Figure 1. Syntax

Atomic Types	T	$::=$	$A \to B$	Function type
		$\mid$	$\top$	Top type
		$\mid$	$\forall \alpha * B.\,A$	Universal quantification
Types	A, B, C, D	$::=$	α	Type variable
		$\mid$	$A \cap B$	Intersection type
		$\mid$	T	Atomic type
Expressions	e	$::=$	x	Variable
		$\mid$	$\top$	Top
		$\mid$	$\lambda(x{:}A).\,e$	Lambda
		$\mid$	$e_1\,e_2$	Application
		$\mid$	$\Lambda\alpha * A.\,e$	Big lambda
		$\mid$	$e\,A$	Type application
		$\mid$	$e_1,,e_2$	Merge
Contexts	Γ	$::=$	ϵ	
		$\mid$	$\Gamma, \alpha * A$	
		$\mid$	$\Gamma, x{:}A$	

Figure 1. Syntax.

Figure 3. Disjointness

$$\boxed{\Gamma \vdash A \bot B}$$

$$\frac{\alpha * B \in \Gamma}{\Gamma \vdash \alpha \bot B}\ \textsc{DisjointRefl} \qquad \frac{\alpha * A \in \Gamma}{\Gamma \vdash A \bot \alpha}\ \textsc{DisjointSym}$$

$$\frac{\Gamma \vdash A \bot C \qquad \Gamma \vdash B \bot C}{\Gamma \vdash A \& B \bot C}\ \textsc{DisjointSub1}$$

$$\frac{\Gamma \vdash A \bot B \qquad \Gamma \vdash A \bot C}{\Gamma \vdash A \bot B \& C}\ \textsc{DisjointSub2}$$

$$\frac{T_1 \not<: T_2 \qquad T_2 \not<: T_1}{\Gamma \vdash T_1 \bot T_2}\ \textsc{DisjointAtomic}$$

Figure 3. Disjointness.

Figure 2. Subtyping

$$\boxed{A <: B\ \hookrightarrow F}$$

$$\frac{}{\alpha <: \alpha\ \hookrightarrow \lambda(x{:}|\alpha|).\,x}\ \textsc{SubVar}$$

$$\frac{}{A <: \top\ \hookrightarrow \lambda(x{:}|A|).\,()}\ \textsc{SubTop}$$

$$\frac{\tau_3 <: \tau_1\ \hookrightarrow C_1 \qquad \tau_2 <: \tau_4\ \hookrightarrow C_2}{\tau_1 \to \tau_2 <: \tau_3 \to \tau_4\ \hookrightarrow \lambda(f{:}|\tau_1 \to \tau_2|).\lambda(x{:}|\tau_3|).\,C_2\,(f\,(C_1\,x))}\ \textsc{SubFun}$$

$$\frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2\ \hookrightarrow C}{\forall \alpha_1.\tau_1 <: \forall \alpha_2.\tau_2\ \hookrightarrow \lambda(f{:}|\forall \alpha_1.\tau_1|).\,\Lambda\alpha.\,C\,(f\,\alpha)}\ \textsc{SubForall}$$

$$\frac{\tau_1 <: \tau_2\ \hookrightarrow C_1 \qquad \tau_1 <: \tau_3\ \hookrightarrow C_2}{\tau_1 <: \tau_2 \cap \tau_3\ \hookrightarrow \lambda(x{:}|\tau_1|).\,(C_1\,x, C_2\,x)}\ \textsc{SubAnd}$$

$$\frac{\tau_1 <: \tau_3\ \hookrightarrow C}{\tau_1 \cap \tau_2 <: \tau_3\ \hookrightarrow \lambda(x{:}|\tau_1 \cap \tau_2|).\,C\,(\mathsf{proj}_1 x)}\ \textsc{SubAnd}_1$$

$$\frac{\tau_2 <: \tau_3\ \hookrightarrow C}{\tau_1 \cap \tau_2 <: \tau_3\ \hookrightarrow \lambda(x{:}|\tau_1 \cap \tau_2|).\,C\,(\mathsf{proj}_2 x)}\ \textsc{SubAnd}_2$$

Figure 2. Subtyping.

Figure 4. Typing

$$\boxed{\Gamma \vdash e : A\ \hookrightarrow E}$$

$$\frac{x{:}A \in \Gamma}{\Gamma \vdash x : A\ \hookrightarrow x}\ \textsc{TyVar} \qquad \frac{}{\Gamma \vdash \top : \top\ \hookrightarrow ()}\ \textsc{TyTop}$$

$$\frac{\Gamma \vdash A\ \text{type} \qquad \Gamma, x{:}A \vdash e : B\ \hookrightarrow E}{\Gamma \vdash \lambda(x{:}A).\,e : A \to B\ \hookrightarrow \lambda(x{:}|A|).\,E}\ \textsc{TyLam}$$

$$\frac{\Gamma \vdash e_1 : A_1 \to A_2\ \hookrightarrow E_1 \qquad \Gamma \vdash e_2 : A_3\ \hookrightarrow E_2 \qquad A_3 <: A_1\ \hookrightarrow C}{\Gamma \vdash e_1\,e_2 : A_2\ \hookrightarrow E_1\,(C\,E_2)}\ \textsc{TyApp}$$

$$\frac{\Gamma, \alpha * B \vdash e : A\ \hookrightarrow E}{\Gamma \vdash \Lambda\alpha * B.\,e : \forall \alpha * B.\,A\ \hookrightarrow \Lambda\alpha.\,E}\ \textsc{TyBLam}$$

$$\frac{\Gamma \vdash e : \forall \alpha * C.\,B\ \hookrightarrow E \qquad \Gamma \vdash A \bot C \qquad \Gamma \vdash A\ \text{type}}{\Gamma \vdash e\,A : [A/\alpha]B\ \hookrightarrow E\,|A|}\ \textsc{TyTApp}$$

$$\frac{\Gamma \vdash e_1 : A\ \hookrightarrow E_1 \qquad \Gamma \vdash e_2 : B\ \hookrightarrow E_2 \qquad \Gamma \vdash A \bot B}{\Gamma \vdash e_1,,e_2 : A \cap B\ \hookrightarrow (E_1, E_2)}\ \textsc{TyMerge}$$

Figure 4. Typing.

$$\epsilon \vdash 1,,2 : (\mathsf{Int} * \mathsf{Int}) \Rightarrow \mathsf{Int} \cap \mathsf{Int}$$

Definition 1. (Disjointness) Two sets S and T are *disjoint* if there does not exist an element x, such that $x \in S$ and $x \in T$.

Definition 2. (Disjointness) Two types A and B are *disjoint* if there does not exist an expression e, which is not a merge, such that $\epsilon \vdash e : A'$, $\epsilon \vdash e : B'$, $A' <: A$, and $B' <: B$.

Definition 3. (Disjointness) $A \perp B = \nexists C.A <: C \wedge B <: C$

Two types A and B are *disjoint* if their least common supertype is $\top$.