$A \times B = B \times A.$

a) $(A \subset B \wedge C \subset D) \quad \Rightarrow \quad (A \cup C \subset B \cup D)$,

$$\underbrace{[x\in A \wedge x\in B \wedge x\in C \wedge x\in D]}_{T} \Rightarrow [(x\in A \vee x\in C) \wedge (x\in B \vee x\in D)]$$

$x\in A \equiv T \rightarrow x\in A$

$x\in B \equiv T \rightarrow x\in B$

$x\in C \equiv T \rightarrow x\in C$

$x\in D \equiv T \rightarrow x\in D$

Nie może być fałszem skoro x należy do wszystkich wspomnianych zbiorów

$$[x\in A \Rightarrow x\in B \wedge x\in C \Rightarrow x\in D] \Rightarrow$$

$$[x\in A \vee x\in C \Rightarrow x\in B \vee x\in D]$$

$$\underbrace{(x\notin A \vee x\in B)\wedge(x\notin C \vee x\in D)}_{\text{założymy } T}$$

$$\underbrace{(x\notin A \wedge x\notin C)}_{F} \vee \underbrace{(x\in B \vee x\in D)}$$

$$\underbrace{x\in B \overset{F}{\vee} x\in D}_{T}$$

9. Naszkicuj na płaszczyźnie zbiory $A \times B$ i $B \times A$ dla:

a) $A = \{y \in \mathbb{R}\colon -1 < y < 1\}$, $B = \{x \in \mathbb{R}\colon 0 < x \leqslant 1\}$,

b) $A = \mathbb{Z}$, $B = \langle 1, 2)$,

c) $A = \{x \in \mathbb{R}\colon x^2 + x - 2 \geqslant 0\}$, $B = \{b \in \mathbb{N}\colon 2^b < 11\}$,

d) $A = \{x \in (0, \infty)\colon \frac{x-1}{x+1} < 0\}$, $B = \{x \in \mathbb{R}\colon x^2 \leqslant 4\}$.

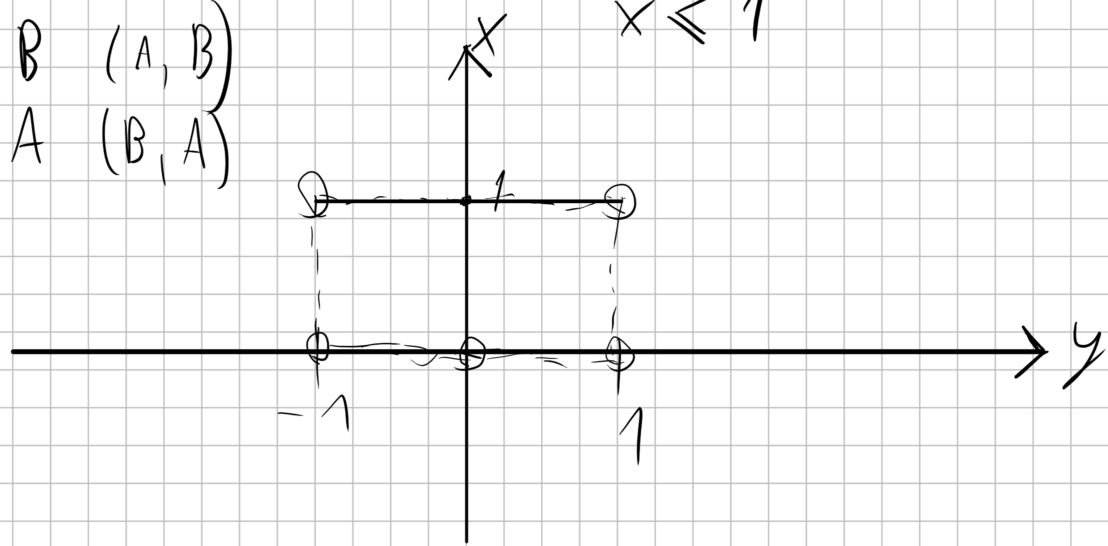
$A \times B$ a) $A = \{y \in R: -1 < y < 1\}$ $B = \{x \in R, 0 < x \leq 1\}$

$y > -1 \wedge y < 1$ $x > 0 \wedge$

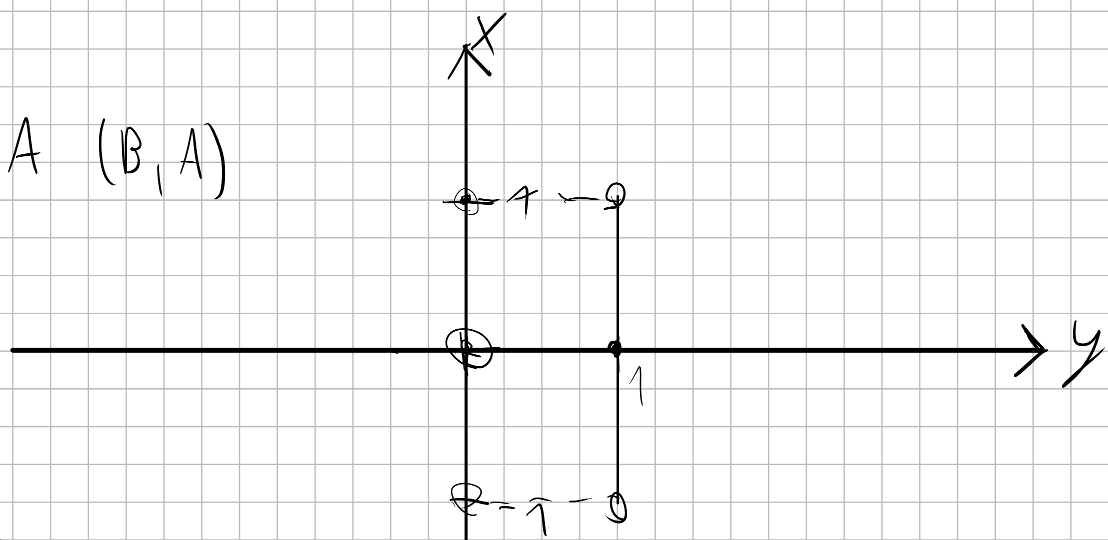
$x \leq 1$

$A \times B$ (A, B)

$B \times A$ (B, A)



$B \times A$ (B, A)

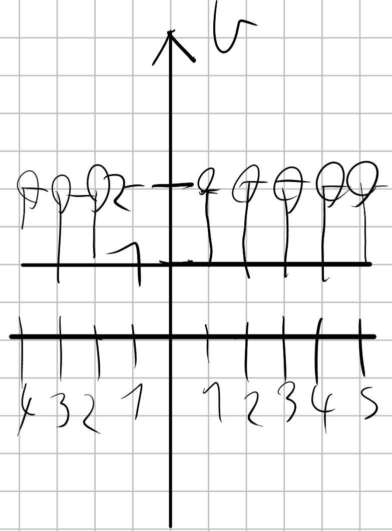


$(A \times B) \times C = ((a, b), c)$ $a \in A, b \in B, c \in C$

$A \times (B \times C) = (a, (b, c))$ $a \in A, b \in B, c \in C$

b) $A = \mathbb{Z}$, $B = \langle 1, 2)$,

$A \times B$



$$\underbrace{(A \cup B) \times C}_{L} = \underbrace{(A \times C) \cup (B \times C)}_{R}$$

$(x, y) \in L$ $\qquad$ $(x, y) \in R \wedge (z, y) \in R$

$(x \in A \vee x \in B) \wedge y \in C$ $\qquad$ $x \in A$, $y \in C$ $\qquad$ $z \in B$, $y \in C$

$(x \in A \wedge y \in C) \vee (x \in B \wedge y \in C)$

$[(x, y) \in A \times C \vee (x, y) \in B \times C]$

11. Niech $A_1, \ldots, A_n$ będą dowolnymi zbiorami. Zdefiniujmy $\mathcal{A}$ jako najmniejszy zbiór, dla którego

$$\bigwedge_{k \in \{1,\ldots,n\}} A_k \in \mathcal{A} \qquad \text{oraz} \qquad X \in \mathcal{A} \wedge Y \in \mathcal{A} \Rightarrow X \cup Y \in \mathcal{A}.$$

Ile maksymalnie elementów ma zbiór $\mathcal{A}$? Podaj przykład takiego zbioru.

$A_1 \ldots A_n$

$\bigwedge_{k \in \{1, 2, \ldots, n\}} A_k \in A$: $X \in A \wedge Y \in A \Rightarrow X \cup Y \in A$

$$\bigwedge_{k \in \{1,2,\ldots,n\}} A_k$$

$$\frac{X \in A \wedge Y \in A \Rightarrow X \cup Y \in A}{T}$$

$$X \in A \quad Y \in A$$

$$A_1 \wedge A_2 \Rightarrow A_1$$

A_k – różne niepuste

A

1 2 4 8

$$2^n - 1 = \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots + \binom{n}{n}$$

$A_1 \; A_2 \; A_3 \; A_4$

$\binom{n}{2}$

$A_1 \vee A_2$

$A_1 \vee A_3$

$A_1 \vee A_4$

$A_2 \vee A_3$

$$\frac{n!}{k!(n-k)!}$$

$$\frac{n!}{1(n-1)!} + \frac{n!}{2(n-2)!} + \frac{n!}{6(n-3)!} + \ldots \frac{n!}{n!(n-n)!}$$

$$\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots + \binom{n}{n-1} \quad \frac{n!}{n!(n-n)!} = 1$$

$\shortparallel$

$2n$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

Dwumian Newtona

$x = 1$

To ten zjebany trójkąt?

AHHHHHHHHH!

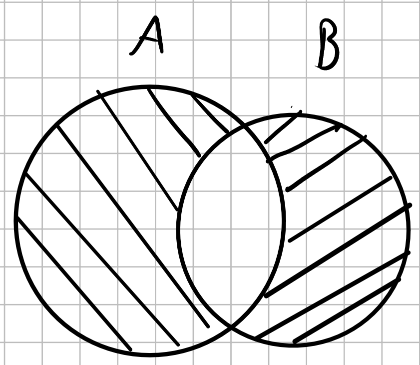
						1						
					1		1					
				1		2		1				
			1		3		3		1			
		1		4		5		4		1		
	1		5		9		9		5		1	
1		6		14		18		14		6		1

INDUKCJA

$$\left[p(1) \wedge \bigwedge_{n} p(n) \Rightarrow p(n+1)\right] \Rightarrow \left[\bigwedge_{n} p(n)\right]$$

efekt domina

$A \triangle B \quad (A \cup B) - (A \cap B)$



2. Wyznacz iloczyn kartezjański $A \times B$ i $B \times A$ dla zbiorów:

a) $A = \{0,1\}$, $B = \{1,2\}$,

b) $A = \{0,1,2\}$, $B = \{2,3\}$,

c) $A = \emptyset$, $B = \{1,2,3\}$.

a1)

$$A \times B = \{(0,1);\ (0,2);\ (1,1);\ (1,2)\}$$

a2)

$$B \times A = \{(1,0);\ (1,1);\ (2,0);\ (2,1)\}$$

b1)

$$A \times B = \{(0,2);\ (0,3);\ (1,2);\ (1,3);\ (2,2);\ (2,3)$$

b2)

$$B \times A = \{(2,0);\ (2,1);\ (2,2);\ (3,0);\ (3,1);\ (3,2)\}$$

$$A \times B = B \times A \Rightarrow \frac{A = B}{F}$$

$$(a_i, b_i) = (b_i, a_i)$$

$$a_i = b_i$$

skoro zbiór elementów jest różny poszczególne elementy będą różne

5. Wyznacz zbiór potęgowy dla zbiorów:

a) $\{1,2,3\}$,

b) $\emptyset$,

c) $\{\emptyset\}$,

d) $\{\emptyset, \{\emptyset\}\}$.

a) $A = \{1,2,3\}$

$$2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

$A = \emptyset \qquad A = \{\emptyset\}$

$2^A = \emptyset \qquad 2^A = \{\emptyset, \{\emptyset\}\}$

$A = \{\emptyset, \{\emptyset\}\}$

$2^A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

6. Udowodnij, że dla dowolnych zbiorów A, B, C, D zachodzą równości:

a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$,

b) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$,

c) $(A \setminus B) \cup C = [(A \cup C) \setminus B] \cup (B \cap C)$,

d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$,

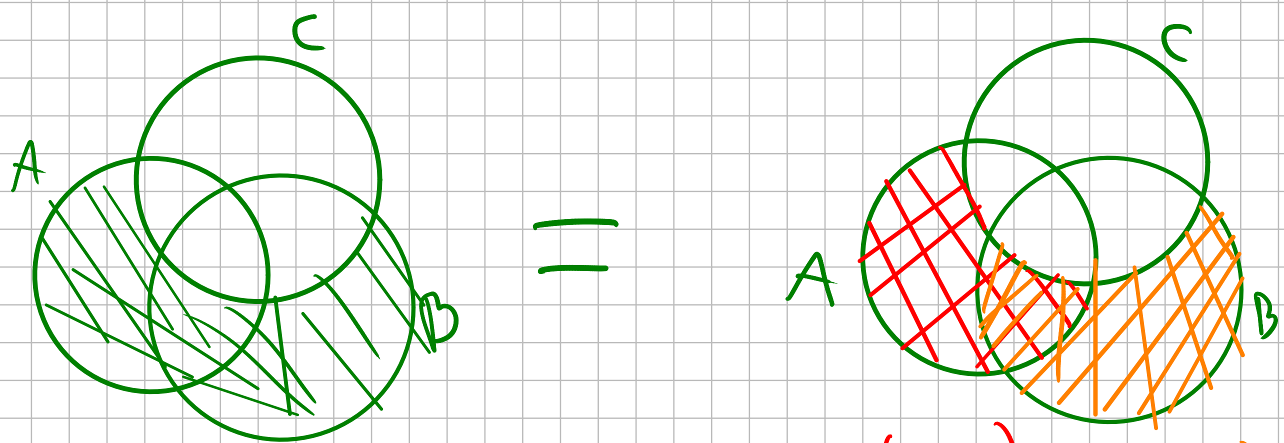
e) $(A \setminus B) \cap (C \setminus D) = (A \cap C) \setminus (B \cup D)$,

f) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$,

g) $(A \triangle B)^c = A^c \triangle B^c$.

a)

$(A \cup B) - C = (A - C) \cup (B-C)$



$A \cup B - C \qquad (A - C) \cup (B - C)$

b)

$A - (B - C) = (A - B) \cup (A \cap C)$

$x \in A \wedge (x \notin B \vee x \in C)$

$(x \in A \wedge x \notin B) \wedge (x \in C \wedge x \in A)$

$(x \in A \wedge x \notin B) \vee (x \in A \wedge x \in C)$

$x \in A \wedge (x \notin B \vee x \in C)$

6. Udowodnij, że dla dowolnych zbiorów A, B, C, D zachodzą równości:

a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$,
b) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$,
c) $(A \setminus B) \cup C = [(A \cup C) \setminus B] \cup (B \cap C)$,
d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$,
e) $(A \setminus B) \cap (C \setminus D) = (A \cap C) \setminus (B \cup D)$,
f) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$,
g) $(A \triangle B)^c = A^c \triangle B^c$.

c)

$$(A-B) \cup C = [(A \cup C) - B] \cup (B \cap C)$$

$$(x \in A \wedge x \notin B) \vee x \in C$$

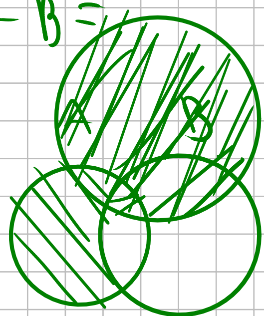
$$[(x \in A \vee x \in C) \wedge x \notin B] \vee (x \in B \wedge x \in C)$$

$$(x \in A \wedge x \notin B) \vee (x \in C \wedge x \notin B) \vee (x \in B \wedge x \in C)$$

$$(x \in A \vee x \in C) \wedge (\underline{x \notin B} \vee \underline{x \in C})$$

$$(A-B) \cup C = [(A \cup C) - B] \cup (B \cap C)$$

$A - B =$



$$(A-B) \cup (C-B) \cup (B \cap C)$$

$$C - B = C - B \cap C$$

$$(A-B) \cup (C - \cancel{(B \cap C)}) \cup \cancel{(B \cap C)}$$

$x \notin$ $x \in$

$$[A - (A \cap B)] \cup C$$

$$[A - (A \cap B)] \cup (C \cap A) \cup C$$

która $B \cap C$ należy to $C - B \cap C = C$

$$(A-B) \cup C$$

6. Udowodnij, że dla dowolnych zbiorów A, B, C, D zachodzą równości:

a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$,

b) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$,

c) $(A \setminus B) \cup C = [(A \cup C) \setminus B] \cup (B \cap C)$,

d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$,

e) $(A \setminus B) \cap (C \setminus D) = (A \cap C) \setminus (B \cup D)$,

f) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$,

g) $(A \triangle B)^c = A^c \triangle B^c$.

$$(A - B) \cap (C - D) = (A \cap C) - (B \cup D)$$

$$\underbrace{x \in A}_{p} \land \underbrace{x \notin B}_{r} \land \underbrace{x \in C}_{q} \land \underbrace{x \notin D}_{s} \Leftrightarrow$$

$$\underbrace{x \in A}_{p} \land \underbrace{x \in C}_{q} \land \underbrace{x \notin B}_{r} \land \underbrace{x \notin D}_{s}$$

$$A \triangle (B \triangle C) = (A \triangle B) \triangle C$$

$$A \triangle B \Leftrightarrow (A \cup B) - (A \cap B)$$

$$(A \cup Q) - (A \cap Q)$$

$$Q = (B \cup C) - (B \cap C)$$

$$(Z \cup C) - (Z \cap C)$$

$$Z = (A \cup B) - (A \cap B)$$

$$(A \cup B \cup C) - (B \cap C) - (A \cap B \cap C$$

KURWA

MIAŁEM TYLE DO ZROBIENIA I ZROBIŁEM NIC