$$\boxed{\tau \perp \tau} \qquad \frac{\alpha_1 \neq \alpha_2}{\alpha_1 \perp \alpha_2}\ \textsc{Orth/Var} \qquad \frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_4}{\tau_1 \to \tau_2 \perp \tau_3 \to \tau_4}\ \textsc{Orth/Fun} \qquad \frac{\tau_1 \perp [\alpha_1/\alpha_2]\tau_2}{\forall \alpha_1 .\, \tau_1 \perp \forall \alpha_2 .\, \tau_2}\ \textsc{Orth/Forall}$$

$$\frac{\tau_1 \perp \tau_3 \qquad \tau_2 \perp \tau_3}{\tau_1 \& \tau_2 \perp \tau_3}\ \textsc{Orth/And1} \qquad \frac{\tau_1 \perp \tau_2 \qquad \tau_1 \perp \tau_3}{\tau_1 \perp \tau_2 \& \tau_3}\ \textsc{Orth/And2} \qquad \frac{l_1 \neq l_2}{\{l_1 : \tau_1\} \perp \{l_2 : \tau_2\}}\ \textsc{Orth/RecLab}$$

$$\frac{\tau_1 \perp \tau_2}{\{l : \tau_1\} \perp \{l : \tau_2\}}\ \textsc{Orth/Rec}$$

Figure 1. Orthogonality between types.

$$\boxed{\gamma \tau} \qquad \frac{\alpha \in \gamma}{\gamma \alpha}\ \color{red}{wf}\textsc{var} \qquad \frac{}{\gamma \top}\ \color{red}{wf}\textsc{top} \qquad \frac{\gamma \tau_1 \qquad \Gamma \tau_2}{\gamma \tau_1 \to \tau_2}\ \color{red}{wf}\textsc{fun} \qquad \frac{\gamma, \alpha \tau}{\gamma \forall \alpha .\, \tau}\ \color{red}{wf}\textsc{forall} \qquad \frac{\gamma \tau_1 \qquad \Gamma \tau_2 \qquad \tau_1 \perp \tau_2}{\gamma \tau_1 \& \tau_2}\ \color{red}{wf}\textsc{and}$$

$$\frac{\gamma \tau}{\gamma \{l : \tau\}}\ \color{red}{wf}\textsc{rec}$$

Figure 2. Well-formedness of types.

$$\text{Values} \quad v \quad ::= \quad \top \mid \lambda(x{:}\tau).\, e \mid \Lambda \alpha . e \mid v_1 ,, v_2 \mid \{l = e\}$$

Figure 3. Values.

$$\frac{\tau_1 <: \tau}{(\tau)(v : \tau_1) \hookrightarrow v}\ \textsc{Cast/UpCast} \qquad \frac{(\tau)(v_1 : \tau_1) \hookrightarrow v}{(\tau)(v_1 ,, v_2 : \tau_1 \& \tau_2) \hookrightarrow v}\ \textsc{Cast/TakeLeft} \qquad \frac{(\tau)(v_2 : \tau_2) \hookrightarrow v}{(\tau)(v_1 ,, v_2 : \tau_1 \& \tau_2) \hookrightarrow v}\ \textsc{Cast/TakeRight}$$

Figure 4. Casts.

$$\frac{}{v \Downarrow v}\ \textsc{Dyn/Val} \qquad \frac{e_1 \Downarrow \lambda(x{:}\tau).\, e \qquad e_2 \Downarrow v_2 \qquad (\tau)(v_2 : \tau_2) \hookrightarrow v_3 \qquad [v_3/x]e \Downarrow v}{e_1\ (e_2 : \tau_2) \Downarrow v}\ \textsc{Dyn/App}$$

$$\frac{e_1 \Downarrow \forall \alpha .\, e \qquad [\tau/\alpha]e \Downarrow v}{e_1\ \tau \Downarrow v}\ \textsc{Dyn/TApp} \qquad \frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2}{e_1 ,, e_2 \Downarrow v_1 ,, v_2}\ \textsc{Dyn/Merge}$$

Figure 5. Dynamic semantics.

$$\boxed{|\tau| = \mathsf{T}}$$

$$\begin{aligned} |\alpha| &= \alpha \\ |\top| &= () \\ |\tau_1| \to |\tau_2| &= |\tau_1| \to |\tau_2| \\ |\forall \alpha .\, \tau| &= \forall \alpha .\, |\tau| \\ |\tau_1 \& \tau_2| &= (|\tau_1|, |\tau_2|) \\ |\{l : \tau\}| &= |\tau| \end{aligned}$$

Figure 6. Type translation.

$$\boxed{\tau <: \tau \hookrightarrow C} \qquad \frac{}{\alpha <: \alpha \hookrightarrow \lambda(x{:}|\alpha|).\,x} \qquad \frac{}{\tau <: \top \hookrightarrow \lambda(x{:}|\tau|).\,()}$$

$$\frac{\tau_3 <: \tau_1 \hookrightarrow C_1 \qquad \tau_2 <: \tau_4 \hookrightarrow C_2}{\tau_1 \to \tau_2 <: \tau_3 \to \tau_4 \hookrightarrow \lambda(f{:}|\tau_1 \to \tau_2|).\,\lambda(x{:}|\tau_3|).\,C_2\ (f\ (C_1\ x))} \qquad \frac{\tau_1 <: [\alpha_1/\alpha_2]\tau_2 \hookrightarrow C}{\forall\alpha_1.\,\tau_1 <: \forall\alpha_2.\,\tau_2 \hookrightarrow \lambda(f{:}|\forall\alpha_1.\,\tau_1|).\,\Lambda\alpha.C\ (f\ \alpha)}$$

$$\frac{\tau_1 <: \tau_2 \hookrightarrow C_1 \qquad \tau_1 <: \tau_3 \hookrightarrow C_2}{\tau_1 <: \tau_2\&\tau_3 \hookrightarrow \lambda(x{:}|\tau_1|).\,(C_1\ x, C_2\ x)} \qquad \frac{\tau_1 <: \tau_3 \hookrightarrow C}{\tau_1\&\tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1\&\tau_2|).\,C\ (\mathtt{proj}_1 x)}$$

$$\frac{\tau_2 <: \tau_3 \hookrightarrow C}{\tau_1\&\tau_2 <: \tau_3 \hookrightarrow \lambda(x{:}|\tau_1\&\tau_2|).\,C\ (\mathtt{proj}_2 x)} \qquad \frac{\tau_1 <: \tau_2 \hookrightarrow C}{\{l : \tau_1\} <: \{l : \tau_2\} \hookrightarrow \lambda(x{:}|\{l : \tau_1\}|).\,C\ x}$$

Figure 7. Elaboration subtyping.

$$\boxed{\gamma \vdash e : \tau \hookrightarrow E} \qquad \frac{(x, \tau) \in \gamma}{\gamma \vdash x : \tau \hookrightarrow x}\ \textsc{S/var} \qquad \frac{}{\gamma \vdash \top : \top \hookrightarrow ()}\ \textsc{S/top} \qquad \frac{\gamma, x{:}\tau \vdash e : \tau_1 \hookrightarrow E \qquad \gamma\tau}{\gamma \vdash \lambda(x{:}\tau).\,e : \tau \to \tau_1 \hookrightarrow \lambda(x{:}|\tau|).\,E}\ \textsc{S/lam}$$

$$\frac{\gamma \vdash e_1 : \tau_1 \to \tau_2 \hookrightarrow E_1 \qquad \gamma \vdash e_2 : \tau_3 \hookrightarrow E_2 \qquad \tau_3 <: \tau_1 \hookrightarrow C}{\gamma \vdash e_1\ e_2 : \tau_2 \hookrightarrow E_1\ (C\ E_2)}\ \textsc{S/app} \qquad \frac{\gamma, \alpha \vdash e : \tau \hookrightarrow E}{\gamma \vdash \Lambda\alpha.e : \forall\alpha.\,\tau \hookrightarrow \Lambda\alpha.E}\ \textsc{S/blam}$$

$$\frac{\gamma \vdash e : \forall\alpha.\,\tau_1 \hookrightarrow E \qquad \gamma\tau}{\gamma \vdash e\ \tau : [\tau/\alpha]\tau_1 \hookrightarrow E\ |\tau|}\ \textsc{S/tapp} \qquad \frac{\gamma \vdash e_1 : \tau_1 \hookrightarrow E_1 \qquad \gamma \vdash e_2 : \tau_2 \hookrightarrow E_2 \qquad \tau_1 \perp \tau_2}{\gamma \vdash e_1,, e_2 : \tau_1\&\tau_2 \hookrightarrow (E_1, E_2)}\ \textsc{S/merge}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E}{\gamma \vdash \{l = e\} : \{l : \tau\} \hookrightarrow E}\ \textsc{S/rec-construct} \qquad \frac{\gamma \vdash e : \tau \hookrightarrow E \qquad \tau \bullet l = \tau_1 \hookrightarrow C}{\gamma \vdash e.l : \tau_1 \hookrightarrow C\ E}\ \textsc{S/rec-select}$$

$$\frac{\gamma \vdash e : \tau \hookrightarrow E \qquad \tau \setminus l = \tau_1 \hookrightarrow C}{\gamma \vdash e \setminus l : \tau_1 \hookrightarrow C\ E}\ \textsc{S/rec-restrict}$$

$$\boxed{\tau_1 \bullet l = \tau_2 \hookrightarrow C} \qquad \frac{}{\{l : \tau\} \bullet l = \tau \hookrightarrow \lambda(x{:}|\{l : \tau\}|).\,x}\ \mathtt{select} \qquad \frac{\tau_1 \bullet l = \tau \hookrightarrow C}{\tau_1\&\tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1\&\tau_2|).\,C\ (\mathtt{proj}_1 x)}\ \mathtt{select}_1$$

$$\frac{\tau_2 \bullet l = \tau \hookrightarrow C}{\tau_1\&\tau_2 \bullet l = \tau \hookrightarrow \lambda(x{:}|\tau_1\&\tau_2|).\,C\ (\mathtt{proj}_2 x)}\ \mathtt{select}_2$$

$$\boxed{\tau_1 \setminus l = \tau_2 \hookrightarrow C} \qquad \frac{}{\{l : \tau\} \setminus l = \top \hookrightarrow \lambda(x{:}|\{l : \tau\}|).\,()}\ \mathtt{restrict}$$

$$\frac{\tau_1 \setminus l = \tau \hookrightarrow C}{\tau_1\&\tau_2 \setminus l = \tau\&\tau_2 \hookrightarrow \lambda(x{:}|\tau_1\&\tau_2|).\,(C\ (\mathtt{proj}_1 x), \mathtt{proj}_2 x)}\ \mathtt{restrict}_1$$

$$\frac{\tau_2 \setminus l = \tau \hookrightarrow C}{\tau_1\&\tau_2 \setminus l = \tau_1\&\tau \hookrightarrow \lambda(x{:}|\tau_1\&\tau_2|).\,(\mathtt{proj}_1 x, C\ (\mathtt{proj}_2 x))}\ \mathtt{restrict}_2$$

Figure 8. Elaboration typing.