

Types	$A, B, C$	$::=$	$\alpha$ $\top$ $A \rightarrow B$ $\forall \alpha. A$ $A \cap B$ $(A * B) \Rightarrow C$	Type variable Top type Function type Universal quantification Intersection type Disjoint constraint
Expressions	$e$	$::=$	$x$ $\top$ $\lambda(x:A). e$ $e_1 e_2$ $\Lambda \alpha. e$ $e A$ $e_1, e_2$ $\text{assume}(A * B). e$ $e \_$	Variable Top Lambda Application Big lambda Type application Merge Disjointness assumption Disjointness check
Contexts	$\Gamma$	$::=$	$\epsilon$ $\Gamma, \alpha$ $\Gamma, x:A$ $\Gamma, \alpha_1 * \alpha_2$	

**Figure 1.** Syntax.

**Definition 1.** (Disjointness) Two sets  $S$  and  $T$  are *disjoint* if there does not exist an element  $x$ , such that  $x \in S$  and  $x \in T$ .

**Definition 2.** (Disjointness) Two types  $A$  and  $B$  are *disjoint* if there does not exist an expression  $e$ , which is not a merge, such that  $\epsilon \vdash e : A'$ ,  $\epsilon \vdash e : B'$ ,  $A' <: A$ , and  $B' <: B$ .

**Definition 3.** (Disjointness) Two types  $A$  and  $B$  are *disjoint* if their least common supertype is  $\top$ .

$\frac{x:A \in \Gamma}{\Gamma \vdash x : A \hookrightarrow x}$	TY/VAR	$\frac{}{\Gamma \vdash \top : \top \hookrightarrow ()}$	TY/TOP
$\frac{\Gamma, x:A \vdash e : B \hookrightarrow E \quad \Gamma \vdash A}{\Gamma \vdash \lambda(x:A). e : A \rightarrow B \hookrightarrow \lambda(x: A ). E}$	TY/LAM		
$\frac{\Gamma \vdash e_1 : A_1 \rightarrow A_2 \hookrightarrow E_1 \quad \Gamma \vdash e_2 : A_3 \hookrightarrow E_2 \quad A_3 <: A_1 \hookrightarrow C}{\Gamma \vdash e_1 e_2 : A_2 \hookrightarrow E_1 (C E_2)}$	TY/APP		
$\frac{\Gamma, \alpha \vdash e : A \hookrightarrow E}{\Gamma \vdash \Lambda \alpha. e : \forall \alpha. A \hookrightarrow \Lambda \alpha. E}$	TY/BLAM		
$\frac{\Gamma \vdash e : \forall \alpha. B \hookrightarrow E \quad \Gamma \vdash A}{\Gamma \vdash e A : [A/\alpha]B \hookrightarrow E [A]}$	TY/TAPP		
$\frac{\Gamma \vdash e_1 : A \hookrightarrow E_1 \quad \Gamma \vdash e_2 : B \hookrightarrow E_2 \quad \Gamma \vdash A * B}{\Gamma \vdash e_1, e_2 : A \cap B \hookrightarrow (E_1, E_2)}$	TY/MERGE		
$\frac{\Gamma, A_1 * A_2 \vdash e : B \hookrightarrow E}{\Gamma \vdash \text{assume}(A_1 * A_2). e : B \hookrightarrow E}$	TY/DISJOINTASSUME		
$\frac{\Gamma \vdash e : (A_1 * A_2) \Rightarrow B \hookrightarrow E \quad \Gamma \vdash A_1 * A_2}{\Gamma \vdash e \_ : B \hookrightarrow E}$	TY/DISJOINTCHECK		

**Figure 2.** Typing.