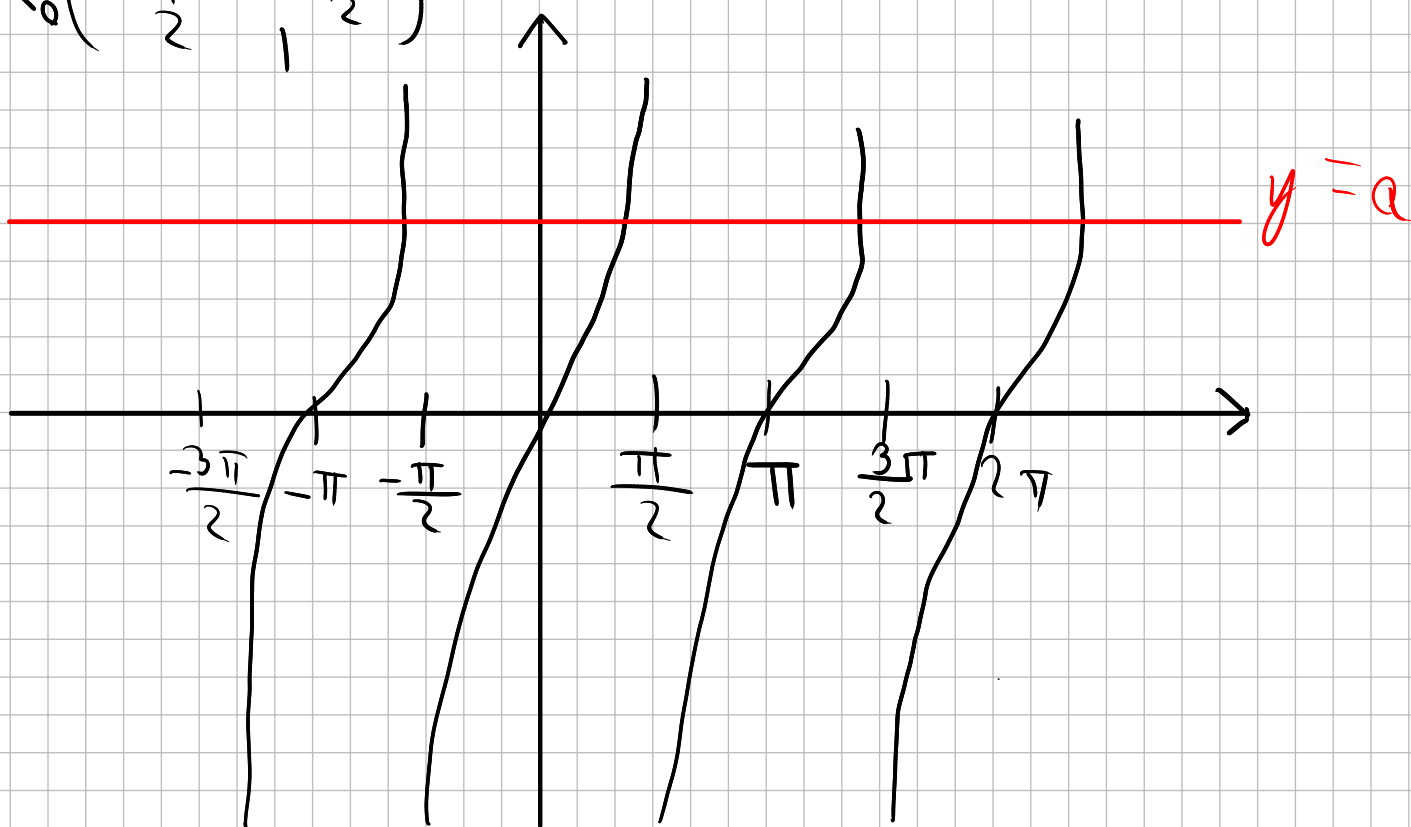Wstęp do matematyki

$$\operatorname{tg} x = a \qquad a \in \mathbb{R}$$

$$x = x_0 + k\pi$$

$$x_0 \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$
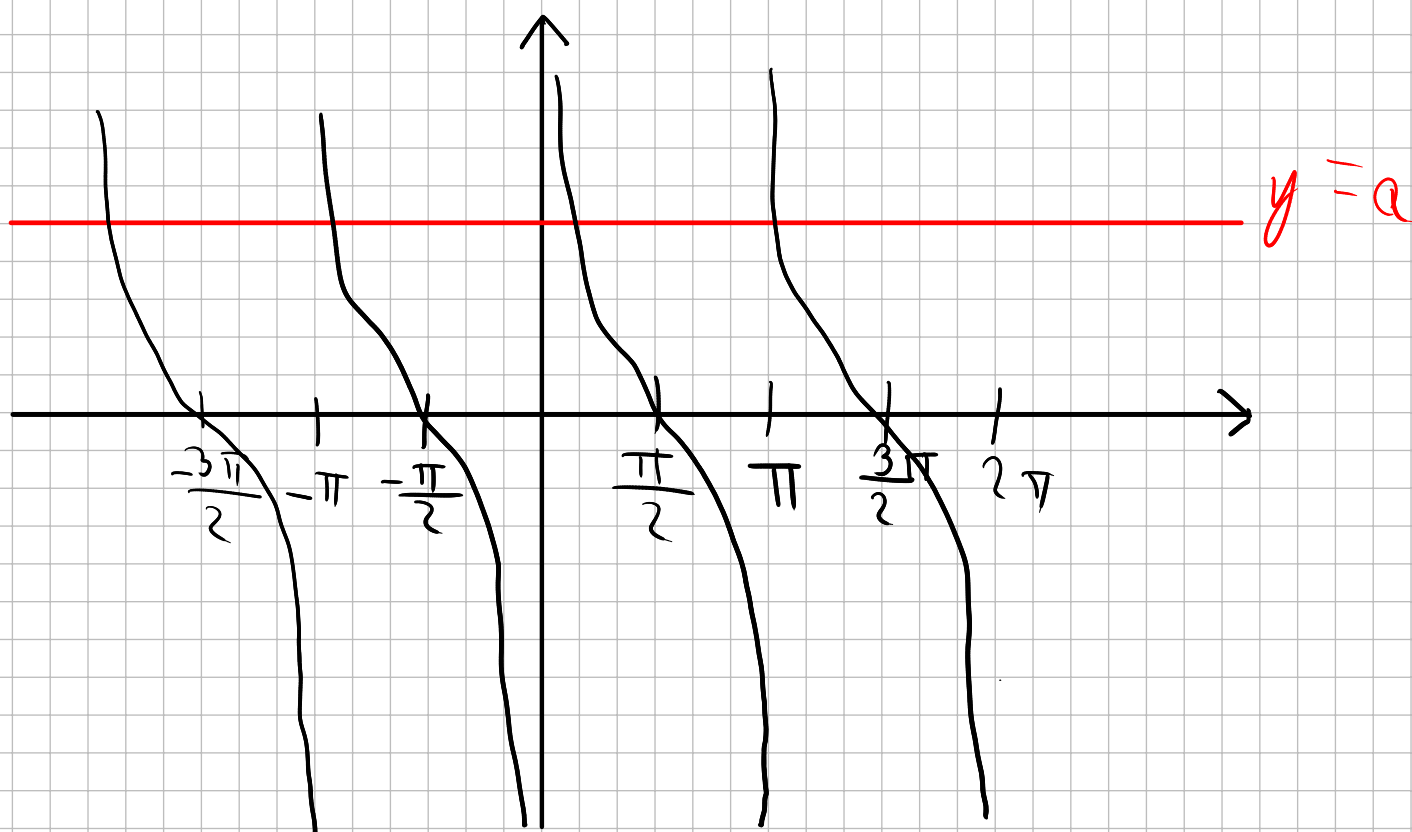

$$\operatorname{tg} x = a \wedge a \in \mathbb{R} \Rightarrow x = x_0 + k\pi \wedge k \in \mathbb{Z}$$
$$\wedge x_0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \qquad \operatorname{tg} x_0 = a$$

$$D = \mathbb{R} - \left\{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\right\}$$

$$\text{ctg } x = a \quad a \in R$$
$$x = x_0 + k\pi$$
$$k \in Z \quad \text{ctg } x_0$$

$y = a$

$$\text{ctg } x = a \land a \in R \Longrightarrow x_0 + k\pi \land k \in Z \land x_0 \in (0, \pi)$$

$$D = R - \{k\pi : k \in Z\}$$

$$3\sin x = 2\cos^2 x$$

$$3\sin x = 2(1 - \sin^2 x)$$

$$2\sin^2 x - 3\sin x - 2 = 0$$

$$\Delta = 9 - 4 \cdot 2 \cdot 2 = 25$$

$$t_1 = \frac{-3 - 5}{4} = -2 \quad \lor \quad t_2 = \frac{-3 + 5}{4} = -\frac{1}{2}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} + 2k\pi \quad \vee \quad \cancel{\frac{\pi}{6} = \frac{1}{2}}$$

$$\sin^2 2x = 1 - \sin^2 x$$

$$(2\sin x \cos x)^2 = \cos^2 x$$

$$4\sin^2 x \cos^2 x - \cos^2 x = 0$$

$$\cos^2 x (4\sin^2 x - 1) = 0$$

$$\cos^2 x = 0 \vee 4\sin^2 x = 1$$

$$\cos x = 0 \vee \quad \sin x = \frac{1}{2} \vee \sin x = -\frac{1}{2}$$

$$\cos x = 0$$

$$\left(x = \frac{\pi}{2} + k\pi \wedge k \in \mathbb{Z}\right) \vee \left(x = \frac{\pi}{6} + 2k\pi \wedge k \in \mathbb{Z}\right) \overset{\vee}{\left(x = \frac{5}{6}\pi + 2k\pi\right)}$$

$$\left[\left(x = \frac{\pi}{2} + k\pi\right) \vee \left(x = -\frac{\pi}{6} + 2k\pi\right) \vee \left(x = \frac{5}{6}\pi + 2k\pi\right)\right.$$
$$\left.\vee \left(x = -\frac{\pi}{6} + 2k\pi\right) \vee \left(x = \frac{7}{6}\pi + 2k\pi\right)\right] \wedge k \in \mathbb{Z} \wedge$$

$$\frac{7}{6}\pi = \frac{\pi}{6} + \pi$$

$$-\frac{\pi}{6} = \frac{5}{6} - \pi$$

$$\left\{ x: x = \frac{\pi}{6} + k\pi \right\} \cup \left\{ x: x = \frac{5}{6} + 2k\pi \right\}$$

$$\left\{ x: x = -\frac{\pi}{6} + k\pi \right\}$$

$$x \in \left\{ -\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi \right\} \quad k \in Z$$

nierówności

$$\sin x > -\frac{\sqrt{3}}{2}$$

a) $\quad x \langle -\pi, 2\pi \rangle$

b) $\quad R$

e)

$$\sin x = \frac{\sqrt{3}}{2}$$

$$\left(x = -\frac{\pi}{3} + 2k\pi \right) \vee \left(x = -\frac{4}{3}\pi + 2k\pi \right) \wedge x \in \langle -\pi, 2\pi \rangle$$

$$\left(x = -\frac{\pi}{3} \vee^{x = \frac{5}{3}\pi} \vee x = \frac{4}{3}\pi \vee x = -\frac{2}{3}\pi \right) \wedge x \in \langle -\pi, 2\pi \rangle$$
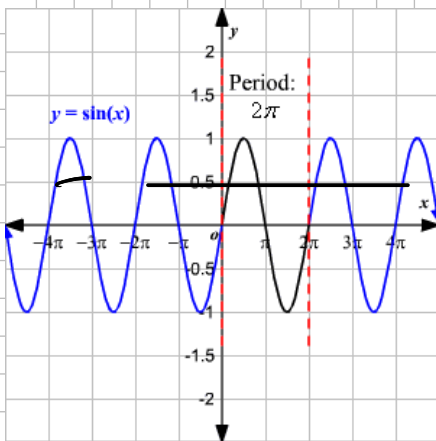
$$\sin x > -\frac{\sqrt{3}}{2}$$

$$x \in \langle -\pi, -\frac{2}{3}\pi) \cup \left(-\frac{\pi}{3}, \frac{4}{3}\pi \right)$$

$$\cup \left(\frac{5}{3}\pi, 2\pi \rangle$$

R $\quad \sin x = -\frac{\sqrt{3}}{2} \wedge x \in R$

$\left(-\frac{\pi}{3} + 2k\pi, \frac{4}{3}\pi + 2k\pi\right) \quad k \in Z$

$\sin x > -\frac{\sqrt{3}}{2} \wedge x \in R \Rightarrow x \in \bigcup\limits_{k \in Z}\left(-\frac{\pi}{3} + 2k\pi, \frac{4}{3}\pi + 2k\pi\right)$

$\left[\dfrac{\cos x}{1 - \cos x} \geq 0 \quad \wedge x \in D\right]$
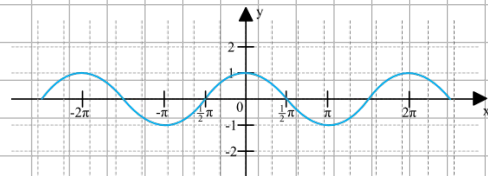
$-\dfrac{2\cos x - 1}{1 - \cos x} \geq 0 \wedge x \in D$

D
$\cos x \neq 1$
$x \neq 1 + k\pi$

$\cos x \geq \frac{1}{2}$

$\cos x = \frac{1}{2}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

$x \in \left(\bigcup\limits_{x \in Z}\left\langle -\frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + 2k\pi\right) \wedge \right.$

$x \in D \big)$ $\qquad x \in \left(\bigcup\limits_{x \in Z}\left\langle -\frac{\pi}{3} + 2k\pi, 2k\pi\right) \cup \left(2k\pi, \right.\right.$
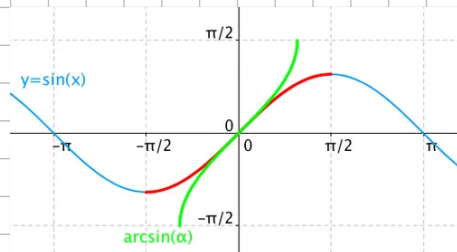
Funkcje Cyklometryczne

odwrotne do funkcji trygonometrycznych
ograniczonych do pewnych przedziałów.

arcus sinus

$$y = \arcsin x \iff x = \sin y, \quad x \in \langle -1, 1 \rangle$$

$$y \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2} \right\rangle$$

$$y = \arccos x$$

$$\implies x = \cos y \quad x \in \langle -1, 1 \rangle$$

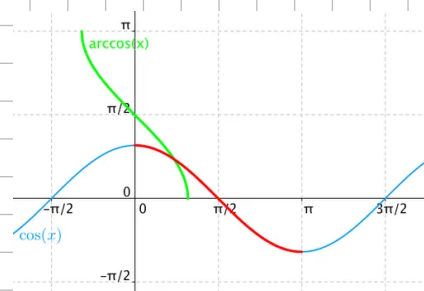$$y \in \langle 0, \pi \rangle$$

$$D = \langle -1, 1 \rangle$$

$$Z_W \langle 0, \pi \rangle$$

ściśle malejąca

$$\text{tg } x \quad \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$\bigwedge_{x \in \mathbb{R}} \text{tg}(\text{arctg} x) = x$$

$$\bigwedge_{x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)} \operatorname{arc} tg\left(tg(x)\right) = x$$

daj pan spokój

$$\operatorname{arc} sin \frac{\sqrt{2}}{2} = \alpha \equiv sin\alpha \frac{\sqrt{2}}{2} \wedge \alpha \in \left\langle -\frac{\pi}{2}, \frac{\pi}{13}\right\rangle$$

$$\Rightarrow \alpha = \frac{\pi}{4}$$

$$\operatorname{arc} ctg(-1) = \alpha$$

2) Wyznaczyć dziedzinę naturalną

$$f(x) = \operatorname{arc} sin\left(\log_{\frac{1}{3}} x\right)$$

$$D = \left\{ x \in R: x > 0 \wedge \log_{\frac{1}{3}} 3 \leq \log_{\frac{1}{3}} x \leq \log_{\frac{1}{3}} \frac{1}{3}\right\}$$

$$D = \left\{ x \in R: x > 0 \wedge x \in \left\langle \frac{1}{3}, 3 \right\rangle \right\} = \left\langle \frac{1}{3}, 3 \right\rangle$$

3) Rozwiąż nierówność

$$\operatorname{arc} \cos x \leq \frac{5}{6}\pi$$

$$D = \langle -1, 1 \rangle \qquad \bigwedge_{x \in \langle 0, \pi \rangle} \operatorname{arc} \cos(\cos x) = x$$

z własności funkcji

$$\frac{5}{6}\pi = \arccos\left(\cos\frac{5\pi}{6}\right)$$

$$\arccos x \le \frac{5}{6}\pi \wedge x \in \langle -1, 1 \rangle$$

$$\arccos x \le \arccos\left(\cos\frac{5}{6}\pi\right)$$

$$x \ge \cos\frac{5}{6}\pi \qquad \text{bo malejąca } (\text{arc}^{\cos}x)$$

$$x - \cos\frac{5}{6}\pi \wedge x \in \langle -1, 1 \rangle$$

$$x \ge \underbrace{-\cos\frac{\pi}{6}}_{\text{redukcja}} \quad\text{---} \quad\text{'' ---}$$

$$x \ge -\frac{\sqrt{3}}{2} \quad\text{---}\quad\text{'' ---}$$

$$x \in \left\langle -\frac{\sqrt{3}}{2}, 1 \right\rangle$$

$$\arcsin(-x) = -\arcsin x$$
$$\operatorname{arc tg}(-x) = -\operatorname{arc tg} x$$
$$\arccos(-x) = \pi - \cos x$$
$$\operatorname{arc ctg}(-x) = \pi - \operatorname{ctg} x$$

$$2 \mid n \Rightarrow n^2 + \left\lfloor \frac{n+1}{2} \right\rfloor \cdot n + n$$
$$\sim(2 \mid n) \Rightarrow n^2 + \left\lfloor \frac{n}{2} \right\rfloor \cdot (n+1) + n$$
$$2 \mid n \Rightarrow n\left(n + \left\lfloor \frac{n+1}{2} \right\rfloor + 1\right)$$

$$1024 = n^2 + \frac{n+1}{2} \cdot n + n$$

$$2n^2 + n^2 + n + n - 1024 = 0$$

$$3n^2 + 2n - 1024 = 0$$

$$\Delta = 4 - 4 \cdot 3 \cdot -1024$$

$$\sqrt{\Delta} = 110,87$$

$$x_1 = \frac{-2 + 110,87}{6} = 18$$

$$x_2 = \frac{-2 - 110,87}{6} = -19$$

$$(0,125)^x \cdot (\sqrt{2})^{x+1}$$

$$\left(\frac{1}{8}\right)^x \cdot \left(2^{\frac{1}{2}}\right)^{x+1}$$

$$2^{-3x} \cdot 2^{\frac{x+1}{2}} = 2^{-3x + \frac{x+1}{2}}$$

b) $(0.125)^x \cdot (\sqrt{2})^{x+1} = \left(\frac{4}{\sqrt[3]{2}}\right)^{3x}$

$$2^{-3x + \frac{x+1}{2}} = \left(\frac{4}{\sqrt[3]{2}}\right)^{3x} \qquad \left(2^2 : 2^{\frac{1}{3}}\right)^{3x} = 2^{\frac{5}{3} \cdot 3x}$$

$$2^{-3x + \frac{x+1}{2}} = 2^{5x} \qquad \text{różnowartościowość}$$

$$-3v + \frac{x+1}{2} = 5x$$

$$\frac{x+1}{2} = 8x$$

$$x + 1 = 16y$$

$$x = \frac{1}{15}$$

24 kalokwium

21.

c) $21 \cdot 3^x - 25 \cdot 5^x = 81 \cdot 3^x - 125 \cdot 5^x$,

$$7 \cdot 3 \cdot 3^x - 5^2 \cdot 5^x = 3^4 \cdot 3^x - 5^3 \cdot 5^x$$

$$7 \cdot 3^{x+1} - 5^{x+2} = 3^{x+4} - 5^{x+3}$$

$$7 \cdot 3^{x+1} - 3^{x+4} = 5^{x+2} - 5^{x+3}$$

$$7 \cdot 3^{x+1} - 27 \cdot 3^{x+1}$$

$$-20 \cdot 3^{x+1} = 5^{x+2} - 5 \cdot 5^{x+2}$$

$$-20 \cdot 3^{x+1} = -4 \cdot 5^{x+2}$$

$$5 \cdot 3^{x+1} = 5^{x+2}$$

$$5 \cdot 3^{x+1} = 5 \cdot 5^{x+1}$$

$$3^{x+1} = 5^{x+1}$$

i) $\left(\frac{1}{2}\right)^{3x} + 64 < \left(\frac{1}{2}\right)^{x-2} \cdot (1 + 2^{2-x})$,

$$\left(\frac{1}{2}\right)^{3x} + 64 < \left(\frac{1}{2}\right)^{x-2} \cdot (1 + 2^{2-x})$$

$$2^{-3x} + 2^6 < 2^{2-x} + 2^{2-x} \cdot 2^{2-x}$$

$$2^{-3x} + 2^6 < 2^{2-x} + 2^{4-2x}$$

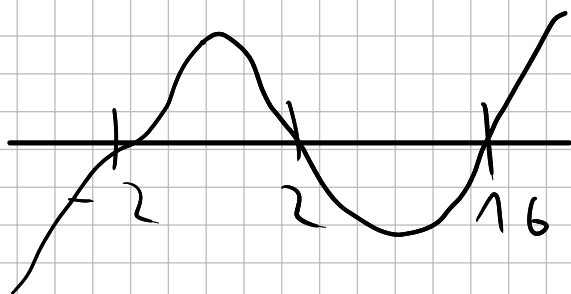$$(2^{-x})^3 + 64 < 4 \cdot 2^{-x} + 16 \cdot (2^{-x})^2$$

$$(2^{-x})^3 - 4 \cdot 2^{-x} - 16 \cdot (2^{-x})^2 + 64 < 0$$

$$t^3 - 4t - 16t^2 + 64 < 0$$
$$t(t^2 - 4) - 16(t^2 - 4) < 0$$
$$(t - 16)(t - 4)(t + 2) \qquad \land t > 0 \land t \in \mathbb{D}$$
$$t = 16 \lor t = 2 \lor t = -2$$

$$t \in (-\infty, -2) \lor (2, 16) \land t > 0 \land t \in$$

$$t \in (2, 16)$$

funkcja
malejąca

$$2^{-x} \in (2, 16)$$

$$2 < 2^{-x} < 16$$

$$2^1 < 2^{-x} \qquad\qquad 2^{-x} < 2^4 \quad \begin{matrix}1\\2\\4\\8\\16\end{matrix}$$

$$1 < -x \qquad\qquad -x < 4$$

$$x < -1 \qquad\qquad x > -4$$

$$x \in (-4, -1)$$

$$\frac{1}{3^x - 1} > \frac{9}{9 - 3^x} \qquad 3^x \neq 1 \quad x \neq 0 \land x \neq 2$$
$$D_f = \mathbb{R} - \{0, 2\}$$

$$9 \cdot 3^x - 9 > 9 - 3^x \land x < 3$$

$$3^x \cdot 3^2 - 3^2 - 3^2 - 3^x > 0 \land x < 3$$

$$3^x \cdot 3^2 - 2 \cdot 3^2 - 3^x > 0 \land x \in D_f$$

$$(3^x - 2) \cdot 3^2 - 3^3 > 0 \land x \in D_f$$

$$3^x < 3^1$$

$$x < 1$$

$$3^2(3^x{-}2{-}1) > 0$$
$$3^2(3^x{-}3) > 0 \;/\!/\cdot 3^2$$
$$3^x > +3^1$$
$$x > 1$$

$$x^5 > x^2 \wedge 5 > 2$$
$$\text{rosnąca}$$

$$x \in (1,2) \vee (2,3) \cup (0,1)$$

ZADANIE 10. Obliczyć

a) $\left(\sqrt[3]{125}\right)^{2(1-\log_5 4)}$, b) $16^{-\log_2 \sqrt{2}} \cdot 3^{-\log_{\frac{1}{3}} 5} \cdot 4^{\log_2 \sqrt{8}} \cdot \frac{1}{2}$, c) $\log_2 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8$.

$$\sqrt[3]{125}^{\,2(1-\log_5 4)}$$

$$5^{2-\log_5 16}$$

$$5^x = 16$$

$$\frac{5^2}{16} = \frac{25}{16} = 1,5625 \qquad 5^{5^x=16}$$

$$5^x = 16$$

b) $16^{-\log_2 \sqrt{2}} \cdot 3^{-\log_{\frac{1}{3}} 5} \cdot 4^{\log_2 \sqrt{8}} \cdot \frac{1}{2}$,

$$1 \atop 2 \atop 4 \atop 8 \atop 16$$

$$16^{-\log_2 \sqrt{2}} \cdot 3^{-\log_{\frac{1}{3}} 5} \cdot 4^{\log_2 \sqrt{8}} \cdot \frac{1}{2}$$

$$2^{-\log_2 4\sqrt{2}} \cdot \frac{1}{3}^{\log_{\frac{1}{3}} 5} \cdot 2^{\log_2 4\sqrt{2}} \cdot \frac{1}{2}$$

$$1 \qquad\qquad 1$$

$$8 \atop 4 \atop 2 \atop 1 \Bigg| \begin{matrix} 2 \\ 2 \\ 2 \end{matrix}$$

$$5 : 2 = 2,5$$

11)

$$2^{\log_3 5} = 5^{\log_5 2 \cdot \log_3 5}$$

$$\log_a b \cdot \log_b c = \log_a c$$

12)

ZADANIE 12. Obliczyć $\log_{a^2} \frac{1}{b}$ wiedząc, że $\log_a b = \sqrt{2}$, gdzie a,b są liczbami dodatnimi i $a \neq 1$.

$$\log_{a^2} \frac{1}{b} \qquad \log_a b = \sqrt{2}$$

$$\log_{a^2} \frac{1}{b} = \log_a \sqrt{\frac{1}{b}}$$

$$\log_a b^{-2} = -2\log_a b = -2\sqrt{2}$$

ZADANIE 13. Określić znak liczby:

a) $\log_a b$, jeżeli $a > 1$ i $b > 1$,

b) $\log_a b$, jeżeli $a \in (0,1)$ i $b \in (0,1)$,

c) $\log_a b$, jeżeli $a \in (0,1)$ i $b > 1$.

$$\log_a b \qquad a > 1 \wedge b > 1$$

a)
$$+$$
$$\log_2 2 = 1$$

b)
$$\log_{\frac{1}{2}} \frac{1}{4} = 2 \quad +$$

c) $\log_{\frac{1}{2}} 2 = -1 \quad -$

$$f(x) = \log_x (x^2 - 4)$$

$$x > 0 \wedge x \neq 1$$

$$\log_{x^2-1} \frac{2x+1-\frac{7}{x}}{} \qquad \frac{6-15}{ect\ldots}$$