

## แบบฝึกหัด 6 LEAST-SQUARES REGRESSION

### 1. LINEAR REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

1.1 จงใช้ LINEAR REGRESSION ในการคำนวณหา  $f(x) = a_0 + a_1x_1$  และค่า  $f(65)$

1.2 จงเขียน code

### 2. POLYNOMIAL REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

2.1 จงใช้ POLYNOMIAL REGRESSION order  $m = 2$  ในการสร้างสมการ  $f(x) = a_0 + a_1x + a_2x^2$

2.2 จงเขียน code

### 3. MULTIPLE LINEAR REGRESSION

$x_1$	$x_2$	$x_3$	Y
1	0	1	4
0	1	3	-5
2	4	1	-6
3	2	2	0
4	1	5	-1
2	3	3	-7
1	6	4	-20

3.1 จงใช้ MULTIPLE LINEAR REGRESSION ในการสร้างสมการ  $f(x) = a_0 + a_1x_1 + a_2x_2 + a_3x_3$

3.2 จงเขียน code

## 1. LINEAR REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

1.1 จงใช้ LINEAR REGRESSION ในการคำนวณหา  $f(x) = a_0 + a_1x_1$  และค่า  $f(65)$

1.2 จงเขียน code

$x_i$	$y_i$	$x_i^2$	$x_i y_i$		
10	5	100	50	$a_0 = \frac{(\sum_{i=1}^n y_i)(\sum_{i=1}^n x_i^2) - (\sum_{i=1}^n x_i y_i)(\sum_{i=1}^n x_i)}{n(\sum_{i=1}^n x_i^2) - (\sum_{i=1}^n x_i)^2}$	$a_1 = \frac{n(\sum_{i=1}^n x_i y_i) - (\sum_{i=1}^n x_i)(\sum_{i=1}^n y_i)}{n(\sum_{i=1}^n x_i^2) - (\sum_{i=1}^n x_i)^2}$
15	9	225	135	$= \frac{(215)(20625) - (1165)(395)}{9(22825) - (975)^2}$	$= \frac{9(1165) - (975)(215)}{9(20625) - (975)^2}$
20	15	400	300	$= 1.313333$	$= 0.519999$
30	18	900	540		
40	22	1600	880		
50	30	2500	1500		
60	35	3600	2100	$f(65) = 1.313333 + 0.519999x$	
70	38	4900	2660	$= 36.340000$	
80	43	6400	3440		
$\Sigma$	395	215	20625	1165	

```
1 const math = require('mathjs');
2
3 function pow_sum(x){
4   let sum=0;
5   for(let i=0; i<x.length; i++){
6     sum += Math.pow(x[i],2);
7   }
8   return sum;
9 }
10
11 function linear_regression(points,x,y,fx){
12   let A = [
13     [points.length,math.sum(x)],
14     [math.sum(x),pow_sum(x)]
15   ];
16   A = math.inv(A)
17
18   let B = [math.sum(y),math.multiply(x,y)];
19
20   let a = math.multiply(A,B);
21
22   return "f("+fx+") = "+parseFloat(a[0]).toFixed(6)+" + "+parseFloat(a[1]).toFixed(6)+"x"+"ln"+
23     " | "f("+fx") = "+((a[0] + (a[1] * fx)).toFixed(6));
24 }
25
26 let points = [
27   {x: 10, y: 5},
28   {x: 15, y: 9},
29   {x: 20, y: 15},
30   {x: 30, y: 18},
31   {x: 40, y: 22},
32   {x: 50, y: 30},
33   {x: 60, y: 35},
34   {x: 70, y: 38},
35   {x: 80, y: 43}
36 ]
37
38 let x = points.map(points => points.x);
39 let y = points.map(points => points.y);
40 let fx = 65;
41
42 console.log(linear_regression(points,x,y,fx));
```

## 2. POLYNOMIAL REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

2.1 จงใช้ POLYNOMIAL REGRESSION order m = 2 ในการสร้างสมการ  $f(x) = a_0 + a_1x + a_2x^2$

2.2 จงเขียน code

$x_i$	$y_i$	$x_i^2$	$x_i^3$	$x_i^4$	$x_i \cdot f(x_i)$	$x_i^2 \cdot f(x_i)$
10	5	100	1000	10000	50	500
15	9	225	3375	50625	135	2025
20	15	400	8000	160000	300	6000
30	18	900	27000	810000	540	16200
40	22	1600	64000	2560000	880	35200
50	30	2500	125000	6250000	1500	75000
60	35	3600	216000	12960000	2100	126000
70	38	4900	343000	24010000	2660	186200
80	43	6400	512000	40960000	3440	275200
$\Sigma$	395	215	20625	1161975	11605	783525

$$y_i = a_0 + a_1x_i + a_2x_i^2 \quad f(x) = a_0 + a_1x + a_2x^2$$
$$\begin{bmatrix} 11 & \sum x_i & \sum x_i^2 \\ \sum x_i y_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 y_i & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum f(x_i) \\ \sum x_i \cdot f(x_i) \\ \sum x_i^2 \cdot f(x_i) \end{bmatrix}$$
$$\begin{bmatrix} 11 & 395 & 20625 \\ 11605 & 20625 & 1161975 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 215 \\ 783525 \\ 1296000 \end{bmatrix}$$
$$A \cdot B$$
$$\begin{bmatrix} -0.700930 \\ 0.641524 \\ -0.001943 \end{bmatrix}$$

$$f(x) = -0.700930 + 0.641524x + (-0.001943)x^2$$
$$= 36.340000$$

```
1 const math = require('mathjs'); //import math.js
2
3 function polynomial_regression(x,y,fx){
4   let sumx2=0,sumx3=0,sumx4=0,sumxy=0,sumx2y=0,n=x.length;
5
6   for(let i=0;i<n;i++){
7     sumx2 += Math.pow(x[i],2);
8     sumx3 += Math.pow(x[i],3);
9     sumx4 += Math.pow(x[i],4);
10    sumxy += x[i] * y[i];
11    sumx2y += Math.pow(x[i],2) * y[i];
12  }
13
14  let A = [
15    [n,math.sum(x),sumx2],
16    [math.sum(x),sumx2,sumx3],
17    [sumx2,sumx3,sumx4]
18  ];
19  A = math.inv(A); //matrix inverse
20
21  let B = [math.sum(y),sumxy,sumx2y];
22
23  let a = math.multiply(A,B); //multiply matrix A and B
24
25  return "f(x) = "+parseFloat(a[0]).toFixed(6)+" + "+parseFloat(a[1]).toFixed(6)+"x"+ " + "+parseFloat(a[2]).toFixed(6)+"x^2"+"ln"+
26    " | "f("fx") = "+parseFloat(a[0] + a[1] * fx + a[2] * Math.pow(fx,2)).toFixed(6);
27 }
28
29 let points = [
30   {x: 10, y: 5},
31   {x: 15, y: 9},
32   {x: 20, y: 15},
33   {x: 30, y: 18},
34   {x: 40, y: 22},
35   {x: 50, y: 30},
36   {x: 60, y: 35},
37   {x: 70, y: 38},
38   {x: 80, y: 43}
39 ]
40
41 let x = points.map(points => points.x);
42 let y = points.map(points => points.y);
43 let fx = 65;
44
45 console.log(polynomial_regression(x,y,fx));
```

### 3. MULTIPLE LINEAR REGRESSION

X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	Y	x <sub>1</sub> x <sub>1</sub>	x <sub>1</sub> x <sub>2</sub>	x <sub>1</sub> x <sub>3</sub>	x <sub>1</sub> x <sub>2</sub>	x <sub>1</sub> x <sub>3</sub>	x <sub>2</sub> x <sub>3</sub>	x <sub>1</sub> y	x <sub>2</sub> y	x <sub>3</sub> y
1	0	1	4	1	0	1	0	1	0	4	0	4
0	1	3	-5	0	1	3	0	0	3	0	-5	-15
2	4	1	-6	4	16	1	8	2	4	-12	-24	-6
3	2	2	0	9	4	4	6	6	4	0	0	0
4	1	5	-1	16	1	25	4	20	5	-4	-1	-5
2	3	3	-7	4	9	9	6	6	9	-14	-21	-21
1	6	4	-20	1	36	16	6	4	24	-10	-120	-80
Σ	13	17	19	-35	35	67	65	30	39	49	-46	-171

3.1 จงใช้ MULTIPLE LINEAR REGRESSION ในการสร้างสมการ  $f(x) = a_0 + a_1x_1 + a_2x_2 + a_3x_3$

3.2 จงเขียน code

$$\begin{bmatrix} n & \sum_{i=1}^n x_{1i} & \sum_{i=1}^n x_{2i} & \sum_{i=1}^n x_{3i} \\ \sum_{i=1}^n x_{1i} & \sum_{i=1}^n x_{1i}^2 & \sum_{i=1}^n x_{1i}x_{2i} & \sum_{i=1}^n x_{1i}x_{3i} \\ \sum_{i=1}^n x_{2i} & \sum_{i=1}^n x_{1i}x_{2i} & \sum_{i=1}^n x_{2i}^2 & \sum_{i=1}^n x_{2i}x_{3i} \\ \sum_{i=1}^n x_{3i} & \sum_{i=1}^n x_{1i}x_{3i} & \sum_{i=1}^n x_{2i}x_{3i} & \sum_{i=1}^n x_{3i}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_{1i}y_i \\ \sum_{i=1}^n x_{2i}y_i \\ \sum_{i=1}^n x_{3i}y_i \end{bmatrix}$$

$$\begin{bmatrix} 7 & 13 & 17 & 19 \\ 13 & 35 & 30 & 39 \\ 17 & 30 & 67 & 49 \\ 19 & 39 & 49 & 65 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -35 \\ -46 \\ -171 \\ -123 \end{bmatrix}$$

$$A^{-1} \cdot B$$

$$\begin{bmatrix} 4.000000 \\ 2.000000 \\ -1.000000 \end{bmatrix}$$

$$f(x) = 4.000000 + 2.000000x_1 + (-1.000000)x_2 + (-1.000000)x_3$$

```

1  const math = require('mathjs');
2
3  function multiple_linear_regression(x1,x2,x3,y,fx){
4      let A = [
5          [x1.length,math.sum(x1),math.sum(x2),math.sum(x3)],
6          [math.sum(x1),math.multiply(x1,x1),math.multiply(x1,x2),math.multiply(x1,x3)],
7          [math.sum(x2),math.multiply(x1,x2),math.multiply(x2,x2),math.multiply(x2,x3)],
8          [math.sum(x3),math.multiply(x1,x3),math.multiply(x2,x3),math.multiply(x3,x3)]
9      ]
10
11      let B = [math.sum(y),math.multiply(x1,y),math.multiply(x2,y),math.multiply(x3,y)];
12
13      A = math.inv(A); //matrix inverse
14
15      let a = math.multiply(A,B); //multiply matrix A and B
16
17      return "f(x) = "+parseFloat(a[0]).toFixed(6)+" + "+parseFloat(a[1]).toFixed(6)+"x1"+" + "+parseFloat(a[2]).toFixed(6)+"x2"+" + "+parseFloat(a[3]).toFixed(6)+"x3";
18  }
19
20  let points = [
21      {x1: 1,x2: 0,x3: 1,y: 4},
22      {x1: 0,x2: 1,x3: 3,y: -5},
23      {x1: 2,x2: 4,x3: 1,y: -6},
24      {x1: 3,x2: 2,x3: 2,y: 0},
25      {x1: 4,x2: 1,x3: 5,y: -1},
26      {x1: 2,x2: 3,x3: 3,y: -7},
27      {x1: 1,x2: 6,x3: 4,y: -20},
28  ]
29
30  let x1 = points.map(points => points.x1);
31  let x2 = points.map(points => points.x2);
32  let x3 = points.map(points => points.x3);
33  let y = points.map(points => points.y);
34  let fx = 65;
35
36  console.log(multiple_linear_regression(x1,x2,x3,y,fx));

```