$$\boxed{\tau \perp \tau} \qquad \frac{\alpha_1 \neq \alpha_2}{\alpha_1 \perp \alpha_2}\ \textsc{Orth::Var} \qquad \frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_4}{\tau_1 \to \tau_2 \perp \tau_3 \to \tau_4}\ \textsc{Orth::Fun} \qquad \frac{\tau_1 \perp [\alpha_1/\alpha_2]\tau_2}{\forall \alpha_1 . \tau_1 \perp \forall \alpha_2 . \tau_2}\ \textsc{Orth::Forall}$$

$$\frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_3}{\tau_1 \,\&\, \tau_2 \perp \tau_3}\ \textsc{Orth::And1} \qquad \frac{\tau_1 \perp \tau_2 \qquad \tau_1 \perp \tau_3}{\tau_1 \perp \tau_2 \,\&\, \tau_3}\ \textsc{Orth::And2} \qquad \frac{l_1 \neq l_2}{\{l_1 : \tau_1\} \perp \{l_2 : \tau_2\}}\ \textsc{Orth::RecLab}$$

$$\frac{\tau_1 \perp \tau_2}{\{l : \tau_1\} \perp \{l : \tau_2\}}\ \textsc{Orth::Rec}$$

Figure 1. Orthogonality between types.

$$\boxed{\gamma \vdash \tau} \qquad \frac{\alpha \in \gamma}{\gamma \vdash \alpha}\ wf\textsc{var} \qquad \frac{}{\gamma \vdash \top}\ wf\textsc{top} \qquad \frac{\gamma \vdash \tau_1 \qquad \Gamma \vdash \tau_2}{\gamma \vdash \tau_1 \to \tau_2}\ wf\textsc{fun} \qquad \frac{\gamma, \alpha \vdash \tau}{\gamma \vdash \forall \alpha . \tau}\ wf\textsc{forall}$$

$$\frac{\gamma \vdash \tau_1 \qquad \Gamma \vdash \tau_2 \qquad \tau_1 \perp \tau_2}{\gamma \vdash \tau_1 \,\&\, \tau_2}\ wf\textsc{and} \qquad \frac{\gamma \vdash \tau}{\gamma \vdash \{l : \tau\}}\ wf\textsc{rec}$$

Figure 2. Well-formedness of types.

$$\text{Values} \quad v \quad ::= \quad \top \mid \lambda(x{:}\tau).\, e \mid \Lambda \alpha . e \mid v_1 ,, v_2 \mid \{l = e\}$$

Figure 3. Values.

$$\begin{array}{lcl}
\texttt{fields}(v_1 ,, v_2) & = & \texttt{fields}(v_1) +\!\!+ \texttt{fields}(v_2) \\
\texttt{fields}(\{l = e\}) & = & [(l, e)] \\
\texttt{fields}(v) & = & []
\end{array}$$

Figure 4. `fields`.

$$\begin{array}{lcll}
\texttt{remove}(\{l = e\}, l) & = & \top & \\
\texttt{remove}(\{l = e\} ,, v_2, l) & = & v_2 & \\
\texttt{remove}(\{l = e\} ,, v_2, l') & = & \{l = e\} ,, \texttt{remove}(v_2, l') & (l \neq l') \\
\texttt{remove}(v_1 ,, \{l = e\}, l) & = & v_1 & \\
\texttt{remove}(v_1 ,, \{l = e\}, l') & = & \texttt{remove}(v_1, l') ,, \{l = e\} & (l \neq l') \\
\texttt{remove}(v, l) & = & v &
\end{array}$$

Figure 5. `remove`.

$$\frac{}{v \Downarrow v}\ \textsc{Dyn::Val} \qquad \frac{e_1 \Downarrow \lambda(x{:}\tau).\, e \qquad e_2 \Downarrow v_2 \qquad [v_2/x]e \Downarrow v}{e_1\ e_2 \Downarrow v}\ \textsc{Dyn::App} \qquad \frac{e_1 \Downarrow \forall\alpha.\, e \qquad [\tau/\alpha]e \Downarrow v}{e_1\ \tau \Downarrow v}\ \textsc{Dyn::TApp}$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2}{e_1 ,, e_2 \Downarrow v_1 ,, v_2}\ \textsc{Dyn::Merge} \qquad \frac{e \Downarrow v \qquad (l, e_1)\ \text{`}\mathtt{uniqueElem}\text{`}\ \mathtt{fields}(v) \qquad e_1 \Downarrow v_1}{e.l \Downarrow v_1}\ \textsc{Dyn::RecSelect}$$

$$\frac{e \Downarrow v \qquad (l, e_1)\ \text{`}\mathtt{uniqueElem}\text{`}\ \mathtt{fields}(v)}{e \setminus l \Downarrow v\ \text{`}\mathtt{remove}\text{`}\ l}\ \textsc{Dyn::RecRestrict}$$

Figure 6. Dynamic semantics.

$\boxed{|\tau| = T}$

$$
\begin{aligned}
|\alpha| &= \alpha \\
|\top| &= () \\
|\tau_1| \to |\tau_2| &= |\tau_1| \to |\tau_2| \\
|\forall\alpha.\, \tau| &= \forall\alpha.\, |\tau| \\
|\tau_1 \,\&\, \tau_2| &= (|\tau_1|, |\tau_2|) \\
|\{l : \tau\}| &= |\tau|
\end{aligned}
$$

Figure 7. Type translation.

$\boxed{\tau <: \tau \hookrightarrow C}$

$$\frac{}{\alpha <: \alpha \hookrightarrow \lambda(x{:}|\alpha|).\, x}\ \mathsf{sub}\textsc{var} \qquad \frac{}{\tau <: \top \hookrightarrow \lambda(x{:}|\tau|).\, ()}\ \mathsf{sub}\textsc{top}$$

$$\frac{\tau_3 <: \tau_1 \hookrightarrow C_1 \qquad \tau_2 <: \tau_4 \hookrightarrow C_2}{\tau_1 \to \tau_2 <: \tau_3 \to \tau_4 \hookrightarrow \lambda(f{:}|\tau_1 \to \tau_2|).\, \lambda(x{:}|\tau_3|).\, C_2\ (f\ (C_1\ x))}\ \mathsf{sub}\textsc{fun}$$

$$\frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2 \hookrightarrow C}{\forall\alpha_1.\, \tau_1 <: \forall\alpha_2.\, \tau_2 \hookrightarrow \lambda(f{:}|\forall\alpha_1.\, \tau_1|).\, \Lambda\alpha.C\ (f\ \alpha)}\ \mathsf{sub}\textsc{forall} \qquad \frac{\tau_1 <: \tau_2 \hookrightarrow C_1 \qquad \tau_1 <: \tau_3 \hookrightarrow C_2}{\tau_1 <: \tau_2 \,\&\, \tau_3 \hookrightarrow \lambda(x{:}|\tau_1|).\, (C_1\ x, C_2\ x)}\ \mathsf{sub}\textsc{and}$$

$$\frac{\tau_1 <: \tau_3 \hookrightarrow C}{\tau_1 \,\&\, \tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathtt{proj}_1 x)}\ \mathsf{sub}\textsc{and}_1 \qquad \frac{\tau_2 <: \tau_3 \hookrightarrow C}{\tau_1 \,\&\, \tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathtt{proj}_2 x)}\ \mathsf{sub}\textsc{and}_2$$

$$\frac{\tau_1 <: \tau_2 \hookrightarrow C}{\{l : \tau_1\} <: \{l : \tau_2\} \hookrightarrow \lambda(x{:}|\{l : \tau_1\}|).\, C\ x}\ \mathsf{sub}\textsc{rec}$$

Figure 8. Elaboration subtyping.

$$\boxed{\gamma \vdash e : \tau \hookrightarrow E} \qquad \frac{(x, \tau) \in \gamma}{\gamma \vdash x : \tau \hookrightarrow x}\ \textsc{EVar} \qquad \frac{}{\gamma \vdash \top : \top \hookrightarrow ()}\ \textsc{ETop} \qquad \frac{\gamma, x{:}\tau \vdash e : \tau_1 \hookrightarrow E \qquad \gamma \vdash \tau}{\gamma \vdash \lambda(x{:}\tau).\, e : \tau \to \tau_1 \hookrightarrow \lambda(x{:}|\tau|).\, E}\ \textsc{ELam}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \to \tau_2 \hookrightarrow E_1 \qquad \gamma \vdash e_2 : \tau_3 \hookrightarrow E_2 \qquad \tau_3 <: \tau_1 \hookrightarrow C}{\gamma \vdash e_1\ e_2 : \tau_2 \hookrightarrow E_1\ (C\ E_2)}\ \textsc{EApp} \qquad \frac{\gamma, \alpha \vdash e : \tau \hookrightarrow E}{\gamma \vdash \Lambda\alpha.e : \forall\alpha.\,\tau \hookrightarrow \Lambda\alpha.E}\ \textsc{EBLam}$$

$$\frac{\gamma \vdash e : \forall\alpha.\,\tau_1 \hookrightarrow E \qquad \gamma \vdash \tau}{\gamma \vdash e\ \tau : [\tau/\alpha]\tau_1 \hookrightarrow E\ |\tau|}\ \textsc{ETApp} \qquad \frac{\gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \qquad \gamma \vdash e_2 : \tau_2 \hookrightarrow E_2 \qquad \tau_1 \perp \tau_2}{\gamma \vdash e_1 ,, e_2 : \tau_1 \,\&\, \tau_2 \hookrightarrow (E_1, E_2)}\ \textsc{EMerge}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E}{\gamma \vdash \{l = e\} : \{l : \tau\} \hookrightarrow E}\ \textsc{ERec-construct} \qquad \frac{\gamma \vdash e : \tau \hookrightarrow E \qquad \tau \bullet l = \tau_1 \hookrightarrow C}{\gamma \vdash e.l : \tau_1 \hookrightarrow C\ E}\ \textsc{ERec-select}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E \qquad \tau \setminus l = \tau_1 \hookrightarrow C}{\gamma \vdash e \setminus l : \tau_1 \hookrightarrow C\ E}\ \textsc{ERec-restrict}$$

$$\boxed{\tau_1 \bullet l = \tau_2 \hookrightarrow C} \qquad \frac{}{\{l : \tau\} \bullet l = \tau \hookrightarrow \lambda(x{:}|\{l : \tau\}|).\, x}\ \mathsf{select}$$

$$\frac{\tau_1 \bullet l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathtt{proj}_1 x)}\ \mathsf{select}_1 \qquad \frac{\tau_2 \bullet l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, C\ (\mathtt{proj}_2 x)}\ \mathsf{select}_2$$

$$\boxed{\tau_1 \setminus l = \tau_2 \hookrightarrow C} \qquad \frac{}{\{l : \tau\} \setminus l = \top \hookrightarrow \lambda(x{:}|\{l : \tau\}|).\, ()}\ \mathsf{restrict}$$

$$\frac{\tau_1 \setminus l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \setminus l = \tau \,\&\, \tau_2 \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, (C\ (\mathtt{proj}_1 x), \mathtt{proj}_2 x)}\ \mathsf{restrict}_1$$

$$\frac{\tau_2 \setminus l = \tau \hookrightarrow C}{\tau_1 \,\&\, \tau_2 \setminus l = \tau_1 \,\&\, \tau \hookrightarrow \lambda(x{:}|\tau_1 \,\&\, \tau_2|).\, (\mathtt{proj}_1 x, C\ (\mathtt{proj}_2 x))}\ \mathsf{restrict}_2$$

Figure 9. Elaboration typing.