

$$\begin{array}{c}
\boxed{\tau \perp \tau} \quad \frac{\alpha_1 \neq \alpha_2}{\alpha_1 \perp \alpha_2} \text{orthogvar} \quad \frac{\tau_1 \perp \tau_3 \quad \tau_2 \perp \tau_4}{\tau_1 \rightarrow \tau_2 \perp \tau_3 \rightarrow \tau_4} \text{orthogfun} \quad \frac{\tau_1 \perp [\alpha_1/\alpha_2]\tau_2}{\forall \alpha_1. \tau_1 \perp \forall \alpha_2. \tau_2} \text{orthogforall} \\
\\
\frac{\tau_1 \perp \tau_3 \quad \tau_2 \perp \tau_3}{\tau_1 \& \tau_2 \perp \tau_3} \text{orthogand}_1 \quad \frac{\tau_1 \perp \tau_2 \quad \tau_1 \perp \tau_3}{\tau_1 \perp \tau_2 \& \tau_3} \text{orthogand}_2 \quad \frac{l_1 \neq l_2}{\{l_1:\tau_1\} \perp \{l_2:\tau_2\}} \text{orthogreclab} \\
\\
\frac{\tau_1 \perp \tau_2}{\{l:\tau_1\} \perp \{l:\tau_2\}} \text{orthogrec}
\end{array}$$

Figure 1. Orthogonality between types.

$$\begin{array}{c}
\boxed{\gamma \vdash \tau} \quad \frac{\alpha \in \gamma}{\gamma \vdash \alpha} \text{wfvar} \quad \frac{}{\gamma \vdash \top} \text{wftop} \quad \frac{\gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2}{\gamma \vdash \tau_1 \rightarrow \tau_2} \text{wffun} \quad \frac{\gamma, \alpha \vdash \tau}{\gamma \vdash \forall \alpha. \tau} \text{wffforall} \\
\\
\frac{\gamma \vdash \tau_1 \quad \Gamma \vdash \tau_2 \quad \tau_1 \perp \tau_2}{\gamma \vdash \tau_1 \& \tau_2} \text{wfand} \quad \frac{\gamma \vdash \tau}{\gamma \vdash \{l:\tau\}} \text{wffrec}
\end{array}$$

Figure 2. Well-formedness of types.

$$\boxed{|\tau| = \top}$$

$$\begin{aligned}
|\alpha| &= \alpha \\
|\top| &= () \\
|\tau_1| \rightarrow |\tau_2| &= |\tau_1| \rightarrow |\tau_2| \\
|\forall \alpha. \tau| &= \forall \alpha. |\tau| \\
|\tau_1 \& \tau_2| &= (|\tau_1|, |\tau_2|) \\
|\{l:\tau\}| &= |\tau|
\end{aligned}$$

Figure 3. Type translation.

$$\begin{array}{c}
\boxed{\tau <: \tau \hookrightarrow C} \quad \frac{}{\alpha <: \alpha \hookrightarrow \lambda(x:|\alpha|). x} \text{subvar} \quad \frac{}{\tau <: \top \hookrightarrow \lambda(x:|\tau|). ()} \text{subtop} \\
\\
\frac{\tau_3 <: \tau_1 \hookrightarrow C_1 \quad \tau_2 <: \tau_4 \hookrightarrow C_2}{\tau_1 \rightarrow \tau_2 <: \tau_3 \rightarrow \tau_4 \hookrightarrow \lambda(f:|\tau_1 \rightarrow \tau_2|). \lambda(x:|\tau_3|). C_2 (f (C_1 x)))} \text{subfun} \\
\\
\frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2 \hookrightarrow C}{\forall \alpha_1. \tau_1 <: \forall \alpha_2. \tau_2 \hookrightarrow \lambda(f:|\forall \alpha_1. \tau_1|). \Lambda \alpha. C (f \alpha)} \text{subforall} \quad \frac{\tau_1 <: \tau_2 \hookrightarrow C_1 \quad \tau_1 <: \tau_3 \hookrightarrow C_2}{\tau_1 <: \tau_2 \& \tau_3 \hookrightarrow \lambda(x:|\tau_1|). (C_1 x, C_2 x)} \text{suband} \\
\\
\frac{\tau_1 <: \tau_3 \hookrightarrow C}{\tau_1 \& \tau_2 <: \tau_3 \hookrightarrow \lambda(x:|\tau_1 \& \tau_2|). C (\text{proj}_1 x)} \text{suband}_1 \quad \frac{\tau_2 <: \tau_3 \hookrightarrow C}{\tau_1 \& \tau_2 <: \tau_3 \hookrightarrow \lambda(x:|\tau_1 \& \tau_2|). C (\text{proj}_2 x)} \text{suband}_2 \\
\\
\frac{\tau_1 <: \tau_2 \hookrightarrow C}{\{l:\tau_1\} <: \{l:\tau_2\} \hookrightarrow \lambda(x:|\{l:\tau_1\}|). C x} \text{subrec}
\end{array}$$

Figure 4. Elaboration subtyping.

$$\boxed{\gamma \vdash e : \tau \hookrightarrow E} \quad \frac{(x, \tau) \in \gamma}{\gamma \vdash x : \tau \hookrightarrow x} \text{Evar} \quad \frac{}{\gamma \vdash \top : \top \hookrightarrow ()} \text{Etop} \quad \frac{\gamma, x : \tau \vdash e : \tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash \lambda(x : \tau). e : \tau \rightarrow \tau_1 \hookrightarrow \lambda(x : |\tau|). E} \text{Elam}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_3 \hookrightarrow E_2 \quad \tau_3 <: \tau_1 \hookrightarrow C}{\gamma \vdash e_1 \ e_2 : \tau_2 \hookrightarrow E_1 \ (C \ E_2)} \text{Eapp} \quad \frac{\gamma, \alpha \vdash e : \tau \hookrightarrow E}{\gamma \vdash \Lambda \alpha. e : \forall \alpha. \tau \hookrightarrow \Lambda \alpha. E} \text{Eblam}$$

$$\frac{\gamma \vdash e : \forall \alpha. \tau_1 \hookrightarrow E \quad \gamma \vdash \tau}{\gamma \vdash e \ \tau : [\tau/\alpha] \tau_1 \hookrightarrow E \ |\tau|} \text{Etapp} \quad \frac{\gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \quad \gamma \vdash e_2 : \tau_2 \hookrightarrow E_2 \quad \tau_1 \perp \tau_2}{\gamma \vdash e_1, , e_2 : \tau_1 \ \& \ \tau_2 \hookrightarrow (E_1, E_2)} \text{Emerge}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E}{\gamma \vdash \{l = e\} : \{l : \tau\} \hookrightarrow E} \text{Erec-construct} \quad \frac{\gamma \vdash e : \tau \hookrightarrow E \quad \tau \bullet l = \tau_1 \hookrightarrow C}{\gamma \vdash e.l : \tau_1 \hookrightarrow C \ E} \text{Erec-select}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E \quad \tau \setminus l = \tau_1 \hookrightarrow C}{\gamma \vdash e \setminus l : \tau_1 \hookrightarrow C \ E} \text{Erec-restrict}$$

$$\boxed{\tau_1 \bullet l = \tau_2 \hookrightarrow C} \quad \frac{}{\{l : \tau\} \bullet l = \tau \hookrightarrow \lambda(x : |\{l : \tau\}|). x} \text{select} \quad \frac{\tau_1 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_1 x)} \text{select}_1$$

$$\frac{\tau_2 \bullet l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \bullet l = \tau \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). C \ (\text{proj}_2 x)} \text{select}_2$$

$$\boxed{\tau_1 \setminus l = \tau_2 \hookrightarrow C} \quad \frac{}{\{l : \tau\} \setminus l = \top \hookrightarrow \lambda(x : |\{l : \tau\}|). ()} \text{restrict}$$

$$\frac{\tau_1 \setminus l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \setminus l = \tau \ \& \ \tau_2 \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). (C \ (\text{proj}_1 x), \text{proj}_2 x)} \text{restrict}_1$$

$$\frac{\tau_2 \setminus l = \tau \hookrightarrow C}{\tau_1 \ \& \ \tau_2 \setminus l = \tau_1 \ \& \ \tau \hookrightarrow \lambda(x : |\tau_1 \ \& \ \tau_2|). (\text{proj}_1 x, C \ (\text{proj}_2 x))} \text{restrict}_2$$

Figure 5. Elaboration typing.