COMS21103: String Matching

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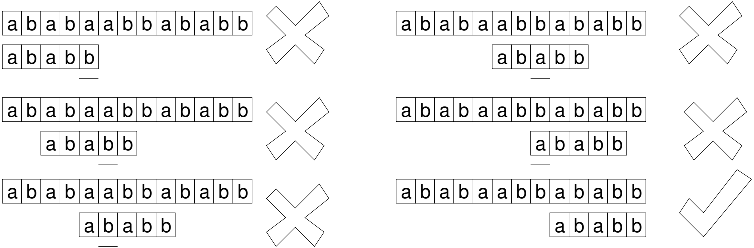
Introduction - String Matching

In this lecture we look at string matching algorithms.



In simple terms, we want to find all the occurrences of some string P in a larger string T.

Introduction

- This seems like a very simple problem, and it is:
  - The real problem is one of efficiency in time and space.
  - Doing many matching operations demands a better than naive approach.
- High performance string matching is vital in many applications:
  - In web searches or databases:
    - We might search stored text for a keyword supplied by the user.
  - In Unix tools like grep:
    - We need to match regular expressions against input text streams.
  - In DNA matching:
    - We match a small DNA strand against a large corpus
    - Here, as in many situations, inexact matching is also required

Introduction

- We look at four different exact string matching methods:
  - The naive, obvious method.
  - The Knuth-Morris-Pratt (KMP) algorithm.
  - The Boyer-Moore-Horspool (BMH) algorithm.
  - The Finite State Machine (FSM) algorithm.
- To compare each different approach, we count the number of comparisons they do:
  - Performance is mostly determined by how many comparisons are performed.
  - It is therefore a good candidate as our computational step.
  - To make absolutely sure, we should check that the run-time is linearly related to the number of comparisons in each case (it is).

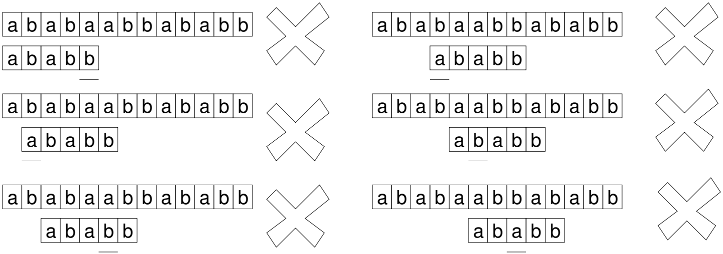
Naive Method – Algorithm

- The basic idea is this:
  - Match pattern string against input string character by character.
  - When there is a mismatch, shift the whole input string down by one character in relation to the pattern string, and start again at the beginning.

Input: Strings P and T
$n \leftarrow |T|$;
$m \leftarrow |P|$;
for $i \leftarrow 0$ *to* $n - m$ **do**
  if $P[1, \ldots, m] = T[i + 1, \ldots, i + m]$ **then**
    $\textsc{Output}(i + 1)$;
  end
end

Naive Method – Example

- Consider matching some example arrays where we set $T = \texttt{ababaabbababb}$ and $P = \texttt{ababb}$



- The underlined characters are where the match fails.
- This performs a total of 23 comparisons to find a match.

Naive Method – Summary

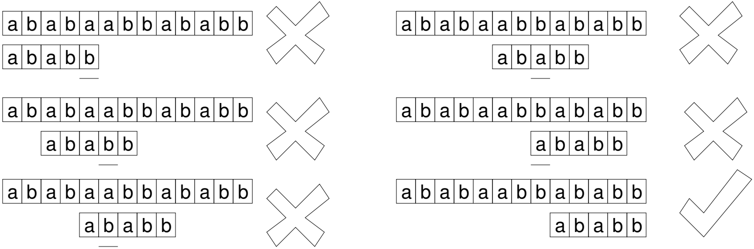
- However, consider a worst case example:
  - The input text is `aaa...b` and of total length n.
  - That is, $n - 1$ `a` characters followed by one `b`.
  - The pattern is `aaa...b` and of total length m.
  - That is, $m - 1$ `a` characters followed by one `b`.
- Using the naive method, we match up to the m-th character and after each mismatch, restart at the first character:
  - The mismatch occurs $n - m$ times.
  - The match succeeds at position $n - m + 1$.
  - The total number of comparisons is therefore $(n - m + 1) \cdot m \in \Theta(nm)$.

Knuth-Morris-Pratt – Algorithm

- When a mismatch occurs at index j in the naive method, we have found $j-1$ characters that do match.
- We can take advantage of this when deciding where to restart the match:
  - Imagine a case where the string `ababb` mismatches at the 5th character.
  - The matched text consists of `abab?` where `?` is unknown.
  - We restart the match by comparing the 3rd character, an `a`, against `?`.
- In short, since we know the pattern beforehand we can work out where to restart the match.

Knuth-Morris-Pratt – Example

- Consider matching the same example arrays where we set $T = \texttt{ababaabbababb}$ and $P = \texttt{ababb}$:



- Note that we are shifting the pattern either by one or a distance from a precomputed prefix table.
- Now we only perform a total of 17 comparisons.

Knuth-Morris-Pratt – Algorithm

- The basic idea is this:
  - First compute the prefix table of the pattern which tells us where to restart.
  - Then when we run the KMP matcher, use the table when a mismatch occurs.

  What is a prefix table? The prefix table tells us how far we can shift the pattern along at each turn without missing any matches. Let $P = ababb$.
  - The jth element of the prefix table for P is the length of the longest prefix of $P[1, \ldots, j]$ that is also a proper suffix of $P[1, \ldots, j]$
  - $P[1, 1] = a$. There is no proper suffix of a so the first element of the prefix table is 0
  - $P[1, 2] = ab$. The only proper suffix of ab is b which is not a prefix of ab. Therefore the second element of the prefix table is 0
  - $P[1, 3] = aba$. The proper suffices are ba and a. a is a prefix of aba so the third element of the prefix table is 1.
  - The fourth and fifth elements of the prefix table are therefore 2 and 0.

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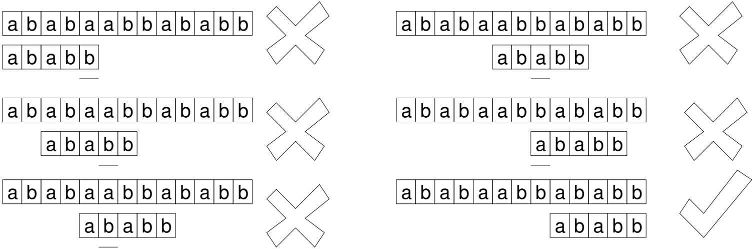
Knuth-Morris-Pratt – Algorithm

The overall algorithm structure is as follows.

```
KMP-MATCHER(T, P)
begin
  n ← |T|
  m ← |P|
  Π ← KMP-PREFIX(P)
  i ← 0
  for j = 1 upto n step 1 do
    while i > 0 and P[i + 1] ≠ T[j] do
      i ← Π[i]            ▷ Skip using prefix table
    if P[i + 1] = T[j] then
      i ← i + 1           ▷ Next character matches
    if i = m then
      OUTPUT(j − m)       ▷ Pattern at shift j − m
      i ← Π[i]            ▷ Look for next match
end
```

Knuth-Morris-Pratt – Example

- Let's look at the example again. $T = \texttt{ababaabbababb}$ and $P = \texttt{ababb}$:
  - We calculate the prefix table as $\Pi = \{0, 0, 1, 2, 0\}$.



- We use the prefix table to shift the pattern as required.

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Knuth-Morris-Pratt – Running Time

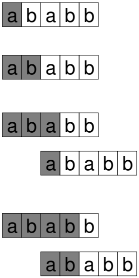
What is the largest number of comparison KMP can take for a pattern of length m and a text of length n? Recall that i is the current index in the pattern and j is the current index in the text.

- There can be at most $n-1$ successful comparisons
- There can be at most $n-1$ failed comparisons since the number of times we decrease index i cannot exceed the number of times we increment i
- At max there are $2n-2$ comparisons
- KMP takes $O(n)$ time assuming that the prefix table is available

Knuth-Morris-Pratt – Prefix Table

We can compute the prefix table by comparing the pattern against itself.

- For each $j \leq m$ we compute the length of the longest prefix of $P[1, \ldots, j]$ that is also a proper suffix of $P[1, \ldots, j]$.
- This can be done cleverly in $O(m)$ time



KMP-PREFIX(P)
begin
  $m \leftarrow |P|$
  $\Pi[1] \leftarrow 0$
  $i \leftarrow 0$
  for $j = 2$ **upto** m **step** 1 **do**
    while $i > 0$ **and** $P[i+1] \neq P[j]$ **do**
      $i \leftarrow \Pi[i]$
    if $P[i+1] = P[j]$ **then**
      $i \leftarrow i + 1$
    $\Pi[j] \leftarrow i$
  return Π
end

Knuth-Morris-Pratt – Summary

- The KMP algorithm never needs to back-track in the input text:
  - This is an advantage if the text is streamed rather than in an array since we don't have to maintain a buffer for the stream.
- The worst case performance is $O(n)$ comparisons.
- However, it doesn't improve much on the average case:
  - Best performance when alphabet is small since this means higher chance of repeated substrings in the input and pattern.

Boyer-Moore-Horspool – Algorithm

- The main goals of BMH are simplicity and improving on the weakness of KMP over large alphabets
  - Use the alphabet that makes up the pattern and input text to skip large distances.
  - Make comparisons starting on the right of the pattern rather than the left, this finds the rightmost mismatch.
- The trick is to consider the alphabet over which the text is defined:
  - The alphabet is the set of characters $C_0, C_1, \ldots, C_{k-1}$.
- We build a table Γ which tells us the rightmost occurrence of each letter in the pattern.
  - For each C_i in the pattern string, set $\Gamma[i]$ to be the position of the rightmost occurrence of C_i in the pattern.

Boyer-Moore-Horspool – Example

- Say we want to search for the pattern `lean` in the following string:

c	a	r	p	e	t	s		n	e	e	d		c	l	e	a	n	i	n	g
l	e	a	n																	

- The characters `n` and `p` mismatch, also `p` doesn't appear anywhere in the pattern so we can move all the way past `p` and restart:

c	a	r	p	e	t	s		n	e	e	d		c	l	e	a	n	i	n	g
				l	e	a	n	l	e	a	n	l	e	a	n					

- The characters `n` and `e` mismatch, but now `e` occurs in the pattern so line them up:
- Only 8 comparisons needed in this example. KMP would need 18.

c	a	r	p	e	t	s		n	e	e	d		c	l	e	a	n	i	n	g
														l	e	a	n			

Boyer-Moore-Horspool – Summary

- In the worst case no better then naive matching:
  - The Horspool heuristic is very fast in practice
- The method works best when you have little repetition within the input text.
  - The worst case is $O(nm)$ comparisons. Consider $p = ba^{m-1}$ and $t = a^n$.
  - The best case is now $O(n/m)$ comparisons.
  - Fast in practice.

Finite State Machines (1)

- The previous methods are quite efficient but also quite simplistic in what they can do:
  - What happens if we want to consider regular expressions ?
- This is exactly the problem faced by parsers in a compiler:
  - One defines the syntactic tokens as regular expressions:

    ```
    value := [0-9]+
    ident := [a-z][a-z0-9]*
    ```
  - The goal is to match these patterns against the input text.
- Using the KMP or Boyer-Moore methods is problematic:
  - Pre-computing the tables can't be done since the pattern is not finite.

Finite State Machines (2)

- We can solve this problem by using a fourth type of matching method.
- A Finite State Machine (FSM) is formally defined by the following parts.
  - Q, a finite set of states.
  - $q_0 \in Q$, a start state.
  - $A \subseteq Q$, a set of accepting states.
  - Σ, an input alphabet.
  - δ, a function from $Q \times \Sigma$ into Q which we call the transition function.
- More simply, it is just a graph where moving between nodes means consuming input characters.

Finite State Machines (3)

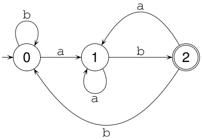
- Here are a few examples of basic regular expressions:
  `a+` – one or more repetitions of `a`.
  `a*` – zero or more repetitions of `a`.
  `.` – any character.
- The hard bit is constructing the automaton.
- However, once this is done implementing the matching algorithm is quite easy.

FSM-MATCHER(T, P)
begin
  $n \leftarrow |T|$
  $\delta \leftarrow$ FSM-BUILD(P)
  $s \leftarrow 0$
  for $i = 1$ **upto** n **step** 1 **do**
    $s \leftarrow \delta(s, T[i])$
    if s is an accepting state **then**
      OUTPUT(i)
end

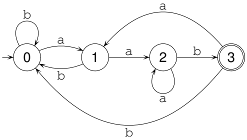
Finite State Machines (4)

- Consider some example FSM constructions:
  - Take the input string a character at a time and move along edges which match each character.
  - The empty edge denotes the start state, double circled nodes are accepting states which signal a match.

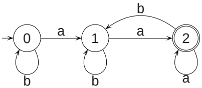
pattern=`ab`



pattern=`aab`



pattern=`ab*a`



- Question: How do we move through the states for the string `aababaaba` ?

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Comparison

- To get an idea of which algorithm is best, we can compare their complexities:

	Pre-computation	Matching
Naive		$\Theta(nm)$
Knuth-Morris-Pratt	$\Theta(m)$	$\Theta(n)$
Boyer-Moore-Horspool	$O(m + \|\Sigma\|)$	$\Theta(nm)$
Finite State Machine	$O(m\|\Sigma\|)$	$\Theta(n)$

- Some points to note from this analysis are:
  - For one-off matches on short strings, the naive method isn't so bad.
  - The methods that require pre-computation may also require extra memory.

Conclusions

- String matching sounded like a trivial problem:
  - Hopefully you can see there is a little more to it than the naive method.
- As a general rule, selecting the right algorithm is done as follows:
  - If you need to consider complex matching like regular expressions, use the FSM method.
  - Otherwise, the choice depends on the alphabet size:
    - For large alphabets, like natural language, use the Boyer-Moore-Horspool method.
    - For small alphabets, use the Knuth-Morris-Pratt method.
- However, there are some even more complicated methods than these.
- See `http://tinyurl.com/eolt7` for a long list with example code.

Further Reading

- Introduction to Algorithms
  T.H. Cormen, C.E. Leiserson, R.L. Rivest and C. Stein.
  MIT Press/McGraw-Hill, ISBN: 0-262-03293-7.
  - Chapter 32 – String Matching.