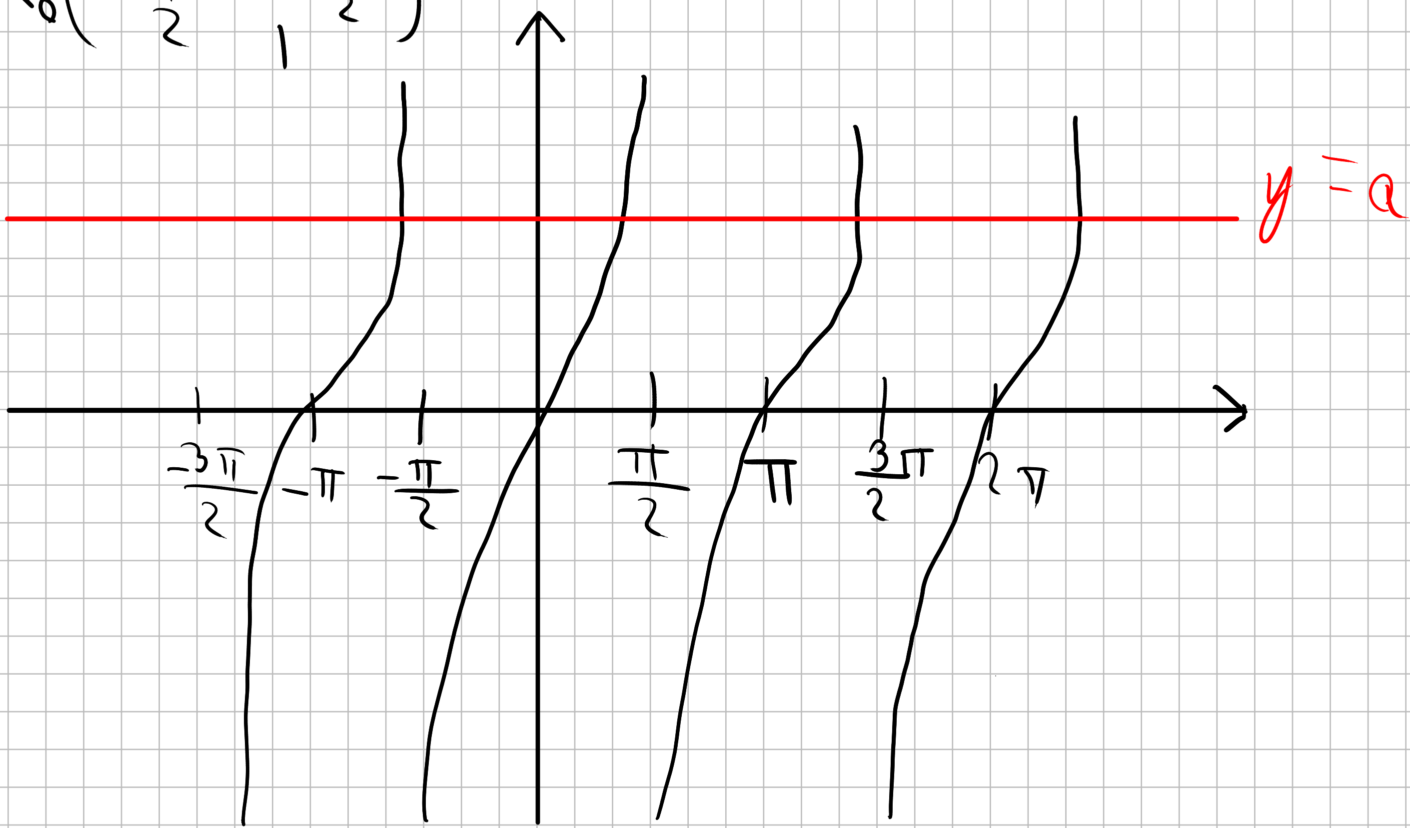
Wstęp do matematyki

$\operatorname{tg} x = a \quad a \in R$

$x = x_0 + k\pi$

$x_0 \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



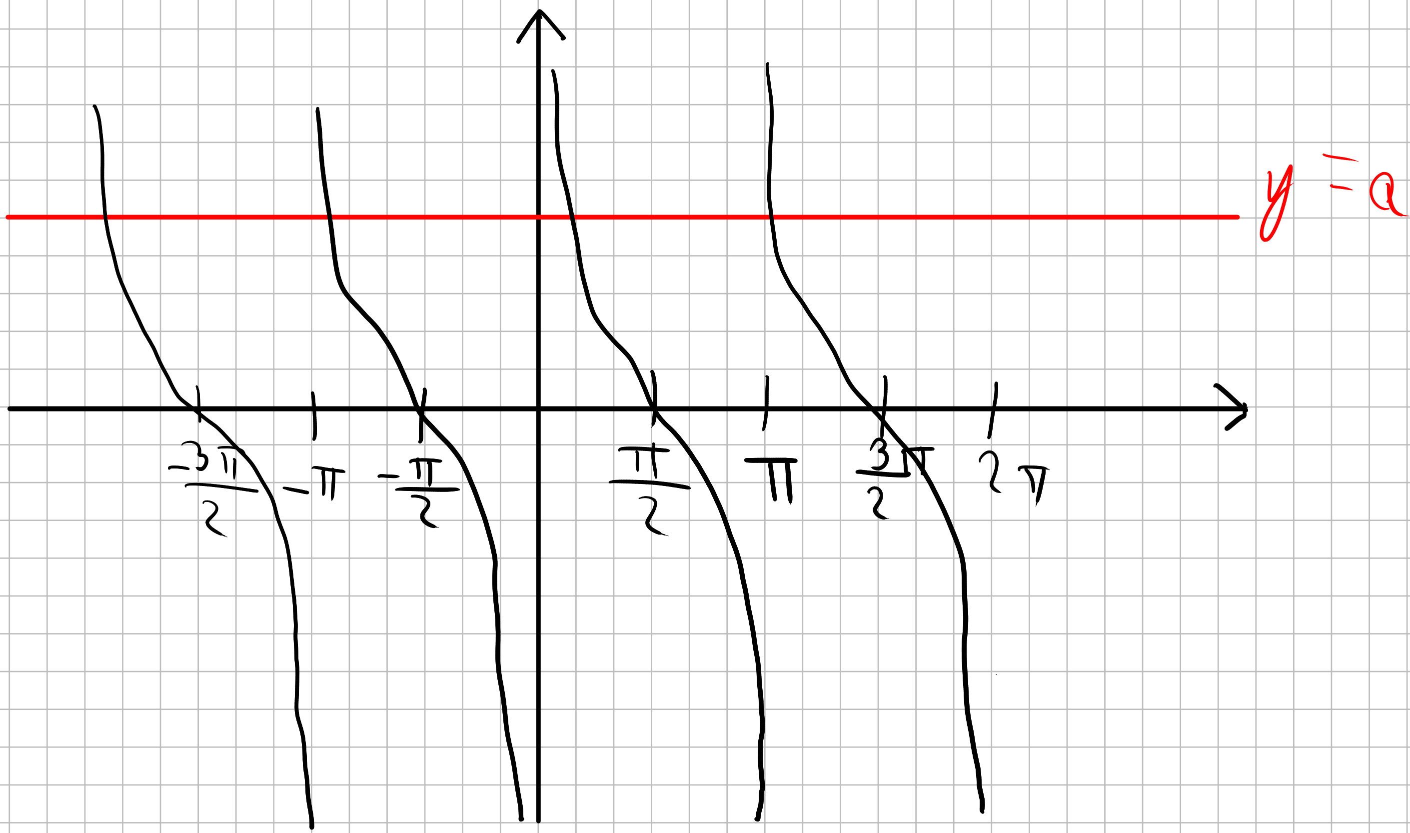
$\operatorname{tg} x = a \wedge a \in R \Rightarrow x = x_0 + k\pi \wedge k \in \mathbb{Z}$
$\wedge\, x_0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \qquad \operatorname{tg} x_0 = a$

$D = \mathbb{R} - \left\{\frac{\pi}{2} + k\pi : k \in \mathbb{Z}\right\}$

$\operatorname{ctg} x = a \quad a \in R$

$x = x_0 + k\pi$

$k \in Z \quad \operatorname{ctg} x_0$



$\operatorname{ctg} x = a \wedge a \in R \Longrightarrow x_0 + k\pi \wedge k \in Z \wedge x_0 \in (0, \pi)$

$D = R - \{k\pi : k \in \mathbb{Z}\}$

$3 \sin x = 2 \cos^2 x$

$3 \sin x = 2(1 - \sin^2 x)$

$2 \sin^2 x - 3 \sin x - 2 = 0$

$\Delta = 9 - 4 \cdot 2 \cdot 2 = 25$

$t_1 = \frac{-3-5}{4} = -2 \vee t_2 = \frac{-3+5}{4} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} + 2k\pi \quad \vee \quad \cancel{\frac{\pi}{6} = \frac{1}{2}}$

$\sin^2 2x = 1 - \sin^2 x$

$(2 \sin x \cos x)^2 = \cos^2 x$

$4\sin^2 x \cos^2 x - \cos^2 x = 0$

$\cos^2 x (4 \sin^2 x - 1) = 0$

$\cos^2 x = 0 \vee 4 \sin^2 x = 1$

$\cos x = 0 \vee \quad \sin x = \frac{1}{2} \vee \sin x = -\frac{1}{2}$

$\cos x = 0$

$(x = \frac{\pi}{2} + k\pi \wedge k \in Z) \vee (x = \frac{\pi}{6} + 2k\pi \wedge k \in Z) \vee (x = \frac{5}{6}\pi + 2k\pi)$

$[(x = \frac{\pi}{2} + k\pi) \vee (x = \frac{\pi}{6} + 2k\pi) \vee (x = \frac{5}{6}\pi + 2k\pi)$
$\vee (x = -\frac{\pi}{6} + 2k\pi) \vee (x = \frac{7}{6}\pi + 2k\pi)] \wedge k \in Z \wedge x \in R$

$\frac{7}{6}\pi = \frac{\pi}{6} + \pi$

$-\frac{\pi}{6} = \frac{5}{6} - \pi$

$\{x: x = \frac{\pi}{6} + k\pi\} \vee \{x: x = \frac{5}{6} + 2k\pi\}$

$\{x: x = -\frac{\pi}{6} + k\pi\}$

$x \in \{-\frac{\pi}{6} + k\pi, \frac{\pi}{6} + k\pi, \frac{\pi}{2} + k\pi\} \quad k \in Z$

nierówności

$\sin x > -\frac{\sqrt{3}}{2}$

a) $x \in \langle -\pi, 2\pi \rangle$

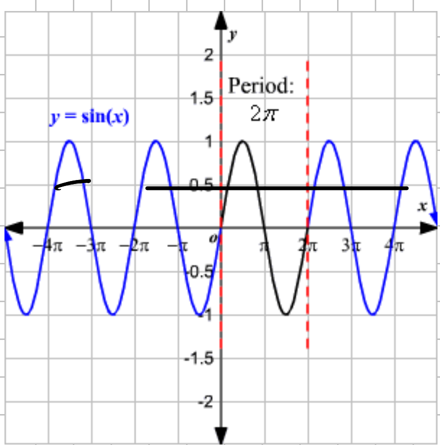
b) R

c)

$\sin x = \frac{\sqrt{3}}{2}$

$(x = -\frac{\pi}{3} + 2k\pi) \vee (x = -\frac{4}{3}\pi + 2k\pi) \wedge x \in \langle -\pi, 2\pi \rangle$

$(x = -\frac{\pi}{3} \vee x = \frac{5}{3}\pi \vee x = \frac{4}{3}\pi \vee x = -\frac{2}{3}\pi) \wedge x \in \langle -\pi, 2\pi \rangle$



$\sin x > -\frac{\sqrt{3}}{2}$

$x \in \langle -\pi, -\frac{2}{3}\pi) \vee (-\frac{\pi}{3}, \frac{4}{3}\pi) \vee (\frac{5}{3}\pi, 2\pi \rangle$

R $\quad \sin x = -\frac{\sqrt{3}}{2} \wedge x \in R$

$$\left(-\frac{\pi}{3} + 2k\pi, \frac{4}{3}\pi + 2k\pi\right) \quad k \in Z$$

$$\sin x > -\frac{\sqrt{3}}{2} \wedge x \in R \iff x \in \bigcup_{k \in Z} \left(-\frac{\pi}{3} + 2k\pi, \frac{4}{3}\pi + 2k\pi\right)$$

$$\left[\frac{\cos x}{1 - \cos x} \geq 0 \quad \wedge x \in D\right]$$

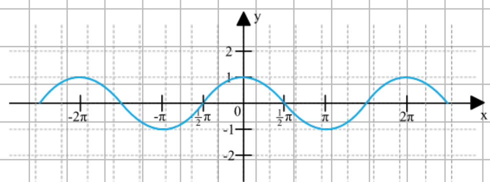
$$-\frac{2\cos x - 1}{1 - \cos x} \geq 0 \wedge x \in D$$

D

$\cos x \neq 1$

$x \neq 1 + k\pi$

$\cos x \geq \frac{1}{2}$



$\cos x = \frac{1}{2}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

$$x \in \left(\bigcup_{x \in Z} \left\langle -\frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + 2k\pi \right\rangle \wedge x \in D\right)$$

$$x \in \left(\bigcup_{x \in Z} \left\langle -\frac{\pi}{3} + 2k\pi, 2k\pi\right) \cup \left(2k\pi, \right.\right.$$

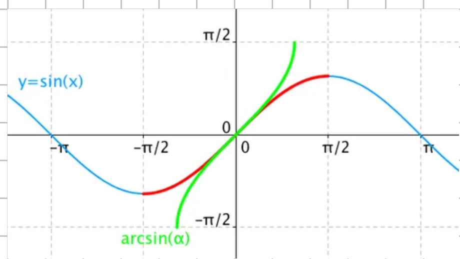
Funkcje Cyklometryczne

odwrotne do funkcji trygonometrycznych ograniczonych do pełnych przedziałów.

arcus sinus

$y = \arcsin x \iff x = \sin y,\ x \in \langle -1, 1 \rangle$

$y \in \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$



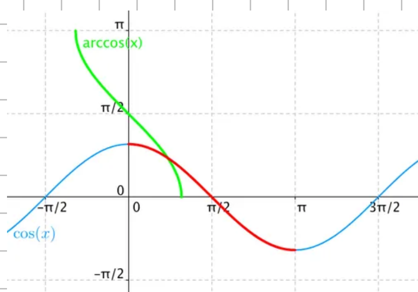
$y = \arccos x$

$\iff x = \cos y \quad x \in \langle -1, 1 \rangle$

$y \in \langle 0, \pi \rangle$

$D = \langle -1, 1 \rangle$

$ZW \langle 0, \pi \rangle$



ściśle malejąca

$tg\, x \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\bigwedge_{x \in R} tg(arctg\, x) = x$

$$\bigwedge_{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)} \operatorname{arc\,tg}(\operatorname{tg}(x)) = x$$

daj pan spokój

$$\arcsin\frac{\sqrt{2}}{2} = \alpha \equiv \sin\alpha = \frac{\sqrt{2}}{2} \wedge \alpha \in \left\langle -\frac{\pi}{2}, \frac{\pi}{2}\right\rangle$$

$$\Rightarrow \alpha = \frac{\pi}{4}$$

$$\operatorname{arc\,ctg}(-1) = \alpha$$

2) Wyznaczyć dziedzinę naturalną

$$f(x) = \arcsin(\log_{\frac{1}{3}} x)$$

$$D = \{x \in \mathbb{R} : x > 0 \wedge \log_{\frac{1}{3}} 3 \leq \log_{\frac{1}{3}} x \leq \log_{\frac{1}{3}} \frac{1}{3}$$

$$D = \{x \in \mathbb{R} : x > 0 \wedge x \in \left\langle \frac{1}{3}, 3 \right\rangle\} = \left\langle \frac{1}{3}, 3 \right\rangle$$

3) Rozwiąż nierówność

$$\arccos x \leq \frac{5}{6}\pi$$

$$\bigwedge_{x \in \langle 0, \pi \rangle} \arccos(\cos x) = x$$

$$D = \langle -1, 1 \rangle$$

z własności funkcji

$\frac{5}{6}\pi = \arccos\left(\cos\frac{5\pi}{6}\right)$

$\arccos x \leq \frac{5}{6}\pi \wedge x \in \langle -1, 1\rangle$

$\arccos x \leq \arccos\left(\cos\frac{5}{6}\pi\right)$

$x \geq \cos\frac{5}{6}\pi$ — bo malejąca ($\arccos x$)

$x - \cos\frac{5}{6}\pi \wedge x \in \langle -1, 1\rangle$

$x \geq \overset{\text{redukcja}}{-\cos\frac{\pi}{6}}$ —— " ——

$x \geq -\frac{\sqrt{3}}{2}$ —— " ——

$x \in \langle -\frac{\sqrt{3}}{2}, 1\rangle$

$\arcsin(-x) = -\arcsin x$

$\operatorname{arctg}(-x) = -\operatorname{arctg} x$

$\arccos(-x) = \pi - \cos x$

$\operatorname{arcctg}(-x) = \pi - \operatorname{ctg} x$

KILL ME

PLEASE