tau \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, (\mathtt{proj}_1 x, C\ (\mathtt{proj}_2 x))}\ \mathtt{restrict}_2$$

Figure 5. Elaboration typing.