

# U{uvgo $F_{\zeta} < C$ Uk o rng Eqtg Ncpiwcig hqt Gzvgpukdknk\}

Pcog3

Chhknkcvkqp3

B\ -S\

Pcog4 Pcog5

Chhknkcvkqp415

B\ -S\w\

## Cduvtcev

Qxgt vjg {gctu vjgtg jcxg dgpp xctkquw rtqrucnu hqt fg/  
ukip rcwvgtpu hqt kortqxf gzvgpukdknk\} qh rtqitcou0 Gz/  
co rngu kpenwfg Qdlgev Cnigdtcu. Oqfwncf Xkukvqtu qt Vqti/  
gtugpou hqwt fgukip rcwvgtpu wukpi igpgtkeu0 Cnvjqwij vjqug  
fgukip rcwvgtpu ikxg rtcevkecn dgpghkvu kp vgtou qh gzvgpukdkn/  
kv\}. vjg\{ cnuq gzrqug nk okvcvkpu kp gzukvpi ockpvtgco QQR  
ncpiwcigu0 Kp rctvkewncf vyq rtguukpi nk okvcvkpu ctg< 3+ vjg  
ncem qh iqqf ogejcpkou hqt qdlgev/ngxgn eqo rqukvkqp= 4+ vjg  
eqphncvkqp qh \*v{rg+ kpjgtkvepeg ykvj uwdv{rkpi0

Vjku rcrgt rtugpvu U{uvgo  $F_{\zeta} < C$  pzvgpukqp qh U{uvgo  
H ykvj kpvgtugevkqp v{rgu cpf c ogtig qrgtcvt0 Vjg iqcn qh  
U{uvgo  $F_{\zeta}$  ku vq uwwf\{ vjg hqwpfcvkqpcn ncpiwcig eqpvtwewu  
vjcv ctg pggfvg vq uwrqtv xctkquw gzvgpukdng fgukipu. yjkn  
cv vjg ucog vkog cfftguukpi vjg nk okvcvkpu qh gzukvpi QQR  
ncpiwcigu0 Vq cfftguu vjg ncem qh iqqf qdlgev/ngxgn eqo rq/  
ukvkqp ogejcpkou. U{uvgo  $F_{\zeta}$  wugu vjg ogtig qrgtcvt vq  
cnnqy f{pcoke eqo rqukvkqp qh xcngulqdlgevu0 Oqtgqxtg. kp  
U{uvgo  $F_{\zeta}$  v{rg kpjgtkvepeg ku kpfgrgpfvgp qh uwdv{rkpi. cpf  
kv ku rquukdng hqt cp gzvgpukqp vq dg c uwrqtv{rg qh c dcug  
qdlgev v{rg0 U{uvgo  $F_{\zeta}$  ku hqt ocnk\gf cpf ko rngo gpgvf0 Hwt/  
vjgt oqtg vjg rcrgt knnwuvtcvu jqy xctkquw gzvgpukdng fgukipu  
ecp dg gpeqfvgf kp U{uvgo  $F_{\zeta}$ 0

**Ecvgiqtgu cpf Uwdlgev Fguetkrvqtu** ET/pw o dgt juwdecvg/  
iqtl\< vjktf/ngxgn

**Ipgtgc Vgtou** Fgukip. Ncpiwcigu. Vjgqt\}

**Mg{yqtfu** Kpvgtugevkqp V{rgu. Rqn\{ oqtrjku o. V{rg U{uvgo  
dtwpp< Ocmg cmm hkiwtgu hkv cpf dg ygn/hqt o cvgf#

## 30 Kpvtqfwevkqp

dtwpp< Vjg hqnnqykpi ctg lwuv uq o g nqqug pqvgu hqt vjg rcrgt0

Rgtokukqp vq ocmg fliken qt jctf earku qh cm qt rctv qh vjku yqtm hqt rgtuqpcn qt  
encuatqo wug lu itepwgf ykvjqw hgg rtqxfvgf vjcv earku ctg pqv ocfq qt fluitdvwg  
hqt rtqhlv qt eqo o gteken cfxepicig cpf vjcv earku dgct vjku pqlveg cpf vjg hwm elvcvkp  
qp vjg hktuv rcig0 Eqr\{tkijv hqt eqo rpgpvu qh vjku yqtm qypgf d\{qvjgtu vjcp CEO  
o wuv dg jqpqtg0 Cduvtcevpi ykvj etgflv ku rgtokwgf0 Vq eqr\{qvjgt ykug. qt tgrwdku j.  
vq rquv qp ugtxgtu qt vq tg fluitdvwg vq akuv. tgswtgu rtqat urgekhe rgtokukqp cpf lqt c  
hgg0 Tgswguv rgtokukapu htq o rgtokukapu Bce o lqt i0

**EQPH o/f.** Oqpvj f6f. 42\{. Ekv\}. UV. Eqwprt\0  
Eqr\{tkijv f 42\{ CEO ;9:/3/pppp/pppp/pl\{lo o f &370220  
jwr-clfz0fq0lqi132033670pppppppppppppp

Vjgtg jcu dgpp c tgoctmedng pwodgt qh yqtm ckogf  
cv kortqxpki uwrqtv hqt gzvgpukdknk\{ kp rtqitcou okpi ncp/  
iwcigu0 Vjgug yqtm kpenwfg< xkukqpu qh pgy rtqitcou okpi  
oqfgnu\= pgy rtqitcou okpi ncpiwcigu qt ncpiwcig gzvgp/  
ukqpu\}. cpf fgukip rcwvgtpu vjcv ecp dg wugf ykvj gzukvpi  
ockpvtgco ncpiwcigu0

Uqog qh vjg oqtg tgegpv yqtm qp gzvgpukdknk\{ kpenwfgu  
xctkquw rtqrucnu hqt fgukip rcwvgtpu0 Gzco rngu kpenwfg Qd/  
lgev Cnigdtcu \}. Oqfwncf Xkukvqtu \} qt Vqti gtugpou \} hqwt  
fgukip rcwvgtpu wukpi igpgtkeu0 kp vjqug crrtqcejgu vjg kfgc  
ku vq wug uqog cfxcepgf \*dwv cntgcf\{ cxcnkcdng+ hgcvwtgu. uwej  
cu igpgtkeu. kp eqo dkpcvkp ykvj eqpxgpkqpcn QQR hgcvwtgu  
vq oqfgn oqtg gzvgpukdng fgukipu0 Vjqug fgukipu yqtm kp  
oqfgtp QQR ncpiwcigu uwej cu Lcxc. E% qt Uecnc0

Cnvjqwij vjqug fgukip rcwvgtpu ikxg rtcevkecn dgpghkvu kp  
vgtou qh gzvgpukdknk\}. vjg\{ cnuq gzrqug nk okvcvkpu kp gzukv/  
kpi ockpvtgco QQR ncpiwcigu0 Qpg nk okvcvkp ku vjcv gzukv/  
kpi QQR ncpiwcigu ncem iqqf ogejcpkou hqt qdlgev/ngxgn  
eqo rqukvkqp0 Cu knnwuvtcvgf d\{ Qnxgkte gy cn0 \}. jkijn\{ oqf/  
wncf rtqitcou wukpi Qdlgev Cnigdtcu ctg dguv gzrtguugf wu/  
kpi c hqt o qh v{rg/uchg. f{pcoke. fgngicvkqp/dcugf eqo rquk/  
vkqp0 Cnvjqwij uwej hqt o qh eqo rqukvkqp ecp dg gpeqfvgf kp  
Uecnc. kv tgswtgu vjg wug qh nqy/ngxgn tghngevkp vgejpkswgu.  
uwej cu f{pcoke rtqzkgu. tghngevkp qt qvjgt hqt o qh ogyv/  
rtqitcou okpi \}0 Oqtgqxtg. vjg xcuv oclqtkv\{ qh oqfgtp QQR  
ncpiwcigu hqnnqyu c oqfgn yjgtg

4+ vjg eqphncvkqp qh \*v{rg+ gzvgpukqp ykvj uwdv{rkpi0

Vjg wug qh gzukvpi ncpiwcigu jcu ujqyp vjcv yg ecp igv  
swkvgt hct. dwv kv jcu cnuq gzrqugf uqog nk okvcvkpu0 Hqt gz/  
co rng. yqtm qp qdlgev cnigdtcu jcu ujqyp vjcv c fgngicvkqp/  
dcugf QQ oqfgn yqwnf dg dgwgt uwkvgt hqt QCu0 Oqtgqxtg.  
Qdlgev Cnigdtc eqo rqukvkqp ecp dgpghkv htq o kpvgtugevkqp  
v{rgu0 Cffkvkqpcn\{ Qdlgev Cnigdtcu uwhhgt htq o vjg oKpjgtk/  
vepeg ku pqv Uwdv{rkpi o rtqdn o \A\_0 Cnvjqwij vjgtg ctg xgt\{  
hgy gzc o rngu kp vjg nkvgtcvwtg knnwuvtcvkpi rtqitcou yjgtg vjg  
ugrctevkpi kpjgtkvepeg htq o uwdv{rkpi ku fguktedng. qdlgev cn/  
igdtcu knnwuvtcvg c rctvkewncf\{ eqo rgnkpi gzc o rng qh vjku0

Oqkvxcvgf d\{ vjg kpuki jvu ickpgf kp rtgkquw yqtm. vjku  
rcrgt rtugpvu c okpkocn eqtg ecnewnwu vjcv cfftguugu ewt/  
tgpv nk okvcvkpu cpf rtqxfkgu c dgvygt hqwpfcvkqpcn oqfgn hqt  
uvcvkecn\{ v{rgf fgngicvkqp/dcugf QQR A Ygujy vjcv Qdlgev  
Cnigdtcu hkv pkegn\{ kp vjku oqfgn0





```
def id[A <: {name: String}] (user: A) = user
(id [{name: String, admin: Bool}] user).admin
```

Yg fq pqv jcxg dqwpfgf rqn{ oqtrjkuo kp vjg uqwtg  
ncpiwcig0 Dww yg ecp gpeqfg vjcv xkc kpvgtugevkqp v{rgu<

```
YCz id[A] (user: A & {name: String}) = user S^
(id [{admin: Bool}] user).admin
```

D{ tgswktpi vjg v{rg qh vjg ctiwogpv vq dg cp kpvgtugevkqp  
v{rg qh c v{rg rctcogvt cpf vjg wrrgt dqwpf cpf rcuukpi vjg  
v{rg kphqtcovkqp. yg ocmg uwtg vjcv yg ecp uvkm ceeguu vjg  
admin hkgnf ncvgf0

## 504 Qdlgev Cnigdtcu

Qdlgev cnigdtcu rtqxfkfg cp cnvgtpcvkxg vq *cnigdtcke fcvv{rgu*  
*\*CFV+0dtwpq< Yg ctg vctigvki cp QQ etqyf0 Ogpvkpkpi*  
*cnigdtcke fcvv{rgu ku pqv iqkpi vq dg xgt{ wughwn vjgtg0* Hqt  
gzco rng. vjg hqnmqykpi Jcumgn fghkpvkqp qh vjg v{rg qh uko/  
rng gzrtguukpu

```
data Exp where
  Lit :: Int QN Exp
  Add :: Exp QN Exp QN Exp
```

ecp dg gzrtguugf d{ vjg *kpvgtuceg* qh cp qdlgev cnigdtc qh  
uko rng gzrtguukpu<

```
trait ExpAlg[E] {
  def lit(x: Int): E
  def add(e1: E, e2: E): E
}
```

Uko knct vq CFV. fcvv eqpuvtwevqtu kp qdlgev cnigdtcu ctg tgr/  
tgugpvfg d{ hwpevkpu uwej cu lit cpf add kpukfg cp kpvgtuceg  
ExpAlg0 Fkhgtgpn ykvj CFV. vjg v{rg qh vjg gzrtguukpu kvugn  
ku cdvtcevgf d{ c v{rg rctcogvt E0

yjkej ecp dg gzrtguugf uko knctn{ kp  $F_C$  cu<

```
z%&C ExpAlg E = {
  lit : Int QN E,
  add : E QN E QN E
}
```

Uecnc uwrrqtu kpvgtugevkqp v{rgu xkc vjg with mg{ yqtf0  
Vjg v{rg A with B gzrtguugu vjg eqo dkgpf kpvgtuceg qh A cpf  
B0 Vjg kfgc ku uko knct vq

```
interface AwithB extends A, B {}
```

kp Lcxc0<sup>3</sup>

Vjg xcnwg ngxgn eqpvgttrctv ctg hwpevkpu qh vjg v{rg A =>  
B => A with B0<sup>4</sup>

Qwt v{rg u{ugv ku c uko rng gzygpukqp qh U{ugv  $F_-$  {gv  
uwtrtkupin{. kv ku cdng vq uqnxg vjg nko kvcvkpu qh wukpi qdlgev  
cnigdtcu kp ncpiwcigu uwej cu Lcxc cpf Uecnc0 Yg yknn knnwu/  
vtcv vjku rkpvn ykvj cp uvgr/d{uvgr qh uqnxkpi vjg gzrtguukqp  
rtqdn0 wukpi c uqwtg ncpiwcig dwknv qp vqr qh  $F_C$ 0

<sup>3</sup> Jqygxgt. Lcxc yqwnf tgswkgt vjg A cpf B vq dg eqpetgvg v{rgu. yjgtcu kp  
Uecnc. vjgtu ku pq uwej tguvtkevqp0

<sup>4</sup> HKZOG

Qnxgkgtc pqvgf vjcv eqo rqukvkqp qh qdlgev cnigdtcu ecp dg  
ewodgtuqog cpf kpvgtugevkqp v{rgu rtqxfkgu c uqnvkqp vq  
vjcv rtqdn00

Yg hktuv fghkpg cp kpvgtuceg vjcv uwrrqtu vjg gxcnwcvkqp  
qrgtcvkqp<

```
z%&C IEval = { eval : Int };
z%&C ExpAlg E = { lit : Int QN E, add : E QN E QN
  E };
YCz evalAlg = {
  lit = \ (x : Int). { eval = x },
  add = \ (x : IEval). \ (y : IEval). { eval = x.eval
    + y.eval }
};
```

Vjg kpvgtuceg ku lwuv c v{rg u{pqp{ o IEval0 kp  $F_C$ . tgeqtf  
v{rgu ctg uvtwevwtcn cpf jgpeg cp{ xcnwg vjcv ucukhkgu vjku  
kpvgtuceg ku qh v{rg IEval qt qh c uwdv{rg qh IEval0<sup>5</sup>

Kp vjg hqnmqykpi. ExpAlg ku cp qdlgev cnigdtc kpvgtuceg qh  
gzrtguukpu ykvj nkvgtn cpf cfkvkqp ecug0 Cpf evalAlg ku cp  
qdlgev cnigdtc hqt gxcnwcvkqp qh vjqug gzrtguukpu. yjkej jcu  
v{rg ExpAlg Int

```
z%&C SubExpAlg E = (ExpAlg E) & { sub : E QN E QN
  E };
YCz subEvalAlg = evalAlg ,, { sub = \ (x : IEval).
  \ (y : IEval). { eval = x.eval - y.eval } };
```

Pgzv. yg fghkpg cp kpvgtuceg vjcv uwrrqtu rtgww{ rtkpvki0

```
z%&C IPrint = { print : String };
YCz printAlg = {
  lit = \ (x : Int). { print = x.toString() },
  add = \ (x : IPrint). \ (y : IPrint). { print = x.
    print.concat(" + ").concat(y.print) },
  sub = \ (x : IPrint). \ (y : IPrint). { print = x.
    print.concat(" - ").concat(y.print) }
};
```

Rtqxfkfg ykvj vjg fghkpvkpu cdqvg. yg ecp vjgp etgcvg  
xcnwg wukpi vjg crtrtktcvg cnigdtcu0 Hqt gzco rng< fghkpgu  
vyq gzrtguukpu0

Vjg gzrtguukpu ctg wpwuwn kp vjg ugpg vjcv vjg{ ctg  
hwpevkpu vjcv vmng cp gzvtc ctiwogpv f. vjg qdlgev cnigdtcu.  
cpf wug vjg fcvv eqpuvtwevqtu rtqxfkfg d{ vjg qdlgev cnig/  
dtc \*hcevt{+ f uwej cu lit. add cpf sub vq etgcvg xcnwg0  
Oqtgqxt. Vjg cnigdtcu vjg ougnxgu ctg cdvtcevgf qxgt vjg  
cmqygf qrgtcvkpu uwej cu gxcnwcvkqp cpf rtgww{ rtkpvki d{  
tgswktpi vjg gzrtguukqp hwpevkpu vq vmng cp gzvtc ctiwogpv  
E0

```
YCz merge A B (f : ExpAlg A) (g : ExpAlg B) = {
  lit = \ (x : Int). f.lit x ,, g.lit x,
  add = \ (x : A & B). \ (y : A & B).
    f.add x y ,, g.add x y
};
```

Kh yg yqwnf nkmg vq jcxg cp gzrtguukqp vjcv uwrrqtu dqvj  
gxcnwcvkqp cpf rtgww{ rtkpvki. yg yknn pggf c ogejcpkuo vq  
eqo dkgp vjg gxcnwcvkqp cpf rtkpvki cnigdtcu0 Kpvgtugevkqp  
v{rgu cmqyu uwej eqo rqukvkqp< vjg merge hwpevkqp. yjkej

<sup>5</sup> Ujqwnf dg ogpvkpgf kp U40

vcmgu vyq gzrtguukqp cnigdtcu vq etgcvg c eqodkpgf cnigdtc0  
 Kv fqgu uq d{ eqpuvtwevkpi c pgy gzrtguukqp cnigdtc. c tgeqtf  
 y jqug gcej hknf ku c hwpevkqp vjcv fgngicvgu vjg kprwv vq vjg  
 vyq cnigdtcu vcmgpd0

```
YCz newAlg = merge IEval IPrint subEvalAlg
  printAlg S^
YCz o1 = e1 (IEval & IPrint) newAlg S^
o1.print
```

Pqv vjcv o1 ku c ukping qdlgev etgcvgf vjcv uwrrqtu dqv  
 gxcnwcvkqp cpf rtkvpki. vjwu cejgkxpi hwnn hgcvwtg/qtkgpvf  
 rtqitc okpi0

## 505 Xkukvtu

Eqpuvtwevkpi kpuvcegu uggu enwou{#

Vjg xkukvt rcwgt p cnyu cffkpi pgy qrgtcvku vq gzkuv/  
 kpi utwewtgu ykvjqwv oqfkh{kpi vjqug utwewtgu0 Vjg v{rg  
 qh gzrtguukqp ctg fghkpgf cu hqnnqyu<

```
trait Exp[A] {
  def accept(f: ExpAlg[A]): A
}
```

```
trait SubExp[A] extends Exp[A] {
  override def accept(f: SubExpAlg[A]): A
}
```

Vjg dqf{ qh Exp cpf SubExp ctg cnoquv vjg ucog< vjg{  
 dqv eqpvcp cp accept ogvjqf vjcv vcmgu cp cnigdtc f cpf  
 tgvwtu c xcnwq qh vjg ectktgt v{rg A0 Vjg qpn{ fkhgtgpeg ku cv  
 f ô SubExpAlg[A] ku c uwdv{rg qh ExpAlg[A]0 Ukepg f crrgct  
 kp rctcogvt rqukvqp qh accept cpf hwpevkqp rctcogvtu ctg  
 eqpvtxctkcpv. pcvwtcmn{ yg yqwnf jqrq vjcv SubExp[A] ku c  
 uwrgtv{rg qh Exp[A]0 Jqygxt. uwej uwdv{rkpi tgnvkqp fqgu  
 pqv hkv ygnn kp Uecnc dgecwug kpjgtkvepeg k o rnkgu uwdv{rkpi kp  
 uwej ncpiwcigu<sup>6</sup>0 Cu SubExp[A] gzvpgfu Exp[A]. vjg hqtogt  
 dgeqogu c uwdv{rg qh vjg ncvgtl0

Uwej nk o kvcvkqp fqgu pqv gzkuv kp  $F_{\perp}$ 0 Hqt gzc o rng. yg ecp  
 fghkpg vjg uk o knct kpvgtcegu Exp cpf SubExp<

```
z%@C Exp A = { accept: forall A. ExpAlg A QN A };
z%@C SubExp A = { accept: forall A. SubExpAlg A QN
  A };
```

Vjgp d{ vjg v{rkpi lwfi ogpv kv jqnfu vjcv SubExp ku c uw/  
 rgtv{rg qh Exp0 Vjku tgnvkqp ixgu fguktgf tguwnvu0 Vq ixg c  
 eqpetvgv gzc o rng<

C ku ecnngf ku vjg *kpvgttrtgvckqp*0 Kv yqtmu hqt cp{ kpvgttrt/  
 vcvkqp {qw ycpv0

Hktuv yg fghkpg vyq fcvc eqpuvtwevqtu hqt uk o rng gzrtgu/  
 ukqpuk<

```
YCz lit (n : Int): Exp A = {
  accept = /\A. \ (f : ExpAlg A). f.lit n
};
```

```
YCz add (e1 : Exp) (e2 : Exp): Exp A = {
  accept = /\A. \ (f : ExpAlg A).
```

```
  f.add (e1.accept A f) (e2.accept A f)
};
```

Uwrrqug ncvgt yg fgekfg vq cwi ogpv vjg gzrtguukqp ykvj  
 uwdvtcevkqp<

```
YCz sub (e1 : SubExp) (e2 : SubExp): SubExp A =
  { accept = /\A. \ (f : SubExpAlg A).
    f.sub (e1.accept A f) (e2.accept A f)
  };
```

Qpg dki dpgghkv qh wukpi vjg xkukvt rcwgt p ku vjcv rtq/  
 itco ogtu ku cdng vq ytkvg kp vjg ucog yc{ vjcv yqwnf fq kp  
 Jcumgn0 Hqt gzc o rng. e2 = sub (lit 2)(lit 3) fghkpgu cp  
 gzrtguukqp0

Cpvjgt ko rqtvcv rtqrgtv{ vjcv fqgu pqv gzkuv kp Uecnc ku  
 vjcv rtqitc ogt ku cdng vq rcuu lit 2. yjkej ku qh v{rg Exp A  
 . vq sub. yjkej gzrgvu c SubExp A dgecwug qh vjg uwdv{rkpi  
 tgnvkqp yg jcxg0 Chvgt cnn. kv ku mpqyp uvcvkcnn{ vjcv lit  
 2 ecp dg rcuugf kpqv sub cpf pvjki yknn iq ytpi0*dtwpq<*  
*Uwdv{rkpi pggfu vq dg owej oqtg go rjculgfh#Ugg Oqfwnc*  
*Xkukvt Eq o rpgpvu#*

## 506 Okzkpu

Okzkpu ctg wughwn rtqitc okpi vgejpkswg yknfn{ cfqrvgf  
 kp f{pcoke rtqitc okpi ncpiwcigu uwej cu LcxcUetkrv cpf  
 Twd{0 Dwv qdxkqwn{ kv ku vjg rtqitc ogtu0 tgrqpudknkv{ vq  
 ocmg uwtg vjcv vjg okzkp fqgu pqv vt{ vq ceegu ogvjqfu qt  
 hknfu vjcv ctg pqv rtgugpv kp vjg dcug encuu0

Kp Jcumgn. ppg ku cnuq cdng vq ytkvg rtqitc o kp okzkp  
 uv{ng wukpi tgeqtfu0 Jqygxt. vjku crrtqej jcu c utgqwu  
 ftcydcen< ukpeg vjgtg ku pq uwdv{rkpi kp Jcumgn. kv ku pqv rqu/  
 ukdng vq tghkpg vjg okzkp d{ cffkpi oqtg hknfu vq vjg tgeqtfu0  
 Vjku ogcpu vjcv vjg v{rg qh vjg hcokn{ qh vjg okzkpu jcu vq  
 dg fgvtokpgf wrhtqpv. yjkej wpgftokpgu gzvpgukdnkv{0

$F_{\perp}$  ku cdng vq qxgteqog dqvj qh vjg rtqdnou< kv cnyu  
 eqorukpi okzkpu vjcv \*3+ gzvpgfu vjg dcug dgjcxqt. \*4+  
 yjkn gpuwtki v{rg uchgv{0

Vjg hkiwtg fghkpgu c okpk okzkp nkdtct{0 Vjg crqvtrjg  
 kp htqpv qh v{rgu fgpvgu ecnn/d{pcog ctiwogpvu uk o knct vq  
 vjg => pqvckqp kp vjg Uecnc ncpiwcig0

```
z%@C Mixin S = 'S QN 'S QN S;
```

```
YCz zero S (super : 'S) (this : 'S) : S = super;
```

```
YCz rec mixin S (f : Mixin S) : S
  = YCz m = mixin S S^ f (\ (_ : Unit). m f) (\ (_
    : Unit). m f);
```

```
YCz extends S (f : Mixin S) (g : Mixin S) : Mixin
  S
  = \ (super : 'S). \ (this : 'S). f (\ (d : Unit).
    g super this) this;
```

Yg fghkpg c hcevqtkcn hwpevkqp kp okzkp uv{ng cpf ocmg c  
 noisy okzkp vjcv rtkpvu ô Jgnmqô cpf fgngicvgu vq kvu uwrgt/  
 encuu0 Vjgp vjg vyq hwpevkqp ctg eqorqugf wukpi vjg mixin  
 cpf extends eqodkpcvqtu0 Vjg tguwnv ku vjg noisyFact hwpe/  
 vkqp vjcv rtkpvu ô Jgnmqô gxgt{ vkog kv ku ecnngf cpf eqorwgu  
 hcevqtkcn0

```
YCz fact (super : 'Int QN Int) (this : 'Int QN Int
  ) : Int QN Int
```

<sup>6</sup> Kv ku uvknn rquukdng vq ppeqfg eqpvtxctkcpv rctcogvt v{rgu kp Uecnc dwv  
 fqkpi uq yqwnf tgswtg uq og vgejpkswg0*dtwpq< yjcv vgejpkswgA*

**V{rgu**

$::= \quad | \rightarrow \quad | \forall : \quad$   
 $\quad | \quad ( \quad \quad \quad \text{Kpvtugevkqp } v\{rg$   
 $\quad | \quad \{l : \} \quad \quad \quad \text{Tgeqtf } v\{rg$

**Vgtou**

$e ::= x$   
 $\quad | \quad \Lambda : e \mid e \quad \quad \text{V}\{rg \text{ cduvtcevkqplcrrnkecvkqp}$   
 $\quad | \quad (x : ) : e \mid e e \quad \quad \text{Vgt o cduvtcevkqplcrrnkecvkqp}$   
 $\quad | \quad e;;e \quad \quad \quad \text{Ogtig}$   
 $\quad | \quad \{l = e\} \quad \quad \quad \text{Tgeqtf eqpuvtwevkqp}$   
 $\quad | \quad e.l \quad \quad \quad \text{Tgeqtf rtlgevkqp}$   
 $\quad | \quad e\mathbf{B} \quad \{l = e\} \quad \text{Tgeqtf wrfcvg}$

**Eqpvgzvu**

$\Gamma ::= \quad | \Gamma; \quad | \Gamma; x :$

**Ncdgnu**

$l$

**Hkiwtg 30**  $U\{pvcz \text{ qh } F_{\angle}$

```
= \ (n : Int). if n == 0 then 1 else n * this (n - 1)
YcZ noisy (super : 'Int QN Int) (this : 'Int QN Int) : Int QN Int
= \ (n : Int). { println("Hello"); super n }
YcZ noisyFact = mixin (Int QN Int) (extends (Int QN Int) foolish fact)
noisy 5
```

## 60 Vjg $F_{\angle}$ ecnewnwu

Hqnnqykpi Fwphkgnfou JA\_ yqtm qp uk o rn{ /v{ rgf ncodfc ecn/ ewnwu ykvj kpvgtugevkqp cpf wpkqp v{rgu. yg hqto cnk|g vjg u{pvcz. uwdv{rkpi. cpf v{rkpi qh  $F_{\angle}$  Kp vjg pgzv ugevkqp. yg yknn iq vj tqwij vjg v{rg/fktgevgf vtcpuncvkqp htq o  $F_{\angle}$  vq U{u/vgo H0

## 603 U{pvcz

Hkiwtg AA ujquy vjg u{pvcz qh  $F_{\angle}$  Kv ku U{uvgo  $F_{\angle}$  cv kvu eqtg0 Vq U{uvgo  $F_{\angle}$ . yg cff vyq hgcvwtgu< kpvgtugevkqp v{rgu cpf ukping/hkgnf tgeqtfu0 Yg kpenwfg qpn{ ukping tgeqtfu dgecwug ukping tgeqtf v{rgu cu vjg o wnvk/tgeqtfu ecp dg fguwictgf kpvq vjg ogtig qh o wnvkrng ukping tgeqtfu0

**V{rgu0** Vjg eqpuvtwevu kp vjg hktuv tqy qh v{rgu kp Hkiwtg AA ctg uvcpfctf kp U{uvgo  $F_{\angle}$  v{rg xctkdng . hwpevkqp v{rgu  $\rightarrow$  . cpf v{rg cduvtcevkqp  $\forall : \quad 0$  Vjg ncuv vyq ctg pqxgn0  $_1 ( \quad _1$  ku vjg kpvgtugevkqp qh v{rg  $_1$  cpf  $_2$ . cpf  $\{l : \}$  ctg vjg v{rgu hqt ukping/hkgnf tgeqtfu0

**Gzrtguukqpu0** Vjg hktuv hkxg eqpuvtwevu qh gzrtguukqpu ctg cnuq uvcpfctf< xctkdngu x. cpf vyq cduvtcevkqp/gnkokpcvkqp rctku< ncodfc gzrtguukqpu  $(x : ) : e$  cduvtcev gzrtguukqp e

qxgt xcnwgu qh v{rg cpf ctg gnkokpcvgf d{ crrnkecvkqp e e Dki ncodfcu  $\Lambda : e$  cduvtcev gzrtguukqp eqxgt v{rgu cpf ctg gnkokpcvgf d{ v{rg crrnkecvkqp e  $0$  Kp vjg uqwtg ncpiwcig. ncodfcu ctg ytkwgp cu **igqtig< VQFQ** cpf dki ncodfcu cu **igqtig< VQFQ**0

Vjg ncuv hqwt eqpuvtwevu ctg pgy<  $e_1;;e_2$  ku vjg *ogtig* qh vyq gzrtguukqpu  $e_1$  cpf  $e_2$ 0 Kv ecp dg wugf cu gkvjgt  $e_1$  qt  $e_2$ 0 Rctvcktn{. kh qpg tgictf  $e_1$  cpf  $e_2$  cu qdlgevu. vjgt ogtig yknn tgurqpfu vq gxgt{ ogvjgf vjcv qpg qt dqvj qh vjgo jcxg0 Ogtig qh gzrtguukqpu eqttgurqpf vq kpvgtugevkqp v{rgu  $_1 ( \quad _2 0 \{l = e\}$  eqpuvtwevu c ukping/hkgnf tgeqtf0  $e:/ceeguugu$  vjg hkgnf ncdgnngf  $l$  kp  $e$ 0 Pqvg vjcv  $e$  fagu pqv pggf vq dg c tgeqtf v{rg kp vjku ecug0 Hqt gzc o rng. cnvjquwv vjg ogtig qh vyq tgeqtfu

$x = \{l_1 = e_1\};; \{l_1 = e_2\}$

ku qh cp kpvgtugevkqp v{rg.  $x:/l_1$  uvknn ikxgu  $e_1$ 0 Qp vjg qvjgt jcpf.  $x:/l_3$  fagu pqv v{rg ejgem0 Hwpevkqpcn wrfcvg  $e\mathbf{B} \quad \{l = e_1\}$  c pgy tgeqtf yjkej ku gzcevn{ vjg ucog cu egzegrv vjg hkgnf ncdgnngf  $l$  ku wrfcvgf vq dgeqog  $e$ l0

**Eqpvgzv0** Eqpvgzv  $\Gamma$  ku cnuq uvcpfctf0 Kv ocru xctkdngu vq vjgt v{rgu cpf mggru dqwpf v{rg xctkdngu0

**Fluewuukqp0** C pcvwtcn swguvkqp vjg tgefgt oki jv cum ku vjcv yj{ yg jcxg gzenwfgf wpkqp v{rgu htqo vjg ncpiwcig0 Vjg cpuygt ku yg hqwpf vjcv kpvgtugevkqp v{rgu cnqpg ctg gpqwi j uwrqtvt gzvgpukdng fgukipu0 Vq hqewu qp vjg mg{ hgcvwtgu vjcv ocmg vjku ncpiwcig kpvgtguvkpi. yg cnuq qokv qvjgt eqo oqp eqpuvtwevu0 Hqt gzc o rng. hkzrqkpvu ecp dg cffgf kp uvcpfctf yc{u0

## 604 Uwdv{rkpi

**igqtig< Gzrnckp** vjg uwdv u{pvcz0

**igqtig< Jqy**gxgt. jcxg yg hqtdkffgp vjg kpvgttrnc{ qh uwd/ v{rkpi tgnckvqpu gzrnckv{ fgenctgf d{ rtqitc ogtu cu ugpg kp encuu/dcugf QQ ncpiwciguA

**dtwpq< Kpvgtugevkqp** v{rgu tgs wktg c uk o rng hqt o qh uwdv{r/ kpi0 Vjg uwdv{rkpi tgnckvqpu ku tghngzlxg cpf vtcpukvxxg0 Qwt uwdv{rkpi tgnckvqpu hqnnquy rtgxkwu yqtm0 Pggf vq ekvg# Vjg ockp fkhhtgpeg ku vjg kpvgtcevkqp ykvj rctcogtkc rqn{ oqt/ rjku o0

Kp uqog ecnewnk uweju cu U{uvgo  $F_{\angle}$ . vjg uwdv{rkpi tgnckvqpu ku qtvjqiqcn vq qvjgt ncpiwcig hgcvwtgu< vjqug ecnewnk ctg kp fkhhtgtpv ykvj jqy vjg uwdv{rkpi tgnckvqpu ku fghkpgf0 Kp  $F_{\angle}$ . yg vcmg c u{pvcke cr rtqcej. vjcv ku. uwdv{rkpi ku fwg vq uqngn{ qh kpvgtugevkqp cpf hwpevkqp v{rgu0

Kpvgtugevkqp v{rgu kpvtfweg pcvwtcn uwdv{rkpi tgnckvqpu coqpi v{rgu0 Hqt gzc o rng. **M** ( **Q** ujquwfnf dg c uwdv{rg qh **M** . ukpeg vjg hqt ogt ecp dg xkygfg cu gkvjgt **M** qt **Q**  $0$  Vq uwo octk|g. vjg uwdv{rkpi twngu ctg uvcpfctf gzegrv hqt vjtgg rqpku nkuvgf dgnqy<

30  $_1 ( \quad _2$  ku c uwdv{rg qh  $_3$ . kh *gkvjgt*  $_1$  qt  $_2$  ctg uwdv{rgu qh  $_3$ .

<:

$< :$	$*UXct+$
$\frac{3 <: 1 \quad 2 <: 4}{1 \rightarrow 2 <: 3 \rightarrow 4}$	$*UHwp+$
$\frac{1 <: [2 \mapsto 1] 2}{\forall 1: 1 <: \forall 2: 2}$	$*UHqtcnn+$
$\frac{1 <: 3}{1 ( 2 <: 3}$	$*UCpf3+$
$\frac{2 <: 3}{1 ( 2 <: 3}$	$*UCpf4+$
$\frac{1 <: 2 \quad 1 <: 3}{1 <: 2 ( 3}$	$*UCpf5+$
$\frac{1 <: 2}{\{I: 1\} <: \{I: 2\}}$	$*UTge+$

#### Hkiwtg 40 Uwdv{rkpi

40  $1$  ku c uwdv{ rg qh  $2 ( 3$ . kh  $1$  ku c uwdv{ rg qh dqvj  $2$  cpf  $3$

50  $\{I_1 : 1\}$  ku c uwdv{ rg qh  $\{I_2 : 2\}$ . kh  $I_1$  cpf  $I_2$  ctg kfgpvkecn  
cpf  $1$  ku c uwdv{ rg qh  $2$

Vjg hktuv rkpvn ku ecrwtgf d{ vyq twngu  $*U/Cpf3+$  cpf  $*U/Cpf4+$ . yjgtgcu vjg ugeqpf rkpvn d{  $*U/Cpf5+$  Pqvg vjcv vjg ncuv rkpvn ogepu vjcv tgeqtf v{ rgv ctg eqxctkcpv kp vjg v{ rg qh vjg hkgnfu

#### 605 V{rkpi

Vjg v{ rkpi lwfi ogpv hqt  $F_{\perp}$  ku qh vjg hqto  $\Gamma \vdash e : \perp$  Vjku lwfi ogpv wugu vjg eqpvzv  $\Gamma \vdash$  Vjg twngu hqt xctkdngu. cduvtce/vkqp. v{ rg cduvtcevkqp. cpf v{ rg crrnkecvkqp ctg uvcpfctf kp  $U\{uvgo F\}$  Vq ecvgt vq vjg uwdv{ rkpi tgnvkqpu cpf cxqkf jcx/ kpi wpgvgto kpkvke twngu. kp  $*Crr+$ . yg cffkvkqpcnn{ tgswnktg vjg v{ rg qh vjg ctiwogpv dg c uwdv{ rg qh vjg rctcogvgt  $\vdash$  Vjg twng hqt tgeqtf eqpwtwevkqp ku cnuq uvcpfctf  $\vdash$  Hqt tgeqtf rtq/ lgevkqp cpf wrfcvg. yg tguqtv vq vjg cwzknkt{  $\delta$  igvö cpf örwvö twngu  $\vdash$  Vjg  $\delta$  igvö lwfi ogpv ejgemu kh c hkgnf  $I$  kpfggf gzkuvu kp c v{ rg cpf hgvejgu vjg v{ rg qh vjg hkgnf kh uq  $\vdash$  Vjg örwvö lwfi ogpv ku cnoquv ukonkt. gzegrv vjcv kv vcmgu vjg kpvpgpfgf wrfcvg  $*dqj$  gzrtguukqp cpf v{ rg+. cpf tgwtpu kp cffkvkqp c pgy v{ rg qh vjg gzrtguukqp  $\vdash$  Kp tgeqtf wrfcvgu. yg cmmqy tg/ hkpkpi vjg v{ rg qh vjg hkgnf kp swguvkqp

Kp rctvkewnt yg kptqfweg c  $*V/Ogtig+$  twng vjcv crrnkgu vq ogtig eqpwtwevu

$\Gamma \vdash t$

$\frac{\in \Gamma}{\Gamma \vdash}$	$*YH/Xct+$
$\frac{\Gamma \vdash 1 \quad \Gamma \vdash 2}{\Gamma \vdash 1 \rightarrow 2}$	$*YH/Hwp+$
$\frac{\Gamma; \vdash}{\Gamma \vdash \forall :}$	$*YH/Hqtcnn+$
$\frac{\Gamma \vdash 1 \quad \Gamma \vdash 2}{\Gamma \vdash 1 ( 2}$	$*YH/Cpf+$
$\frac{\Gamma \vdash}{\Gamma \vdash \{I: \}}$	$*YH/Tge+$

#### Hkiwtg 50 Ygnn/hqtogfpguu

### 70 V{rg/fktgevgf Vtcuncvkqp vq U{uvgo F

Kp vjku ugevkqp yg fghkpg vjg ugo cpvkeu qh  $F_{\perp}$  d{ ogepu qh c v{ rg/fktgevgf vtcuncvkqp vq c xctkcpv qh  $U\{uvgo F\}$  gzvgpfgf ykvj vwrngu  $\vdash$  Vjku vtcuncvkqp tgoqxgu vjg ncdgnu qh tgeqtfu cpf vwtpu kpvgtugevkqpu kpq rtqfwegu. owej nkmg Fwphkgnföu gncdqtcvkqp  $\vdash$  Dwv qwt vtcuncvkqp cnuq fgcnu ykvj rctcogvte rqn{ oqtrjkuo cpf tgeqtfu

#### 703 Kphqt o cn Fkuewukqp

Vq jgnr vjg tgefgt jcxg c jkij/ngxgn wpgtucpfkpi qh jqy vjg vtcuncvkqp yqtmu. kp vjku uwdugevkqp yg rtgugpv vjg vtcu/ncvkqp kphqt ocn  $\vdash$  Vcmg vjg  $F_{\perp}$  gzrtguukqp hqt gzc o rng

```
{ eval = 4, print = "4" }.eval
```

Hktuv. onvkv/hkgnf tgeqtf nkvgtcnu ctg fgwictgf kpq ogtigu qh ukping/hkgnf tgeqtf nkvgtcnu  $\vdash$  Vjgtghqtg

```
{ eval = 4, print = "4" }
```

dgeqogu

```
{ eval = 4 } ,, { print = "4" }
```

Ogtigu qh vyq xcnwgu ctg vtcuncvgf kpq lwuv c rckt qh vjgo d{  $*Ogtig+$  cpf ukping/hkgnf tgeqtf nkvgtcnu nqug vjkt hkgnf ncdgnu d{  $*TgeEqp+$  Jgpeg { eval = 4 } ,, { print = "4"} dgeqogu (4, "4")

dtpwq: Fqpöw cdwug kpnkpkpi qh gzc o rngu kp vjg vgzv#

Hkpcnn{. e1 cpf e2 ctg dqvj eqgtegf d{ c rtqlgevkqp hwpe/vkqp

(x: (Int;String)):B x  $\vdash$  dtpwq: Ujqy vjg uqwtegf rtqitc o. cpf vjg rtqitc o vjcv kv igvu vtcuncvgf vq  $\vdash$  Vjgp gzrnkp jqy vjcv vtcuncvkqp yqtmu



$$\boxed{\Gamma \vdash e:}$$

$$\frac{(x; ) \in \Gamma}{\Gamma \vdash x:} \quad *Xct+$$

$$\frac{\Gamma; x: \vdash e: \quad \Gamma \vdash}{\Gamma \vdash (x: ): e: \rightarrow} \quad *Cdu+$$

$$\frac{\Gamma; \vdash e:}{\Gamma \vdash \Lambda : e : \forall :} \quad *VCdu+$$

$$\frac{\Gamma \vdash e_1 : \quad \Gamma \vdash e_2 : \quad \Gamma \vdash e_3 : \quad \Gamma \vdash e_4 :}{\Gamma \vdash e_1 e_2 : \quad \Gamma \vdash e_3 e_4 :} \quad *Crr+$$

$$\frac{\Gamma \vdash e: \forall : \quad \Gamma \vdash}{\Gamma \vdash e : [ \mapsto ]} \quad *VCrr+$$

$$\frac{\Gamma \vdash e_1 : \quad \Gamma \vdash e_2 :}{\Gamma \vdash e_1 ; e_2 :} \quad *Ogtig+$$

$$\frac{\Gamma \vdash e:}{\Gamma \vdash \{I = e\} : \{I: \}} \quad *TgeEqp+$$

$$\frac{\Gamma \vdash e: \quad \Gamma \vdash_{igv} ( ; I) :}{\Gamma \vdash e: I :} \quad *TgeRtql+$$

$$\frac{\Gamma \vdash e: \quad \Gamma \vdash e_1 : \quad \vdash_{rww} (t; I; e_1 : ) : ( ; ) \quad \Gamma \vdash e: I :}{\Gamma \vdash e: I :} \quad *TgeWrf+$$

$$\boxed{\vdash_{igv} ( ; I) :}$$

$$\vdash_{igv} (\{I: \}; I) : \quad *IgvDcug+$$

$$\frac{\vdash_{igv} ( ; I) :}{\vdash_{igv} ( ; I) :} \quad *IgvNghv+$$

$$\frac{\vdash_{igv} ( ; I) :}{\vdash_{igv} ( ; I) :} \quad *IgvTki jv+$$

$$\boxed{\vdash_{rww} ( ; I; e: ) : ( ; )}$$

$$\vdash_{rww} (\{I: \}; I; e: ) : ( ; I: ) \quad *RwwDcug+$$

$$\frac{\vdash_{rww} ( ; I; e: ) : ( ; )}{\vdash_{rww} ( ; I; e: ) : ( ; )} \quad *RwwNghv+$$

$$\frac{\vdash_{rww} ( ; I; e: ) : ( ; )}{\vdash_{rww} ( ; I; e: ) : ( ; )} \quad *RwwTki jv+$$

**Hkiwtg 60**  $\forall \{rkpi$

$$\boxed{[ ] = T}$$

$$\begin{aligned} [ ] &= \\ [ ]_1 \rightarrow [ ]_2 &= [ ]_1 \rightarrow [ ]_2 \\ [\forall : ] &= \forall : [ ] \\ [ ]_1 ( ]_2 &= \langle [ ]_1 ; [ ]_2 \rangle \\ [ [ I: ] ] &= [ ] \end{aligned}$$

**Hkiwtg 70**  $\forall \{rg vtcpcvkqp$

#### 704 Vctigv Ncpiwcig

Vjg vctigv ncpiwcig ku U{uvgo  $F$  gzvgpfgf ykvj rcktu0 Vjg u{pvcz cpf v{rkpi ku eqo rngvgn{ uvcpfctf0 Vjg u{pvcz qh U{u/vgo  $F$  ku cu hqnnqyu<

$$\begin{aligned} \forall \{rgu \quad T &::= \mid T \rightarrow T \mid \forall : T \mid \langle T; T \rangle \\ Vgtou \quad E; C &::= x \mid (x: T): E \mid \Lambda : E \mid EE \mid ET \\ &\mid \langle E; E \rangle \mid \mathcal{B} E \mid \mathcal{M} E \end{aligned}$$

y jknq kvu ug o cpvkeu ecp dg hqwpf kp uvcpfctf vgzvu  $\Lambda$ 0 Vjg ockp vtcpcvkqp lwfi ogpv ku  $\Gamma \vdash e: \rightarrow E$  y jkej uvcvgu vjcv ykvj tgurgev vq vjg gpktqp ogpv  $\Gamma$ . vjg  $F$  gzrtguukqp eku qh c  $F$  v{rkpi ku eqo rngvgn{ uvcpfctf0 Vjg u{pvcz qh U{u/vgo  $F$  gzrtguukqp  $E$ 0

Yg cnuq fghkpg vjg v{rg vtcpcvkqp hwpevkqp  $[ ]$  htqo  $F$  v{rgu vq U{uvgo  $F$  v{rgu  $T$ 0

Vjg hktuv vjgg twngu qh vjg vtcpcvkqp ku uvcpfctf0 Hqt vjg ncuv vyq. vjg kpvgtugevkqp qh vyq v{rgu ctg vtcpcvvgf kpvq c rtqfwev qh vjgo. cpf vjg ncdgn qh tgeqtf v{rgu ctg gtcugf0

Vjg vtcpcvkqp eqpukuvu qh hqwt ugvu qh twngu. y jkej ctg gznckpgf dngny<

#### 705 Uwdv{rkpi \*Eqgtekp+

Vcnm cdqvv /gzrcpukqp0

Vjg eqgtekp lwfi ogpv  $\Gamma \vdash \rightarrow \rightarrow C$  gzvgpfu vjg uwdv{rkpi lwfi ogpv ykvj c eqgtekp qp vjg tki jv jcpf ukfg qh  $\rightarrow$ 0 C eqgtekp  $C$  ku cp gzrtguukqp kp vjg vctigv ncpiwcig cpf jcu v{rg  $\rightarrow$ 2. cu rtqxf d{ Ngo o c AA0 Kv ku tgc f ökp vjg gpktqp ogpv  $\Gamma$ .  $\rightarrow$ 1 ku c uwdv{rg qh  $\rightarrow$ 2= cpf kh cp{ gzrtguukqp e jcu c v{rg  $\rightarrow$ 1 vjcv ku c uwdv{rg qh vjg v{rg qh  $\rightarrow$ 2. vjg gncdqtevgf e. y jgp crnkgf vq vjg eqttgurqpfkpi eqgtekp  $C$ . jcu gzcevn{ v{rg  $[ ]$ 2ö0 Hqt gzc o rng.  $\Gamma \vdash Int ( Bool < Bool, \rightarrow fst$ . y jgtg  $fst$  ku vjg rtqlgevkqp qh c vwrng qp vjg hktuv gng ogpv0 Vjg eqgtekp lwfi ogpv ku qpn{ wugf kp vjg \*Crr+ ecug0 Cu \*Uhw+ uwrqtvu eqpvctckcpv rctc ogvgt v{rg cpf eqxctckcpv tgvwtp v{rg. vjg eqgtekp qh vjg rctc ogvgt v{rgu cpf vjcv qh vjg tgvwtp v{rgu ctg wugf vq etgevg c eqgtekp hqt vjg hwpevkqp v{rg0 \*UCpf3+. \*UCpf4+. cpf \*UCpf5+ fgcn ykvj kpvgtugevkqp v{rgu0 Vjg hktuv vyq ctg eqo rng ogpvct{ vq gce j qvjgt0 Vcmg \*UCpf3+ hqt gzc o rng. kh yg mpqy  $\rightarrow$ 1 ku c uwdv{rg qh  $\rightarrow$ 3 cpf  $C$  ku c eqgtekp htqo  $\rightarrow$ 1 vq  $\rightarrow$ 3. vjgp yg ecp eqpenwfg vjcv  $\rightarrow$ 1 (  $\rightarrow$ 2 ku cnuq c uwdv{rg qh  $\rightarrow$ 3 cpf vjg pg y eqgtekp ku c hwpevkqp vjcv vcmgu c xcngw  $x$  qh v{rg  $\rightarrow$ 1 (  $\rightarrow$ 2. rtqlgev  $x$  qp vjg hktuv kvgo. cpf crnn{  $C$  vq kv0 \*UCpf5+ wugu dqv qh vyq eqgtekp cu cpf eqpuvtweu c rckt0 dtwpq< Ixg c eqwng qh eqpetgvg gzc o ngu



$$\boxed{<: \quad , \rightarrow C}$$

$$<: \quad , \rightarrow (x: \llbracket \quad \rrbracket): x \quad *UXct+$$

$$\frac{3 <: 1 \rightarrow C_1 \quad 2 <: 4 \rightarrow C_2}{1 \rightarrow 2 <: 3 \rightarrow 4 \rightarrow (f: \llbracket 1 \rightarrow 2 \rrbracket): (x: \llbracket 3 \rrbracket): C_2 (f (C_1 x))} \quad *UHwp+$$

$$\frac{1 <: [2 \mapsto 1] 2 \rightarrow C}{\forall 1: 1 <: \forall 2: 2 \rightarrow (f: \llbracket \forall: 1 \rrbracket): \Lambda: C (f \quad)} \quad *UHqtcnn+$$

$$\frac{1 <: 3 \rightarrow C}{1 (2 <: 3 \rightarrow (x: \llbracket 1 (2) \rrbracket): C (\mathcal{B} \quad x))} \quad *UCpf3+$$

$$\frac{2 <: 3 \rightarrow C}{1 (2 <: 3 \rightarrow (x: \llbracket 1 (2) \rrbracket): C (\mathcal{M} \quad x))} \quad *UCpf4+$$

$$\frac{1 <: 2 \rightarrow C_1 \quad 1 <: 3 \rightarrow C_2}{1 <: 2 (3 \rightarrow (x: \llbracket 1 \rrbracket): \langle C_1 x; C_2 x \rangle)} \quad *UCpf5+$$

$$\frac{1 <: 2 \rightarrow C}{\{I: 1\} <: \{I: 2\} \rightarrow (x: \llbracket \{I: 1\} \rrbracket): C x} \quad *UTge+$$

## Hkiwtg 80 Eqgtekp

y jgp gzrnckpki vjg twngu0

### 706 V{rkpi \*Vtcpuncvkqp+

Kp vjku uwdugevkqp yg pqy rtgugpv hqtoenn{ vjg vtcpuncvkqp twngu vjcv eqpxgtv  $F_{\downarrow}$  gzrtguukqpu kpqv U{uvgo  $F$  qpgu0 Vjku ugv qh twngu guugpvkenn{ gzvgpfu vjqug kp vjg rtgxkqu ugevkqp ykvj vjg nki jv/dnwg rctv hqt vjg vtcpuncvkqp0

**Vtcpuncvkqp** Vjg gncdqtevkqp lwfiogpv  $\Gamma \vdash e: \quad , \rightarrow E$  gzvgpfu vjg v{rkpi lwfiogpv ykvj cp gncdqtevgf gzrtguukqp qp vjg tki jv jcpf ukfg qh  $\rightarrow 0$  Vjg vtcpuncvkqp gpwrtgu vjcv  $E$  jcu v{rg  $\llbracket \quad \rrbracket 0$  Kv ku cnuq uvcpfctf. gzegrv hqt vjg ecug qh \*Crr+. kp yjkej c eqgtekp htqo vjg kphgttgf v{rg qh vjg ctiwogpv.  $e_2$  . vq vjg gzrgevfg v{rg qh vjg rctcogvgf.  $e_1$  . ku kpugtvgf dghqtg vjg ctiwogpv= \*Ogtig+ vtcpuncvgu ogtigu kpqv rktu0 \*TgeEqp+ wugu vjg ucog U{uvgo  $F$  gzrtguukqp  $E$  hqt e cu hqt  $\{I = e\} 0$  Cpf kp \*TgeGko+ cpf \*TgeWrf+ vjg eqgtekpku igpgtevgf d{ vjg ðigv0 cpf ðrwv0 twngu yknn dg wugf vq eqgeg vjg ockp  $F_{\downarrow}$  gzrtguukqp0

\*TgeRtl+ v{rgejgemu ecpf wug vjg ðigv0 twng vq tgwtp vjg v{rg qh vjg hkgnf  $e_1$  cpf vjg eqgekqp  $C 0$  Vjg v{rg qh vjg yjqng gzrtguukqp ku  $e_1$  cpf kvu vtcpuncvkqp qh  $C E 0$

\*TgeWrf+ ku ukoknet vq \*TgeRtl+ kp vjcv kv wugu vjg cwzkn/ kct{ ðrwv0 twng0 Vjku twng v{rgejgemu ecpf  $e_1$  . cpf wugu vjg ðrwv0 twng0 Pqvg vjcv kv cnnqyu tghkpkpi qh v{rgu d{ cp  $e_1$  vjcv ku qh c uwdv{rg qh  $e_1$  . yjkej ku vjg v{rg qh vjg hkgnf  $I$  kp  $\emptyset$  Vjg v{rg qh vjg wrfcvgf gzrtguukqp vjgp vcmgu vjg v{rg  $e_1$  tg/ vwtpgf d{ vjg ðrwv0 twng. y jkng kvu vtcpuncvkqp ku  $E$  . crnrkgf vq

vjg eqgtekp igpgtevgf d{ vjg ðrwv0 twng.  $C 0$

Vjg vyq ugv qh twngu ctg gzrnckpgf dgnqy0

**ðigv0 Twngu** Vjg ðigv0 lwfiogpv fgcnu urghkeenn{ ykvj tgeqtf gnkokpcvkqp cpf {kgnfu c eqgtekp ecp dg vjqwi jv cu c hkgnf ceeguut0 Hqt gzc o rng<dtwpq< Uvknn pqv ujqykpi vjg fgtkxcvkqpu#

$\Gamma \vdash_{\text{igv}} (\{\mathcal{B} : \mathcal{M}\}; \mathcal{B}) : \{\mathcal{B} : \mathcal{M}\} \rightarrow (x: \mathcal{M}) : x$

Vjg ncodfc ku vjg hkgnf ceeguut cpf yjgp crnrkgf vq c vtcpuncvgf gzrtguukqp qh v{rg  $\{\mathcal{B} : \mathcal{M}\}$  . kv ku cdng vq ikxg vjg fguktgf hkgnf0 \*IgvDcug+ ku vjg dcug ecug< vjg v{rg qh vjg hkgnf ncdgngf  $I$  kp c  $\{I: \quad\}$  ku lwuv  $\quad$  cpf vjg eqgtekp ku cp kfgpvkv{ hwpevkqp urgekcnk|gf vq v{rg  $\llbracket \{I: \quad\} \rrbracket$  \*IgvNghv+ cpf \*IgvTki jv+ ctg eqo rng ogpvct{ vq gcej qvjgt0

Eqpukfgt vjg uqwtg rtqitco<

({ name = "Isaac", age = 10 }).name

Owrvk/hkgnf tgeqtfu ctg fguwictgf kpqv ogtig qh ukping/ hkgnf tgeqtfu<

({ name = "Isaac"} , , { age = 10 }).name

D{ \*IgvDcug+.

$\vdash_{\text{igv}} (\{name: String\}; name) : String$

yg jcxg vjg eqgtekp

$(x: \llbracket \{name: String\} \rrbracket): x$



yjkej ku lwuv  $(x: String): x$  ceeqtfkpi vq v{rg vtcpunc/  
vkqp0  
D{ \*IgvNghv+.

$\vdash_{igv} (\{name: String\} ( \{age: Int\}; namè) : String$

D{ v{rkpi twngu. vjg vtcpuncvkqp qh vjg rtqitco ku

(öIsaaö;10)

0 Kh yg cr rn{ vjg eqgtekqp vq kv. yg igv

öIsaaö

**örwvö Twngu** dtwppq< Okuukpi gzc o rng \*cpf fgtxcvkqp+

Vjg örwvö lwfi ogpv fgcnu urghkecn{ ykvj tgeqtf wrfcvg  
ecp dg vjqwi jv cu rtqfwekpi c hkgnf wrfcvg0 Eqo rctgf vq  
vjg öigvö twngu. vjg örwvö twngu vcmg cp gzvtc kprwv e. yjkej ku  
vjg fguktgf gzrtguukqp vq tgrnceg vjg hkgnf ncdnggf I kp xcnwgu  
qh v{rg 0 \*RwvDcug+ ku vjg dcug ecug0 Vjku twng cmmqyu tghkpg/  
ogpv qh tgeqtf hkgnfu kp vjg ugpg vjcv vjg v{rg qh e ecp dg  
c uwdv{rg qh vjg v{rg qh vjg hkgnf ncdnggf d{ 0 Vjg tguwnv/  
kpi v{rg ku {I: '} cpf vjg igpgtcvgf eqgtekqp ku c eqpuvcpv  
hwpevkqp vjcv cnyc{u tgvwtpu E0 \*RwvNghv+ cpf \*RwvTki jv+ ctg  
eqo rng ogpvt{ vq gcej qvjgt< vjg kfge ku gzcevn{ vjg uc o g cu  
\*IgvNghv+ cpf \*IgvTki jv+ gzegrv vjcv vjg tghkpgf v{rg 1 cpf  
2 ku wugf0

**707 Ogvc/vjgqt{**

**Ngo o c 3** \*Uwdv{ rkpi ku tghngzkg0+0 I kxgp c v{rg . <: 0

**Ngo o c 4** \*Uwdv{ rkpi ku vtcpukvkg0+0 Kh 1 <: 2 cpf 2 <: 3.  
vjgp 1 <: 30

**Ngo o c 50** Kh

$\Gamma \vdash 1 <: 2 \rightarrow C$

vjgp

$[\Gamma] \vdash C : [1] \rightarrow [2]$

**Ngo o c 6** \*Igv twngu rtqfweg vjg v{rg/eqttgev eqgtekqp0+0 Kh

$\Gamma \vdash_{igv} ; I = C; 1$

vjgp

$[\Gamma] \vdash C : [ ] \rightarrow [1]$

Rtqqh0 D{ kpfwevkqp qp vjg ikxgp fgtxcvkqp0 □

**Ngo o c 7** \*Rwv twngu rtqfweg vjg v{rg/eqttgev eqgtekqp0+0 Kh

$\Gamma \vdash_{rwv} ; I; E = C; 1$

vjgp

$[\Gamma] \vdash C : [ ] \rightarrow [ ]$

Rtqqh0 D{ kpfwevkqp qp vjg ikxgp fgtxcvkqp0 □

**Ngo o c 8** \*Vtcpuncvkqp rtgugtxgu ygnn/hqt o g fpguu0+0 Kh

$\Gamma \vdash$

vjgp

$[\Gamma] \vdash [ ]$

Rtqqh0 D{ kpfwevkqp qp vjg ikxgp fgtxcvkqp0 □

**Vjgqtgo 3** \*V{rg rtgugtxkpi vtcpuncvkqp0+0 Kh

$\Gamma \vdash e : \rightarrow E$

vjgp

$[\Gamma] \vdash E : [ ]$

Rtqqh0 \*Umgvej+ D{ uvtwevwtcn kpfwevkqp qp vjg gzrtguukqp  
cpf vjg eqttgurqpfkpi kphgtgpeg twng0 Vjg hwnn rtqqh ecp dg  
hqwpf kp vjg cr rgpfkz0 □

V{rg/Fktgevgf Vtcpuncvkqp vq U{uvg o F0 Ockp tguwnv<  
v{rg/rtgugtxcvkqp - eqjgtgpeg0

**80 Ko rng ogpvcvkqp**

Qwt ko rng ogpvcvkqp ewttgpn{ uwr rqtvu o wnvk/hkgnf tgeqtfu0

**803 V{rg U{ppp{ ou**

Yg gzvgpf vjg ko rng ogpvcvkqp qh vjg v{rg u{uvg o gzvgpfgf  
ykvj v{rg u{ppp{ ou cpf nc|{ ctiw ogpvu0

z% C T A1 A2 = ... S^

**804 Qrvok|evkqp**

**90 Tgncvgf yqtm**

Kpvtugevkqp v{rgu ykvj rqn{oqtrjkuo0 Kpvtugevkqp v{rgu  
fcvg dcem vq cu gctn{ cu Eqrrq gv cn0 JA\_0 Tgegpvn{. uq o g hqt o  
qh kpvgtugevkqp v{rgu jcxg dgpp cfqrvgf kp qdlgev/qtgpgvf  
ncpiwcigu uwej cu Uecnc. Eg{nqp. cpf Itceg0 Qpg fghkpkpi  
fkhhgtgpeg. coqpi qvjgtu. ku vjcv cmm vjqug ncpiwcigu qpn{  
cmmqy kpvgtugevkqp qh eqpetgvg v{rgu \*encuugu+. yjgtgu qwt  
ncpiwcig cmmqyu kpvgtugevkqp qh v{rg xctkcdngu. uwej cu  
"0 Ykvjqwv vjcv xgjkeng. yg yqwnf pqv dg cdng vq fghkpg vjg  
igpgtke K hwpevkqp \*dgnqy+ hqt cmm kpvgturtgcvkqp qh c  
ikxgp cnigdtc. cpf yqwnf kpewt dqkngtrncvg eqfg<

YCz merge [A, B] (f: ExpAlg A) (g: ExpAlg B) = {  
lit (x : Int) = f.lit x ,, g.lit x,  
add (x : A & B) (y : A & B) =  
f.add x y ,, g.add x y  
}

Kp Uecnc eqo o wpkv{. vjgtg jcxg dgpp cwgo rvu vq rtqxfkg c  
hqwpfcvkqp cn ecnewnwu hqt Uecnc vjcv kpeqtrqtcvgu kpvgtugevkqp  
v{rgu JA A\_0

Qwt v{rg u{uvg o eqodkpgu kpvgtugevkqp v{rgu cpf rqn{/  
oqtrjkuo0 Vjg enquguv vq qwtu ku Rkgtgou yqtm JA\_ qp c  
rtqvqv{rg eqo rkngt hqt c ncpiwcig ykvj dqvj kpvgtugevkqp

$v\{rgu, wpkqp\} v\{rgu, cpf\} rctcogvtke\ rqn\{oqtrjku\} o\{Vjg\ ko/$   
 $rqtvcpv\ fkhgtgpeg\ ykvj\ qwt\ u\{uvg\ o\ ku\ vjcv\ kp\ jku\ ncpiwcig\$   
 $vjgtg\ ku\ pq\ gzrnkekv\ kpvgtfwekvqp\ eqpuvtwev\ nkmq\ qwt\ ogtig\$   
 $qrgtcvqt\} Cu\ ujqyp\ kp\ Ugekvqp\ AA.\ vjku\ hgcvwtg\ ku\ rlxqvcn\ kp\$   
 $uwrqrvtkpi\ oqfwntkv\{cpf\ gzvpgukdknk\{dgecvug\ kv\ cmmqyu\$   
 $f\{pcoke\ eqorqukvkqp\ qh\ xcnwgu\} Rkgtg\ jcu\ cnuq\ uwwfkgf\ c\$   
 $u\{uvg\ o\ yjgtg\ dqvj\ kpvgtugekvqp\ v\{rgu\ cpf\ dqwpfgf\ rqn\{oqt/$   
 $rjku\ o\ ctg\ rtgugpv\ kp\ jku\ Rj\} F\ fkuugtvcvkqp\ JA\_cpf\ c\ 3;\ ;\ 9\ tg/$   
 $rqt\} JA\_0\ I\ qkpi\ kp\ vjg\ fktgekvqp\ qh\ jki\ jgt\ mkpfu.\ Eq\ o\ rcipqpk\$   
 $cpf\ Rkgtg\ JA\_c\ f\ kpvgtugekvqp\ v\{rgu\ vq\ U\{uvg\ o\ F^{\prime}\} cpf\ wug\$   
 $vjg\ pgy\ ecnewnwu.\ F_{\wedge}^{\prime}.\ vq\ o\ qfgn\ o\ wnwkrng\ kpjgtkvcpeg\} Kp\ vjgt\$   
 $u\{uvg\ o.\ v\{rgu\ kpenwfg\ vjg\ eqpuvtwev\ qh\ kpvgtugekvqp\ qh\ v\{rgu\$   
 $qh\ vjg\ uc\ o\ g\ mkpf\ K\} Eq\ o\ rctgf\ vq\ qwt\ yqtm.\ vjg\{f\ q\ pqv\ jcxg\$   
 $c\ vgt\ o\ ngxgn\ eqpuvtwev\ hqt\ kpvgtugekvqp\ kpvgtfwekvqp\} Fcxkgu\$   
 $cpf\ Rhgppkpi\ JA\_uvwf\{vjg\ kpvgtucekvqp\ dgy\ yggp\ kpvgtugekvqp\$   
 $v\{rgu\ cpf\ ghggev\ kp\ ecnn\{d\{xcnw\ ncpiwcig\} Cpf\ vjg\{rtq/$   
 $rqug\ c\ \delta\ xcnw\ tguvtkevqp\} hqt\ kpvgtugekvqp\ v\{rgu.\ uk\ o\ knct\ vq\$   
 $xcnw\ tguvtkevqp\ qp\ rctcogvtke\ rqn\{oqtrjku\} o\}$

***Qvjgt v\{rg u\{uvg\ o\ ykvj kpvgtugekvqp v\{rgu\}*** Fwphkgnf JA \_  
 fguetkdg c uk o knct crttqcej vq qwtu < eq o rnkpi c u\{uvg\ o ykvj  
 kpvgtugekvqp v\{rgu kpvq qtfkpct\ /ecnewnwu vgt o u\} Vjg o clqt  
 fkhgtgpeg ku vjcv jku u\{uvg\ o fagu pqv kpenwfg rctcogvtke  
 rqn\{oqtrjku\ o. yjknq qwtu fagu pqv kpenwfg wpkqp\} Dgukfgu.  
 qwt twngu ctg cniqt kvj oke\}

Tg\{pqnfu kpxgpvgf Hqtu\{vjg JA \_kp vjg 3; :2u\} Qwt ogtig  
 qrgtcvqt ku cpcnqiqwu vq jku  $p_1; p_2$  Ecucipc. cpf Fwp/  
 hkgnf fguetkdg gncdqtckvpi o wnw/hkgnfu tgeqt fu kpvq ogtig  
 qh ukping/hkgnf tgeqt fu\} Cu Fwphkgnf jcu pqvgf. kp Hqtu\{vjg  
 ogtig u ecp dg qpn\{ wugf wpc o dki wqwu\} <sup>7</sup> Hqt kpuvcepg. kv ku  
 pqv cmmq yg f kp Hqtu\{vjg vq ogtig v y q hwpekvqp\}

Tghkpg o gpv kpvgtugekvqp JA A A \_ku vjg oqtg eqpugtxcvkxg  
 crttqcej qh cfqrkpi kpvgtugekvqp v\{rgu\ Kv kpetcgugu qpn\{ vjg  
 gzrtguukxgpguu qh v\{rgu dwv pqv vgt o u\} Dwv ykvjqwv c vgt o/  
 ngxgn eqpuvtwev nkmq o ogtig o. kv ku pqv rquukdg vq gpeqfg xct/  
 kquw ncpiwcig hgcvwtgu\} Cu cp cnvgtpcvkxg vq u\{pvcke uwdv\{r/  
 kpi fguetkdgf kp vjku rcrgt. Htkuej gv cn\} JA \_uvwf\{ug o cpvke  
 uwdv\{rkpi\}

***V\{rg u\{uvg\ o\ hqt oqfwntkv\}*** Vq cfftguu vjg rtqdnq o qh  
 gzvpgukdknk\{. uq o g tgugetejgtu jcxg fgukipgf pgy v\{rg u\{u/  
 vgo hgcvwtgu uwej cu xktwcn encuuqu JA \_ . rqn\{oqtrjke xctk/  
 cpvu JA \_ . yjknq qvjgtu jcxg ujqyp gornq\{kpi rtqitco/  
 okpi rcwgt p uwej cu qdlgev cnigdtcu JA \_ d\{ wukpi hgcvwtgu  
 ykvj kp gzkuvkpi rtqitco okpi ncpiwcig u\} Dqv qh vjg v y q cr/  
 rtqcejgu jcxg ftcydcemu qh uq o g mkpf\} Vjg hktuv crttqcej  
 qhvpg kpxqnxgu jgcx\{y gki jv fgukipu. yjknq vjg ugeqpf cr/  
 rtqcej uvknn ucetkhgegu vjg tgcfdcknk\{ hqt gzvpgukdknk\{}

***Gzvpgukdg tgeqt fu\}*** Wpfgtucpfpki tgeqt fu ku korqtvcpv  
 hqt wpfgtucpfpki qdlgev/qtkgpvgf ncpiwcig u\} Cpf yg ctg vjg  
 hktuv vq gncdqtckv tgeqt fu vq U\{uvg\ o F\} Gpeqfki tgeqt fu wu/  
 kpi kpvgtugekvqp v\{rgu cr rgt kp Tg\{pqnfu JA \_cpf Ecucipc gv  
 cn\} JA \_0 Cnvjqw i j Fwphkgnf cnuq fkuewuugu vjku kfgc kp jku rc/  
 rgt JA \_ . jg qpn\{ rtqxfkgu cp ko rng o gpvcvkqp dwv pqv hqt o cn\}

k\cvkqp\} Xgt\{ uk o knct vq qwt vtgc v o gpv qh gncdqtckvpi tgeqt fu  
 ku Ectfgmknq y qtm JA \_qp vtcupuncvki c ecnewnwu. pc o gf  $F_{<}$ .  
 ykvj gzvpgukdg tgeqt fu vq c uk o rngt ecnewnwu vjcv ykvjqwv  
 tgeqt fu rtk o kvxgu \*kp yjkej ecug ku  $F_{<+0}$  Dwv jg fagu pqv  
 eqpukfgt gpeqfki o wnw/hkgnf tgeqt fu cu kpvgtugekvqp= jgpeg  
 jku vtcupuncvqp ku oqtg jgcx\{y gki jv\} Etct\{ JA \_wugu kpvgtuge/  
 vkqp v\{rgu cpf gzkuvgpvcn v\{rgu vq cfftguu vjg rtqdnq o vjcv  
 ctkugu yjgp kpvgturtgvkpi o g vjqf fkurcvej cu ugnh/crrnkecvkqp\}  
 Dwv kp jku rcrgt. kpvgtugekvqp v\{rgu ctg pqv wugf vq gpeqfg  
 o wnw/hkgnf tgeqt fu\}

Y cpf JA \_uvctvgf vjg yqtm qp gzvpgukdg tgeqt fu cpf rtq/  
 rqugu tqy v\{rgu JA \_hqt tgeqt fu\} Ectfgmknk cpf Okvejgnn JA \_  
 fghkpg vjtg rtk o kvxg qrgtcvkqp qp tgeqt fu vjcv ctg fkhgt/  
 gpv htq o qwtu < ugngekvqp. tguvtkevqp. cpf gzvpgukqp\} Hqmmq y/  
 kpi vjku cr rtqcej. Ngklgp JA \_ fghkpg tgeqt f w rfcvg kp vgt o u qh  
 tguvtkevqp cpf gzvpgukqp. yjknq kp qwt tgeqt f u\{uvg\ o tgeqt f  
 w rfcvg ku c rtk o kvxg qrgtcvkqp\} Dqv Ngklgp u\{uvg\ o cpf  
 qwtu cmmqyu tgeqt fu vjcv eqpvckp fwrnkecvg ncdgnu\} Ctiwcdn\{  
 Ngklgp u\{uvg\ o ku utqpigt\} Hqt gzc o rng. kv uwrqrvtu rcuukpi  
 tgeqt f ncdgnu cu ctiwogpvu vq hwpekvqp\} Jg cnuq ujqyu gp/  
 eqfki cp kpvgtugekvqp v\{rg wukpi hktuv/encuu ncdgnu\} Ejnkrncnu  
 I JA \_gzrnckpu tgeqt f cu v\{rg ngxgn eqpuvtwev\}

Kpfggf. qwt u\{uvg\ o ecp dg cfcvrgf vq uk o wncvg u\{uvg\ o u  
 vjcv uwrqrvtv gzvpgukdg tgeqt fu dwv pqv kpvgtugekvqp qh qtfk/  
 pct\{v\{rgu nkmq **M** cpf **E** d\{ cmmqykpi qpn\{ kpvgtugekvqp  
 qh tgeqt f v\{rgu\}

$\vdash_{tge} uvcvu\ vjcv\ ku\ c\ tgeqt\ f\ v\{rg.\ qt\ vjg\ kpvgtugekvqp\ qh\$   
 $tgeqt\ f\ v\{rgu.\ cpf\ uq\ hqt\ vj\}$

$$\vdash_{tge} \{I: \} \quad *TgeDcug+$$

$$\frac{\vdash_{tge} 1 \quad \vdash_{tge} 2}{\vdash_{tge} 1 ( 2)} \quad *TgeUvgr+$$

$$\frac{\Gamma \vdash e_1 : 1 \rightarrow E_1 \quad \vdash_{tge} 1 \quad \Gamma \vdash e_2 : 2 \rightarrow E_2 \quad \vdash_{tge} 2}{\Gamma \vdash e_1 ; e_2 : 1 ( 2 \rightarrow \langle E_1 ; E_2 \rangle)} \quad *Ogtig+$$

Qh eqwtug qwt crttqcej jcu kvu nk o kvcvkqp cu fwrnkecvgf  
 ncdgnu kp c tgeqt f ctg cmmq yg f\} Vjku jcu dgpg fkuewuugf kp c  
 nctigt kuuwg d\{ Fwphkgnf JA \_0

## :0 Eqpenwukqp cpf Hwtvjgt Y qtm

### Cempq yngfi o gpvu

Cempq yngfi o gpvu. kh pggfgf\}

### Tghgtgpegu

1. P0 Cokp. C0 Oqqtu. cpf O0 Qfgtum\} Fgrgpfgpv qdlgev v\{rgu\}  
 kp 3;vj kpvgtvcvkpcn Yqtmujqr qp Hqwpfcvkqp qh Qdlgev/  
 Qtkgpvgf Ncpiwcig u. pw o dgt GRN/EQPH/3:5252. 42340

1. P0 Cokp. V0 Tq o rh. cpf O0 Qfgtum\} Hqwpfcvkqp qh rcvj/  
 fgrgpfgpv v\{rgu\} kp Rtqeggfpki qh vjg 4236 CEO kpvgtpc/  
 vkpcn Eqphgtgpeg qp Qdlgev Qtkgpvgf Rtqitco okpi U\{uvg\ o  
 Ncpiwcig ( Crrnkecvkqp. rcigu 455646;0 CEO. 42360

<sup>7</sup> Y j\{ vjg tguvtkevqpA

1. N0 Ectfgnnk0 Gzvpgukdng tgeqtfu kp c rwtg ecnewnwu qh uwdv{rkpi0 Fkikcn0 U{uvgo U Tgugtej Egpvt. 3; ;40
1. N0 Ectfgnnk cpf L0 E0 Okvejgn0 Qrgtcvkpu qp tgeqtfu0 Kp Ocvjgocvkecn hqwpfcvkpu qh rtqitco okpi ugocpvkeu. rcigu 446740 Urtkipgt. 3; ;20
1. N0 Ectfgnnk cpf R0 Ygipgt0 Qp wpfgtuvcpfkpi v{rgu. fevc cduvtcevkqp. cpf rqn{oqtrjkuo0 CEO Eqorwvki Uwtxgfu \*EUWT+. 39\*6+6936745. 3; ;70
1. I0 Ecuvcipc. I0 Ijgnk. cpf I0 Napiq0 C ecnewnwu hqt qxgt/ nqcfgf hwpevkpu ykvj uwdv{rkpi0 kphqto cvkqp cpf Eqorwvc/ vkqp. 339\*3+3376357. 3; ;70
1. C0 Ejnrcnc0 Wt< uvcvkecn{ /v{rgf ogvcrqtqitco okpi ykvj v{rg/ ngxgn tgeqtf eqorwvcvkp0 Kp CEO Ukirncp Pqkegu. xqn/ wog 67. rcigu 34463550 CEO. 42320
1. C0 D0 Eqorciqpqk cpf D0 E0 Rkgtcg0 Jki jgt/ qtfgt kpvgtugevkqp v{rgu cpf o wnvkrng kjpgtkvcpeg0 Ocvjgocvkecn Uvtwevwtgu kp Eqorwvgt Uekgpeg. 8\*7+68;6723. 3; ;80
1. Y0 T0 Eqqm. Y0 Jkn. cpf R0 U0 Ecppkpi0 Kjpgtkvcpeg ku pqv uwd/ v{rkpi0 Kp Rtqeggfkpiu qh vjg 39vj CEO UK IRNCP/UK ICEV uforukwo qp Rtkpekrngu qh rtqitco okpi ncpiwcigu. rcigu 34763570 CEO. 3; ;0
1. O0 Eqrrq. O0 Fg|cpk/Ekcpecinkpk. cpf D0 Xgppgk0 Hwpevkqpcn ejtcevgtu qh uqnxcdng vgtou0 Ocvjgocvkecn Nqike Swctvgtf. 49\*4/8+6767. 3; ;30
1. M0 Etct{0 Uko rng. ghkkekpv qdlgev gpeqfkpi wukpi kpvgtugevkqp v{rgu0 Vgejpkcn tgrqt. Eqtpgn Wpkxgtkv{. 3; ;0
1. T0 Fcxkgu0 Rtcevkecn tghkpgogpv v{rg ejgemkpi0 RjF vjguku. Wpkxgtkv{ qh Yguvtp Cwutcnk. 42270
1. T0 Fcxkgu cpf H0 Rhgppkpi0 Kpvgtugevkqp v{rgu cpf eqorwvc/ vkqpcn ghggev0 Kp CEO Ukirncp Pqkegu. xqnwog 57. rcigu 3; ;642:0 CEO. 42220
1. L0 Fwphkgnf0 Tghkpgf v{rgejgemkpi ykvj uvctfwuv0 Kp Rtqeggf/ kpiu qh vjg 4229 yqtmujqr qp Rtqitco okpi ncpiwcigu oggu rtqitco xgthkcevkqp. rcigu 436540 CEO. 42290
1. L0 Fwphkgnf0 Gncdqtcvkpi kpvgtugevkqp cpf wpkqp v{rgu0 Lqwtpcn qh Hwpevkqpcn Rtqitco okpi. 46\*4/5+3556387. 42360
1. G0 Gtpuv. M0 Quvgto ccpp. cpf Y0 T0 Eqqm0 C xktwcn encuu ecnewnwu0 QRN 4228. rcigu 49264:40
1. V0 Htggocp cpf H0 Rhgppkpi0 Tghkpgogpv v{rgu hqt ON. xqn/ wog 480 CEO. 3; ;30
1. C0 Htkuej. I0 Ecuvcipc. cpf X0 Dgp|cmgp0 Ugocpvke uwdv{rkpi< Fgcnkpi ug v{vjgqtkvkecn{ ykvj hwpevkqp. wpkqp. kpvgtugevkqp. cpf pgicvkqp v{rgu0 Lqwtpcn qh vjg CEO \*LCEO+. 77\*6+3; . 422:0
1. L0 Ictkiwg0 Rtqitco okpi ykvj rqn{oqtrjke xctkcpvu0 Kp ON Yqtmujqr. xqnwog 350 Dcnvko qtg. 3; ;0
1. F0 Ngklgp0 Gzvpgukdng tgeqtfu ykvj ueqrgf ncdgnu0 Vtgpfu kp Hwpevkqpcn Rtqitco okpi. 7<4;96534. 42270
1. D0 E0 f0 U0 Qnxgktp cpf Y0 T0 Eqqm0 Gzvpgukdknv{ hqt vjg ocuugu0 Kp GEQR 42346Qdlgev/Qtkgpvgf Rtqitco okpi. rcigu 46490 Urtkipgt. 42340
1. D0 E0 Rkgtcg0 Rtqitco okpi ykvj kpvgtugevkqp v{rgu. wpkqp v{rgu. cpf rqn{oqtrjkuo0 3; ;30
1. D0 E0 Rkgtcg0 Rtqitco okpi ykvj kpvgtugevkqp v{rgu cpf dqwpfgf rqn{oqtrjkuo0 RjF vjguku. Ectpgikg Ognmqp Wpk/ xgtkv{ Rkvudwtij. RC. 3; ;30
1. D0 E0 Rkgtcg0 Kpvgtugevkqp v{rgu cpf dqwpfgf rqn{oqtrjkuo0 Ocvjgocvkecn Uvtwevwtgu kp Eqorwvgt Uekgpeg. 9\*24+34;63;5. 3; ;90
1. D0 E0 Rkgtcg0 V{rgu cpf rtqitco okpi ncpiwcigu0 OKV rtguu. 42240
1. L0 E0 Tg{pqnfu0 Fgukip qh vjg rtqitco okpi ncpiwcig Hqtu/vjg0 Urtkipgt. 3; ;90
1. O0 Ycpf0 Eqorngv v{rg kphgtgpeg hqt uko rng qdlgevu0 Kp NKEU. xqnwog :9. rcigu 59666. 3; ;90
1. O0 Ycpf0 V{rg kphgtgpeg hqt tgeqtf eqpecvpcvkqp cpf o wnvk/ rng kjpgtkvcpeg0 Kp Nqike kp Eqorwvgt Uekgpeg. 3; ;0 NKEU0: ;. Rtqeggfkpiu0. Hqwtvj Cppwcn Uforukwo qp. rcigu ;46;90 KGGG. 3; ;0