

$$f(x) = \frac{x^3}{x^2-4}$$

$$x^2 - 4 \neq 0$$

$$x^2 = 4$$

$$x \neq 2 \vee x \neq -2$$

$$\begin{array}{r} x \\ \hline x^3 : x^2 - 4 \\ -x^3 + 5x \\ \hline 4x \end{array}$$

$$x \frac{x^2-4}{x^2-4} + 4x = x \left(\frac{4x}{x^2-4} \right)$$

a) $\frac{x}{(x+1)(x+2)(x-3)}$

$$\text{Nur } > 2$$

$$\text{Nur } > 1$$

$$x \neq -1$$

$$x \neq -2$$

$$x \neq 3$$

$$D = \mathbb{R} - \{-2, -1, 3\}$$

$$\frac{x}{(x+1)(x+2)(x-3)}$$

$$\frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-3}$$

$$\frac{x}{(x+1)(x+2)(x-3)} = \frac{A(x-3)(x+2) + B(x+1)(x-3) + C(x+1)(x+2)}{(x+1)(x+2)(x-3)}$$

$$x = A(x^2 + 2x - 3x - 6) + B(x^2 - 3x + x - 3) + C(x^2 + 2x + x + 2)$$

$$x = \underline{A x^2} - \underline{A x} - \underline{6A} + \underline{B x^2} - \underline{2B x} - \underline{3B} + \underline{C x^2} + \underline{3C x} + \underline{2C}$$

$$x = x^2(A+B+C) + x(-A-2B+3C) - 6A-3B+2C$$

$$A+B+C=0$$

$$-A-2B+3C=1$$

$$-6A-3B+2C=0$$

$$C = -A-B$$

$$-3A-A-2B-3B=1$$

$$-4A-5B=1$$

$$\left[\begin{array}{l} (x+1)^2 \quad \frac{A}{x+1} + \frac{B}{(x+1)^2} \\ (x^2 + 1 + 2) \quad \frac{A+B}{x^2 + x + 2} \end{array} \right]$$

b) $\frac{1}{x^2 - 3x + 2}$

$$\frac{1}{x^2 - 3x + 2} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

$$\Delta = 9 - 4 \cdot 2 \cdot 1 = 1$$

$$x_1 = \frac{3-1}{2} = 1$$

$$x_2 = \frac{3+1}{2} = 2$$

$$Ax - 2A + Bx - B$$

$$x(A+B) - 2A - B$$

$$-2A - B = 1$$

$$A + B = 0$$

$$A = -B$$

$$+2B - B = 1$$

$$\underline{\underline{B = 1 \wedge A = -1}}$$

$$[-2(-1) - B]$$

$$-2A - A = 1$$

$$-A = 1$$

$$A = -1$$

$$\frac{1}{x-1} + \frac{-1}{x-2} = \frac{1}{x^2 - 3x + 2}$$

f) $\frac{x^2+4x+6}{x+4} > 3,$

$$\frac{x^2+4x+6}{x+4}$$

$$\frac{x^2+4x+6-3x-12}{x+4} > 0$$

$$D \in \mathbb{R} - \{-4\}$$

$$\frac{x^2+x-6}{x+4} > 0$$

$$\stackrel{\text{BRUH}}{(x+4)(x^2+x-6)} > 0$$

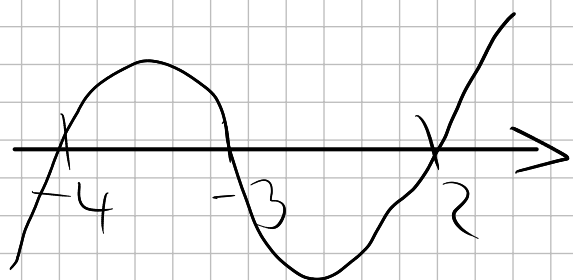
$$x^2+x-6$$

$$\Delta = 1 - 4 \cdot 1 \cdot -6 = 25$$

$$(x+4)(x+3)(x-2) > 0$$

$$\frac{-1+5}{2} = 2$$

$$\frac{-1-5}{2} = -3$$



$$x \in (-4, -3) \cup (2, \infty)$$

c) $\frac{1}{2-x} + \frac{1}{2+x} < 1,$

$$\frac{2+x+2-x-(2-x)(2+x)}{(2-x)(2+x)} < 0$$

$$\cancel{4} - x^2$$

$$D = \mathbb{R} - \{-2, 2\}$$

$$(2-x)(2+x) - x^2 < 0$$

$$4 - x^2 - x^2 < 0$$

$$4 - 2x^2 < 0$$

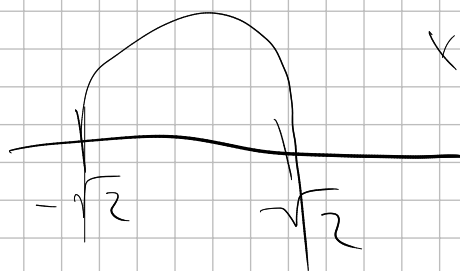
$$2 - x^2 < 0$$

$$(\sqrt{2}-x)(\sqrt{2}+x)$$

$$x = \sqrt{2} \quad x = -\sqrt{2}$$

$$x^2 > 2$$

$$x > \sqrt{2} \vee x < -\sqrt{2}$$



$$x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$$

a) $\frac{x^5 - x^3 + 1}{x(x+1)^3(x^2-1)^2}$

$$\frac{x(x+1)^3(x-1)(x-1)(x+1)(x+1)}{x(x+1)^5(x-1)^2}$$

$$\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3} + \frac{E}{(x+1)^4} + \frac{F}{(x+1)^5} + \frac{G}{(x-1)} + \frac{H}{(x-1)^2}$$

ZADANIE 16. Dana jest funkcja określona wzorem $f(x) = \frac{1}{x}$. Rozwiązać nierówność:

$$f(x) - f\left(\frac{1}{x}\right) < f(x^3) - f\left(\frac{1}{x^3}\right)$$

czw 16:30

$$\frac{1}{x} - x < \frac{1}{x^3} - x^3$$

$$\frac{1}{x} - x <$$

$$(p \wedge (q \Rightarrow r)) \Rightarrow [(p \Rightarrow r) \wedge (q \Rightarrow r)]$$

$$\sim ((p \wedge \sim q) \vee (p \wedge r)) \vee [p \vee r \wedge \sim q \vee r]$$

$$(\sim p \vee q) \wedge (\sim p \vee r) \vee (\sim p \vee r) \wedge (\sim q \vee r)$$

$$\begin{array}{l} p = T \\ q = F \\ r = F \end{array} \quad (q \wedge r) \vee \sim p$$

d) $\sum_{k=1}^n \frac{1}{(4k-3)(4k+1)} = \frac{n}{4n+1}$

$$\sum_{k=1}^n \frac{1}{(4k-3)(4k+1)} = \frac{n}{4n+1}$$

$$\sum_{k=1}^n \frac{1}{(4k-3)(4k+1)}$$

acc = 0

for k in range(1, n):

$$\text{acc} += 1 / (4 \times k - 3) (4k + 1)$$

return acc

$$\frac{1}{(4-3)(4+1)} + \frac{1}{(4 \cdot 2 - 3)(4 \cdot 2 + 1)}$$

$$\frac{1}{5} + \frac{1}{45} + \frac{1}{117}$$

zakładamy że wzór

$$\sum_{k=1}^n \frac{1}{(4k-3)(4k+1)}$$

jest prawdziwy dla n dowolnego

$$Z \Rightarrow T$$

$$n^2 + Z\left(\frac{n+1}{2}\right) + n$$

$$4 \rightarrow 6$$

$$3 \rightarrow 3$$

$$2 \rightarrow 1$$

$$1 \rightarrow 0$$

$$2|n \Rightarrow n \cdot (n+1)/2$$

$$\sim (2|n) \Rightarrow n/2 \cdot (n+1)$$

$$n^2 + Z\left(\frac{n+1}{2}\right) + n$$

$$2|n \Rightarrow n^2 + n \cdot \left\lfloor \frac{n+1}{2} \right\rfloor + n$$

$$\sim (2|n) \Rightarrow n^2 + \left\lfloor \frac{n}{2} \right\rfloor \cdot (n+1) + n$$

c) $10^n + 4^n - 2$ jest podzielna przez 3.

$$① \quad 10^1 + 4^1 - 2 = 12 \quad 12 \mid 3$$

$$x(1) \equiv T$$

②

$$\frac{10^n + 4^n - 2}{T} \Rightarrow 10^{n+1} + 4^{n+1} - 2 \quad n \in \mathbb{N}$$

$$3q \wedge q \in \mathbb{Z}$$

$$10^n + 4^n - 2 = 3q$$

$$10^{n+1} + 4^{n+1} - 2$$

$$10^n \cdot 10 + 4^n \cdot 4 - 2$$

$$(3q - 4^n + 2) \cdot 10 + 4^n \cdot 4 - 2$$

$$30q - 10 \cdot 4^n + 20 + 4^n \cdot 4 - 2$$

$$30q - 6 \cdot 4^n + 18$$

$$\underline{\underline{3(10q - 3 \cdot 4^n + 6)}}$$

$$n \in \mathbb{N} \Rightarrow 3 \mid 3 \cdot n$$

$$2^n > n^2 \quad \wedge n \in \mathbb{N}$$

1
2
4
8
16
32
64
128
256
512 1024

1
1
4
9

$$\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}, \quad n \geq 2.$$

$$\frac{4^2}{3} < \frac{2 \cdot 3 \cdot 4}{4}$$

$$\frac{16}{3} < 18$$

$$\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$$

$$\frac{4^{n+1}}{n+2} < \frac{(2n+2)!}{((n+1)!)^2}$$

$$\frac{4^n \cdot 4}{n+2} < \frac{(2n+2)!}{((n+1) \cdot n!)^2}$$

wzrostka dodatnie

$$(4^n \cdot 4) \cdot ((n+1) \cdot n!)^2 < (n+2)(2n+2)!$$

$$4^{n+1} \cdot [(n+1) \cdot n!]^2 < (n+2)(2n+2)!$$

$$4^{n+1} \cdot [(n+1) \cdot n!]^2 - (n+2)(2n+2)! < 0$$

$$4^{n+1} \cdot (n+1)^2 \cdot n!^2 - (n+2)(2n+2)!$$

$$4^n \cdot 4 \cdot (n+1)^2 \cdot n!^2 < (n+2)(n+2)(2n+1)(2n)!$$

$$4^n \cdot 4(n^2 + 2n + 2) \cdot n!^2 < (n+2)(2n+2)(2n+1)(2n)!$$

8. Znajdź zwartą postać sumy

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n!$$

dla wszystkich $n \in \mathbb{N}$.

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n!$$

$$\sum_{k=1}^n k \cdot k! \quad \begin{array}{l} 1 = 1 \\ 1 + 4 = 5 = 3! - 1 \\ 18 \end{array}$$

$$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 6 = 23 \quad 4! - 1$$

$$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 6 + 4 \cdot 24 = 119 \quad 5! - 1$$

$$\sum_{k=1}^n (k+1)! \quad (n+1)! - 1$$

$$2! + 3! + 4! + 5!$$

$$2 + 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 2 \cdot 3 \cdot 4 \cdot 5$$

$$2(1 + 3 + 3 \cdot 4 + 3 \cdot 4 \cdot 5)$$

$$2(1 + 3(1 + 4 + 4 \cdot 5))$$

$$2(1 + 3(1 + 4(1 + 5)))$$

$$\sum_{k=1}^n (k+1)! = (n+1)! - 1$$

n kąt wypukły

$$1 = 1$$

$$(n+1)! - 1 = (n+2)! - 1$$

$$(n+1)! - 1 = (n+1)!(n+2) - 1$$





