$f(x)=\frac{x^3}{x^2-4}$ $\quad x^2-4\neq 0$

$x^2=4$

$x\neq 2 \vee x\neq -2$

$$\begin{array}{l} \quad x \\ \hline x^3 : x^2-4 \\ -x^3+4x \\ \hline = \quad 4x \end{array}$$

$$\frac{x(x^2-4)+4x}{x^2-4} \quad x\left(\frac{4x}{x^2-4}\right)$$

$$\frac{x^3}{x^2-4} \quad x+\left(\frac{4x}{x^2-4}\right)$$

a) $\frac{x}{(x+1)(x+2)(x-3)}$,

lnr > 2

kalr v > 1

$x\neq -1$

$x\neq -2$

$x\neq 3$

$D=\mathbb{R}-\{-2,-1,3\}$

$$\frac{x}{(x+1)(x+2)(x-3)} \quad \frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x-3}$$

$$\cancel{\frac{x}{(x-3)(x+1)(x+2)}}=\frac{A(x-3)(x+2)+B(x+1)(x-3)+C(x+1)(x+2)}{\cancel{(x+1)(x+2)(x-3)}}$$

$$x=A(x^2+2x-3x-6)+B(x^2-3x+x-3)+C(x^2+2x+x+2)$$

$$x=Ax^2-Ax-A6+Bx^2-B2x-B3+Cx^2+C3x+C2$$

$$x=x^2(A+B+C)+x(-A-2B+3C)-6A-3B+2C$$

$A+B+C=0$ $\quad C=-A-B$

$-A-2B+3C=1$ $\quad -3A-A-2B-3B=1$

$-6A-3B+2C=0$ $\quad -4A-5B=1$

$(x+1)^2 \quad \frac{A}{x+1} + \frac{B}{(x+1)^2}$

$(x^2+x+2) \quad \frac{Ax+B}{x^2+x+2}$

b) $\frac{1}{x^2-3x+2}$,

$$\frac{1}{x^2-3x+2} = \frac{A}{(x-1)} + \frac{B}{(x-2)}$$

$\Delta = 9 - 4 \cdot 2 \cdot 1 = 1$

$x_1 = \frac{3-1}{2} = 1$

$x_2 = \frac{3+1}{2} = 2$

$Ax - 2A + Bx - B$

$x(A+B) - 2A - B$

$-2A - B = 1$

$A + B = 0$

$A = -B$

$+2B - B = 1$

$\underline{B = 1 \wedge A = -1}$

$\boxed{-2(-B) - B}$

$-2A - A = 1$

$-A = 1$

$A = -1$

$$\frac{1}{x-1} + \frac{-1}{x-2} = \frac{1}{x^2-3x+2}$$

f) $\frac{x^2+4x+6}{x+4} > 3,$

$\frac{x^2+4x+6}{x+4}$ $\quad$ $\frac{x^2+4x+6-3x-12}{x+4} > 0$

$D \in R - \{-4\}$

BRUH

$\frac{x^2+x-6}{x+4} > 0$ $\quad$ $(x+4)(x^2+x-6) > 0$

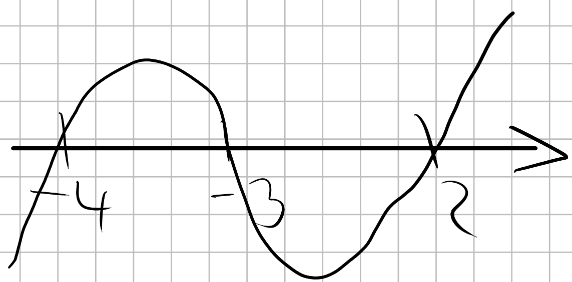
x^2+x-6

$\Delta = 1-4\cdot 1\cdot -6 = 25$

$\frac{-1+5}{2} = 2$

$(x+4)(x+3)(x-2) > 0$

$\frac{-1-5}{2} = -3$



$x \in (-4,-3) \cup (2,\infty)$

c) $\frac{1}{2-x}+\frac{1}{2+x} < 1,$

$\frac{2+\cancel{x}+2-\cancel{x}-(2-x)(2+x)}{(2-x)(2+x)} < 0$

$D = R - \{-2, 2\}$

$\cancel{4}\ \cancel{4}-x^2$

$(2-x)(2+x)-x^2 < 0$

$4-x^2-x^2 < 0$

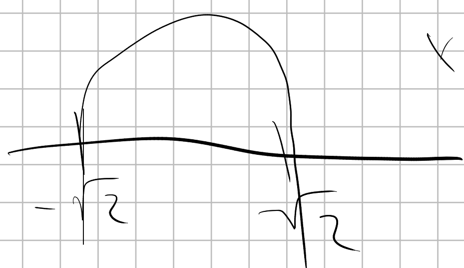
$4-2x^2 < 0$ $\quad$ $2-x^2 < 0$

$(\sqrt{2}-x)(\sqrt{2}+x)$ $\quad x^2>2$

$x=\sqrt{2} \quad x=-\sqrt{2}$

$x>\sqrt{2} \vee x<-\sqrt{2}$

$x\in(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$



a) $\dfrac{x^5-x^3+1}{x(x+1)^3(x^2-1)^2}$,

$x(x+1)^3(x-1)(x-1)\underline{(x+1)(x+1)}$

$x(x+1)^5(x-1)^2$

$$\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}+\frac{D}{(x+1)^3}+\frac{E}{(x+1)^4}+\frac{E}{(x+1)^5}+\frac{G}{(x-1)}+\frac{H}{(x-1)^2}$$

ZADANIE 16. Dana jest funkcja określona wzorem $f(x)=\frac{1}{x}$. Rozwiązać nierówności:

$$f(x)-f\left(\frac{1}{x}\right)<f(x^3)-f\left(\frac{1}{x^3}\right).$$

czw 16:30

$\frac{1}{x}-x<\frac{1}{x^3}-x^3$

$\frac{1}{x}-x<$

$(p \wedge (q \Rightarrow r)) \Rightarrow [(p \Rightarrow r) \wedge (q \Rightarrow r)]$

$\sim((p \wedge \sim q) \vee (p \wedge r)) \vee [\sim p \vee r \wedge \sim q \vee r]$

$(\sim p \vee q) \wedge (\sim p \vee r) \vee (\sim p \vee r) \wedge (\sim q \vee r)$

$p = T \quad (q \wedge r) \vee \sim p$

$q = F$

$r = F$

d) $\displaystyle\sum_{k=1}^{n} \frac{1}{(4k-3)(4k+1)} = \frac{n}{4n+1}.$

$$\sum_{k=1}^{n} \frac{1}{(4k-3)(4+1)} = \frac{n}{4n+1}$$

$$\sum_{k=1}^{n} \frac{1}{(4k-3)(4k+1)}$$

```
acc=0
for k in range(1,n):
    acc += 1/(4*k-3)(4*k+1)

return acc
```

$$\frac{1}{(4-3)(4+1)} + \frac{1}{(4\cdot 2-3)(4\cdot 2+1)}$$

$$\frac{1}{5} + \frac{1}{45} + \frac{1}{117}$$

zakładamy że wzór

$$\sum_{k=1}^{n} \frac{1}{(4k-3)(4k+1)}$$

jest prawdziwy dla n [illegible] n

$$n^2 + Z\left(\frac{n+1}{2}\right) + n$$

$4 \to 6$

$3 \to 3$

$2 \to 1$

$1 \to 0$

$$Z \Rightarrow T$$

$$2 | n \Rightarrow n \cdot (n+1)/2$$

$$\sim(2|n) \Rightarrow n/2 \cdot (n+1)$$

$$n^2 + Z\left(\frac{n+1}{2}\right) + n$$

$$2|n \Rightarrow n^2 + n \cdot \left\lfloor \frac{n+1}{2} \right\rfloor + n$$

$$\sim(2|n) \Rightarrow n^2 + \left\lfloor \frac{n}{2} \right\rfloor \cdot (n+1) + n$$

c) $10^n + 4^n - 2$ jest podzielna przez 3.

1

$10^1 + 4^1 - 2 = 12 \quad 12 | 3$

$x(1) \equiv T$

2

$n \in N$

$\dfrac{10^n + 4^n - 2}{T} \Rightarrow 10^{n+1} + 4^{n+1} - 2$

$3q \wedge q \in Z$

$10^n + 4^n - 2 = 3q$

$10^{n+1} + 4^{n+1} - 2$

$10^n \cdot 10 + 4^n \cdot 4 - 2$

$(3q - 4^n + 2) \cdot 10 + 4^n \cdot 4 - 2$

$30q - 10 \cdot 4^n + 20 + 4^n \cdot 4 - 2$

$30q - 6 \cdot 4^n + 18$

$\underline{\underline{3}}(10q - 3 \cdot 4^n + 6)$

$n \in N \Rightarrow 3 | 3 \cdot n$

$2^n > n^2 \wedge n \in N$

1	1
2	1
4	4
8	9
16	
32	
64	
128	
256	
512 1024	

$$\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}, \qquad n \geqslant 2.$$

$$\frac{4^2}{3} < \frac{2\cdot 3\cdot \cancel{4}}{\cancel{4}}$$

$$\frac{16}{\cancel{3}} < 18$$

$$\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$$

T

$$\frac{4^{n+1}}{n+2} < \frac{(2n+2)!}{((n+1)!)^2}$$

$$\frac{4^n\cdot 4}{n+2} < \frac{(2n+2)!}{((n+1)\cdot n!)^2}$$

wszystko dodatnie

$$(4^n\cdot 4)\cdot((n+1)\cdot n!)^2 < (n+2)(2n+2)!$$

$$4^{n+1}\cdot[(n+1)!]^2 < (n+2)(2n+2)!$$

$$4^{n+1}\cdot[(n+1)\cdot n!]^2-(n+2)(2n+2)! < 0$$

$$4^{n+1}\cdot(n+1)^2\cdot n!^2-(n+2)(2n+2)!$$

$$4^n\cdot 4(n+1)^2\cdot n!^2 < (n+2)(2n+2)(2n+1)(2n)!$$

$$4^n\cdot 4(n^2+2n+2)\cdot n!^2 < (n+2)(2n+2)(2n+1)(2n)!$$

8. Znajdź zwartą postać sumy

$$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n!$$

dla wszystkich $n \in \mathbb{N}$.

$1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \ldots + n \cdot n!$

$1 = 1$

$\sum_{k=1}^{n} k \cdot k!$ $\quad 1 + 4 = 5 \quad 3! - 1$

$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 6 = 23 \quad 4! - 1$

$1 \cdot 1 + 2 \cdot 2 + 3 \cdot 6 + 4 \cdot 4! = 119 \quad 5! - 1$

$\sum_{k=1}^{n} (k+1)! \quad (n+1)! - 1$

$2! + 3! + 4! + 5!$

$2 + 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 2 \cdot 3 \cdot 4 \cdot 5$

$2(1 + 3 + 3 \cdot 4 + 3 \cdot 4 \cdot 5)$

$2(1 + 3(1 + 4 + 4 \cdot 5)$

$2(1 + 3(1 + 4(1 + 5)))$

$\sum_{k=1}^{n} (k+1)! = (n+1)! - 1$

n kąt wypukły

T

$1 = 1$

$(n+1)! - 1 = (n+2)! - 1$

$(n+1)! - 1 = (n+1)!(n+2) - 1$