Towards Optimal Variance Reduction in Online Experiments

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Randomized experiments in tech companies

- A/B testing, randomized experiments, online controlled experiments...
  - Units are randomly assigned to treated / control groups
  - Measure outcomes after a period
  - Evaluate and compare the outcomes in two groups
  - If the effect is significantly positive, then adopt the new feature

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- Desire estimator of treatment effects with smaller variance
  - What treatment effect?
  - How to reduce variance?
  - What is the best effort?

Overview of the work

- A rigorous statistical framework for variance reduction of count and ratio metrics
- Methodology of unbiased variance reduction with flexible machine learning tools and large numbers of covariates
- Optimality in the sense of semiparametric efficiency of all procedures
- Performance on (simulated and) real data

Potential outcome framework

- i.i.d. units $i = 1, \ldots, n$ from $\mathbb{P}$.
- Potential outcomes $Y_i(1)$, $Y_i(0)$.
- Treatment $T_i \overset{\text{i.i.d.}}{\sim} \text{Bernoulli}(p)$.
- SUTVA: observe $Y_i(1)$ for $T_i = 1$ and $Y_i(0)$ for $T_i = 0$.
- $n_t = \sum_i T_i$ size of treated group, $n_c = n - n_t$ size of control group.

Randomization and metrics

- Case 1: Count metric $\tau = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$
- Default estimator: difference-in-mean

$$\widehat{\tau}_{\text{DIM}} = \frac{1}{n_t} \sum_{i \text{ treated}} Y_i - \frac{1}{n_c} \sum_{i \text{ control}} Y_i .$$

Randomization and metrics

- Case 2: ratio metric

$$\frac{\sum_{i \text{ treated}} Y_i}{\sum_{i \text{ treated}} Z_i} = \frac{\frac{1}{n_t}\sum_{i \text{ treated}} Y_i}{\frac{1}{n_t}\sum_{i \text{ treated}} Z_i}, \quad \frac{\sum_{i \text{ control}} Y_i}{\sum_{i \text{ control}} Z_i} = \frac{\frac{1}{n_c}\sum_{i \text{ control}} Y_i}{\frac{1}{n_c}\sum_{i \text{ control}} Z_i},$$

where i stands for a cluster, Y_i is the aggregated outcome, Z_i is the size of cluster.

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- If SDA holds, the population quantity is $\delta' = \frac{\mathbb{E}[Y_i(1)]}{\mathbb{E}[Z_i]} - \frac{\mathbb{E}[Y_i(0)]}{\mathbb{E}[Z_i]}$.

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- If SDA holds, the population quantity is $\delta' = \frac{\mathbb{E}[Y_i(1)]}{\mathbb{E}[Z_i]} - \frac{\mathbb{E}[Y_i(0)]}{\mathbb{E}[Z_i]}$.
- If SDA does not hold, the population quantity is $\delta = \frac{\mathbb{E}[Y_i(1)]}{\mathbb{E}[Z_i(1)]} - \frac{\mathbb{E}[Y_i(0)]}{\mathbb{E}[Z_i(0)]}$.

Methods in the literature

- For count metrics...
- Earlier: linear adjustment [Lin, 2013], CUPED [Deng et al., 2013]..
  - linearity is restrictive
  - cannot handle large numbers of covariates
- Recently: machine learning, large numbers of covariates [Guo et al., 2021]
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Methods in the literature

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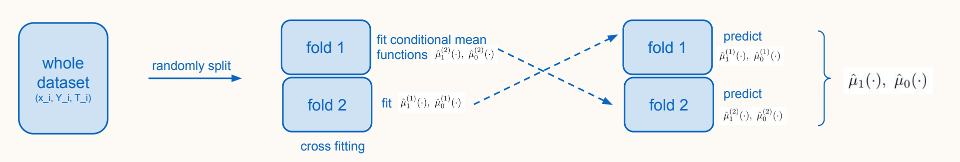
- For ratio metrics...
  - less studied, lack of rigorous statistical framework and guarantee
  - CUPED extension cannot use general covariates
  - optimality of procedures?

High-level idea: fit-then-debias

- Why is diff-in-mean estimator not efficient?
  - Decreased sample size (half of n units are treated / control)
  - Unpaired comparison (variance is the sum of treated and control)
  - Ideal estimator: $\frac{1}{n}\sum_{i=1}^{n}[Y_i(1) - Y_i(0)]$ (for count metrics)
- General idea:
  - with covariates X_i, use ML estimators to predict the missing outcome and plug in to perform pairwise comparison $\Rightarrow \frac{1}{n}\sum_{i=1}^{n}[\widehat{\mu}_1(X_i) - \widehat{\mu}_0(X_i)]$?
  - Bias can be larger than variance!
- De-biasing techniques
  - use cross-fitting to fit the estimators
  - add a de-biasing term to correct for the bias of $\widehat{\mu}_w$

Procedure for count metrics



- Step 1: sample splitting into $\mathcal{D}^{(k)}$, $k = 1, \ldots, K$.
- Step 2: use $\mathcal{D} \backslash \mathcal{D}^{(k)}$ to estimate $\widehat{\mu}_1^{(k)}$ and $\widehat{\mu}_0^{(k)}$.
- Step 3: Plug into $\mathcal{D}^{(k)}$ to obtain $\widehat{\mu}_1(X_i) = \widehat{\mu}_1^{(k)}(X_i)$, $\widehat{\mu}_0(X_i) = \widehat{\mu}_0^{(k)}(X_i)$ for all $i \in \mathcal{D}^{(k)}$.
- Step 4: Estimator

$$\widehat{\theta} = \frac{1}{n}\sum_{i=1}^{n}\left(\widehat{\mu}_1(X_i) - \widehat{\mu}_0(X_i)\right) + \frac{1}{n_t}\sum_{i \text{ treated}}\left(Y_i(1) - \widehat{\mu}_1(X_i)\right) - \frac{1}{n_c}\sum_{i \text{ control}}\left(Y_i(0) - \widehat{\mu}_0(X_i)\right),$$

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- Valid inference when $\|\widehat{\mu}_w^{(k)} - \mu_w^*\|_2 \xrightarrow{P} 0$ for deterministic functions μ_w^*, $w \in \{0,1\}$.
- Semiparametrically efficient when $\mu_w^*(x) = \mathbb{E}[Y(w) \,|\, X = x]$, $w \in \{0,1\}$.

Count metrics: implications on optimality

- When estimators converge in L_2 to true conditional mean functions,

$$\mathrm{Var}(\widehat{\theta}) \approx \underbrace{\frac{1}{n}\mathrm{Var}\left(\mu_1(X_i) - \mu_0(X_i)\right)}_{\text{(i) predictable part}} + \underbrace{\frac{1}{n_t}\mathrm{Var}\left(Y_i(1) - \mu_1(X_i)\right) + \frac{1}{n_c}\mathrm{Var}\left(Y_i(0) - \mu_0(X_i)\right)}_{\text{(ii) irreducible variance}}$$

  - (i) is the best efforts in predicting $Y(1) - Y(0)$ given X.
  - (ii) is the intrinsic uncertainty that cannot be eliminated.

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    $\Rightarrow$ This method is better than CUPED when there is nonlinearity

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- Take-away message on optimality:
  - Should target for conditional mean functions
    $\Rightarrow$ This method is better than CUPED when there is nonlinearity
  - Estimate for two groups $\mathbb{E}[Y(1) \mid X = x]$ and $\mathbb{E}[Y(0) \mid X = x]$ separately
    $\Rightarrow$ This method is better than one single estimator when there is treatment heterogeneity

Ratio metrics: optimal procedures

- We develop two procedures for δ and δ' under different conditions of denominators.
- For the target $\delta = \frac{\mathbb{E}[Y_i(1)]}{\mathbb{E}[Z_i(1)]} - \frac{\mathbb{E}[Y_i(0)]}{\mathbb{E}[Z_i(0)]}$ without SDA
  - fit-then-debias for $\mathbb{E}[Y(w) \mid X = x]$, $\mathbb{E}[Z(w) \mid X = x]$ respectively, $w \in \{0, 1\}$.
- For the target $\delta' = \frac{\mathbb{E}[Y_i(1)]}{\mathbb{E}[Z_i]} - \frac{\mathbb{E}[Y_i(0)]}{\mathbb{E}[Z_i]}$ under SDA,
  - pool all Z_i to estimate $\mathbb{E}[Z]$
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  - pool all Z_i to estimate $\mathbb{E}[Z]$
  - fit-then-debias for $\mathbb{E}[Y(w) \mid X = x, Z = z]$, $w \in \{0, 1\}$.
- Valid inference when estimators converge to deterministic functions (L_2 distance in probability)
- Optimality when they converge to true conditional mean functions

Ratio metrics: optimal procedures

- Ratio metric: target quantity $\delta = \frac{\mathbb{E}[Y_i(1)]}{\mathbb{E}[Z_i(1)]} - \frac{\mathbb{E}[Y_i(0)]}{\mathbb{E}[Z_i(0)]}$

$$\widehat{\delta} = \frac{\sum_{i=1}^n A_i}{\sum_{i=1}^n B_i} - \frac{\sum_{i=1}^n C_i}{\sum_{i=1}^n D_i},$$

$$\text{where } A_i = \widehat{\mu}_1^Y(X_i) + \frac{T_i}{\widehat{p}}(Y_i - \widehat{\mu}_1^Y(X_i)), \quad B_i = \widehat{\mu}_1^Z(X_i) + \frac{T_i}{\widehat{p}}(Z_i - \widehat{\mu}_1^Z(X_i)),$$

$$C_i = \underbrace{\widehat{\mu}_0^Y(X_i)}_{\text{regressor + plug-in}} + \underbrace{\frac{1-T_i}{1-\widehat{p}}(Y_i - \widehat{\mu}_0^Y(X_i))}_{\text{de-bias}}, \quad D_i = \widehat{\mu}_0^Z(X_i) + \frac{1-T_i}{1-\widehat{p}}(Z_i - \widehat{\mu}_0^Z(X_i)).$$

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$$\widehat{\delta}' = \frac{\sum_{i=1}^n \Gamma_i}{\sum_{i=1}^n Z_i}, \quad \text{where } \Gamma_i = \underbrace{\widehat{\mu}_1(X_i, Z_i) - \widehat{\mu}_0(X_i, Z_i)}_{\text{regressor + plug-in}} + \underbrace{\frac{T_i}{\widehat{p}}(Y_i - \widehat{\mu}_1(X_i, Z_i)) - \frac{1-T_i}{1-\widehat{p}}(Y_i - \widehat{\mu}_0(X_i, Z_i))}_{\text{de-bias}},$$

Real data performance

- Count metric: LinkedIn Feed experiment, revenue metric
  - $n = 400,000$ subsample of users
  - Our estimator with random forest from `scikit-learn` python library
  - Incorporate user covariates
  - Reduces 22.22% of variance compared to diff-in-mean, while CUPED reduces 15.91%

Real data performance

- Ratio metric: enterprise experiment in LinkedIn Learning, 'learning engagement' metric
  - $n = 10,299$ enterprise accounts
  - Our estimator with `XGBoost` python library
  - Incorporate enterprise covariates
  - For δ, reduces 12.35% of variance compared to diff-in-mean, while CUPED reduces 1.76%
  - For δ', reduces 83.6% of variance compared to diff-in-mean, while CUPED reduces 76.62%

Thanks!

Our paper: https://arxiv.org/abs/2110.13406