

$$A \times B = B \times A.$$

... trybami, do jakich należąca jest A, B, C, D, E, F

a) $(A \subset B \wedge C \subset D) \Rightarrow (A \cup C \subset B \cup D),$

$$\boxed{x \in A \wedge x \in B \wedge x \in C \wedge x \in D} \Rightarrow \boxed{(x \in A \vee x \in C) \wedge (x \in B \vee x \in D)}$$

T

$$\begin{array}{l} x \in A \equiv T \rightarrow x \in A \\ x \in B \equiv T \rightarrow x \in B \\ x \in C \equiv T \rightarrow x \in C \\ x \in D \equiv T \rightarrow x \in D \end{array}$$

Nie może być fałszem
skoro x należy do wszystkich
wymienionych zbiorów

$$\boxed{x \in A \Rightarrow x \in B \wedge x \in C \Rightarrow x \in D} \Rightarrow$$

$$\boxed{x \in A \vee x \in C \Rightarrow x \in B \vee x \in D}$$

$$\boxed{(x \in A \vee x \in B) \wedge (x \in C \vee x \in D)}$$

złożimy T

$$\boxed{(x \notin A \wedge x \notin C) \vee (x \in B \vee x \in D)}$$

F

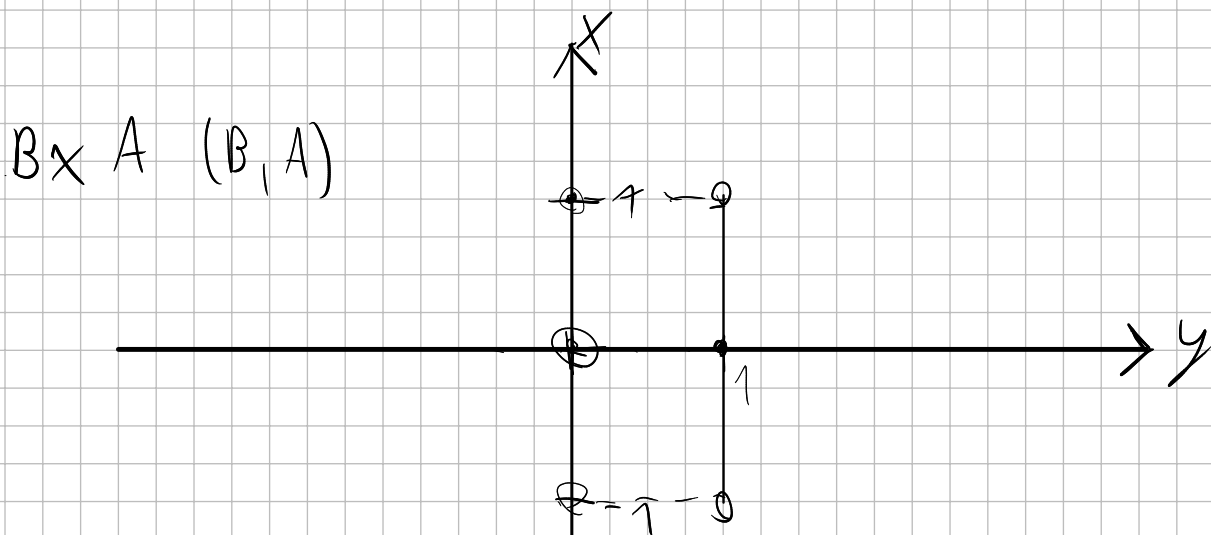
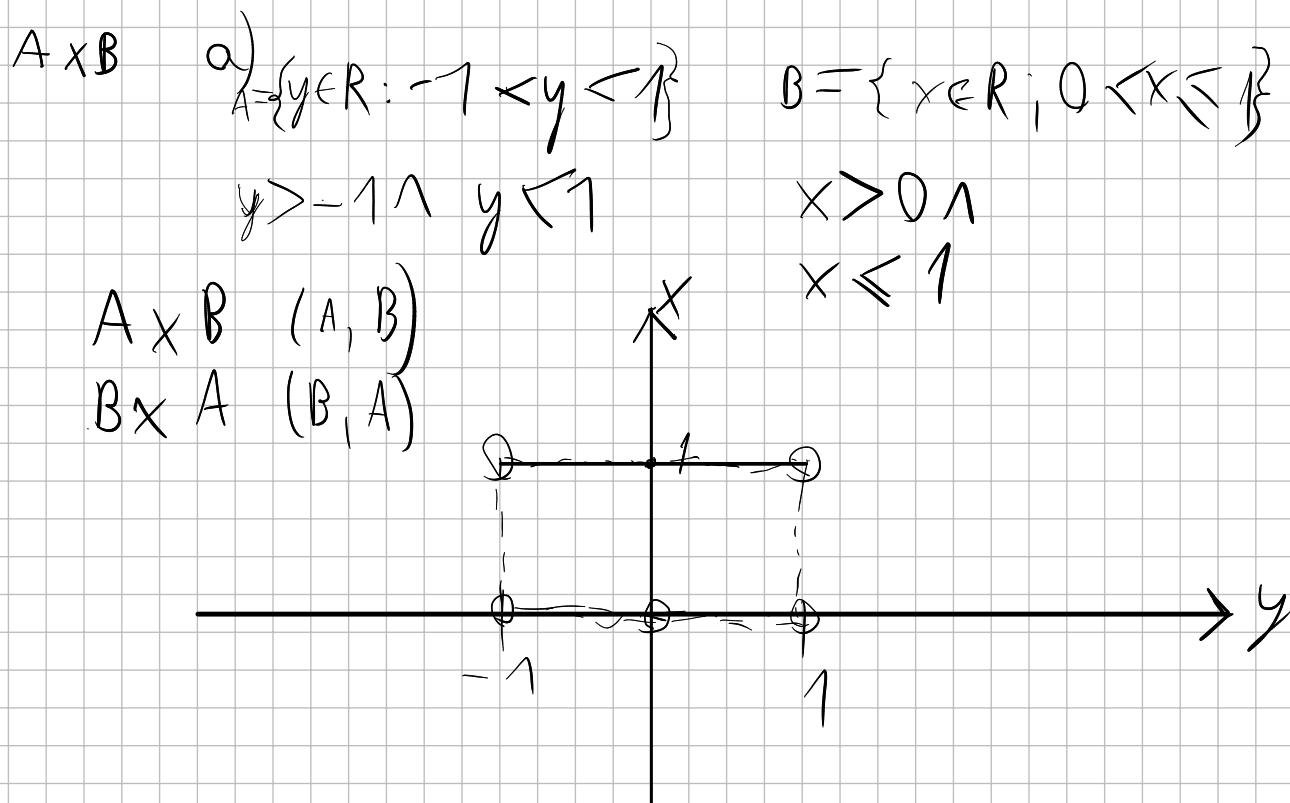
$$\boxed{x \in B \vee x \in D}$$

T

9. Naskicuj na płaszczyźnie zbiory $A \times B$ i $B \times A$ dla:

- a) $A = \{y \in \mathbb{R} : -1 < y < 1\}$, $B = \{x \in \mathbb{R} : 0 < x \leq 1\}$,
 b) $A = \mathbb{Z}$, $B = \langle 1, 2 \rangle$,
 c) $A = \{x \in \mathbb{R} : x^2 + x - 2 \geq 0\}$, $B = \{b \in \mathbb{N} : 2^b < 11\}$,
 d) $A = \{x \in \langle 0, \infty \rangle : \frac{x-1}{x+1} < 0\}$, $B = \{x \in \mathbb{R} : x^2 \leq 4\}$.

1

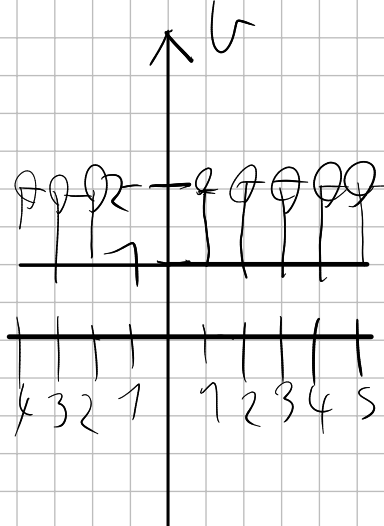


$$(A \times B) \times C = (a, (b, c)) \quad a \in A, b \in B, c \in C$$

$$A \times (B \times C) = (a, (b, c)) \quad a \in A, b \in B, c \in C$$

b) $A = \mathbb{Z}, B = \langle 1, 2 \rangle,$

$A \times B$



$$\underbrace{(A \cup B) \times C}_L = \underbrace{(A \times C) \cup (B \times C)}_R$$

$$\begin{aligned} (x, y) \in L & \quad (x, y) \in R \wedge (z, y) \in R \\ (x \in A \vee x \in B) \wedge y \in C & \quad \begin{array}{l} x \in A \\ y \in C \end{array} \quad \begin{array}{l} z \in B \\ y \in C \end{array} \\ (x \in A \wedge y \in C) \vee (x \in B \wedge y \in C) & \end{aligned}$$

$$[(x, y) \in A \times C \vee (x, y) \in B \times C]$$

11. Niech A_1, \dots, A_n będą dowolnymi zbiorami. Zdefiniujmy \mathcal{A} jako najmniejszy zbiór, dla którego

$$\bigwedge_{k \in \{1, \dots, n\}} A_k \in \mathcal{A} \quad \text{oraz} \quad X \in \mathcal{A} \wedge Y \in \mathcal{A} \Rightarrow X \cup Y \in \mathcal{A}.$$

Ile maksymalnie elementów ma zbiór \mathcal{A} ? Podaj przykład takiego zbioru.

$$A_1, \dots, A_n$$

$$\bigwedge_{k \in \{1, 2, \dots, n\}} A_k \in \mathcal{A}: X \in \mathcal{A} \wedge Y \in \mathcal{A} \Rightarrow X \cup Y \in \mathcal{A}$$

$$\bigwedge_{k \in \{1, 2, \dots, n\}} A_k$$

$$\frac{X \in A \wedge Y \in A}{\vdash} \Rightarrow X \cup Y \in A$$

$$X \in A \quad Y \in A$$

A_k - részei megvannak

$$A_1 \wedge A_2 \Rightarrow A_1$$

A

$$A_1 A_2 A_3 A_4 A_1 \cup A_2$$

$$\binom{n}{2}$$

$$A_1 \cup A_3$$

$$A_1 \cup A_4$$

$$A_2 \cup A_3$$

1 2 4 8

$$2^n - 1 = \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$$

$$\frac{n!}{k!(n-k)!}$$

$$\frac{n!}{1(n-1)!} + \frac{n!}{2(n-2)!} + \frac{n!}{6(n-3)!} + \dots + \frac{n!}{n!(n-n)!}$$

$$\binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n-1} + \frac{n!}{n!(n-n)!} = 1$$

11
2n

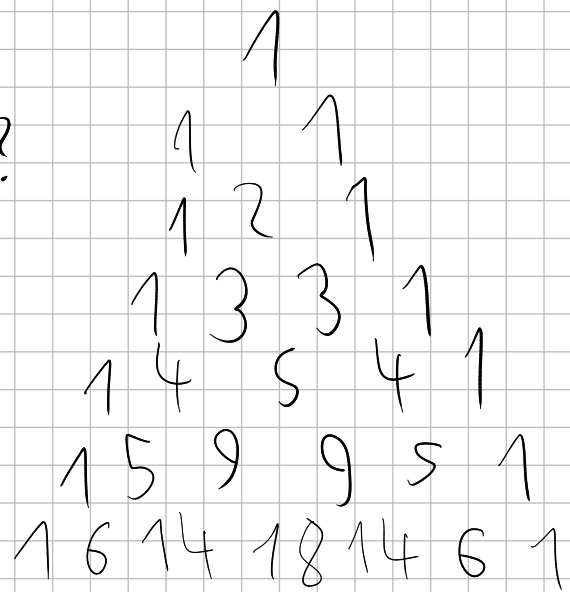
$$(x+y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$$

Binomial Newtona

$$x=1$$

To ten zjevný trojúhelník?

AHHHHHHH/!

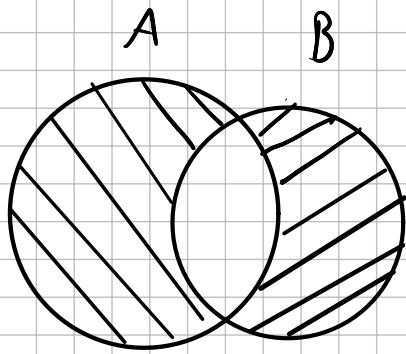


INDUKCJA

$$[p(1) \wedge \bigwedge_n p(n) \Rightarrow p(n+1)] \Rightarrow \left[\bigwedge_n p(n) \right]$$

efekt domina

$$A \Delta B \quad (A \cup B) - (A \cap B)$$



2. Wyznacz iloczyn kartezjański $A \times B$ i $B \times A$ dla zbiorów:

- a) $A = \{0, 1\}, B = \{1, 2\}$,
- b) $A = \{0, 1, 2\}, B = \{2, 3\}$,
- c) $A = \emptyset, B = \{1, 2, 3\}$.

a1)

$$A \times B = \{(0, 1); (0, 2); (1, 1); (1, 2)\}$$

a2)

$$B \times A = \{(1, 0); (1, 1); (2, 0); (2, 1)\}$$

b1)

$$A \times B = \{(0, 2); (0, 3); (1, 2); (1, 3); (2, 2); (2, 3)\}$$

b2)

$$B \times A = \{(2, 0); (2, 1); (2, 2); (3, 0); (3, 1); (3, 2)\}$$

$$A \times B = B \times A \Rightarrow \frac{A = B}{F}$$

$$(a_i, b_i) = (b_i, a_i)$$

$$a_i = b_i$$

skoro zbiór elementów jest różny
poszczególne elementy będą różne

5. Wyznacz zbiór potęgowy dla zbiorów:

- a) $\{1, 2, 3\}$,
- b) \emptyset ,
- c) $\{\emptyset\}$,
- d) $\{\emptyset, \{\emptyset\}\}$.

a)

$$A = \{1, 2, 3\}$$

$$2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

$$A = \emptyset$$

$$A = \{\emptyset\}$$

$$2^A = \emptyset$$

$$2^A = \{\emptyset, \{\emptyset\}\}$$

$$A = \{\emptyset, \{\emptyset\}\}$$

$$2^A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$

6. Udowodnij, że dla dowolnych zbiorów A, B, C, D zachodzą równości:

a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$,

b) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$,

c) $(A \setminus B) \cup C = [(A \cup C) \setminus B] \cup (B \cap C)$,

d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$,

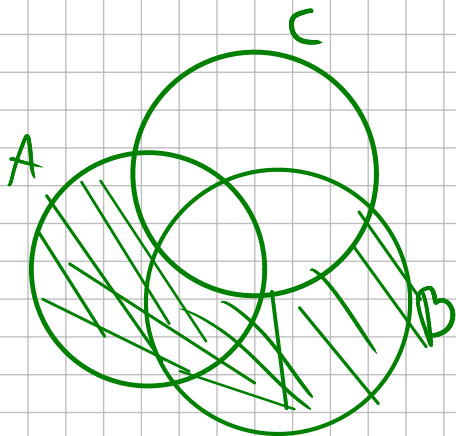
e) $(A \setminus B) \cap (C \setminus D) = (A \cap C) \setminus (B \cup D)$,

f) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$,

g) $(A \triangle B)^c = A^c \triangle B^c$.

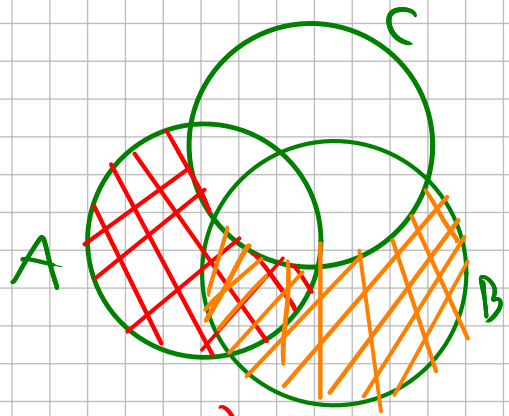
a)

$$(A \cup B) - C = (A - C) \cup (B - C)$$



$$A \cup B - C$$

=



$$(A - C) \cup (B - C)$$

g)

$$A - (B - C) = (A - B) \cup (A \cap C)$$

$$x \in A \wedge (x \notin B \vee x \in C)$$

$$(x \in A \wedge x \notin B) \wedge (x \in C \wedge x \in A)$$

$$(x \in A \wedge x \notin B) \vee (x \in A \wedge x \in C)$$

$$x \in A \wedge (x \notin B \vee x \in C)$$

6. Udowodnij, że dla dowolnych zbiorów A, B, C, D zachodzą równości:

- a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$,
- b) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$,
- c) $(A \setminus B) \cup C = [(A \cup C) \setminus B] \cup (B \cap C)$,
- d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$,
- e) $(A \setminus B) \cap (C \setminus D) = (A \cap C) \setminus (B \cup D)$,
- f) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$,
- g) $(A \Delta B)^c = A^c \Delta B^c$.

c)

$$(A - B) \cup C = [(A \cup C) - B] \cup (B \cap C)$$

$$(x \in A \wedge x \notin B) \vee x \in C$$

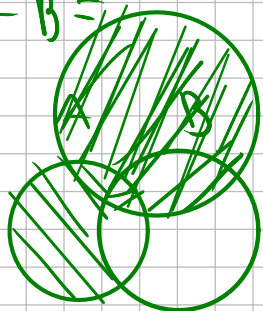
$$[(x \in A \vee x \in C) \wedge x \notin B] \vee (x \in B \wedge x \in C)$$

$$(x \in A \wedge x \notin B) \vee (x \in C \wedge x \notin B) \vee (x \in B \wedge x \in C)$$

$$(x \in A \vee x \in C) \wedge (x \notin B \vee x \in C)$$

$$(A - B) \cup C = [(A \cup C) - B] \cup (B \cap C)$$

$$A - B = (A - B) \cup (C - B) \cup (B \cap C)$$



$$C - B = C - B \cap C$$

$$(A - B) \cup (C - \cancel{B \cap C}) \cup \cancel{B \cap C}$$

$x \notin$

nie ma $x \in B \cap C$

noting

to $C - B \cap C = C$

$$[A - (A \cap B)] \cup C$$

$$[A - (A \cap B)] \cup (C \cap A) \cup C$$

$$(A - B) \cup C$$

6. Udowodnij, że dla dowolnych zbiorów A, B, C, D zachodzą równości:

- a) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$,
- b) $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$,
- c) $(A \setminus B) \cup C = [(A \cup C) \setminus B] \cup (B \cap C)$,
- d) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$,
- e) $(A \setminus B) \cap (C \setminus D) = (A \cap C) \setminus (B \cup D)$,
- f) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$,
- g) $(A \Delta B)^c = A^c \Delta B^c$.

$$(A - B) \cap (C - D) = (A \cap C) - (B \cup D)$$

$$\begin{array}{c} \frac{x \in A \wedge x \notin B}{\sim} \wedge \frac{x \in C \wedge x \notin D}{\sim} \Leftrightarrow \\ \frac{x \in A \wedge x \in C}{\sim} \wedge \frac{x \notin B \wedge x \notin D}{\sim} \end{array}$$

$$A \Delta (B \Delta C) = (A \Delta B) \Delta C$$

$$A \Delta B \Leftrightarrow (A \cup B) - (A \cap B)$$

$$\frac{(A \cup Q) - (A \cap Q)}{\quad}$$

$$\frac{(Z \cup C) - (Z \cap C)}{\quad}$$

$$Q = (B \cup C) - (B \cap C)$$

$$Z = (A \cup B) - (A \cap B)$$

$$(A \cup B \cup C) - (B \cap C) - (A \cap B \cap C)$$

KURWA

MIAŁEM TYŁE DO ZROBIENIA I ZROBIŁEM
NIC