Homework 2: Theory (20 pts)
Programming Languages (CSCI 3300), Fall 2018
Due: Thursday, Sept. 20 by 5:15pm

The following defines the Iffy Language:

Syntax:

$$(\text{Bool}) \quad b \quad ::= \quad 1 \mid 0 \mid b \wedge b \mid b \vee b \mid \text{if } b \text{ then } b \text{ else } b$$

Single-step reduction:

$$\frac{}{(1 \wedge 1) \rightsquigarrow 1} \text{ AndTrue} \qquad \frac{}{(0 \wedge 0) \rightsquigarrow 0} \text{ AndFalse} \qquad \frac{}{(0 \wedge 1) \rightsquigarrow 0} \text{ AndFalse1}$$

$$\frac{}{(1 \wedge 0) \rightsquigarrow 0} \text{ AndFalse2} \qquad \frac{b_1 \rightsquigarrow b_1'}{(b_1 \wedge b_2) \rightsquigarrow (b_1' \wedge b_2)} \text{ And1} \qquad \frac{b_2 \rightsquigarrow b_2'}{(b_1 \wedge b_2) \rightsquigarrow (b_1 \wedge b_2')} \text{ And2}$$

$$\frac{}{(1 \vee 1) \rightsquigarrow 1} \text{ OrTrue} \quad \frac{}{(1 \vee 0) \rightsquigarrow 1} \text{ OrTrue1} \quad \frac{}{(0 \vee 1) \rightsquigarrow 1} \text{ OrTrue2} \quad \frac{}{(0 \vee 0) \rightsquigarrow 0} \text{ OrFalse}$$

$$\frac{b_1 \rightsquigarrow b_1'}{(b_1 \vee b_2) \rightsquigarrow (b_1' \vee b_2)} \text{ Or1} \qquad \frac{b_2 \rightsquigarrow b_2'}{(b_1 \vee b_2) \rightsquigarrow (b_1 \vee b_2')} \text{ Or2} \qquad \frac{}{\text{if } 1 \text{ then } b_1 \text{ else } b_2 \rightsquigarrow b_1} \text{ IfTrue}$$

$$\frac{}{\text{if } 0 \text{ then } b_1 \text{ else } b_2 \rightsquigarrow b_2} \text{ IfFalse} \qquad \frac{b \rightsquigarrow b'}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b' \text{ then } b_1 \text{ else } b_2} \text{ If1}$$

$$\frac{b_1 \rightsquigarrow b_1'}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b \text{ then } b_1' \text{ else } b_2} \text{ If2} \qquad \frac{b_2 \rightsquigarrow b_2'}{\text{if } b \text{ then } b_1 \text{ else } b_2 \rightsquigarrow \text{if } b \text{ then } b_1 \text{ else } b_2'} \text{ If3}$$

1. (5 pts) Show that the following is derivable:

$$(0 \vee (0 \wedge 1)) \rightsquigarrow 0 \vee 0$$

2. (15 pts) Show that the following is derivable:

if $(0 \wedge 1 \wedge (1 \vee 0))$ then $0 \vee (\text{if } 0 \text{ then } 1 \wedge 1 \text{ else } 0)$ else $(0 \vee 1) \rightsquigarrow$ if $(0 \wedge 1 \wedge (1 \vee 0))$ then $0 \vee (\text{if } 0 \text{ then } 1 \text{ else } 0)$ else $(0 \vee 1)$