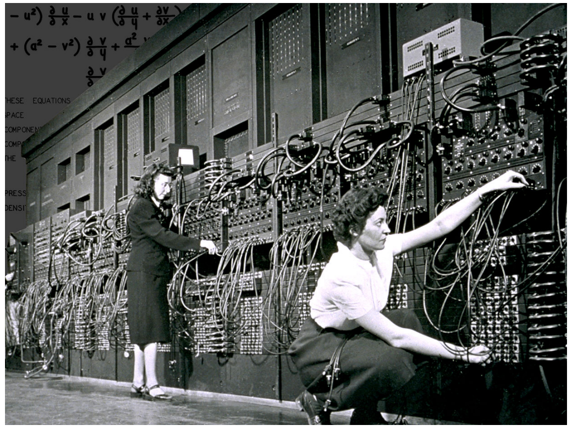
Staging with Class

A Specification of Typed Template Haskell

Ningning Xie

UNIVERSITY OF CAMBRIDGE

2021.12.17 EPFL

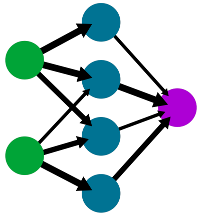


ENIAC

1945

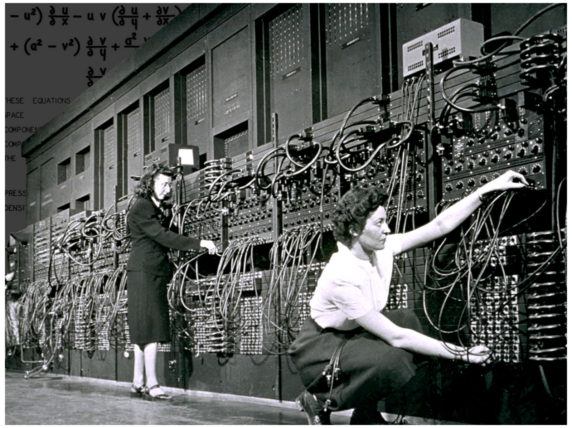
today

web development



machine learning

app development



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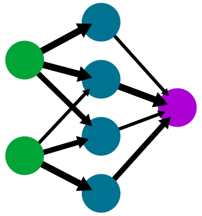
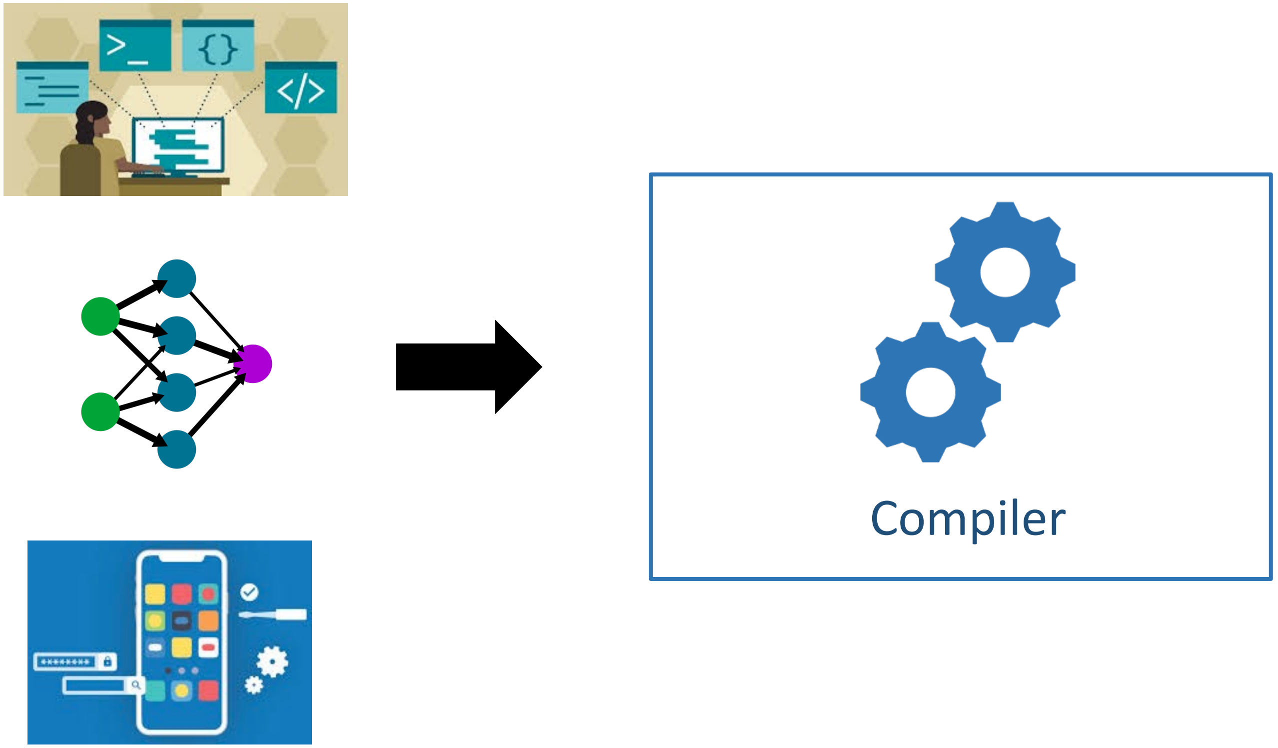
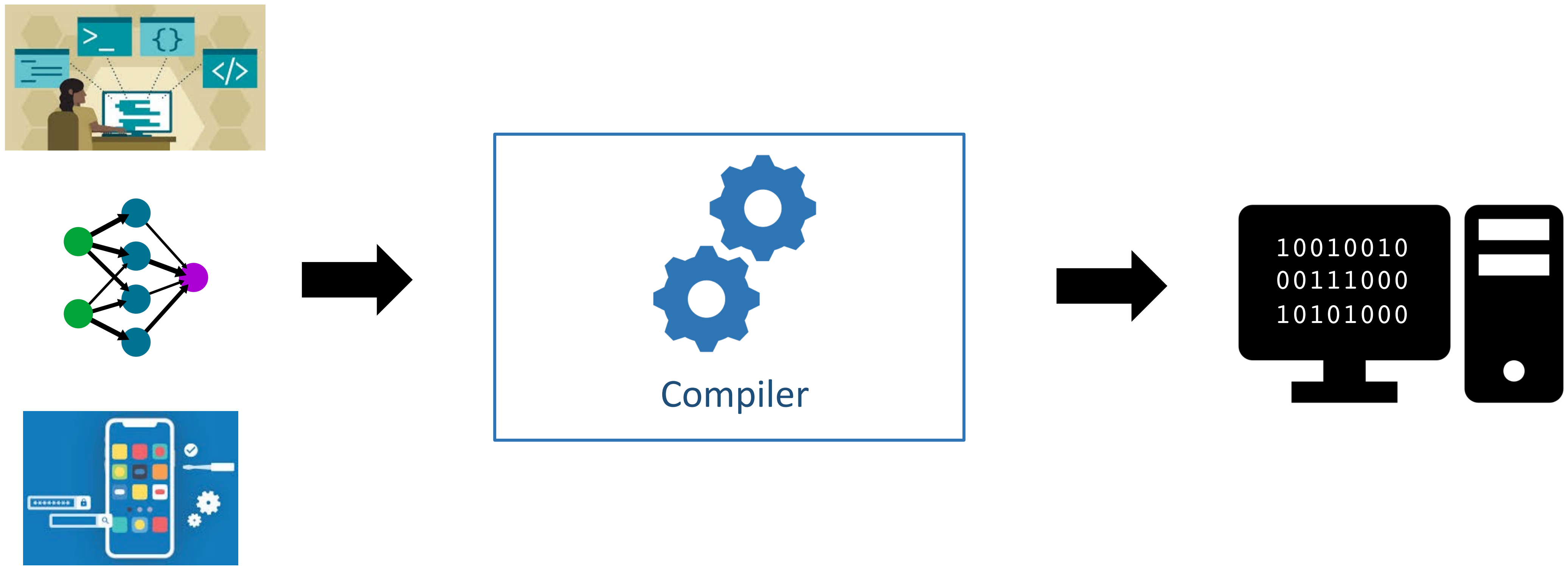
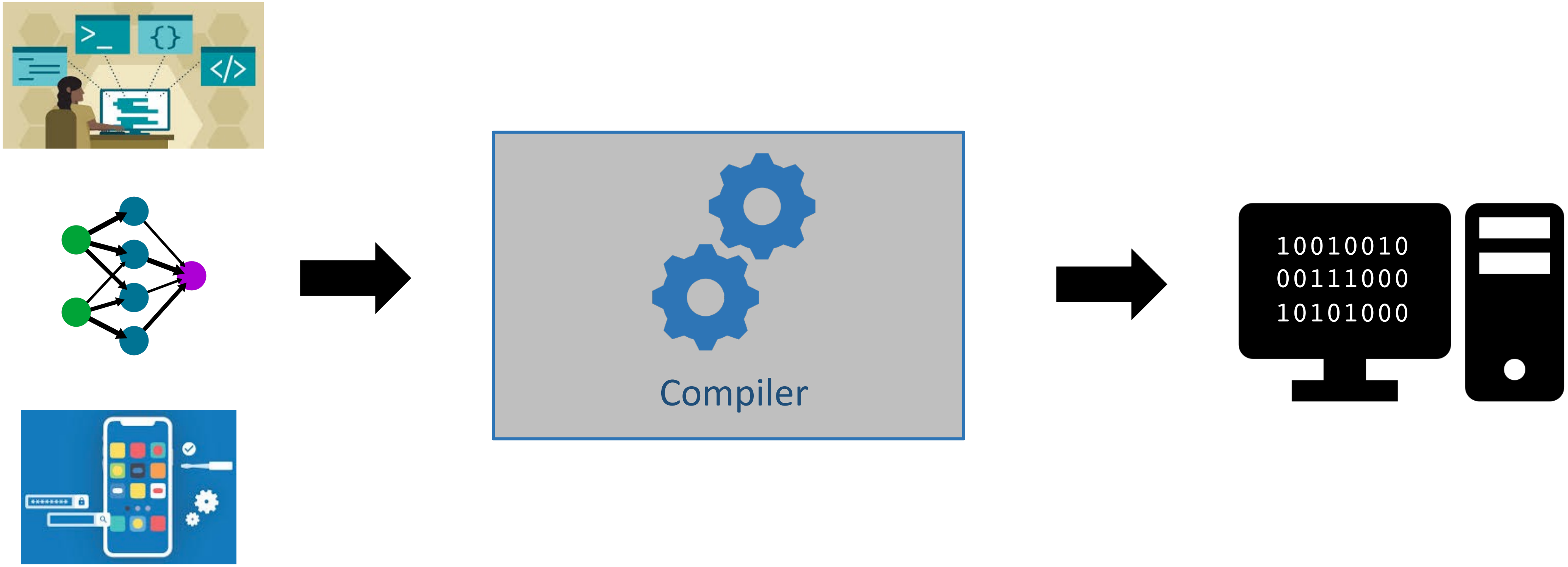



Compiler


Compiler
10010010
00111000
10101000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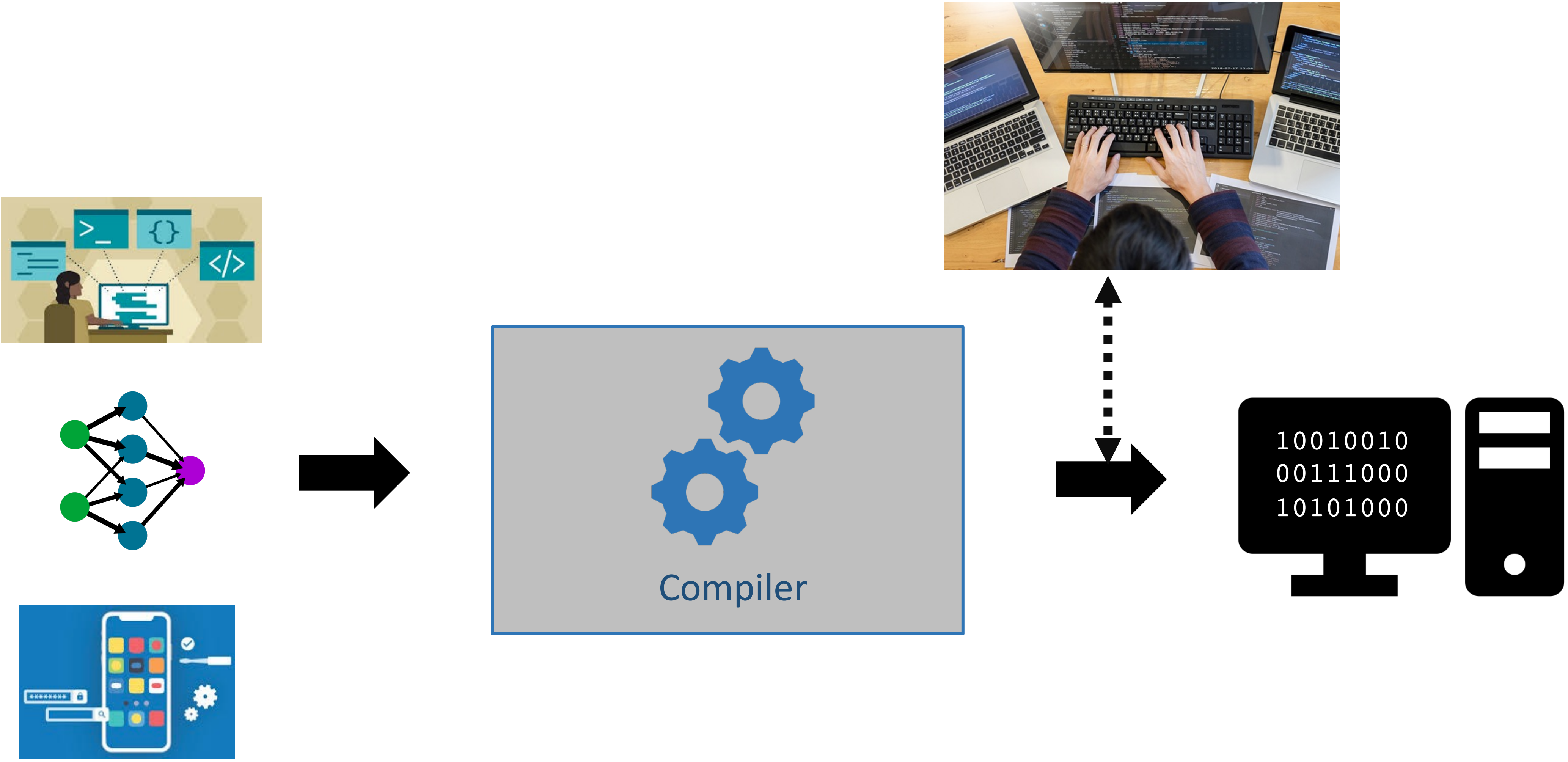
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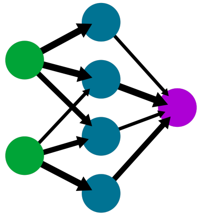
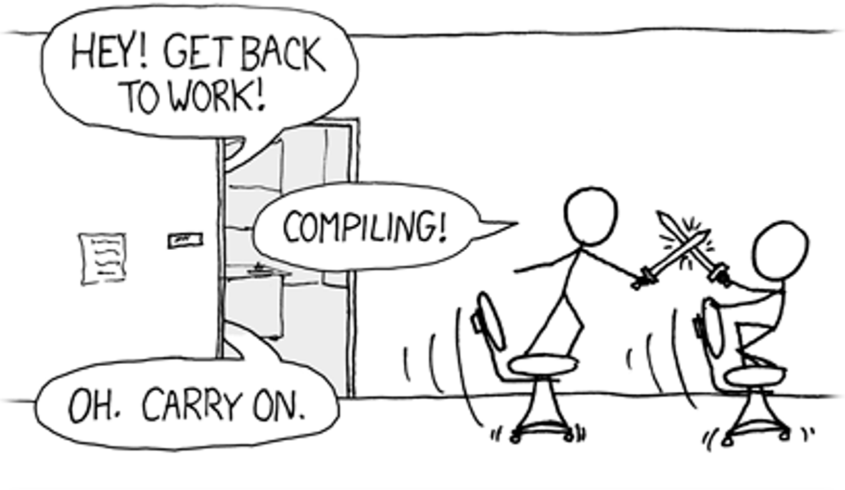
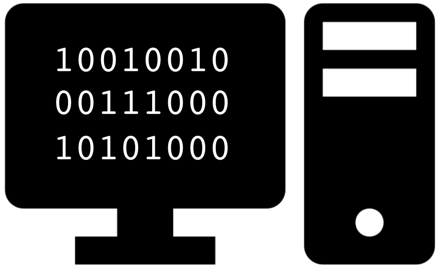



HEY! GET BACK TO WORK!
COMPILING!
OH. CARRY ON.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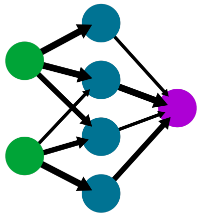
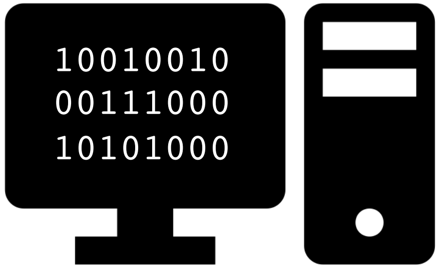
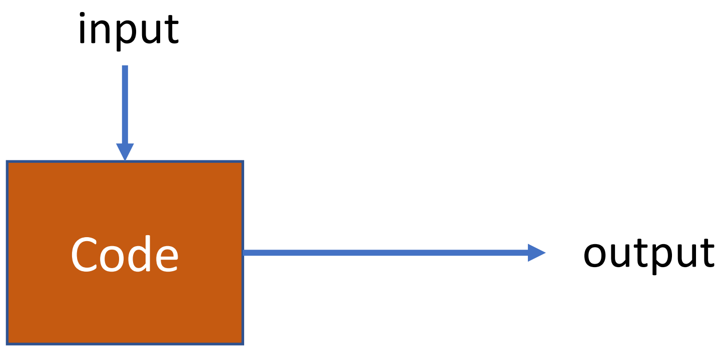


How can we write high-level programs with predictable low-level efficiency?



Multi-stage programming



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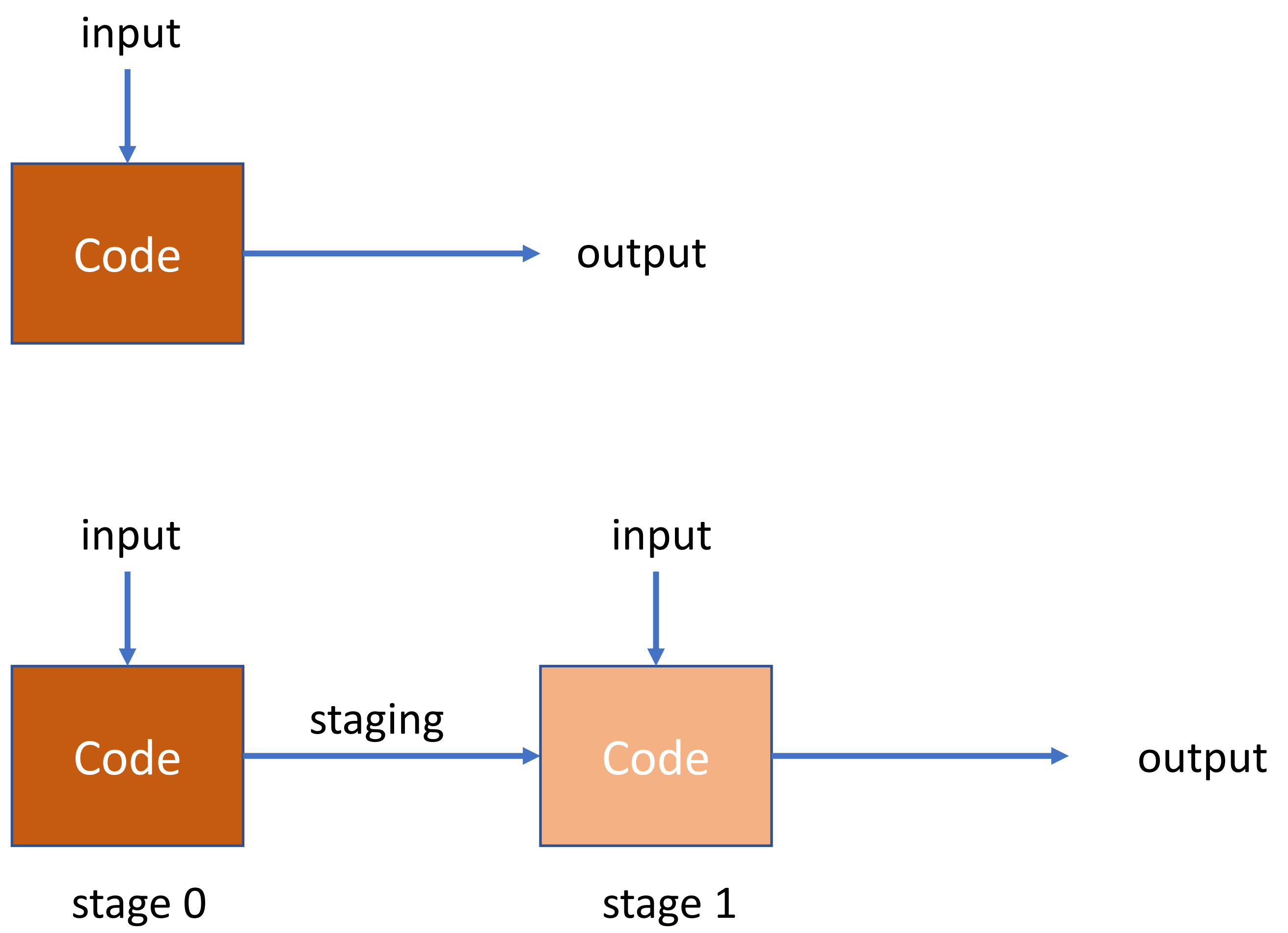

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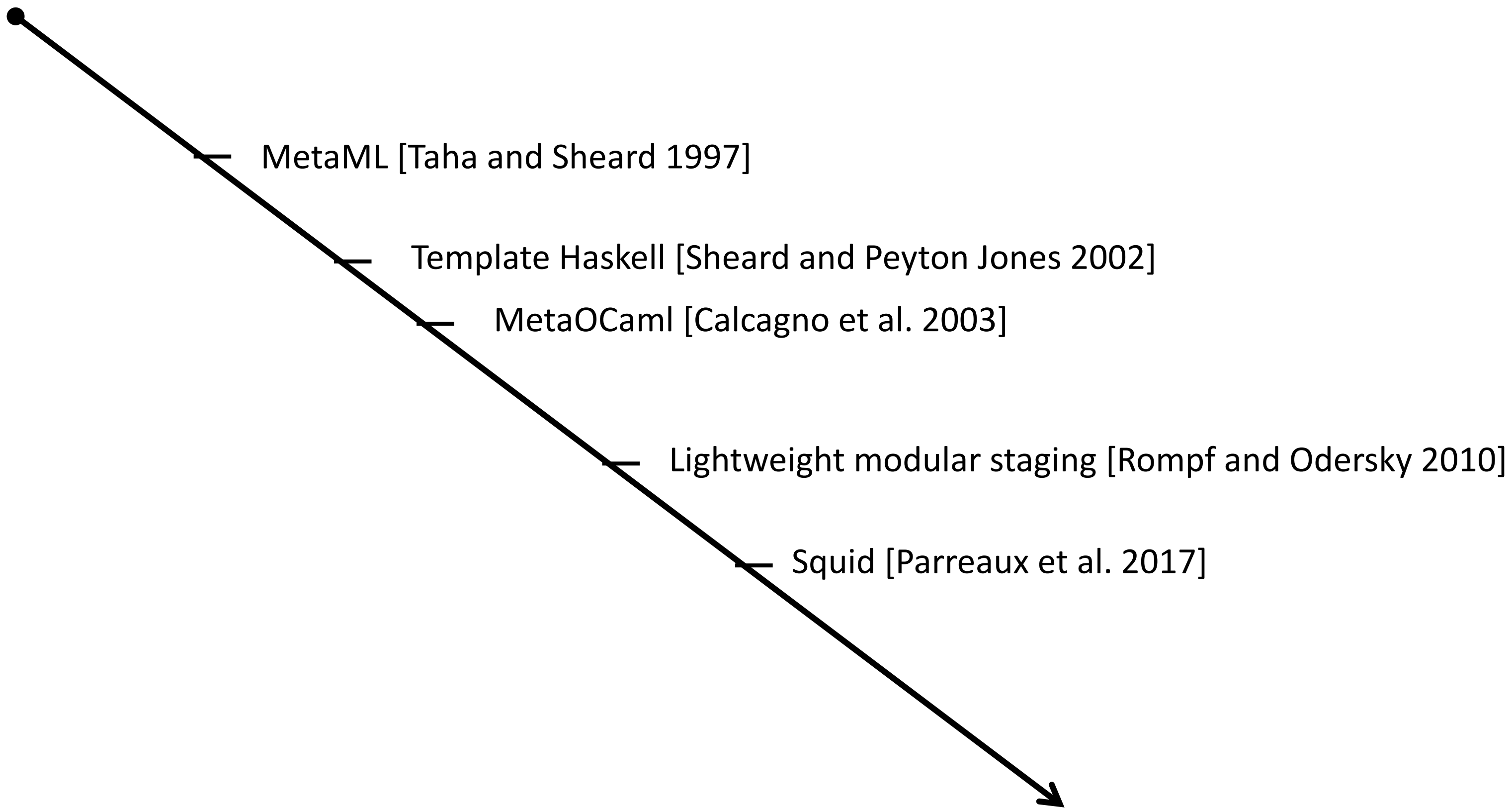

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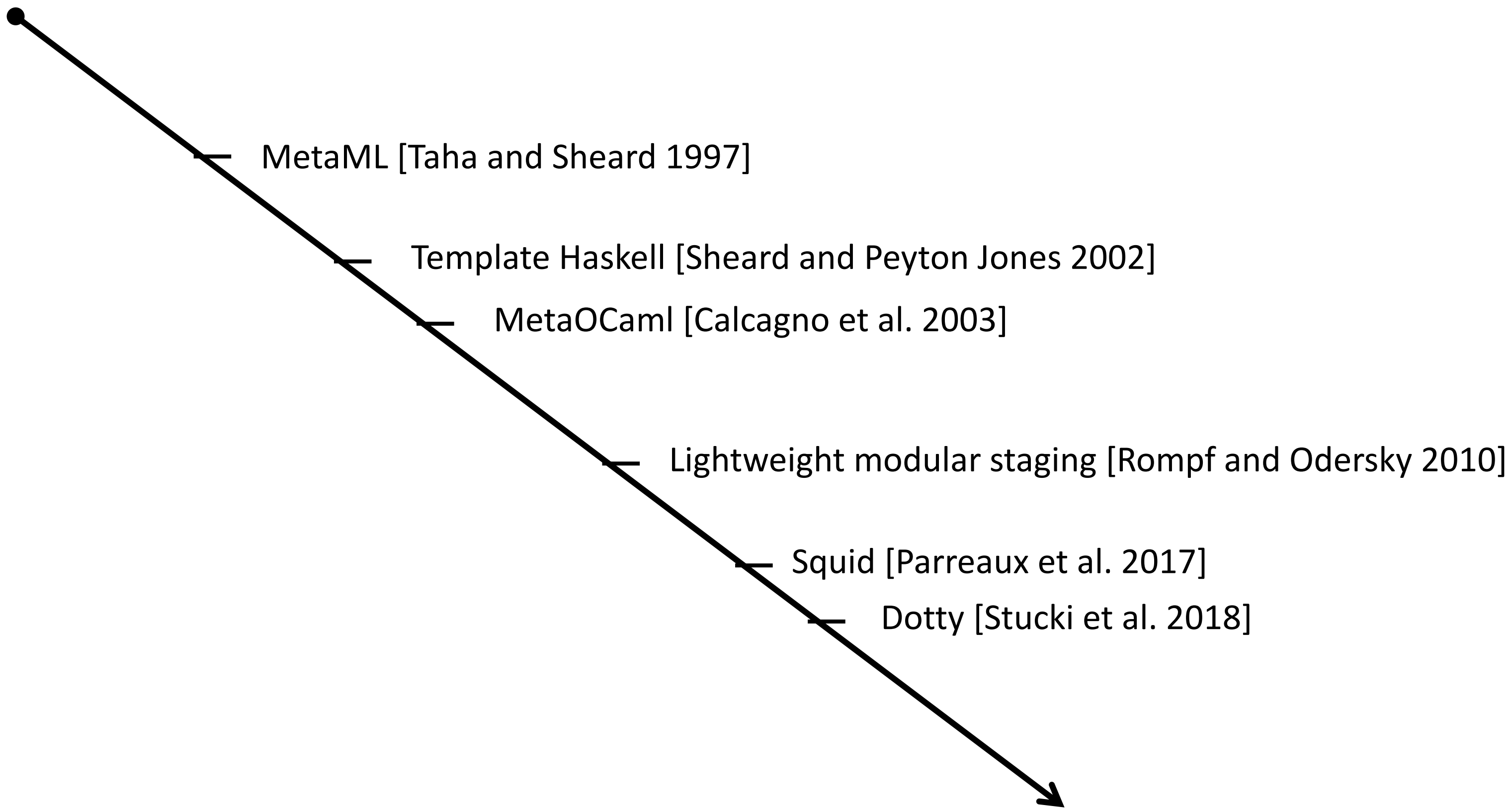

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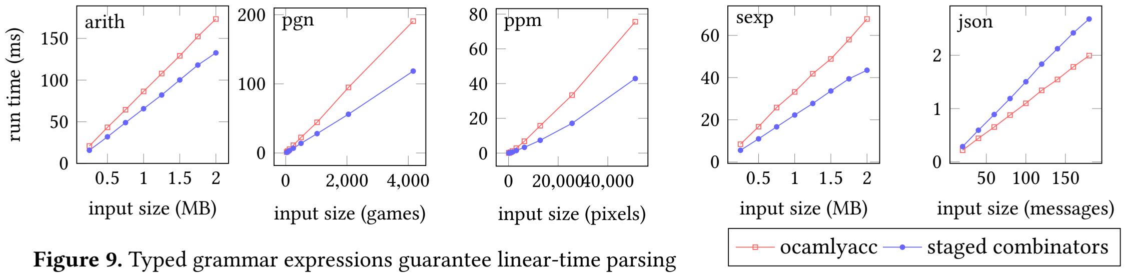

Figure 9. Typed grammar expressions guarantee linear-time parsing

[Krishnaswami and Yallop 2019]

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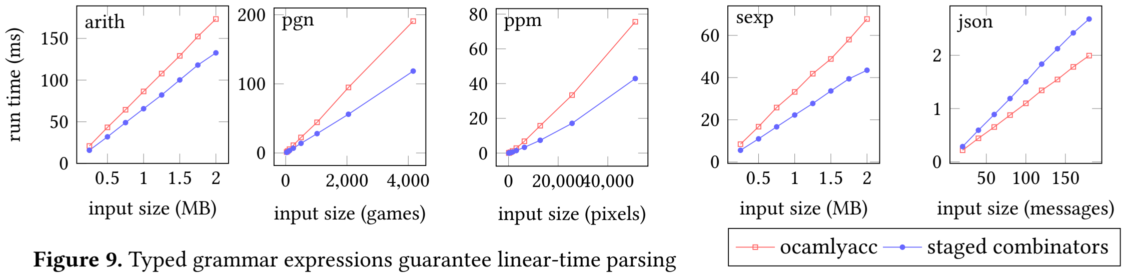

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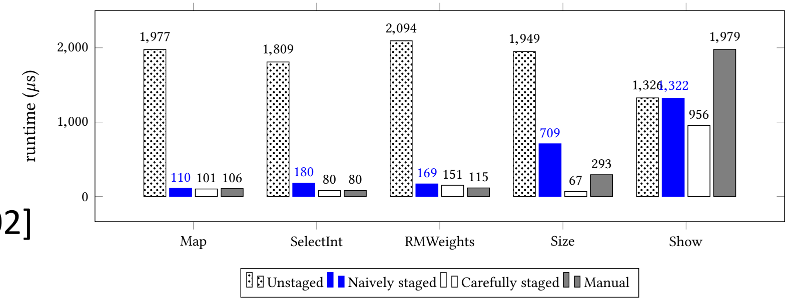

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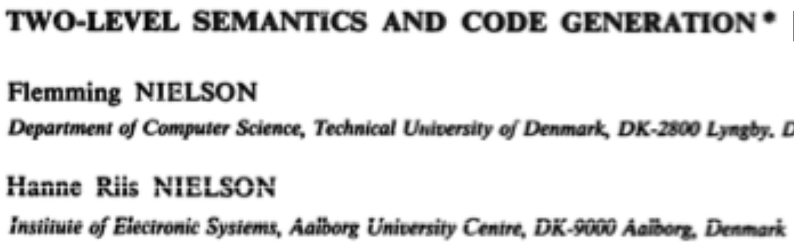
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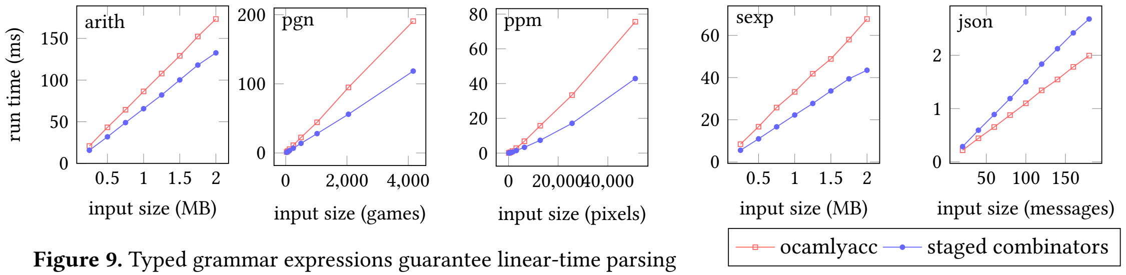

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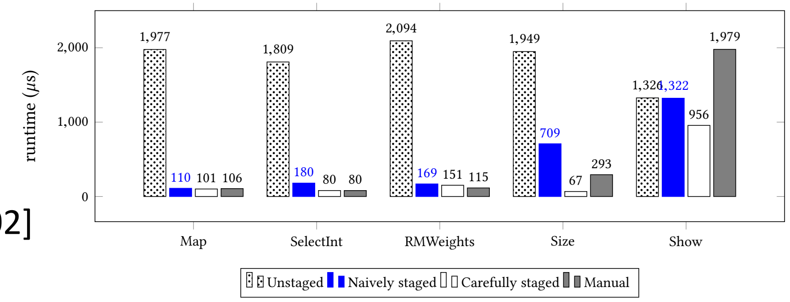

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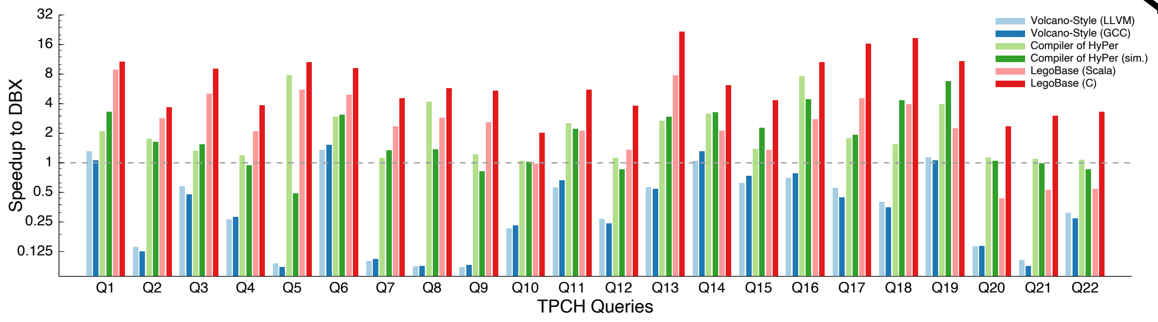

Figure 11: Performance comparison of LegoBase (C and Scala programs) with the code generated by the query compiler of [15].

[Klonatos et al. 2014]

Quotations and splices

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`Code`: program fragment in a future stage

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a representation of the expression as program fragment in a future stage

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a representation of the expression as program fragment in a future stage

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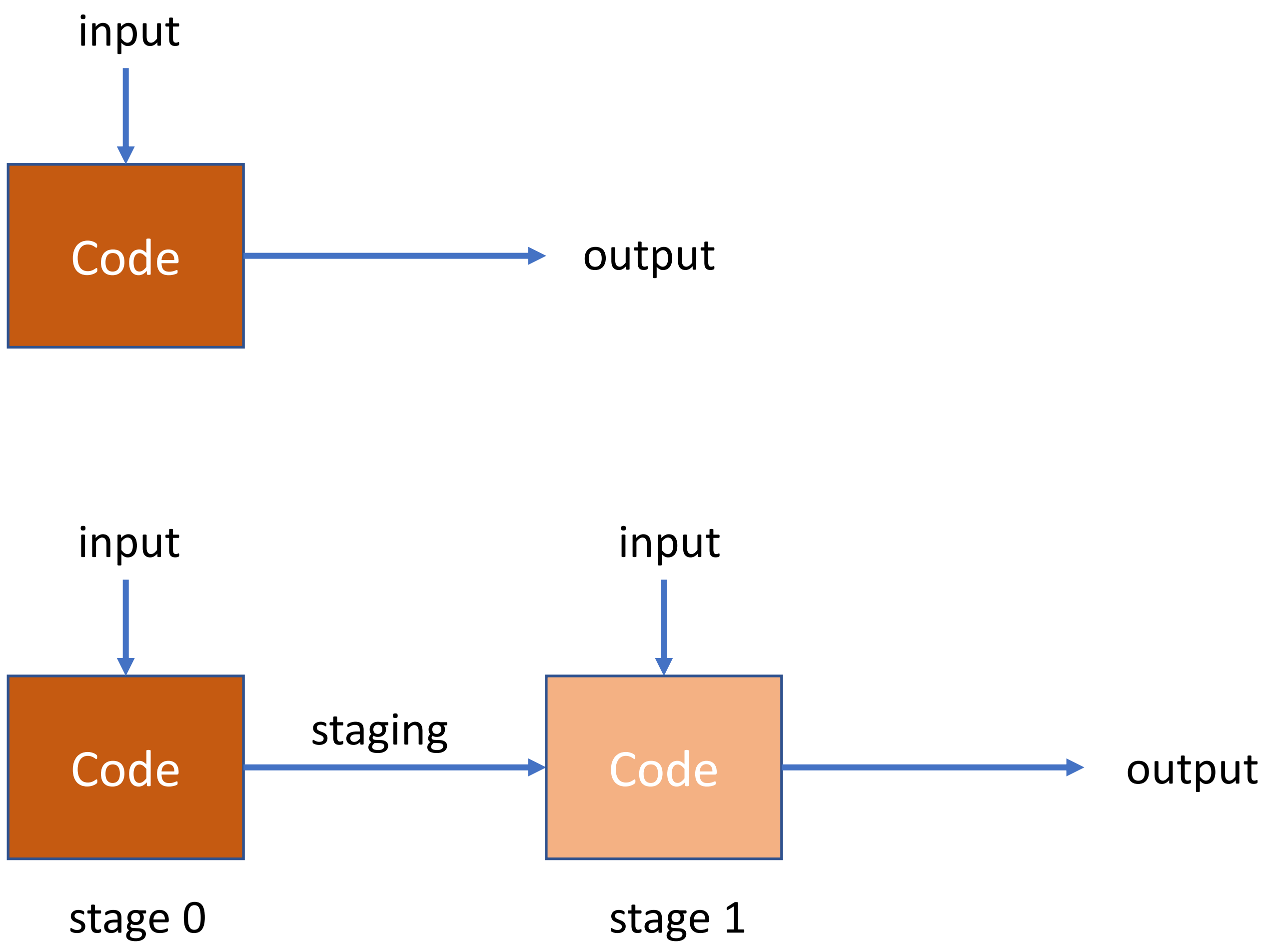
Splice

extracts the expression from its representation

`e :: Code Int   ⟹   $e :: Int`

Multi-stage programming: example



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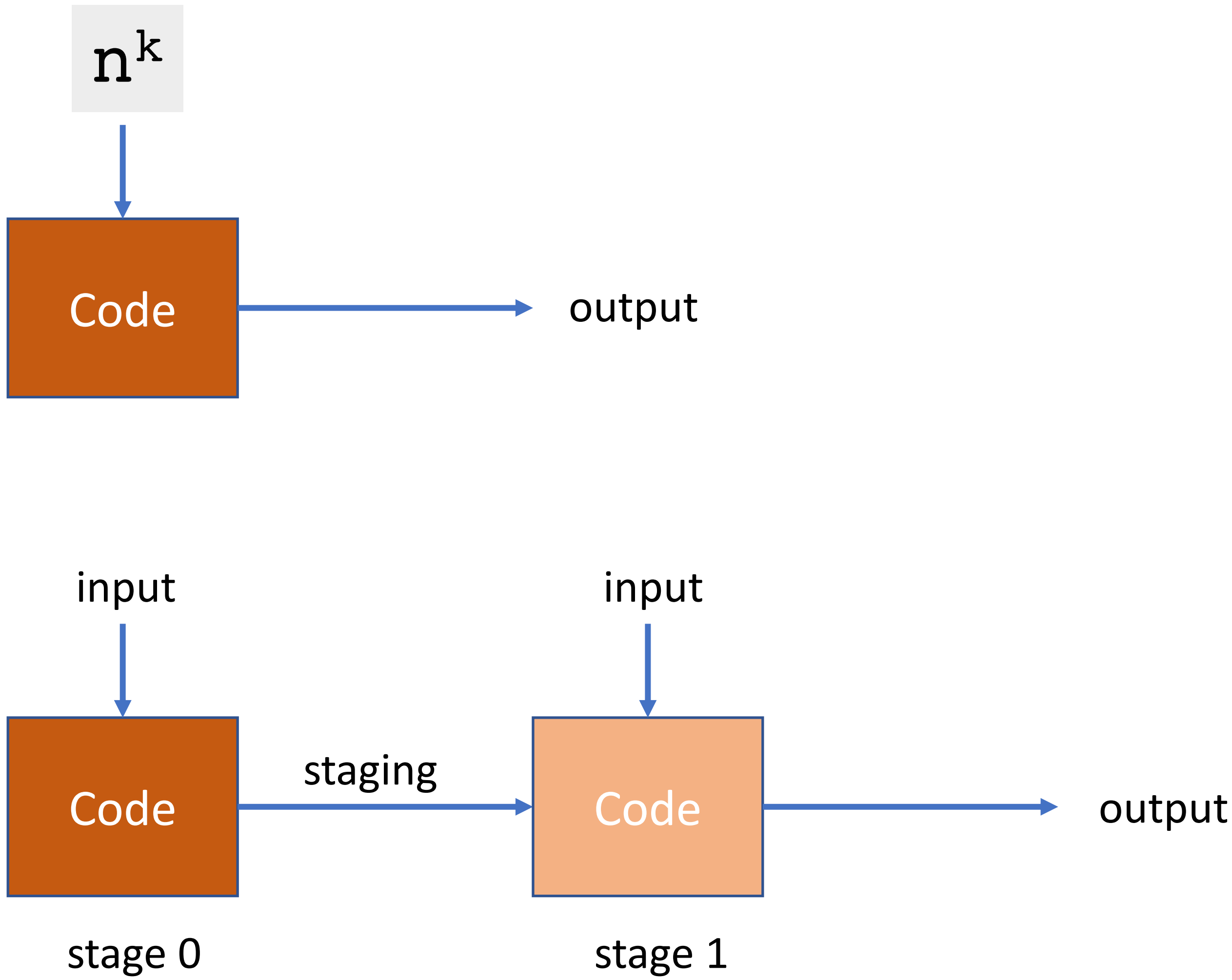

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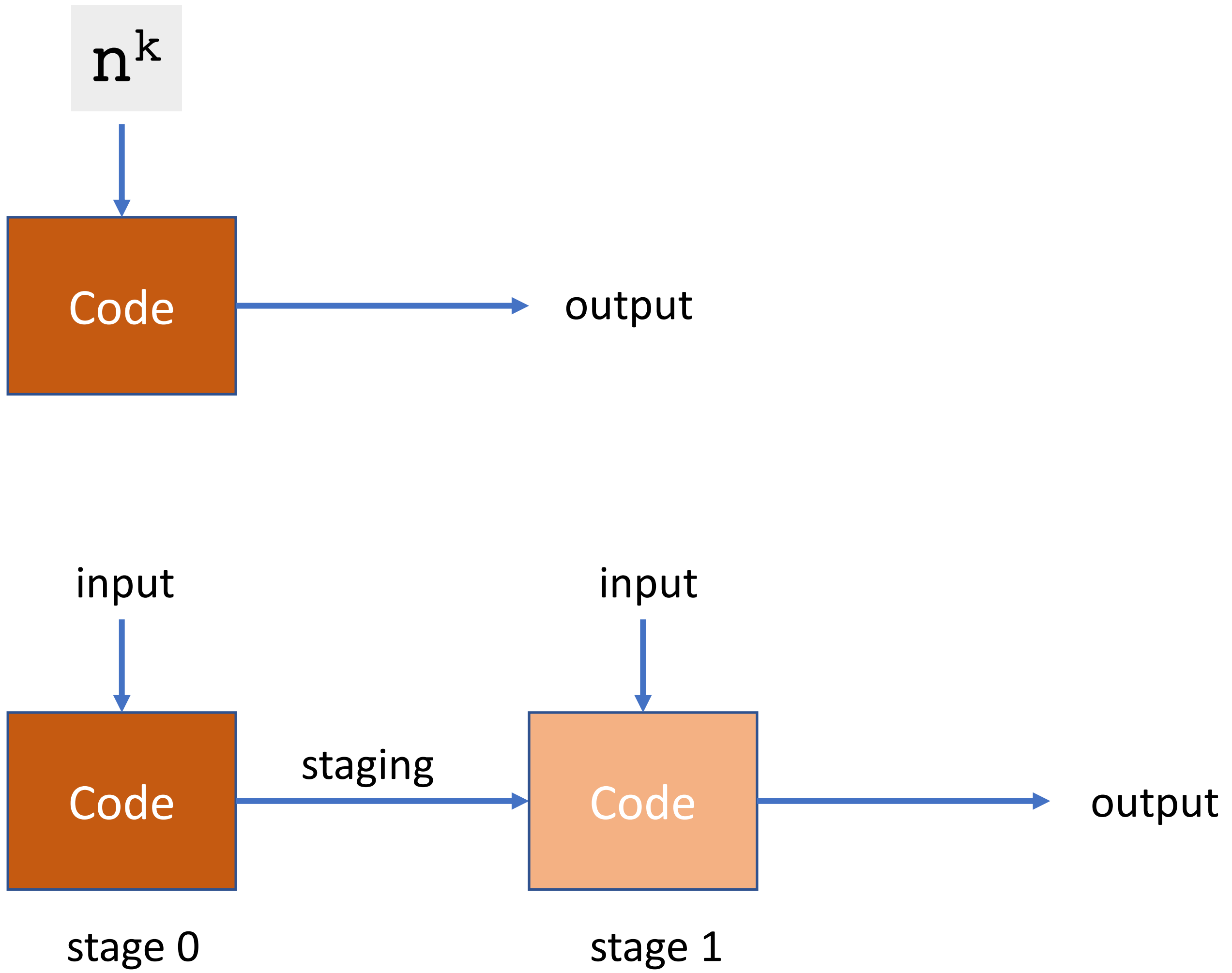

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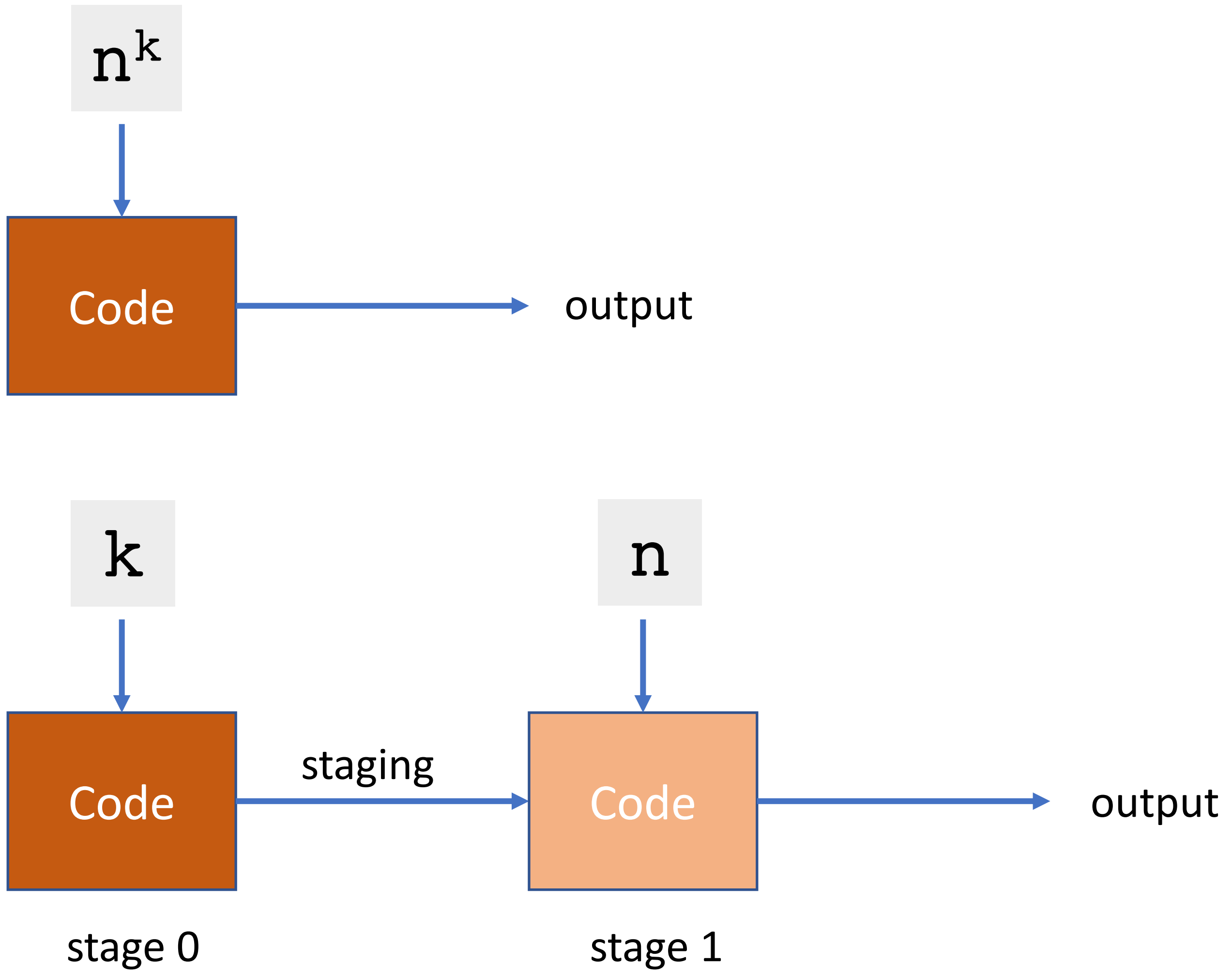

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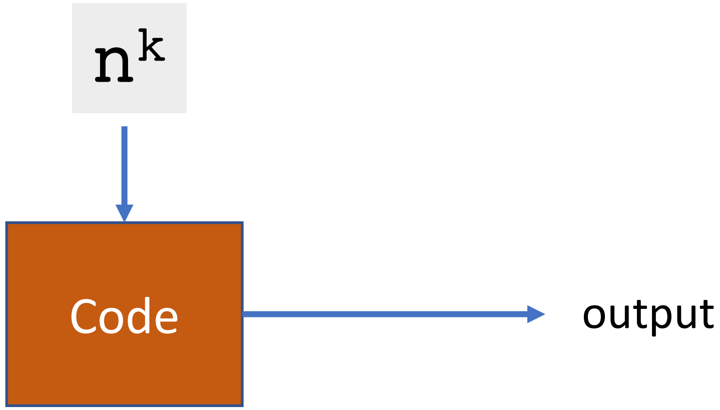


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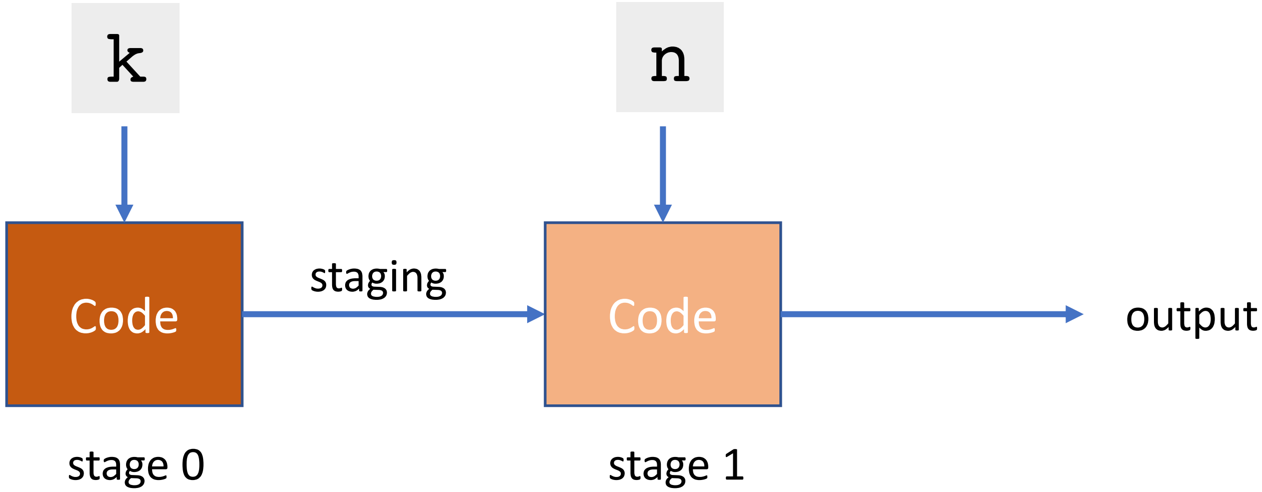

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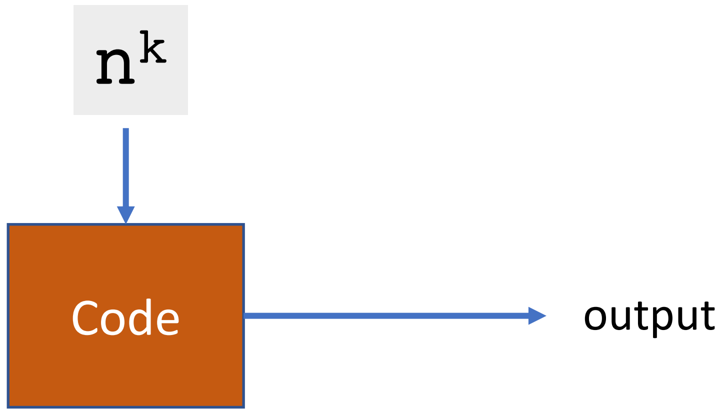


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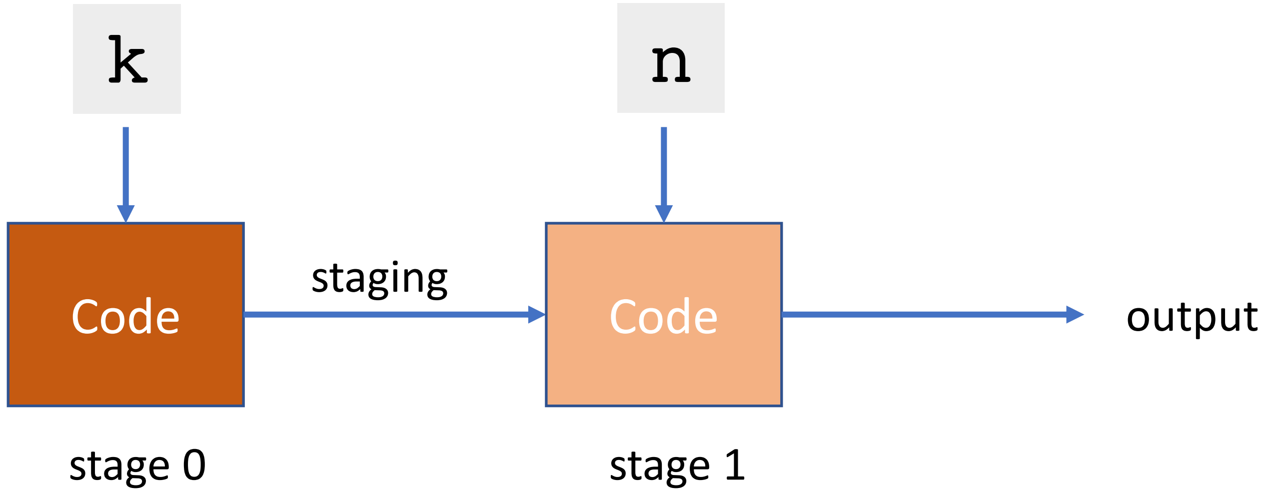

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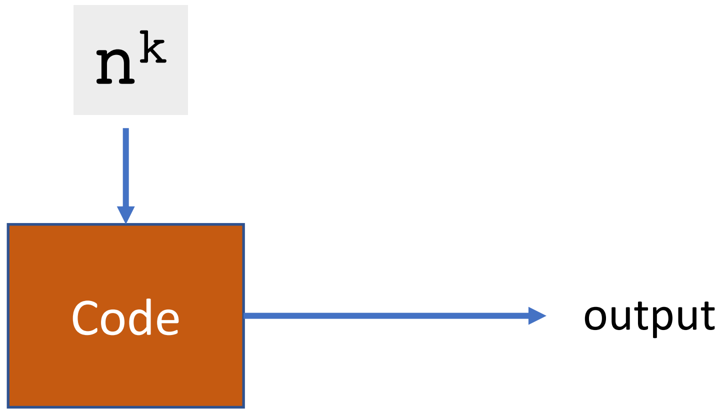


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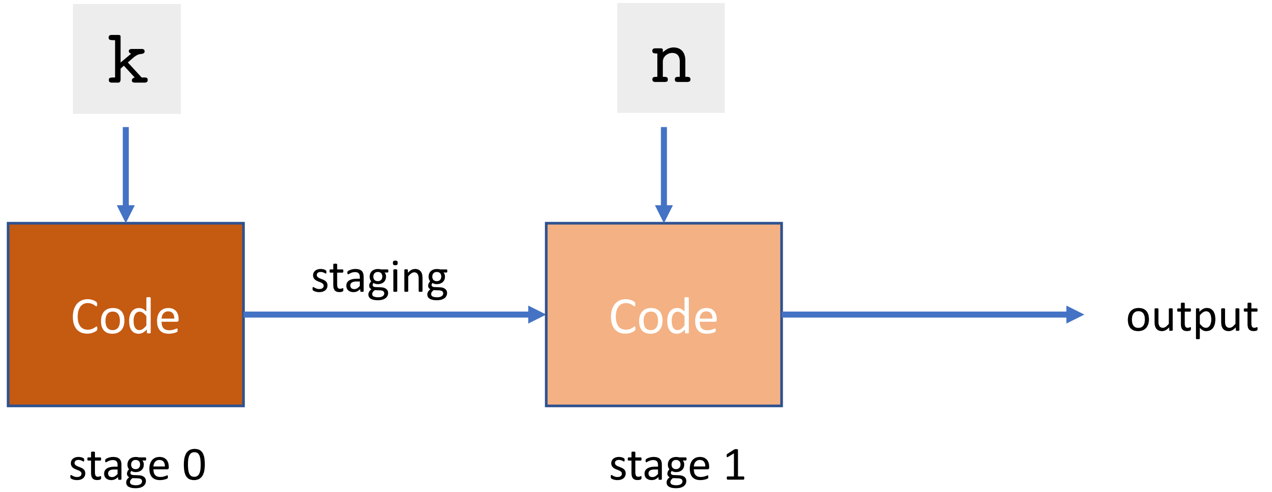

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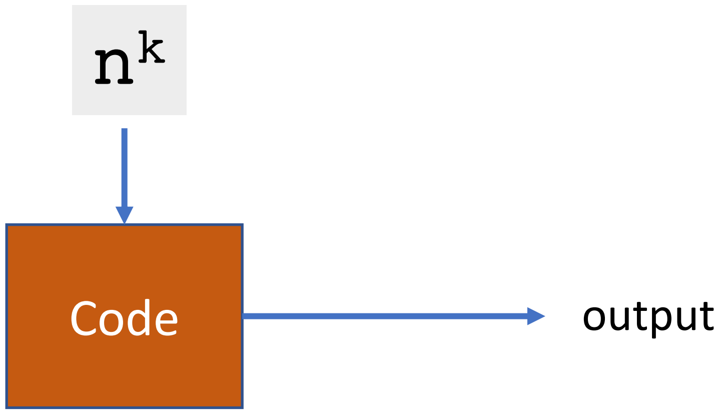


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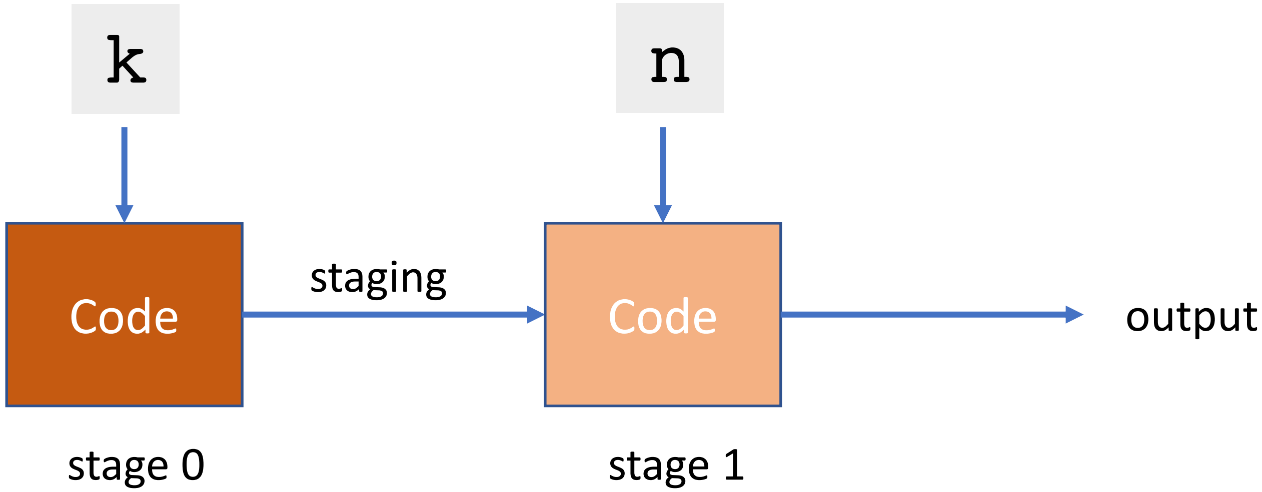

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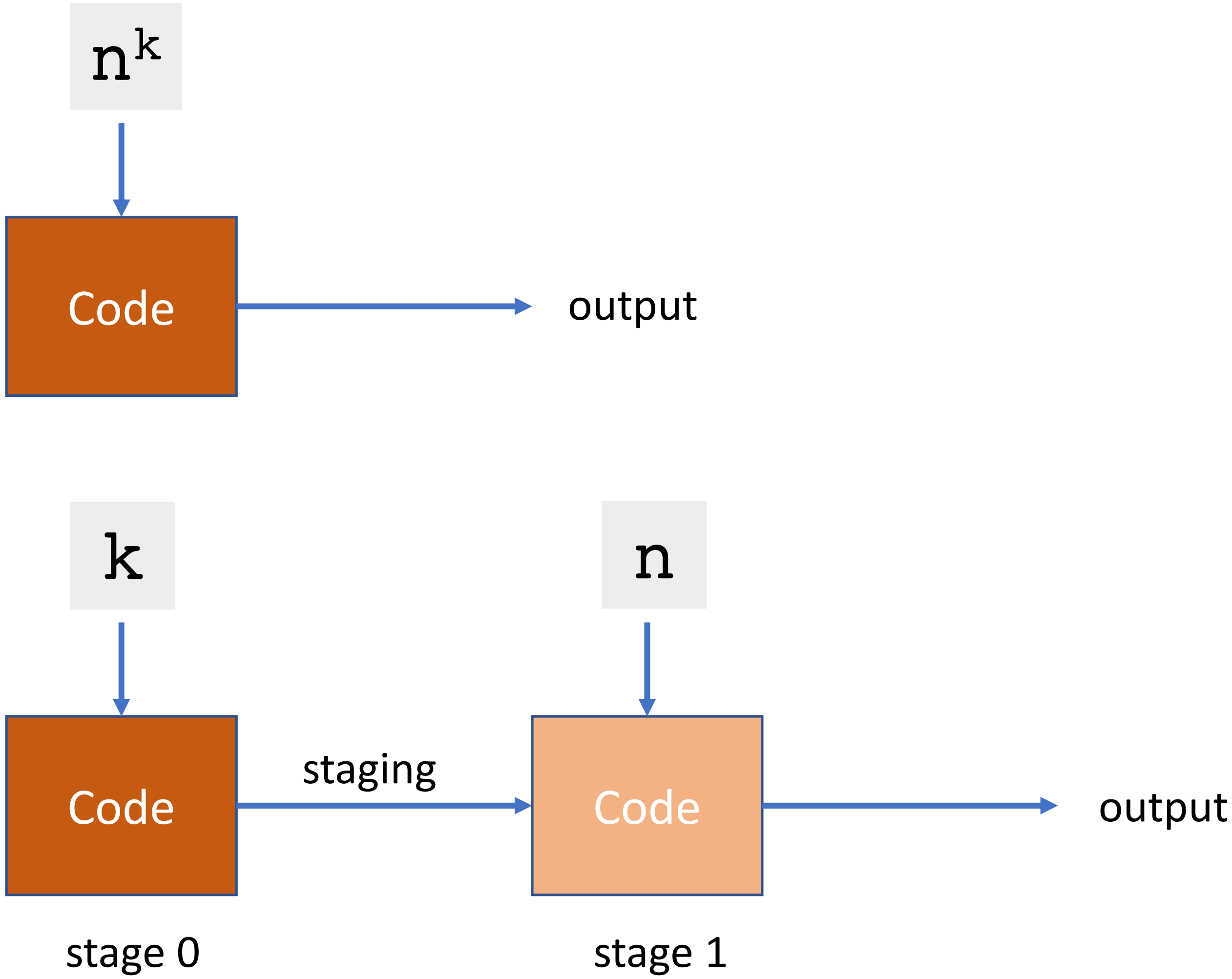

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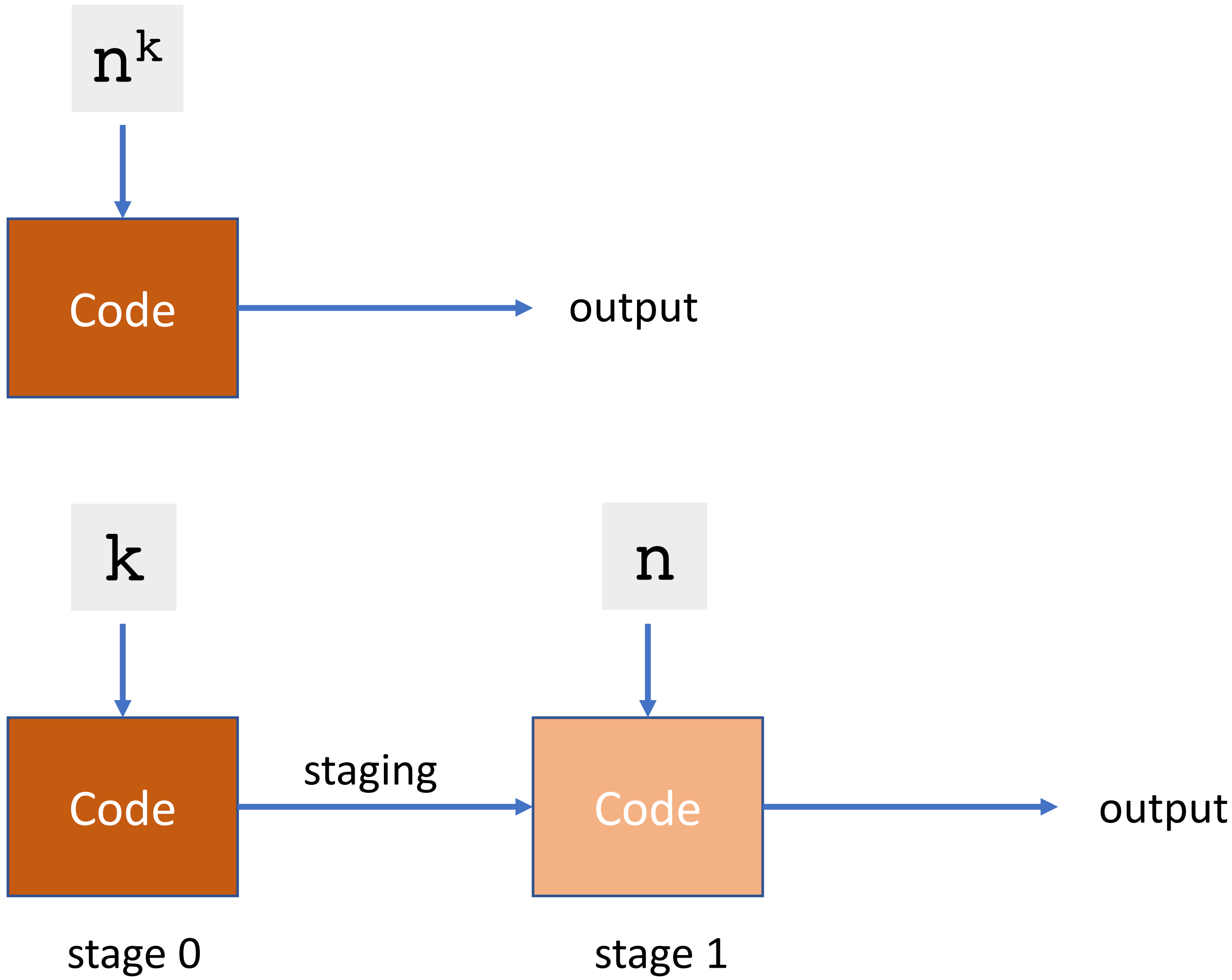

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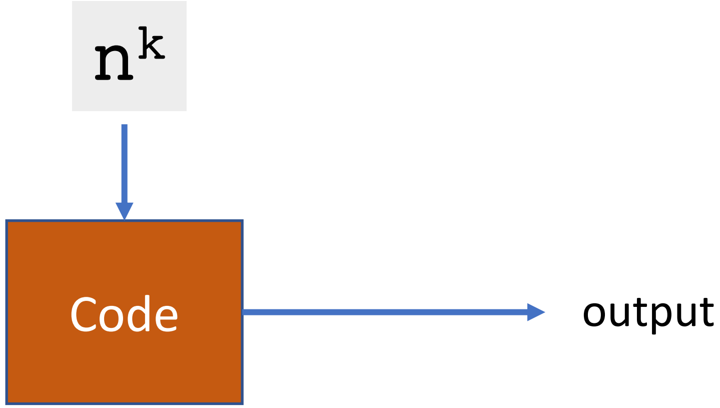


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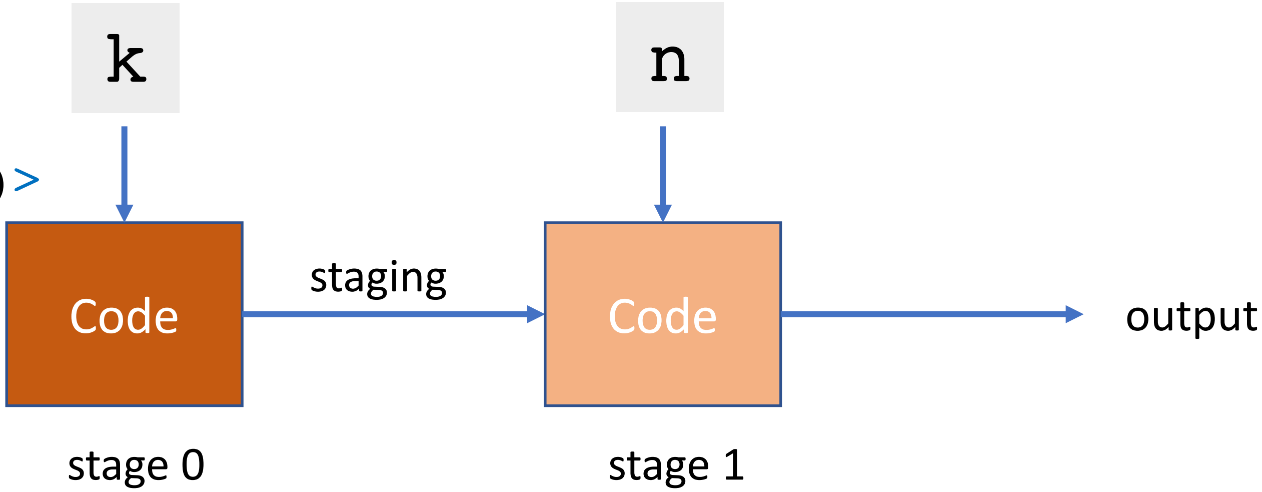

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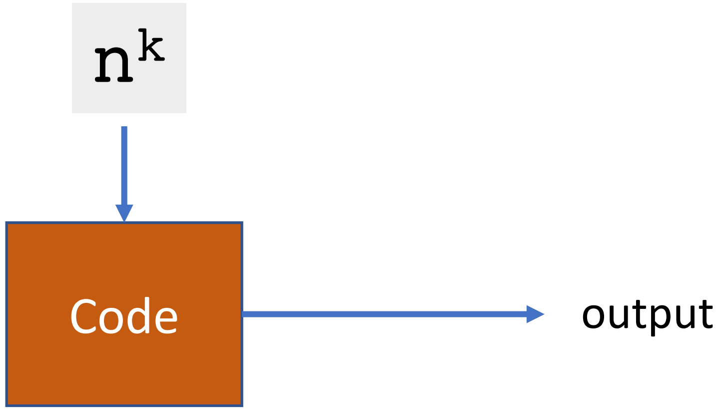


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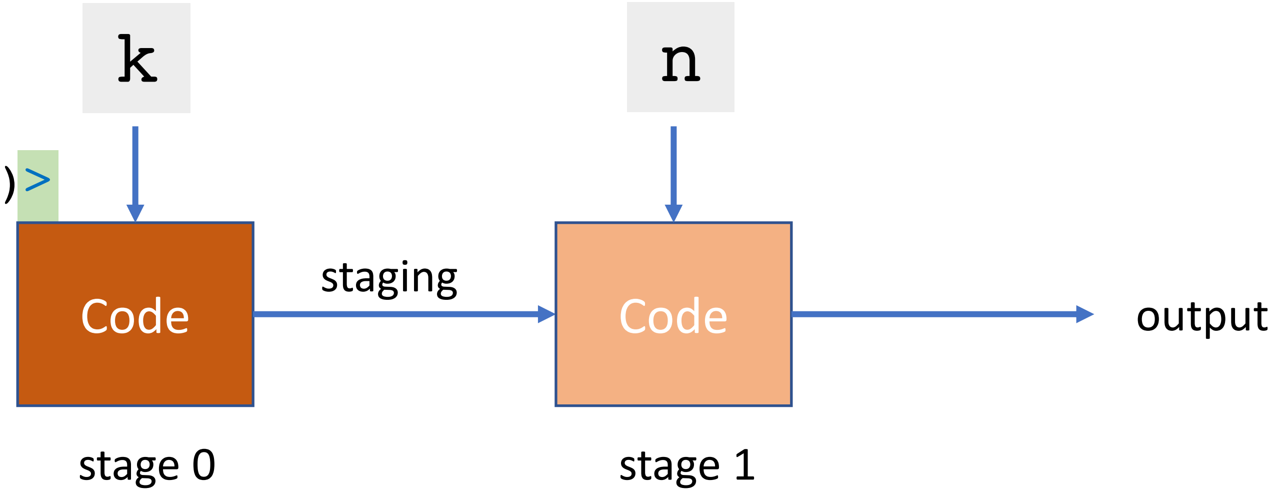

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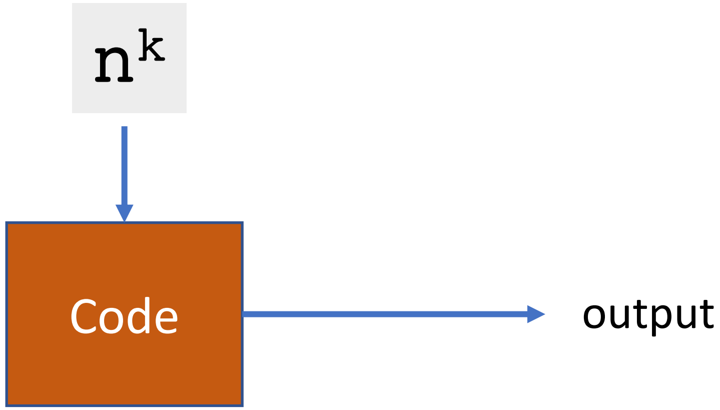


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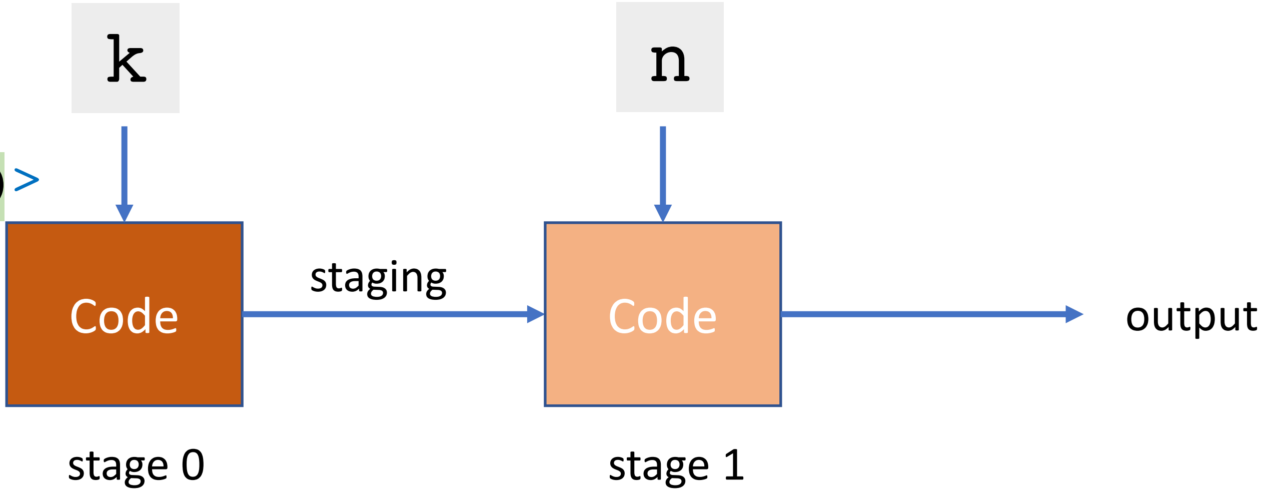

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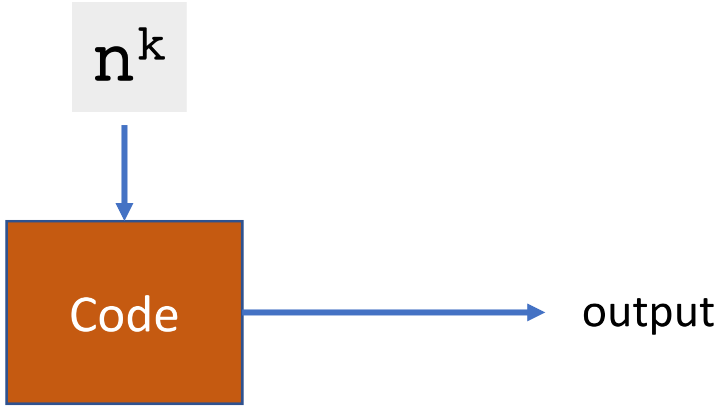


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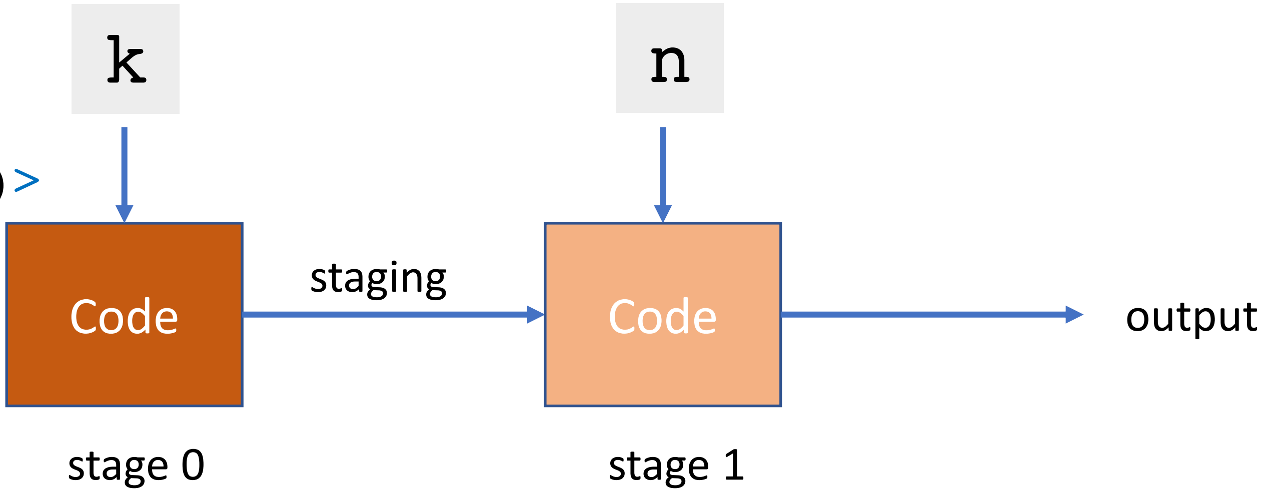

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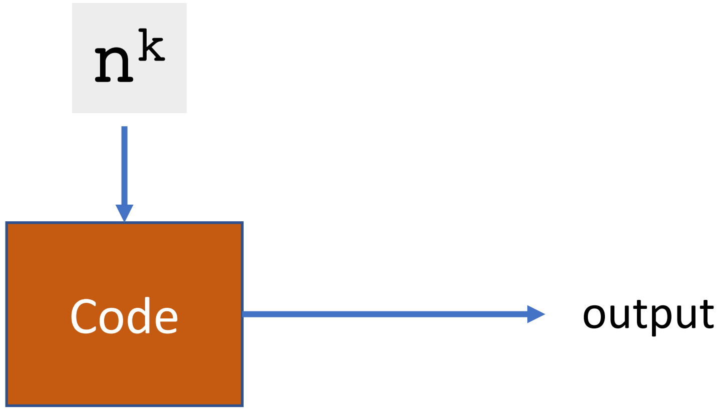


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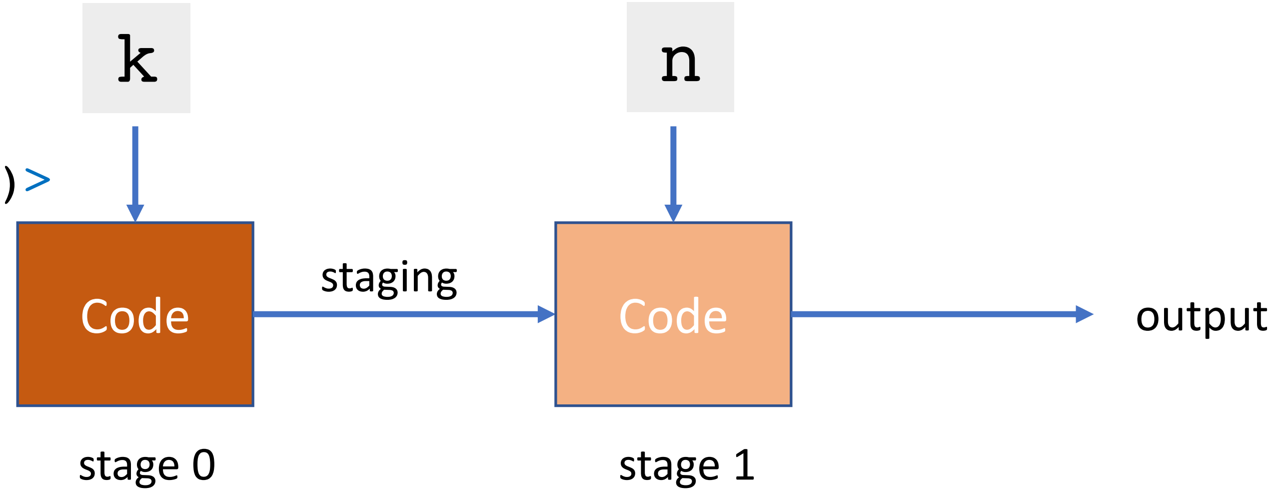
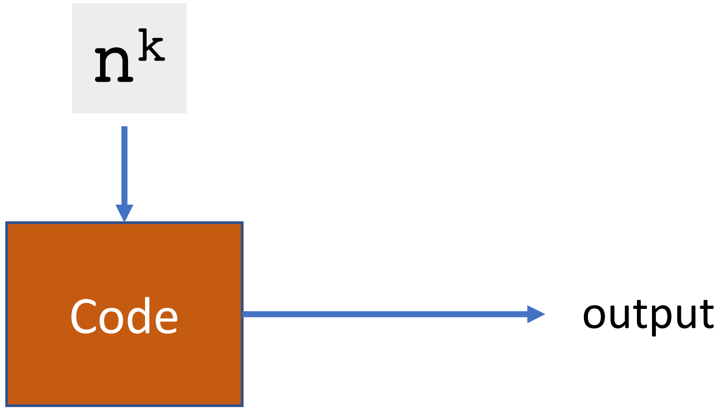
```



Multi-stage programming: example

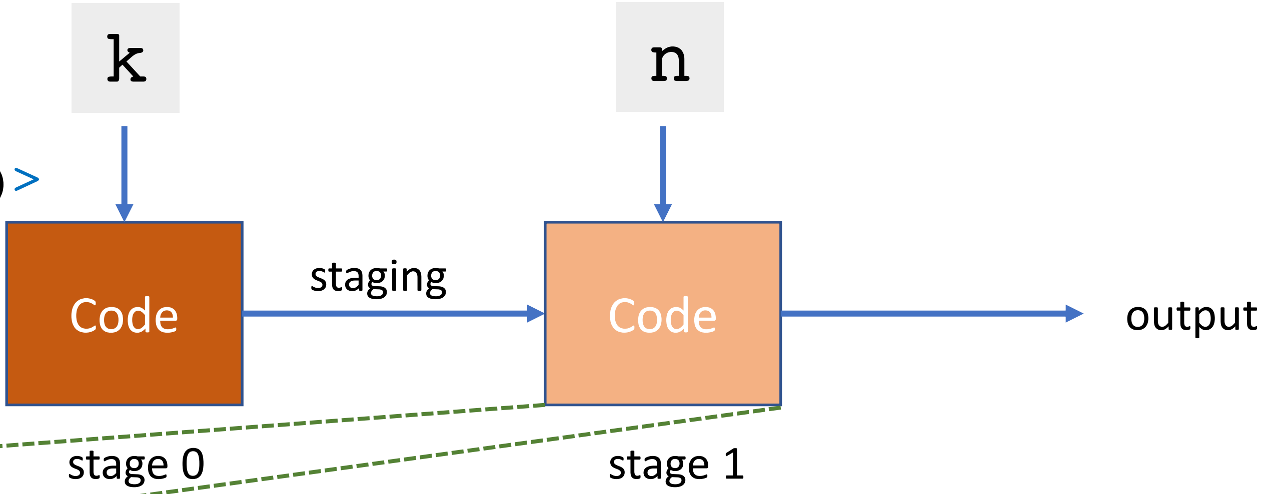
```
power :: Int -> Int -> Int
power 0 n = 1
power k n = n * power (k – 1) n

powerFive :: Int -> Int
powerFive n = power 5 n
```




```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```




```
qpowerFive n = n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```

```
⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```

⟶
```
n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
           ⟶  $(                                  )



           ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
             ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

             ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```

```
             ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)
```

```
             ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
             ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

             ⟶  $(<n>) * $(qpower (5 – 1) <n>)



             ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
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qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
           ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

           ⟶  $(<n>) * $(qpower (5 – 1) <n>)



           ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
             ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

             ⟶  $(<n>) * $(qpower (5 – 1) <n>)



             ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
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             ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

             ⟶  $(<n>) * $(qpower (5 – 1) <n>)

             ⟶  n * $(qpower (5 – 1) <n>)



             ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
           ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

           ⟶  $(<n>) * $(qpower (5 – 1) <n>)

           ⟶  n * $(qpower (5 – 1) <n>)


           ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
               ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

               ⟶  $(<n>) * $(qpower (5 – 1) <n>)

               ⟶  n * $(qpower (5 – 1) <n>)

               ⟶  n * $(qpower 4 <n>)


               ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
          ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

          ⟶  $(<n>) * $(qpower (5 – 1) <n>)

          ⟶  n * $(qpower (5 – 1) <n>)

          ⟶  n * $(qpower 4 <n>)


          ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
         ⟶  $(<$(<n>) * $(qpower (5 – 1) <n>)>)

         ⟶  $(<n>) * $(qpower (5 – 1) <n>)

         ⟶  n * $(qpower (5 – 1) <n>)

         ⟶  n * $(qpower 4 <n>)


         ⟶  n * n * n * n * n * 1
```

Code generation

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```

```
⟶ $(<$(<n>) * $(qpower (5 – 1) <n>)>)

⟶ $(<n>) * $(qpower (5 – 1) <n>)

⟶ n * $(qpower (5 – 1) <n>)

⟶ n * $(qpower 4 <n>)

⟶ ......

⟶ n * n * n * n * n * 1
```

But…

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```

But…

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qpower :: Int -> Code Int -> Code Int
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```
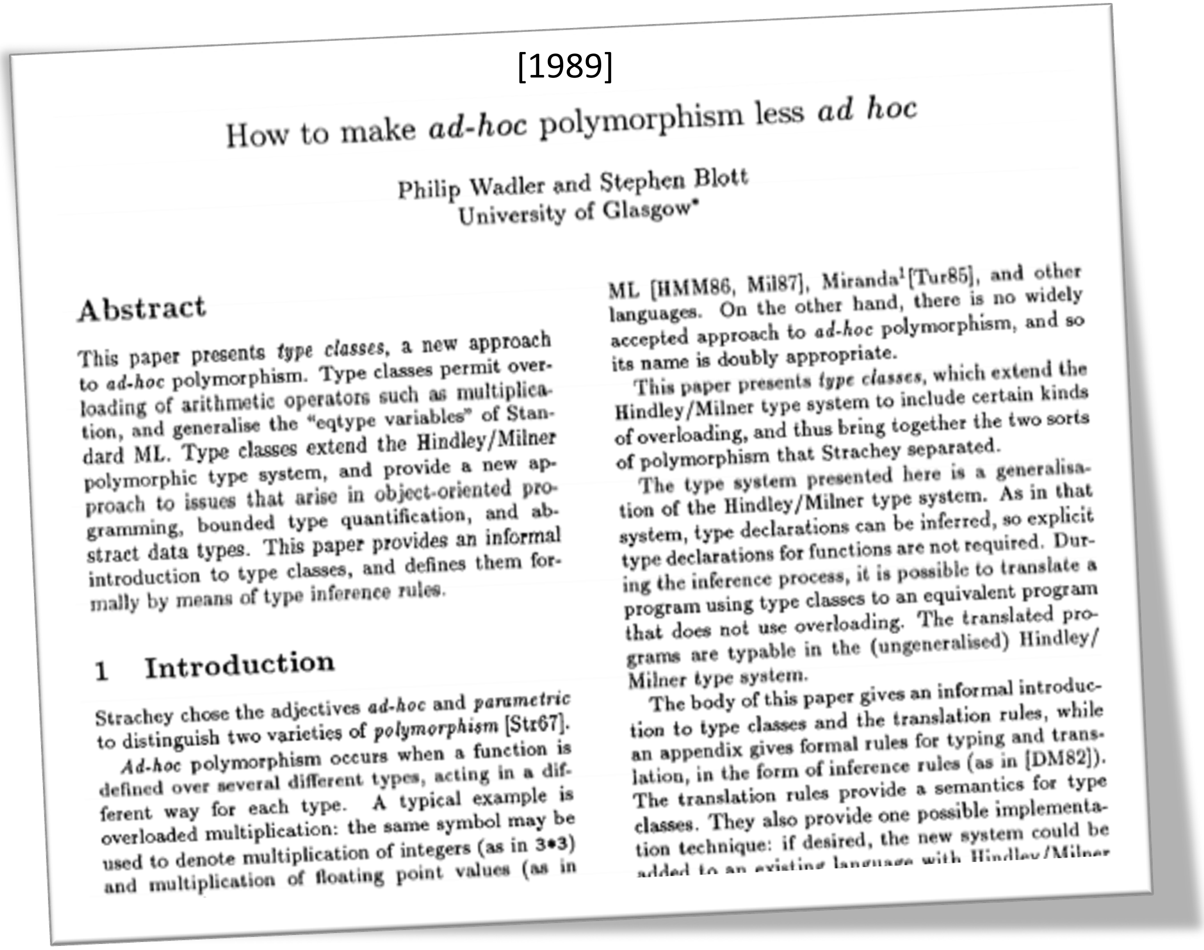

Type classes



[1989]

How to make *ad-hoc* polymorphism less *ad hoc*

Philip Wadler and Stephen Blott
University of Glasgow*

Abstract

This paper presents *type classes*, a new approach to *ad-hoc* polymorphism. Type classes permit overloading of arithmetic operators such as multiplication, and generalise the "eqtype variables" of Standard ML. Type classes extend the Hindley/Milner polymorphic type system, and provide a new approach to issues that arise in object-oriented programming, bounded type quantification, and abstract data types. This paper provides an informal introduction to type classes, and defines them formally by means of type inference rules.

1 Introduction

Strachey chose the adjectives *ad-hoc* and *parametric* to distinguish two varieties of *polymorphism* [Str67].

Ad-hoc polymorphism occurs when a function is defined over several different types, acting in a different way for each type. A typical example is overloaded multiplication: the same symbol may be used to denote multiplication of integers (as in 3*3) and multiplication of floating point values (as in

ML [HMM86, Mil87], Miranda[1][Tur85], and other languages. On the other hand, there is no widely accepted approach to *ad-hoc* polymorphism, and so its name is doubly appropriate.

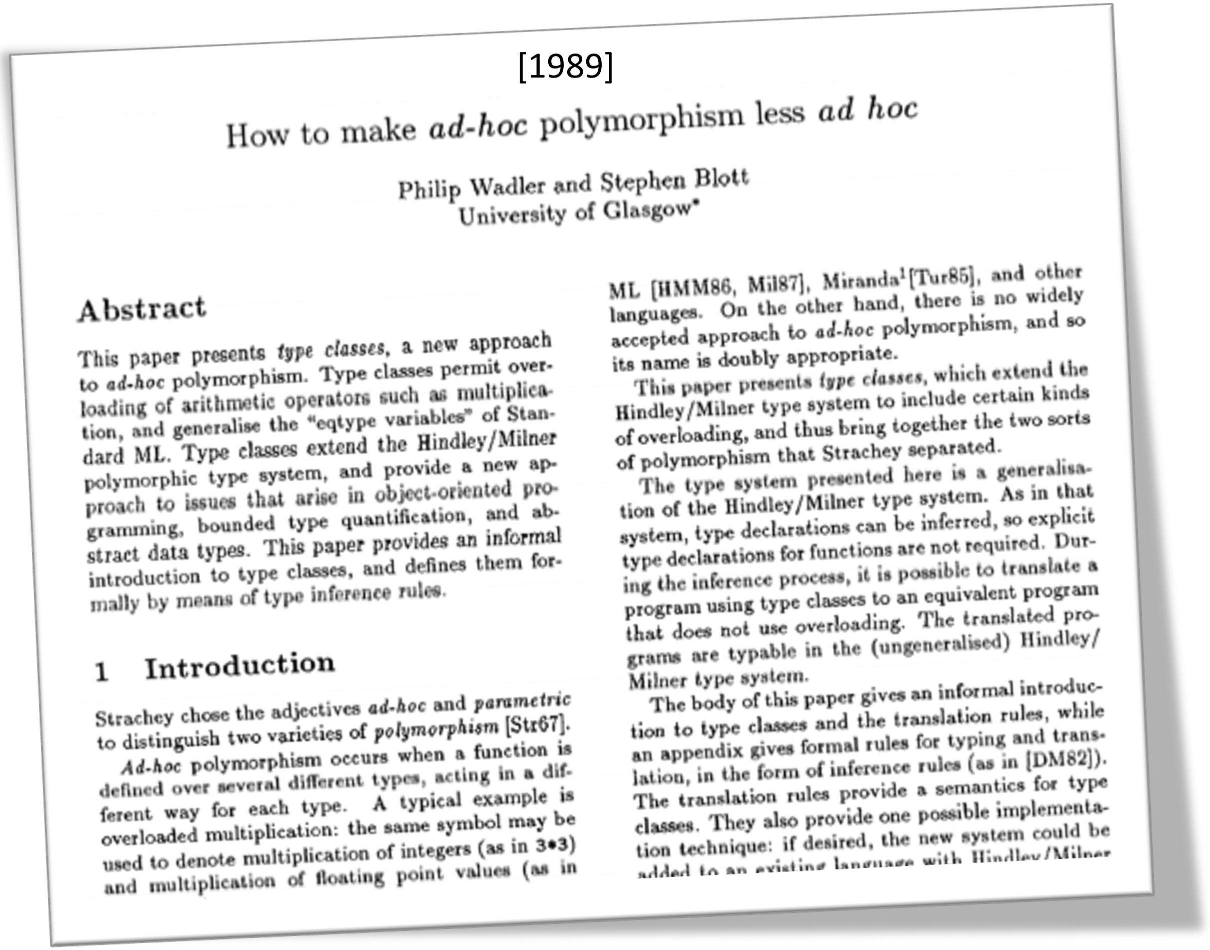
This paper presents *type classes*, which extend the Hindley/Milner type system to include certain kinds of overloading, and thus bring together the two sorts of polymorphism that Strachey separated.

The type system presented here is a generalisation of the Hindley/Milner type system. As in that system, type declarations can be inferred, so explicit type declarations for functions are not required. During the inference process, it is possible to translate a program using type classes to an equivalent program that does not use overloading. The translated programs are typable in the (ungeneralised) Hindley/Milner type system.

The body of this paper gives an informal introduction to type classes and the translation rules, while an appendix gives formal rules for typing and translation, in the form of inference rules (as in [DM82]). The translation rules provide a semantics for type classes. They also provide one possible implementation technique: if desired, the new system could be added to an existing language with Hindley/Milner

Type classes

[1989]



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```
class Show a where
  show :: a -> String

instance Show Int where
  show = primShowInt

instance Show Bool where
  show = primShowBool

print :: Show a => a -> String
print x = show x
```

Multi-stage programming and type classes

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
```

Multi-stage programming and type classes

```
qpower :: Num a => Int -> Code a -> Code a
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```

rejected:
  No instance for (Num a) arising from a use of 'qpower’
  In the expression: qpower 5 <n>

Multi-stage programming and type classes

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qpower :: Num a => Int -> Code a -> Code a
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This talk

unsound

inspire

$\lambda^{[\![\Rightarrow]\!]}$

- Type Classes
- Quotations/Splicing
- Staged type class constraint

type-directed

$F^{[\![\,]\!]}$

- Quotations
- Splice environments

This talk

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This talk

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inspire $\longrightarrow$ $\lambda^{[\![\Rightarrow]\!]}$ type-directed $\longrightarrow$ $F^{[\![]\!]}$

$\lambda^{[\![\Rightarrow]\!]}$:

- Type Classes
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$F^{[\![]\!]}$:

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This talk

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✓ A solid theoretical foundation for integrating type classes into multi-stage programs

This talk

unsound

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✓ Easy to implement and stay close to existing implementations

This talk

unsound

$\xrightarrow{\text{inspire}}$ $\lambda^{[\![\Rightarrow]\!]}$ $\xrightarrow{\text{type-directed}}$ $F^{[\![\,]\!]}$

- Type Classes
- Quotations/Splicing
- Staged type class constraint

- Quotations
- Splice environments

✓ A solid theoretical foundation for integrating type classes into multi-stage programs

✓ Easy to implement and stay close to existing implementations

Multi-stage programming: well-typedness

Quotation

a representation of the expression as program fragment in a future stage

```
e :: Int    ⟹   <e> :: Code Int
```

Splice

extracts the expression from its representation

```
e :: Code Int   ⟹   $e :: Int
```

Multi-stage programming: well-typedness

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$$\boxed{\Gamma \vdash e : \tau}$$

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context

Multi-stage programming: well-typedness

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a representation of the expression as program fragment in a future stage

`e :: Int` $\Longrightarrow$ `<e> :: Code Int`

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extracts the expression from its representation

`e :: Code Int` $\Longrightarrow$ `$e :: Int`

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context

`x : int`

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context

expr

```
x : int
```

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$$\Gamma \vdash e : \tau$$

context — expr — type

`x : int`

Multi-stage programming: well-typedness

Quotation

a representation of the expression as program fragment in a future stage

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \langle e \rangle : \text{Code}\, \tau}$$

Splice

extracts the expression from its representation

$$\frac{\Gamma \vdash e : \text{Code}\, \tau}{\Gamma \vdash \$e : \tau}$$

$$\Gamma \vdash e : \tau$$

context expr type

`x : int`

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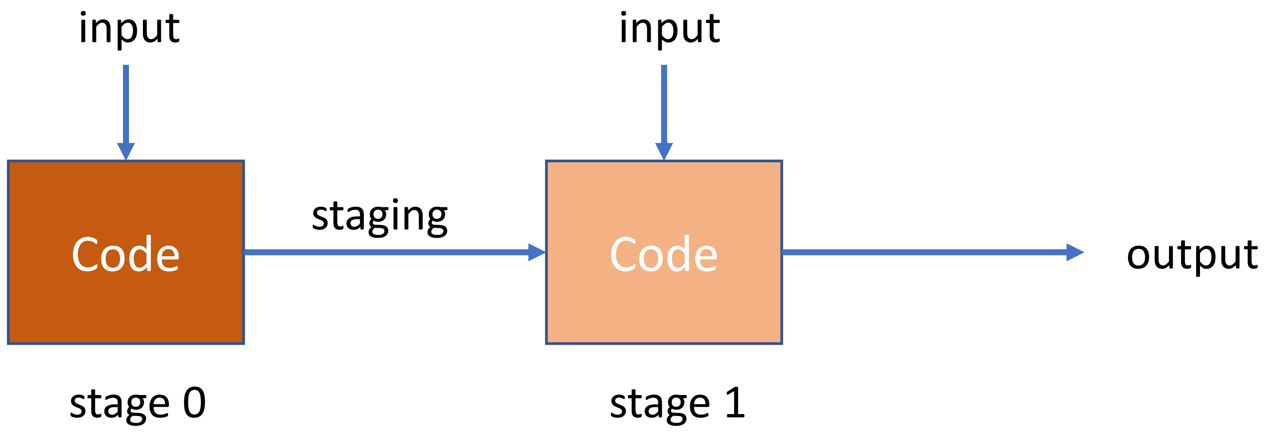

Multi-stage programming: well-typedness

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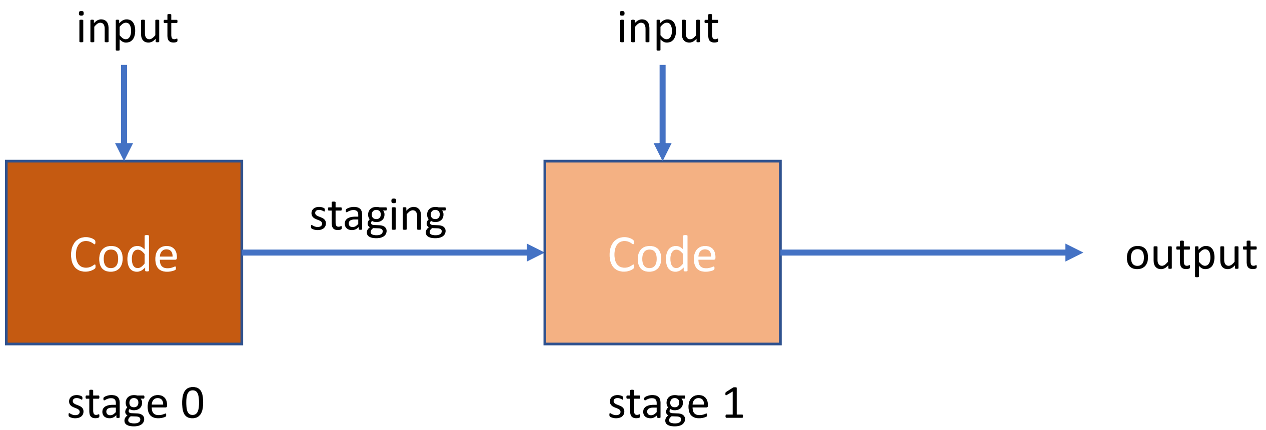
Splice

extracts the expression from its representation

$$\frac{\Gamma \vdash e : \mathrm{Code}\ \tau}{\Gamma \vdash \$e : \tau}$$

```
qpowerN :: Int -> Int
qpowerN n = $(qpower n <n>)
```



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qpowerN :: Int -> Int
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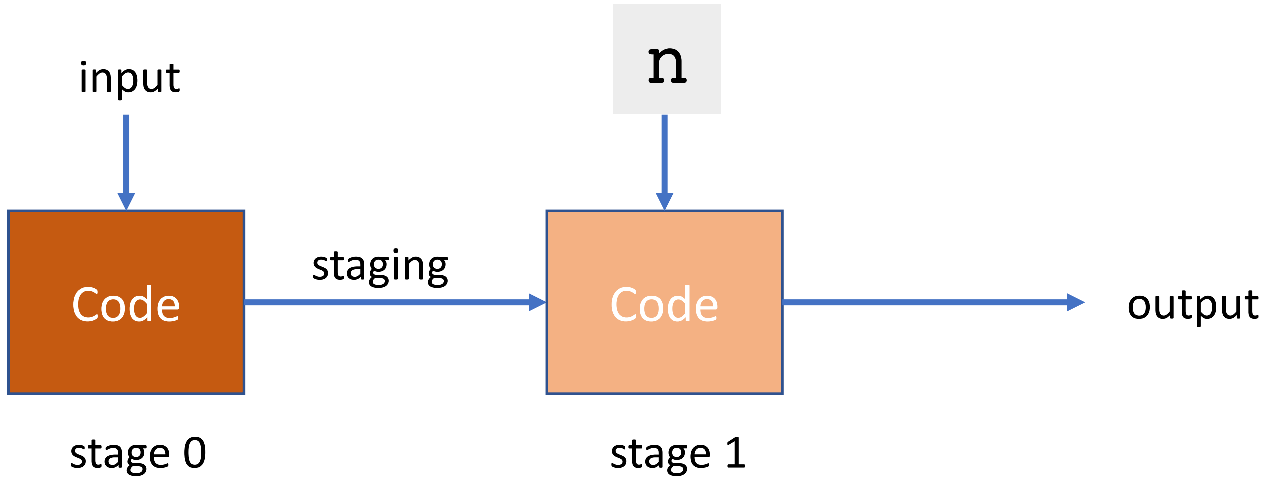

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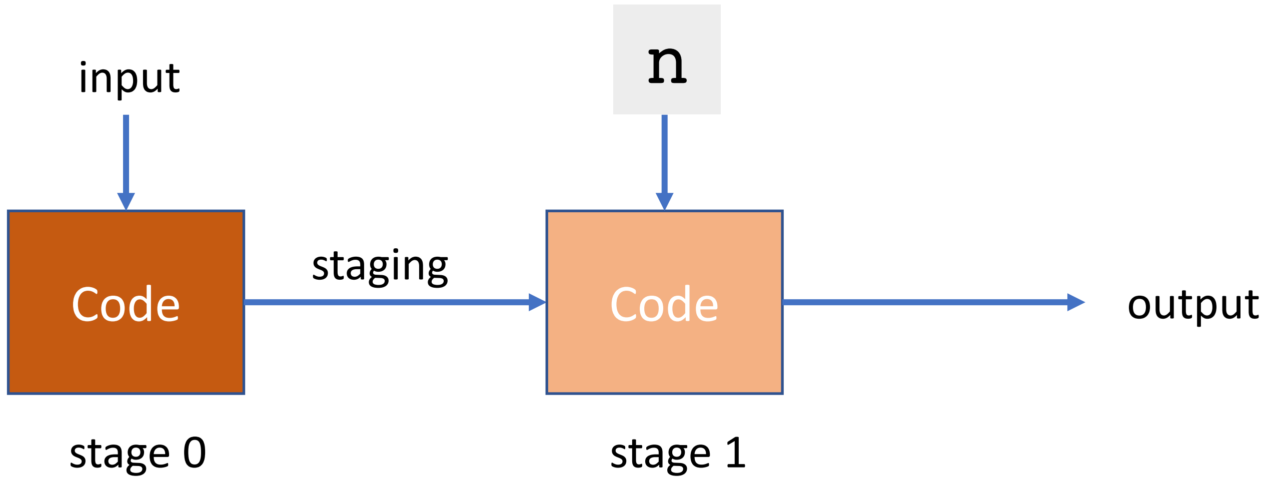
$$\frac{\Gamma \vdash e : \mathrm{Code}\,\tau}{\Gamma \vdash \$e : \tau}$$

```
qpowerN :: Int -> Int
qpowerN n = $(qpower n <n>)
    ⟶  n * $(qpower (n – 1) <n>)
```



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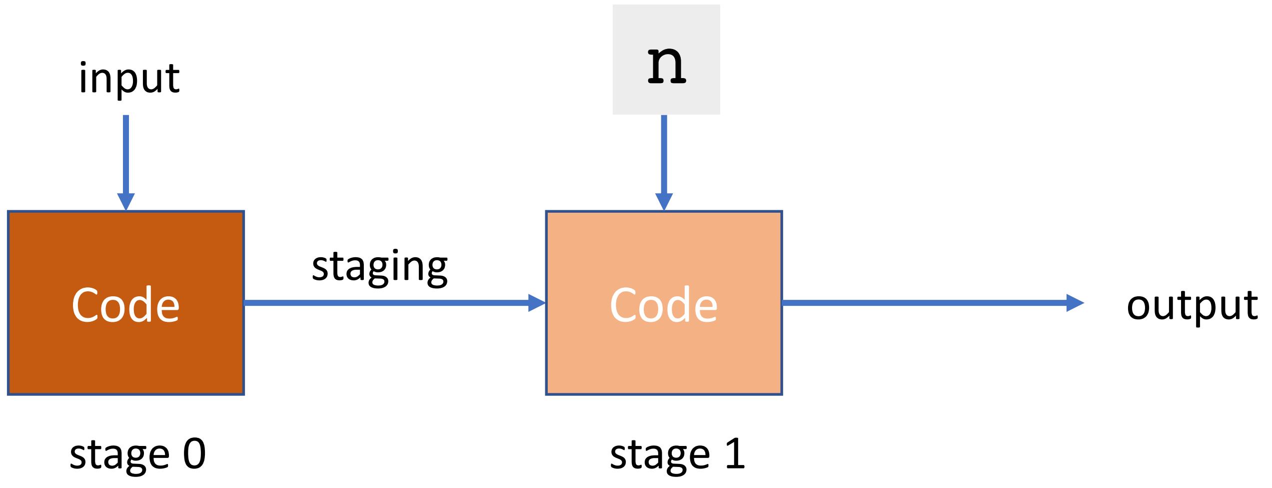
```
qpowerN :: Int
qpo        $(qpower n <n>)
   ⟶  n * $(qpower (n – 1) <n>)
```

rejected!



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extracts the expression from its representation

$$\frac{\Gamma \vdash e : \mathrm{Code}\,\tau}{\Gamma \vdash \$e : \tau}$$

$$\boxed{\underbrace{\Gamma}_{\text{context}} \vdash \underbrace{e}_{\text{expr}} : \underbrace{\tau}_{\text{type}}}$$

context expr type

`x : int`

Well-stagedness: the level of an expression

Quotation

a representation of the expression as program fragment in a future stage

$$\frac{\Gamma \vdash e : \tau}{\Gamma \vdash \langle e \rangle : \mathrm{Code}\ \tau}$$

Splice

extracts the expression from its representation

$$\frac{\Gamma \vdash e : \mathrm{Code}\ \tau}{\Gamma \vdash \$e : \tau}$$

level: evaluation order of expressions

$$\boxed{\Gamma \vdash^{n} e : \tau}$$

context expr type

`x : int`

Well-stagedness: the level of an expression

Quotation

a representation of the expression as program fragment in a future stage

$$\frac{\Gamma \vdash^{n+1} e : \tau}{\Gamma \vdash^{n} \langle e \rangle : \mathrm{Code}\ \tau}$$

Splice

extracts the expression from its representation

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\ \tau}{\Gamma \vdash^{n} \$e : \tau}$$

level: evaluation order of expressions

$$\boxed{\Gamma \vdash^{n} e : \tau}$$

context expr type

`x : int`

Well-stagedness: the level of an expression

Quotation

a representation of the expression as program fragment in a future stage

$$\frac{\Gamma \vdash^{n+1} e : \tau}{\Gamma \vdash^{n} \langle e \rangle : \mathrm{Code}\,\tau}$$

Splice

extracts the expression from its representation

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\,\tau}{\Gamma \vdash^{n} \$e : \tau}$$

level: evaluation order of expressions

$$\boxed{\Gamma \vdash^{n} e : \tau}$$

leveled context expr type

`x : int`

Well-stagedness: the level of an expression

Quotation

a representation of the expression as program fragment in a future stage

$$\frac{\Gamma \vdash^{n+1} e : \tau}{\Gamma \vdash^{n} \langle e \rangle : \mathrm{Code}\,\tau}$$

Splice

extracts the expression from its representation

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\,\tau}{\Gamma \vdash^{n} \$e : \tau}$$

level: evaluation order of expressions

$$\boxed{\Gamma \vdash^{n} e : \tau}$$

leveled context expr type

`x : (int , 0)`

Well-stagedness: the level restriction

Quotation

a representation of the expression as program fragment in a future stage

$$\frac{\Gamma \vdash^{n+1} e : \tau}{\Gamma \vdash^{n} \langle e \rangle : \mathrm{Code}\ \tau}$$

Splice

extracts the expression from its representation

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\ \tau}{\Gamma \vdash^{n} \$e : \tau}$$

level: evaluation order of expressions

The level restriction: each variable is used only at the level in which it is bound

leveled context expr type

x : (int , 0)

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

```
hasty :: Code Int -> Int
hasty = \y -> $(y)

tardy :: Int -> Code Int
tardy = \z -> < z >

timely :: Code (Int -> Int)
timely = < \x -> x >
```

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

well-typed? well-staged?

```
hasty :: Code Int -> Int
hasty = \y -> $(y)

tardy :: Int -> Code Int
tardy = \z -> < z >

timely :: Code (Int -> Int)
timely = < \x -> x >
```

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged?
`hasty :: Code Int -> Int` `hasty = \y -> $(y)`	✓	
`tardy :: Int -> Code Int` `tardy = \z -> < z >`		
`timely :: Code (Int -> Int)` `timely = < \x -> x >`		

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged?
`hasty :: Code Int -> Int` `hasty = \y -> $(y)`	✓	✗
`tardy :: Int -> Code Int` `tardy = \z -> < z >`		
`timely :: Code (Int -> Int)` `timely = < \x -> x >`		

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged?
`hasty :: Code Int -> Int` `hasty = \y -> $(y)`	✓	✗
`tardy :: Int -> Code Int` `tardy = \z -> < z >`	✓	
`timely :: Code (Int -> Int)` `timely = < \x -> x >`		

Well-stagedness: the level restriction

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`hasty :: Code Int -> Int` `hasty = \y -> $(y)`	✓	✗
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Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

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`hasty :: Code Int -> Int` `hasty = \y -> $(y)`	✓	✗
`tardy :: Int -> Code Int` `tardy = \z -> < z >`	✓	✗
`timely :: Code (Int -> Int)` `timely = < \x -> x >`	✓	

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged?
`hasty :: Code Int -> Int`	✓	✗
`hasty = \y -> $(y)`		
`tardy :: Int -> Code Int`	✓	✗
`tardy = \z -> < z >`		
`timely :: Code (Int -> Int)`	✓	✓
`timely = < \x -> x >`		

Well-stagedness: the level restriction

The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged? (path-based persistence for top-level identifiers)
`hasty :: Code Int -> Int` `hasty = \y -> $(y)`	✓	✗
`tardy :: Int -> Code Int` `tardy = \z -> < z >`	✓	✗
`timely :: Code (Int -> Int)` `timely = < \x -> x >`	✓	✓

Is the problem with qpower well-stageness?

The level restriction: each variable is used only at the level in which it is bound

```
qpower :: Num a => Int -> Code a -> Code a
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Num a => a -> a
qpowerFive n = $(qpower 5 <n>)
```

Is the problem with qpower well-stageness?

The level restriction: each variable is used only at the level in which it is bound

well-typed? well-staged?

```
qpower :: Num a => Int -> Code a -> Code a
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The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged?

`qpower :: Num a => Int -> Code a -> Code a`
`qpower 0 n = <1>`
`qpower k n = <$(n) * $(qpower (k – 1) n)>`
	✓	

`qpowerFive :: Num a => a -> a` `qpowerFive n = $(qpower 5 <n>)`	✓	

Is the problem with qpower well-stageness?

The level restriction: each variable is used only at the level in which it is bound

well-typed? well-staged?

```
qpower :: Num a => Int -> Code a -> Code a
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>
```

✓ ✓

```
qpowerFive :: Num a => a -> a
qpowerFive n = $(qpower 5 <n>)
```

✓ ✓

Is the problem with qpower well-stageness?

The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged?

`qpower :: Num a => Int -> Code a -> Code a`
`qpower 0 n = <1>`
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	✓	?

`qpowerFive :: Num a => a -> a` `qpowerFive n = $(qpower 5 <n>)`	✓	?

Is the problem with qpower well-stageness?

The level restriction: each variable is used only at the level in which it is bound

	well-typed?	well-staged?

`qpower :: Num a => Int -> Code a -> Code a`
`qpower 0 n = <1>`
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	✓	?

`qpowerFive :: Num a => a -> a` `qpowerFive n = $(qpower 5 <n>)`	✓	?

Is the problem with qpower well-stageness?

The level restriction: each variable is used only at the level in which it is bound

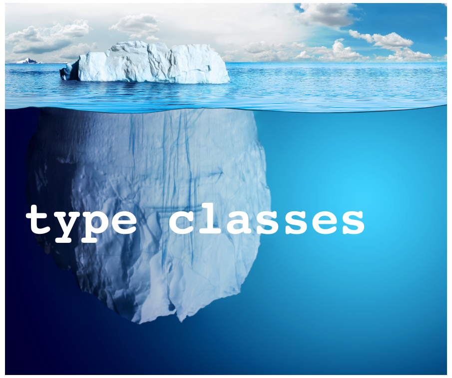
well-typed? well-staged?

```
qpower :: Num a => Int -> Code a -> Code a
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>
```

✓ ?

```
qpowerFive :: Num a => a -> a
qpowerFive n = $(qpower 5 <n>)
```

✓ ?



Type Classes

```
class Show a where
  show :: a -> String

instance Show Int where
  show = primShowInt

instance Show Bool where
  show = primShowBool

print :: Show a => a -> String
print x = show x
```

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class Show a where
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dictionary-passing elaboration

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print :: Show a => a -> String
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```

dictionary-passing elaboration

```
data ShowDict a = ShowDict
  { show :: a -> String }
```

Type Classes

```
class Show a where
  show :: a -> String

instance Show Int where
  show = primShowInt

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print :: Show a => a -> String
print x = show x
```

dictionary-passing elaboration

```
data ShowDict a = ShowDict
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Type Classes

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  { show :: a -> String }

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print :: ShowDict a -> a -> String
print dShow x = show dShow x
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Type Classes

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print :: ShowDict a -> a -> String
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Well-staged type classes

```
qpower :: Num a => Int -> Code a -> Code a
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Num a => a -> a
qpowerFive n = $(qpower 5 <n>)
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qpower :: NumDict a -> Int -> Code a -> Code a
qpower dNum 0 n = <1>
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```

dictionary-passing elaboration

well-staged?

```
qpower :: NumDict a -> Int -> Code a -> Code a
qpower dNum 0 n = <1>
qpower dNum k n = < (*) dNum $(n) $(qpower dNum (k – 1)) n >
```

✗

```
qpowerFive :: NumDict a -> a -> a
qpowerFive dNum n = $(qpower dNum 5 <n>)
```

✗

Key idea: staged type class constraints

$$\lambda^{[\![\Rightarrow]\!]}$$

Key idea: staged type class constraints

unstaged	staged
Int	Code Int
Num a	

Key idea: staged type class constraints

unstaged	staged
Int	Code Int
Num a	CodeC (Num a)

Key idea: staged type class constraints

Key idea: staged type class constraints

```
qpower :: CodeC (Num a) => Int -> Code a -> Code a
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1)) n>

qpowerFive :: Num a => a -> a
qpowerFive n = $(qpower 5 <n>)
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Key idea: staged type class constraints

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qpower :: CodeC (Num a) => Int -> Code a -> Code a
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Key idea: staged type class constraints

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dictionary-passing elaboration

```
qpower :: Code (NumDict a) -> Int -> Code a -> Code a
qpower dNum 0 n = <1>
qpower dNum k n = < (*) $(dNum) $(n) $(qpower dNum (k - 1)) n>

qpowerFive :: NumDict a -> a -> a
qpowerFive dNum n = $(qpower <dNum> 5 <n>)
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qpower :: Code (NumDict a) -> Int -> Code a -> Code a
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dictionary-passing elaboration

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qpower :: Code (NumDict a) -> Int -> Code a -> Code a
qpower dNum 0 n = <1>
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qpower :: CodeC (Num a) => Int -> Code a -> Code a
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dictionary-passing elaboration

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qpower :: Code (NumDict a) -> Int -> Code a -> Code a
qpower dNum 0 n = <1>
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qpowerFive :: NumDict a -> a -> a
qpowerFive dNum n = $(qpower <dNum> 5 <n>)
```

well-staged!

Constraint resolution

```
print :: Show a => a -> String
print x = show x
```

```
print :: ShowDict a -> a -> String
print dShow x = show dShow x
```

Constraint resolution

```
print :: Show a => a -> String
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print dShow x = show dShow x
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Constraint resolution

$$\Gamma \models C \rightsquigarrow e$$

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print :: Show a => a -> String
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print dShow x = show dShow x
```

Constraint resolution

```
print :: Show a => a -> String
print x = show x
```

$$\Gamma \models C \rightsquigarrow e$$

$$\frac{ev : C \in \Gamma}{\Gamma \models C \rightsquigarrow ev}$$

```
print :: ShowDict a -> a -> String
print dShow x = show dShow x
```

Constraint resolution

$$\Gamma \models C \rightsquigarrow e$$

```
print :: Show a => a -> String
print x = show x
```

$$\frac{ev : C \in \Gamma}{\Gamma \models C \rightsquigarrow ev}$$

```
print :: ShowDict a -> a -> String
print dShow x = show dShow x
```

$$\frac{dShow : Show\ a \in \Gamma}{\Gamma \models Show\ a \rightsquigarrow dShow}$$

Level-indexed constraint resolution

$$\Gamma \models^{n} C \rightsquigarrow e$$

```
print :: Show a => a -> String
print x = show x
```

$$\frac{ev : (C, n) \in \Gamma}{\Gamma \models^{n} C \rightsquigarrow ev}$$

```
print :: ShowDict a -> a -> String
print dShow x = show dShow x
```

$$\frac{dShow : Show\ a \in \Gamma}{\Gamma \models Show\ a \rightsquigarrow dShow}$$

Level-indexed constraint resolution

$$\Gamma \models^{n} C \rightsquigarrow e$$

```
print :: Show a => a -> String
print x = show x
```

$$\frac{ev : (C, n) \in \Gamma}{\Gamma \models^{n} C \rightsquigarrow ev}$$

```
print :: ShowDict a -> a -> String
print dShow x = show dShow x
```

$$\frac{dShow : (Show\ a, 0) \in \Gamma}{\Gamma \models^{0} Show\ a \rightsquigarrow dShow}$$

Level-indexed constraint resolution

```
print :: Show a => a -> String
print x = show x
```

```
print :: ShowDict a -> a -> String
print dShow x = show dShow x
```

$$\frac{dShow : (Show\ a, 0) \in \Gamma}{\Gamma \models^{0} Show\ a \leadsto dShow}$$

$$\boxed{\Gamma \models^{n} C \leadsto e}$$

$$\frac{ev : (C, n) \in \Gamma}{\Gamma \models^{n} C \leadsto ev}$$

$$\frac{\Gamma \models^{n+1} C \leadsto e}{\Gamma \models^{n} \mathrm{CodeC}\ C \leadsto \langle e \rangle}$$

Level-indexed constraint resolution

$$\Gamma \models^{n} C \rightsquigarrow e$$

```
print :: Show a => a -> String
print x = show x
```

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print :: ShowDict a -> a -> String
print dShow x = show dShow x
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$$\frac{dShow : (Show\ a, 0) \in \Gamma}{\Gamma \models^{0} Show\ a \rightsquigarrow dShow}$$

$$\frac{ev : (C, n) \in \Gamma}{\Gamma \models^{n} C \rightsquigarrow ev}$$

$$\frac{\Gamma \models^{n+1} C \rightsquigarrow e}{\Gamma \models^{n} \mathrm{CodeC}\ C \rightsquigarrow \langle e \rangle}$$

$$\frac{\Gamma \models^{n-1} \mathrm{CodeC}\ C \rightsquigarrow e}{\Gamma \models^{n} C \rightsquigarrow \$e}$$

This talk

unsound

inspire

$\lambda^{[\![\Rightarrow]\!]}$

- Type Classes
- Quotations/Splicing
- Staged type class constraint

type-directed

$F^{[\![\,]\!]}$

- Quotations
- Splice environments

✓ A solid theoretical foundation for integrating type classes into multi-stage programs

✓ easy to implement and stay close to existing implementations

This talk

unsound $\xrightarrow{\text{inspire}}$ $\lambda^{[\![\Rightarrow]\!]}$ $\xrightarrow{\text{type-directed}}$ $F^{[\![\,]\!]}$

$\lambda^{[\![\Rightarrow]\!]}$:

- Type Classes
- Quotations/Splicing
- Staged type class constraint

$F^{[\![\,]\!]}$:

- Quotations
- Splice environments

✓ A solid theoretical foundation for integrating type classes into multi-stage programs

✓ easy to implement and stay close to existing implementations

How to evaluate staged programs?

How to evaluate staged programs?

$$e_1 \ \langle e_2 \ \$ e_3 \rangle$$

How to evaluate staged programs?

level

$$e_1 \; \langle e_2 \, \$ e_3 \rangle$$

How to evaluate staged programs?

level 0

$$\boxed{e_1 \ \langle e_2 \ \$ e_3 \rangle}$$

How to evaluate staged programs?

level 0 1

$$e_1 \; \langle e_2 \; \$ e_3 \rangle$$

How to evaluate staged programs?

level 0 1 0

$$e_1 \ \langle e_2 \ \$ e_3 \rangle$$

How to evaluate staged programs?

level 0 1 0

$$e_1 \; \langle e_2 \; \$ e_3 \rangle \quad \longrightarrow \quad v_1 \; \langle e_2 \; \$ v_3 \rangle$$

Level-indexed Evaluation

level 0 1 0

$$e_1 \, \langle e_2 \, \$ e_3 \rangle \longrightarrow v_1 \, \langle e_2 \, \$ v_3 \rangle$$

Level-indexed Evaluation

level 0 1 0

$$e_1 \; \langle e_2 \; \$ e_3 \rangle \longrightarrow v_1 \; \langle e_2 \; \$ v_3 \rangle$$

$$\dfrac{e_1 \xrightarrow{0} v_1 \qquad \dfrac{\dfrac{e_2 \xrightarrow{1} e_2 \qquad \dfrac{e_3 \xrightarrow{0} v_3}{\$e_3 \xrightarrow{1} v_3}}{e_2 \; \$e_3 \xrightarrow{1} e_2 \; v_3}}{\langle e_2 \; \$e_3 \rangle \xrightarrow{0} \langle e_2 \; \$v_3 \rangle}}{e_1 \; \langle e_2 \; \$e_3 \rangle \xrightarrow{0} v_1 \; \langle e_2 \; \$v_3 \rangle}$$

Level-indexed Evaluation

level 0 1 0

$$e_1 \langle e_2 \$ e_3 \rangle \longrightarrow v_1 \langle e_2 \$ v_3 \rangle$$

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Adding complexity to implementations

Level-indexed Evaluation

level 0 1 0

$$e_1\ \langle e_2\ \$e_3 \rangle \longrightarrow v_1\ \langle e_2\ \$v_3 \rangle$$

$$\dfrac{e_1 \xrightarrow{0} v_1 \qquad \dfrac{\dfrac{e_2 \xrightarrow{1} e_2 \qquad \dfrac{e_3 \xrightarrow{0} v_3}{\$e_3 \xrightarrow{1} v_3}}{e_2\ \$e_3 \xrightarrow{1} e_2\ v_3}}{\langle e_2\ \$e_3 \rangle \xrightarrow{0} \langle e_2\ \$v_3 \rangle}}{e_1\ \langle e_2\ \$e_3 \rangle \xrightarrow{0} v_1\ \langle e_2\ \$v_3 \rangle}$$

Adding complexity to implementations

Requires inspecting and evaluating inside quotations

Key idea: splice environments

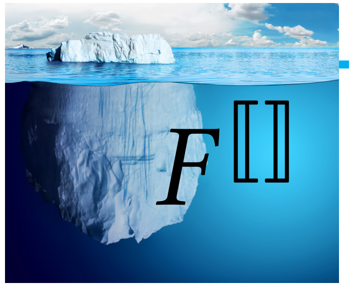
$\lambda^{[\![\Rightarrow]\!]}$

$$e_1 \ \langle e_2 \ \$ e_3 \rangle$$

Key idea: splice environments

$\lambda^{\llbracket\Rightarrow\rrbracket}$

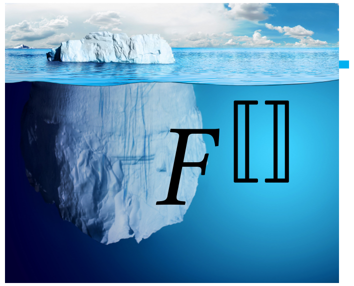
$$e_1 \; \langle e_2 \, \$ e_3 \rangle$$



Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1 \langle e_2 \, \$ e_3 \rangle$$

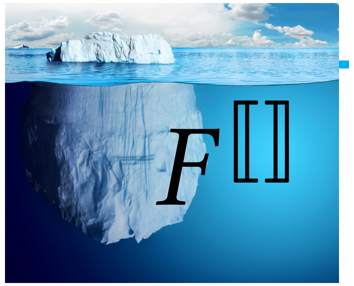


$$e_1 \langle e_2 \quad s \quad \rangle$$

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1 \,\langle e_2 \;\$ e_3 \rangle$$

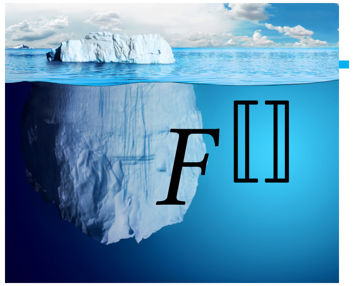


$$e_1 \,\langle e_2 \quad s \quad \rangle_{\bullet \vdash^0 s:\tau = e_3}$$

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1 \, \langle e_2 \, \$ e_3 \rangle$$

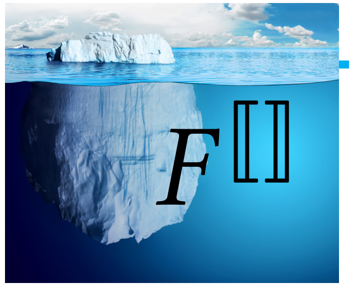


$$e_1 \, \langle e_2 \quad s \quad \rangle_{\bullet \vdash^0 s:\tau = e_3}$$

the spliced expression

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$



$$e_1 \ \langle e_2 \ \$e_3 \rangle$$

the spliced expression

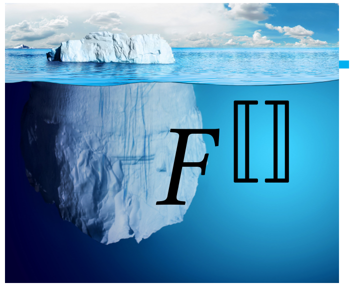
$$e_1 \ \langle e_2 \quad s \ \rangle_{\bullet \vdash^0 s : \tau = e_3}$$

type of s (so the type of e3 is Code τ)

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1 \, \langle e_2 \; \$e_3 \rangle$$



level of the e3 (so level of s is 0 + 1 = 1)

the spliced expression

$$e_1 \, \langle e_2 \quad s \; \rangle_{\bullet \vdash^0 s : \tau = e_3}$$

type of s (so the type of e3 is Code τ)

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

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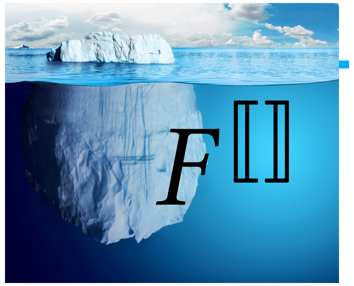


level of the e3 (so level of s is 0 + 1 = 1)

the spliced expression

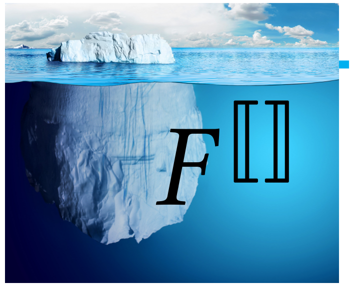
$$e_1 \ \langle e_2 \quad s \ \rangle_{\bullet \vdash^0 s : \tau = e_3}$$

type of s (so the type of e3 is Code τ)

the type context

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$



$$e_1\ \langle e_2\ \$e_3 \rangle$$

level of the quotation

level of the e3 (so level of s is 0 + 1 = 1)

the spliced expression

$$e_1\ \boxed{\langle e_2 \quad s \ \rangle}_{\bullet \vdash^{0} s : \tau = e_3}$$

type of s (so the type of e3 is Code τ)

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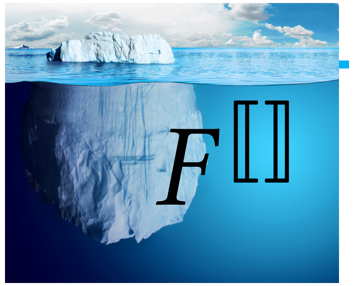
Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1 \, \langle e_2 \, \$e_3 \rangle$$



level of the quotation

a splice is bound to the innermost surrounding quotation at the same level

level of the e3 (so level of s is 0 + 1 = 1)

the spliced expression

$$e_1 \, \boxed{\langle e_2 \quad s \,\rangle} \bullet \vdash^{0} s{:}\tau{=}e_3$$

type of s (so the type of e3 is Code τ)

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Key idea: splice environments

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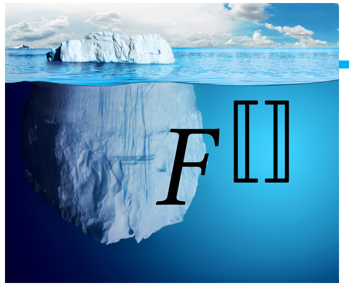

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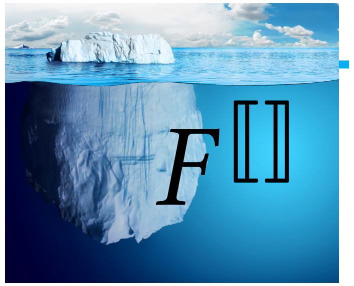
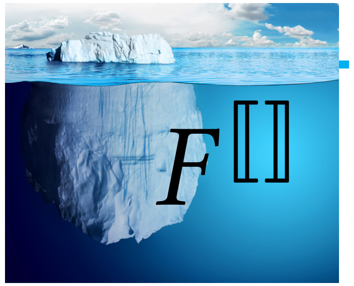


✓ straightforward evaluation

$$e_1 \boxed{\langle e_2 \quad s \quad \rangle} \bullet \vdash^0 s{:}\tau{=}e_3$$

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$



$$e_1 \, \langle e_2 \, \$ e_3 \rangle$$

✓ straightforward evaluation

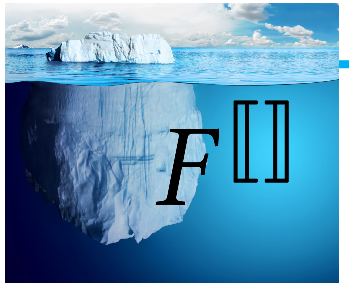
$$\frac{\phi \longrightarrow \phi'}{[\![e]\!]_{\phi} \longrightarrow [\![e]\!]_{\phi'}}$$

$$e_1 \, \boxed{\langle e_2 \quad s \,\rangle}_{\bullet \vdash^0 s:\tau = e_3}$$

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1 \; \langle e_2 \; \$ e_3 \rangle$$



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Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

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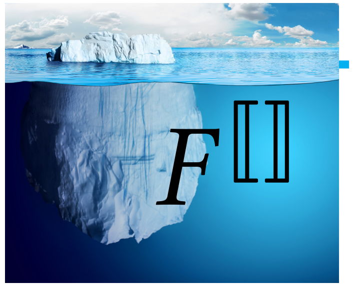

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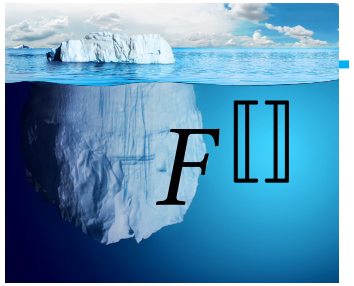
✓ opaque quotations

Key idea: splice environments

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1\ (e_2\ \$e_3\ )$$



$$e_1\ \boxed{\langle e_2 \quad s\ \rangle}\bullet \vdash^{0} s{:}\tau{=}e_3$$

Key idea: splice environments

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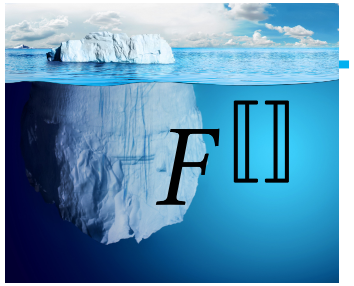
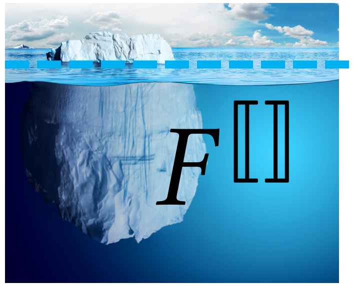


$$e_1\ \boxed{(e_2\quad s\ \ )}$$

Negative levels and top-level splice definitions

$\lambda^{[\![\Rightarrow]\!]}$

$$e_1\ (e_2\ \$e_3\ )$$

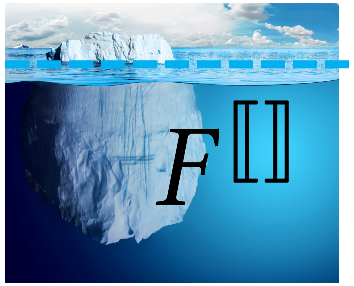


$$e_1\ \boxed{(e_2\quad s\ \ )}$$

Negative levels and top-level splice definitions

$\lambda^{\llbracket\Rightarrow\rrbracket}$

$$e_1\ (e_2\ \$e_3\ )$$

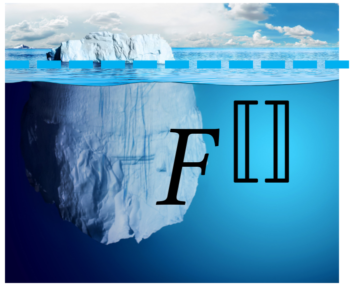


$$\textbf{spdef} \bullet \vdash^{-1} s : \tau = e_3\ ;$$

$$e_1\ \boxed{(\,e_2\quad s\ \ )}$$

Negative levels and top-level splice definitions

$\lambda^{[\![\Rightarrow]\!]}$



$$e_1\ (e_2\ \$e_3\ )$$

compile-time evaluation

$$\mathbf{spdef} \bullet \vdash^{-1} s : \tau = e_3\ ;$$

$$e_1\ \boxed{(e_2\quad s\ )}$$

Type-directed elaboration

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$$\boxed{\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi}$$

Type-directed elaboration

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$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\,\tau \rightsquigarrow e' \mid \phi \quad \Gamma \vdash \tau \rightsquigarrow \tau' \quad \text{fresh}\,s}{\Gamma \vdash^{n} \$e : \tau \rightsquigarrow s \mid \phi, (\bullet \vdash^{n-1} s : \tau' = e')}$$

Type-directed elaboration

$$\boxed{\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi}$$

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Type-directed elaboration

$$\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi$$

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\,\tau \rightsquigarrow \boxed{e'} \mid \phi \quad \Gamma \vdash \tau \rightsquigarrow \tau' \quad \text{fresh } s}{\Gamma \vdash^{n} \$e : \tau \rightsquigarrow s \mid \phi, (\bullet \vdash^{n-1} s : \tau' = \boxed{e'})}$$

$$\frac{\Gamma \vdash^{n+1} e : \tau \rightsquigarrow e' \mid \phi}{\Gamma \vdash^{n} \langle e \rangle : \mathrm{Code}\,\tau \rightsquigarrow \langle e' \rangle_{\phi.n} \mid \lfloor \phi \rfloor^{n}}$$

Type-directed elaboration

$$\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi$$

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\ \tau \rightsquigarrow \boxed{e'} \mid \phi \quad \Gamma \vdash \tau \rightsquigarrow \tau' \quad \text{fresh } s}{\Gamma \vdash^{n} \$e : \tau \rightsquigarrow s \mid \phi, (\bullet \vdash^{n-1} s : \tau' = \boxed{e'})}$$

$$\frac{\Gamma \vdash^{n+1} e : \tau \rightsquigarrow e' \mid \phi}{\Gamma \vdash^{n} \langle e \rangle : \mathrm{Code}\ \tau \rightsquigarrow \langle e' \rangle_{\phi.n} \mid \lfloor \phi \rfloor^{n}}$$

Type-directed elaboration

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get splice
variables at level n

Type-directed elaboration

$$\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi$$

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\ \tau \rightsquigarrow \boxed{e'} \mid \phi \qquad \Gamma \vdash \tau \rightsquigarrow \tau' \qquad \text{fresh } s}{\Gamma \vdash^{n} \$e : \tau \rightsquigarrow s \mid \phi, (\bullet \vdash^{n-1} s : \tau' = \boxed{e'})}$$

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get splice
variables at level n

remove splice
variables at level n

Type-directed elaboration

$$\boxed{\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi}$$

$$\frac{\Gamma \vdash^{n-1} e : \text{Code}\ \tau \rightsquigarrow \boxed{e'} \mid \phi \qquad \Gamma \vdash \tau \rightsquigarrow \tau' \qquad \text{fresh } s}{\Gamma \vdash^{n} \$e : \tau \rightsquigarrow s \mid \phi, (\bullet \vdash^{n-1} s : \tau' = \boxed{e'})}$$

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get splice
variables at level n

remove splice
variables at level n

$$\frac{\Gamma, x : (\tau_1, n) \vdash^{n} e : \tau_2 \rightsquigarrow e \mid \phi_1 \qquad \Gamma \vdash \tau_1 \rightsquigarrow \tau_1' \qquad \phi_1 +\!\!+\, x : (\tau_1', n) \rightsquigarrow \phi_2}{\Gamma \vdash^{n} \lambda x : \tau_1.e : \tau_1 \rightarrow \tau_2 \rightsquigarrow \lambda x : \tau_1'.e \mid \phi_2}$$

Type-directed elaboration

$$\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi$$

$$\frac{\Gamma \vdash^{n-1} e : \text{Code}\ \tau \rightsquigarrow \boxed{e'} \mid \phi \qquad \Gamma \vdash \tau \rightsquigarrow \tau' \qquad \text{fresh } s}{\Gamma \vdash^{n} \$e : \tau \rightsquigarrow s \mid \phi, (\bullet \vdash^{n-1} s : \tau' = \boxed{e'})}$$

$$\frac{\Gamma \vdash^{n+1} e : \tau \rightsquigarrow e' \mid \phi}{\Gamma \vdash^{n} \langle e \rangle : \text{Code}\ \tau \rightsquigarrow \langle e' \rangle_{\phi.n} \mid \lfloor \phi \rfloor^{n}}$$

get splice
variables at level n

remove splice
variables at level n

$$\frac{\Gamma, x : (\tau_1, n) \vdash^{n} e : \tau_2 \rightsquigarrow e \mid \phi_1 \qquad \Gamma \vdash \tau_1 \rightsquigarrow \tau_1' \qquad \phi_1 +\!\!+\, x : (\tau_1', n) \rightsquigarrow \phi_2}{\Gamma \vdash^{n} \lambda x : \tau_1 . e : \tau_1 \to \tau_2 \rightsquigarrow \lambda x : \tau_1'.e \mid \phi_2}$$

Type-directed elaboration

$$\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi$$

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get splice variables at level n

remove splice variables at level n

$$\frac{\Gamma, x : (\tau_1, n) \vdash^{n} e : \tau_2 \rightsquigarrow e \mid \boxed{\phi_1} \qquad \Gamma \vdash \tau_1 \rightsquigarrow \tau_1' \qquad \phi_1 +\!\!+\, x : (\tau_1', n) \rightsquigarrow \phi_2}{\Gamma \vdash^{n} \lambda x : \tau_1 . e : \tau_1 \rightarrow \tau_2 \rightsquigarrow \lambda x : \tau_1' . e \mid \phi_2}$$

Type-directed elaboration

$$\boxed{\Gamma \vdash^{n} \lambda^{[\![\Rightarrow]\!]} \rightsquigarrow F^{[\![]\!]} \mid \phi}$$

$$\frac{\Gamma \vdash^{n-1} e : \mathrm{Code}\ \tau \rightsquigarrow \boxed{e'} \mid \phi \qquad \Gamma \vdash \tau \rightsquigarrow \tau' \qquad \text{fresh } s}{\Gamma \vdash^{n} \$e : \tau \rightsquigarrow s \mid \phi, (\bullet \vdash^{n-1} s : \tau' = \boxed{e'})}$$

$$\frac{\Gamma \vdash^{n+1} e : \tau \rightsquigarrow e' \mid \phi}{\Gamma \vdash^{n} \langle e \rangle : \mathrm{Code}\ \tau \rightsquigarrow \langle e' \rangle_{\phi.n} \mid \lfloor \phi \rfloor^{n}}$$

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get splice
variables at level n

remove splice
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$$\frac{\Gamma, x : (\tau_1, n) \vDash^{n} e : \tau_2 \rightsquigarrow e \mid \boxed{\phi_1} \qquad \Gamma \vdash \tau_1 \rightsquigarrow \tau_1' \qquad \boxed{\phi_1 +\!\!+\, x : (\tau_1', n) \rightsquigarrow \phi_2}}{\Gamma \vDash^{n} \lambda x : \tau_1.e : \tau_1 \rightarrow \tau_2 \rightsquigarrow \lambda x : \tau_1'.e \mid \boxed{\phi_2}}$$

This talk

unsound

inspire $\longrightarrow$ $\lambda^{[\![\Rightarrow]\!]}$ type-directed $\longrightarrow$ $F^{[\![]\!]}$

- Type Classes
- Quotations/Splicing
- Staged type class constraint

- Quotations
- Splice environments

✓ A solid theoretical foundation for integrating type classes into multi-stage programs

✓ easy to implement and stay close to existing implementations

This talk

unsound

$\xrightarrow{\text{inspire}}$ $\lambda^{[\![\Rightarrow]\!]}$ $\xrightarrow{\text{type-directed}}$ $F^{[\![]\!]}$

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- Quotations
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✓ A solid theoretical foundation for integrating type classes into multi-stage programs

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Metatheory: well-typed programs cannot go wrong

A well-typed expression of type τ is either a value, or can reduce to another expression of type τ.

Metatheory: well-typed programs cannot go wrong

$$F^{[\![\,]\!]}$$

$$\begin{array}{lcl} v & ::= & i \mid \lambda x : \tau.e \mid \Lambda a.e \mid \langle e \rangle_{\phi_v} \\ \phi_v & ::= & \bullet \mid \phi_v, \Delta \vDash^n s : \tau = v \end{array}$$

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$$
\begin{aligned}
v &::= i \mid \lambda x : \tau . e \mid \Lambda a . e \mid \langle e \rangle_{\phi_v} \\
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\end{aligned}
$$

A well-typed expression of type τ is either a value, or can reduce to another expression of type τ.

- $\vdash^0 \langle e \rangle_{x:(\mathrm{Code\ Int},0) \vdash^0 s:\mathrm{Int}=x} : \mathrm{Code}\ \tau$

Metatheory: well-typed programs cannot go wrong

$F^{[\![\;]\!]}$

$$\begin{array}{lcl} v & ::= & i \mid \lambda x : \tau.e \mid \Lambda a.e \mid \langle e \rangle_{\phi_v} \\ \phi_v & ::= & \bullet \mid \phi_v, \Delta \vdash^n s : \tau = v \end{array}$$

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$$\bullet \Vdash^0 \langle e \rangle_{x:(\mathrm{Code\ Int},0) \Vdash^0 s:\mathrm{Int}=\boxed{x}} : \mathrm{Code}\ \tau$$

$$\frac{\Gamma, \Delta \Vdash^n e : \mathrm{Code}\ \tau \qquad \Delta \dot{>} n}{\Gamma \vdash \Delta \Vdash^n s : \tau = e}$$

Metatheory: well-typed programs cannot go wrong

$$F^{[\![\,]\!]}$$

$$
\begin{array}{lcl}
v & ::= & i \mid \lambda x : \tau . e \mid \Lambda a . e \mid \langle e \rangle_{\phi_v} \\
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$$\frac{\Gamma, \Delta \vdash^{n} e : \mathrm{Code}\ \tau \qquad \boxed{\Delta \mathrel{\dot{>}} n}}{\Gamma \vdash \Delta \vdash^{n} s : \tau = e}$$

Metatheory: well-typed programs cannot go wrong

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$$
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- $\vdash^0 \langle e \rangle_{x:(\text{Code Int},0) \vdash^0 s:\text{Int}=\boxed{x}} : \text{Code}\ \tau$

- $\vdash^0 \langle\langle e \rangle_{\bullet \vdash^0 s:\text{Int}=(\lambda y:\text{Code Int}.y)\ (\langle 2 \rangle_\bullet)}\rangle_\bullet : \text{Code}\ (\text{Code}\ \tau)$

$$
\frac{\Gamma, \Delta \vdash^{n} e : \text{Code}\ \tau \qquad \boxed{\Delta \dot{>} n}}{\Gamma \vdash \Delta \vdash^{n} s : \tau = e}
$$

Metatheory: well-typed programs cannot go wrong

$F^{[\![\,]\!]}$

$$
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Metatheory: well-typed programs cannot go wrong

$$F^{[\![]\!]}$$

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- $\vdash^{0} \langle e \rangle_{x:(\text{Code Int},0) \vdash^{0} s:\text{Int}=\boxed{x}} : \text{Code } \tau$

- $\vdash^{0} \langle \langle e \rangle_{\bullet \vdash^{0} s:\text{Int}=\boxed{(\lambda y{:}\text{Code Int}.y)\ (\langle 2 \rangle_{\bullet})}} \rangle_{\bullet} : \text{Code } (\text{Code } \tau)$

$$\frac{\Gamma, \Delta \vdash^{n} e : \text{Code } \tau \qquad \boxed{\Delta \mathrel{\dot{>}} n}}{\Gamma \vdash \Delta \vdash^{n} s : \tau = e}$$

$$\frac{\Gamma \vdash^{n} \phi \qquad \Gamma, \phi^{\Gamma} \vdash^{n+1} e : \tau}{\Gamma \vdash^{n} \langle e \rangle_{\phi} : \text{Code } \tau}$$

Metatheory: well-typed programs cannot go wrong

$F^{[\![\,]\!]}$

$$v ::= i \mid \lambda x : \tau.e \mid \Lambda a.e \mid \langle e \rangle_{\phi_v}$$

$$\phi_v ::= \bullet \mid \phi_v, \Delta \Vdash^n s : \tau = v$$

A well-typed expression of type τ is either a value, or can reduce to another expression of type τ.

- $\Vdash^0 \langle e \rangle_{x:(\text{Code Int},0) \Vdash^0 s:\text{Int}=\boxed{x}} : \text{Code } \tau$
- $\Vdash^0 \langle \langle e \rangle_{\bullet \Vdash^0 s:\text{Int}=\boxed{(\lambda y:\text{Code Int}.y)\ (\langle 2 \rangle_\bullet)}} \rangle_\bullet : \text{Code (Code } \tau)$

$$\frac{\Gamma, \Delta \Vdash^n e : \text{Code } \tau \qquad \boxed{\Delta \dot{>} n}}{\Gamma \vdash \Delta \Vdash^n s : \tau = e}$$

$$\frac{\boxed{\Gamma \Vdash^n \phi} \qquad \Gamma, \phi^\Gamma \Vdash^{n+1} e : \tau}{\Gamma \Vdash^n \langle e \rangle_\phi : \text{Code } \tau}$$

Metatheory: well-typed programs cannot go wrong

$\lambda^{[\![\Rightarrow]\!]}$ $\xrightarrow{\text{type-directed}}$ $F^{[\![]\!]}$

$$v ::= i \mid \lambda x : \tau . e \mid \Lambda a.e \mid \langle e \rangle_{\phi_v}$$
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A well-typed expression of type τ is either a value, or can reduce to another expression of type τ.

- $\Vdash^0 \langle e \rangle_{x:(\mathrm{Code\ Int},0) \Vdash^0 s:\mathrm{Int}=\boxed{x}} : \mathrm{Code}\ \tau$
- $\Vdash^0 \langle \langle e \rangle_{\bullet \Vdash^0 s:\mathrm{Int}=\boxed{(\lambda y:\mathrm{Code\ Int}.y)\ (\langle 2 \rangle_\bullet)}} \rangle_\bullet : \mathrm{Code}\ (\mathrm{Code}\ \tau)$

$$\frac{\Gamma, \Delta \Vdash^n e : \mathrm{Code}\ \tau \qquad \boxed{\Delta \dot{>} n}}{\Gamma \vdash \Delta \Vdash^n s : \tau = e}$$

$$\frac{\boxed{\Gamma \Vdash^n \phi} \qquad \Gamma, \phi^{\Gamma} \Vdash^{n+1} e : \tau}{\Gamma \Vdash^n \langle e \rangle_\phi : \mathrm{Code}\ \tau}$$

Metatheory: duality

```
qpower :: Int -> Code Int -> Code Int
qpower 0 n = <1>
qpower k n = <$(n) * $(qpower (k – 1) n)>

qpowerFive :: Int -> Int
qpowerFive n = $(qpower 5 <n>)
             ⟶ $(<$(<n>) * $(qpower (5 – 1) <n>)>)
             ⟶ $(<n>) * $(qpower (5 – 1) n)
             ⟶ n * $(qpower (5 – 1) n)
             ⟶ n * $(qpower 4 n)
             ⟶ ……
             ⟶ n * n * n * n * n * 1
```

Metatheory: duality

$$\lambda^{[\![\Rightarrow]\!]}$$

$$\begin{array}{rcl} \hline \langle \$e \rangle & =_{ax} & e \\ \$\langle e \rangle & =_{ax} & e \\ \hline \end{array}$$

Cancelling splices and quotations out does not change the semantics of programs

Metatheory: duality

$$\lambda^{[\![\Rightarrow]\!]}$$

$$\begin{aligned} \langle \$e \rangle &=_{ax} e \\ \$\langle e \rangle &=_{ax} e \end{aligned}$$

Cancelling splices and quotations out does not change the semantics of programs

$$\dfrac{\dfrac{\Gamma \Vdash^{n} e : \mathrm{Code}\,\tau \rightsquigarrow e \mid \phi}{\Gamma \Vdash^{n+1} \$e : \tau \rightsquigarrow s \mid \phi, \bullet \Vdash^{n} s : \tau = e}}{\Gamma \Vdash^{n} \langle \$e \rangle : \mathrm{Code}\,\tau \rightsquigarrow \langle s \rangle_{\bullet \Vdash^{n} s : \tau = e} \mid \phi}$$

$$\dfrac{\dfrac{\Gamma \Vdash^{n} e : \tau \rightsquigarrow e \mid \phi}{\Gamma \Vdash^{n-1} \langle e \rangle : \mathrm{Code}\,\tau \rightsquigarrow \langle e \rangle_{\phi.n-1} \mid \lfloor \phi \rfloor^{n-1}}}{\Gamma \Vdash^{n} \$\langle e \rangle : \tau \rightsquigarrow s \mid \lfloor \phi \rfloor^{n-1}, \bullet \Vdash^{n-1} s : \tau = \langle e \rangle_{\phi.n-1}}$$

Metatheory: duality

$$\lambda^{[\![\Rightarrow]\!]}$$

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$$\lambda^{[\![\Rightarrow]\!]}$$

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Metatheory: duality

$\lambda^{[\![\Rightarrow]\!]}$

$$\begin{aligned} \langle \$ e \rangle &=_{ax} e \\ \$\langle e \rangle &=_{ax} e \end{aligned}$$

Cancelling splices and quotations out does not change the semantics of programs

$F^{[\![]\!]}$

$$\begin{array}{llll} \langle s \rangle_{\bullet \Vdash^n s:\tau = e} & =_{ax} & e & \\ \langle e_1 \rangle_{\phi_1, \Delta \Vdash^n s:\tau = \langle e \rangle_\phi, \phi_2} & =_{ax} & \langle e_1[s \mapsto e] \rangle_{\phi_1, \phi', \phi_2} & \textit{where } \phi \mathbin{+\!\!+} \Delta \rightsquigarrow \phi' \\ \textbf{spdef } \Delta \Vdash^n s : \tau = \langle e \rangle_\phi ; \rho gm & =_{ax} & \textbf{spdef } \phi' ; \rho gm[s \mapsto e] & \textit{where } \phi \mathbin{+\!\!+} \Delta \rightsquigarrow \phi' \end{array}$$

Metatheory: duality

$\lambda^{[\![\Rightarrow]\!]}$

$$
\begin{aligned}
\langle \$e \rangle &=_{ax} e \\
\$\langle e \rangle &=_{ax} e
\end{aligned}
$$

Cancelling splices and quotations out does not change the semantics of programs

$F^{[\![]\!]}$

$$
\begin{array}{lll}
\langle s \rangle_{\bullet \Vdash^n s:\tau=e} & =_{ax} \ e & \\
\langle e_1 \rangle_{\phi_1, \Delta \Vdash^n s:\tau=\langle e \rangle_\phi, \phi_2} & =_{ax} \ \langle e_1[s \mapsto e] \rangle_{\phi_1, \phi', \phi_2} & \textit{where } \phi \mathbin{+\!\!+} \Delta \rightsquigarrow \phi' \\
\textbf{spdef } \Delta \Vdash^n s : \tau = \langle e \rangle_\phi; \rho gm & =_{ax} \ \textbf{spdef } \phi'; \rho gm[s \mapsto e] & \textit{where } \phi \mathbin{+\!\!+} \Delta \rightsquigarrow \phi'
\end{array}
$$

$$\Theta \vdash \rho gm_1 \simeq_{ctx} \rho gm_2 : \tau$$

Metatheory: duality

$\lambda^{[\![\Rightarrow]\!]}$

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$F^{[\![\,]\!]}$

$$\begin{array}{llll} \langle s \rangle_{\bullet \Vdash^n s:\tau = e} & =_{ax} & e & \\ \langle e_1 \rangle_{\phi_1, \Delta \Vdash^n s:\tau = \langle e \rangle_\phi, \phi_2} & =_{ax} & \langle e_1[s \mapsto e] \rangle_{\phi_1, \phi', \phi_2} & \textit{where } \phi +\!\!+ \Delta \rightsquigarrow \phi' \\ \textbf{spdef } \Delta \Vdash^n s : \tau = \langle e \rangle_\phi ; \rho gm & =_{ax} & \textbf{spdef } \phi' ; \rho gm[s \mapsto e] & \textit{where } \phi +\!\!+ \Delta \rightsquigarrow \phi' \end{array}$$

$$\Theta \vdash \rho gm_1 \simeq_{ctx} \rho gm_2 : \tau$$

Summary

unsound $\xrightarrow{\text{inspire}}$ $\lambda^{[\![\Rightarrow]\!]}$ $\xrightarrow{\text{type-directed}}$ $F^{[\![]\!]}$

$\lambda^{[\![\Rightarrow]\!]}$:

- Type Classes
- Quotations/Splicing
- Staged type class constraint

$F^{[\![]\!]}$:

- Quotations
- Splice environments

✓ A solid theoretical foundation for integrating type classes into multi-stage programs

✓ easy to implement and stay close to existing implementations

Summary

unsound

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- Type Classes
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type-directed →

$F^{[\![\,]\!]}$

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✓ A solid theoretical foundation for integrating type classes into multi-stage programs

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Integration into GHC

Integration into GHC

- Type inference

Integration into GHC

- Type inference
- Local constraints

Integration into GHC

- Type inference
- Local constraints
- Quantified constraints

Integration into GHC

- Type inference
- Local constraints
- Quantified constraints
- Representation of quotations

[POPL 2022]

Staging with Class

A Specification for Typed Template Haskell

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Multi-stage programming using typed code quotation is an established technique for writing optimizing cod generators with strong type-safety guarantees. Unfortunately, quotation in Haskell interacts poorly with typ classes, making it difficult to write robust multi-stage programs.

We study this unsound interaction and propose a resolution, *staged type class constraints*, which we formaliz in a source calculus $\lambda^{[\![\Rightarrow]\!]}$ that elaborates into an explicit core calculus $F^{[\![]\!]}$. We show type soundness of botl calculi, establishing that well-typed, well-staged source programs always elaborate to well-typed, well-staged core programs, and prove beta and eta rules for code quotations.

Our design allows programmers to incorporate type classes into multi-stage programs with confidenc Although motivated by Haskell, it is also suitable as a foundation for other languages that support botl overloading and quotation.

CCS Concepts: • **Software and its engineering** → **Functional languages**; **Semantics**; • **Theory o computation** → **Type theory**.

Matthew Pickering

Andres Löh

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Staging with Class

A Specification of Typed Template Haskell

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