

Wstęp do matematyki

https://moodle3.cs.pollub.pl/pluginfile.php/73589/mod_resource/content/0/fun_tryg_cylkometr_notatka.pdf

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha$$

$$\operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha}$$

$$\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2 \alpha - 1}{2 \operatorname{ctg} \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\operatorname{tg}(\alpha + 2k\pi) = \sin \alpha$$

$$\cos(\alpha + 2k\pi) = \cos \alpha \quad \text{parzysta}$$

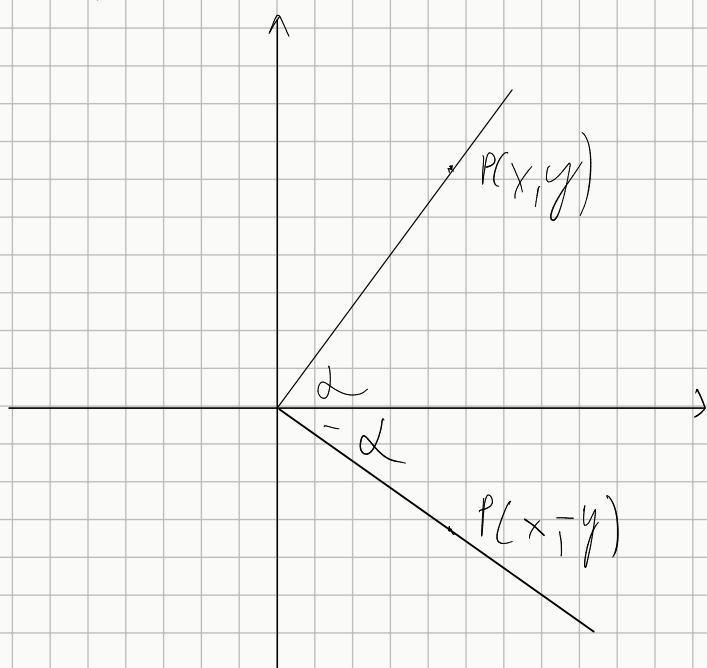
nieparzysta

$$\operatorname{tg}(\alpha + k\pi) = \operatorname{tg} \alpha \quad f(y) = \operatorname{tg} \quad D: \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$$

nieparzysta

$$\operatorname{tg}(\alpha + k\pi) = \operatorname{tg} \alpha \quad D:$$

nieparzysta



Parzysta

$$\bigwedge_{x \in X} x \in X \wedge f(x) = f(x)$$

$$\sin \alpha = \frac{y}{r}$$

$$\cos \alpha = \frac{x}{r}$$

$$\operatorname{tg} \alpha = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

$$\sin \alpha = \frac{y}{r} \quad \sin -\alpha = -\frac{y}{r}$$

$$f(-x) = -f(x)$$

$$\cos \alpha = \frac{x}{r} \quad \cos \alpha = \frac{x}{r} \quad \cos \alpha = \frac{x}{r}$$

$$\tan \alpha = \frac{y}{x} \quad \tan \alpha = \frac{y}{x} \quad \tan \alpha = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2} \quad r = \sqrt{x^2 + y^2} \quad r = \sqrt{x^2 + y^2}$$

sin +	sin +
cos -	cos +
tg -	tg +
ctg -	ctg +

sin -	sin -
cos +	cos +
tg +	tg -
ctg +	ctg -

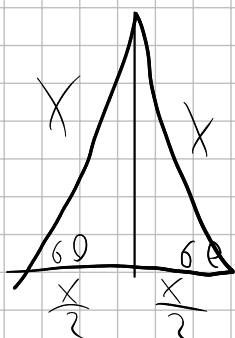
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	x	0	x	0
$\cot \alpha$	x	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	x	0	x

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
	30°	45°	60°	90°	180°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
tg	0	1	$\sqrt{3}$	x	0
ctg	x	1	$\frac{1}{\sqrt{3}}$	0	x

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

$$\frac{0}{1} \cdot \frac{1}{\sqrt{3}} = 0$$

$$\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{1}{3}$$



$$p = \frac{a^2 \sqrt{3}}{4}$$

$$h = \frac{a \sqrt{3}}{2}$$

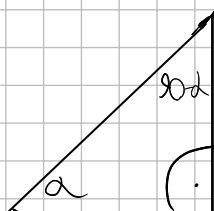
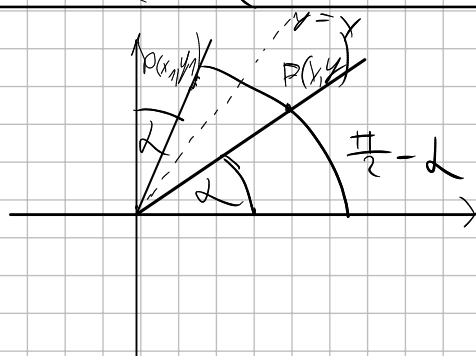
$$90^\circ - \alpha$$

$$\sin(90^\circ - \alpha) = \cos \alpha$$

$$\cos(90^\circ - \alpha) = \sin \alpha$$

$$\tan(90^\circ - \alpha) = \cot \alpha$$

$$\cot(90^\circ - \alpha) = \tan \alpha$$



	$\beta =$						
	$\frac{\pi}{2} - \alpha$	$\frac{\pi}{2} + \alpha$	$\pi - \alpha$	$\pi + \alpha$	$\frac{3\pi}{2} - \alpha$	$\frac{3\pi}{2} + \alpha$	$2\pi - \alpha$
$\sin \beta$	$\cos \alpha$	$\cos \alpha$	$\sin \alpha$	$-\sin \alpha$	$-\cos \alpha$	$-\cos \alpha$	$-\sin \alpha$
$\cos \beta$	$\sin \alpha$	$-\sin \alpha$	$-\cos \alpha$	$-\cos \alpha$	$-\sin \alpha$	$\sin \alpha$	$\cos \alpha$
$\operatorname{tg} \beta$	$\operatorname{ctg} \alpha$	$-\operatorname{ctg} \alpha$	$-\operatorname{tg} \alpha$	$\operatorname{tg} \alpha$	$\operatorname{ctg} \alpha$	$-\operatorname{ctg} \alpha$	$-\operatorname{tg} \alpha$
$\operatorname{ctg} \beta$	$\operatorname{tg} \alpha$	$-\operatorname{tg} \alpha$	$-\operatorname{ctg} \alpha$	$\operatorname{ctg} \alpha$	$\operatorname{tg} \alpha$	$-\operatorname{tg} \alpha$	$-\operatorname{ctg} \alpha$

Na szczęście nie trzeba uczyć się na pamięć powyższej tabeli. Wystarczy zapamiętać poniższy schemat. Niech $\beta \in \langle \frac{\pi}{2}, 2\pi \rangle$. Kąt β przedstawiamy w postaci

$$\beta = n \cdot \frac{\pi}{2} \pm \alpha, \text{ gdzie } \alpha \in \langle 0, \frac{\pi}{2} \rangle \wedge n \in \{1, 2, 3, 4\}.$$

Wówczas

$$f(\beta) = f(n \cdot \frac{\pi}{2} \pm \alpha) = \begin{pmatrix} \text{znak} \\ \text{Tabela nr. 1} \end{pmatrix} \cdot \begin{cases} f(\alpha), & \text{gdy } n - \text{parzyste} \\ cf(\alpha), & \text{gdy } n - \text{nieparzyste} \end{cases}$$

$f(\cdot)$ – funkcja trygonometryczna ($\sin, \cos, \operatorname{tg}, \operatorname{ctg}$). Zaś $cf(\cdot)$ – odpowiadająca funkcji f cofunkcja, wyznaczona według schematu

$$\sin \leftrightarrow \cos; \quad \operatorname{tg} \leftrightarrow \operatorname{ctg}.$$

$$\operatorname{tg}(\frac{3}{2}\pi + 2) \quad 2 \in (0, \pi)$$

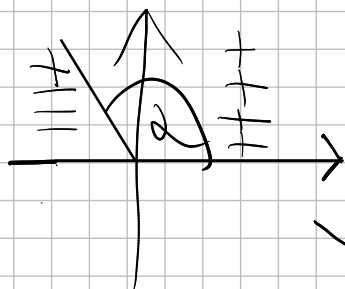
$$\frac{3}{2}\pi = 3 \cdot \frac{\pi}{2}$$

$$\sin \frac{3}{2}\pi = \sin(2\pi - \frac{\pi}{2}) = \sin(4\frac{\pi}{2} - \frac{\pi}{2})$$

$$-\sin \frac{\pi}{2} = -\frac{\sqrt{2}}{2} \quad \text{Co my tu robimy?}$$

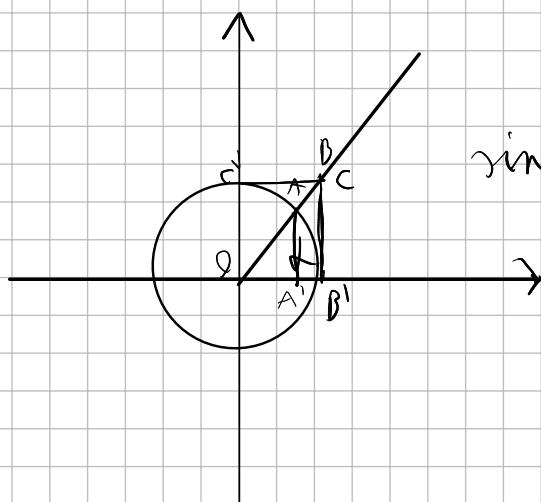
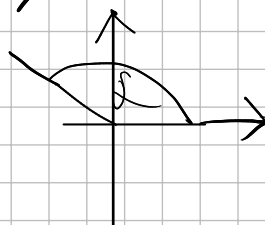
$$\operatorname{tg}(\frac{5}{8}\pi)$$

$$-\frac{\sqrt{3}}{3}$$



$$\sin(-\frac{1742}{3}\pi) = -\sin(580\pi + \frac{2}{3}\pi)$$

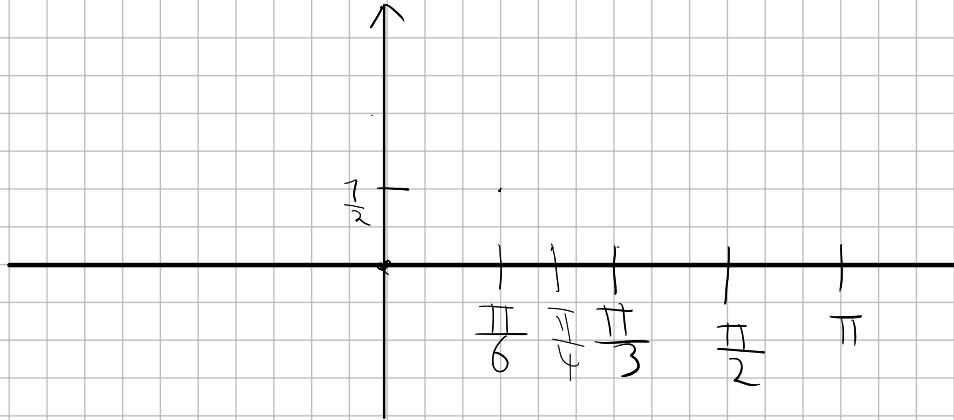
$$= -\sin(\frac{2}{3}\pi) =$$



$$\sin \alpha = \frac{|AA'|}{|OA|}$$

$$\cos \alpha = \frac{|OA'|}{|OA|}$$

tg



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$\sin \alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\operatorname{tg} \alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	x	0	x	0
$\operatorname{ctg} \alpha$	x	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	x	0	x