Wstęp do matematyki

https://moodle3.cs.pollub.pl/pluginfile.php/73589/mod_resource/content/0/fun_tryg_cylkometr_notatka.pdf

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

$\mathrm{tg}\, 2\alpha = \dfrac{2\,\mathrm{tg}\,\alpha}{1-\mathrm{tg}^2\alpha}$

$\mathrm{ctg}\, 2\alpha = \dfrac{\mathrm{ctg}^2\alpha - 1}{2\,\mathrm{ctg}\,\alpha}$

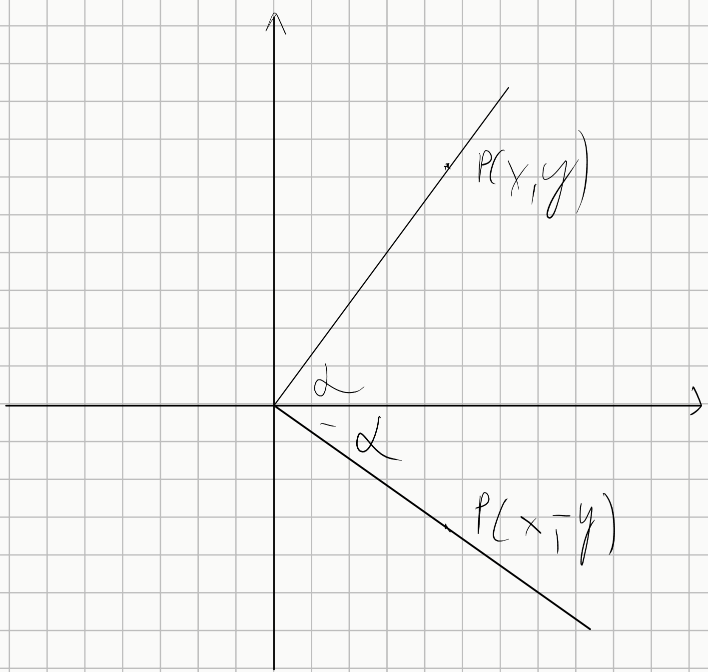
$\boxed{\sin^2\alpha + \cos^2\alpha = 1}$

$\mathrm{tg}(\alpha + 2k\pi) = \sin\alpha$

$\cos(\alpha + 2k\pi) = \cos\alpha$ parzysta — nieparzysta

$\mathrm{tg}(\alpha + k\pi) = \mathrm{tg}\,\alpha$ $f(x)=\mathrm{tg}$ $D: \frac{\pi}{2} + k\pi,\ k\in Z$ — nieparzysta

$\mathrm{tg}(\alpha + k\pi) = \mathrm{tg}\,\alpha$ $D:$ — nieparzysta



Parzysta

$\bigwedge_{x\in X} -x\in X \wedge f(-x) = f(x)$

$\sin\alpha = \frac{y}{r}$

$\cos\alpha = \frac{x}{r}$

$\mathrm{tg}\,\alpha = \frac{y}{x}$

$r = \sqrt{x^2+y^2}$

$\sin\alpha = \frac{y}{r}$ $\sin(-\alpha) = \frac{-y}{r}$

$f(-x) = -f(x)$

$$\cos\alpha = \frac{x}{r} \qquad \cos\alpha = \frac{x}{r} \qquad \cos -\alpha = \frac{x}{r}$$

$$\mathrm{tg}\,\alpha = \frac{y}{x} \qquad \mathrm{tg}\,\alpha = \frac{y}{x} \qquad \mathrm{tg} -\alpha = \frac{-y}{x}$$

$$r = \sqrt{x^2+y^2} \qquad r = \sqrt{x^2+y^2} = r = \sqrt{x^2+(-y)^2}$$

sin +	sin +
cos −	cos +
tg −	tg +
ctg −	ctg +
sin −	sin −
cos −	cos +
tg +	tg −
ctg +	ctg −

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	−1	0	1
$\mathrm{tg}\,\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	x	0	x	0
$\mathrm{ctg}\,\alpha$	x	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	x	0	x

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
	0°	30°	45°	60	90	180
sin	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$	0
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$	−1
tg	0	$\frac{1}{\sqrt{3}}$	1	$\frac{1}{\sqrt{3}}$	—	0
ctg	—	$\sqrt{3}$	1	$\sqrt{3}$	0	—

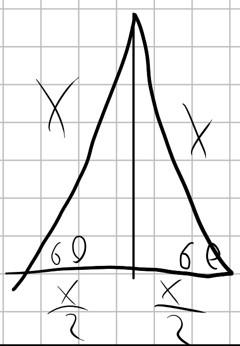
$$\mathrm{tg} = \frac{\sin}{\cos}$$

$$\cancel{\frac{0}{2} \cdot \frac{2}{2} = 0}$$

$$\cancel{\frac{\sqrt{2}}{2}}\ \cancel{\frac{2}{\sqrt{2}}}$$

$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$$



$$P = \frac{a^2\sqrt{3}}{4}$$

$$h = \frac{a\sqrt{3}}{2}$$

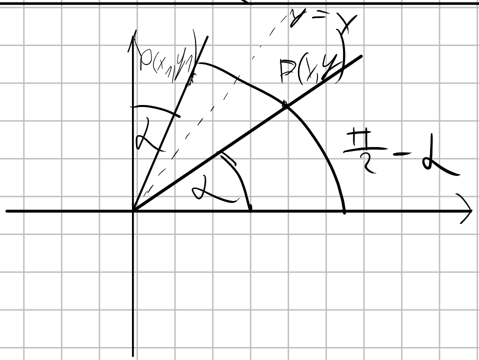
$90° - \alpha$

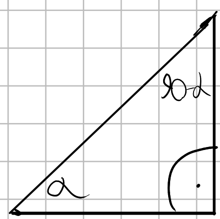
$\sin(90-\alpha) = \cos\alpha$

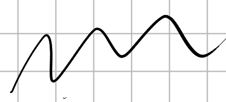
$\cos(90-\alpha) = \sin\alpha$

$\mathrm{tg}(90-\alpha) = \mathrm{ctg}$

$\mathrm{ctg}(90-\alpha) = \mathrm{tg}$







	$\beta =$						
	$\frac{\pi}{2} - \alpha$	$\frac{\pi}{2} + \alpha$	$\pi - \alpha$	$\pi + \alpha$	$\frac{3\pi}{2} - \alpha$	$\frac{3\pi}{2} + \alpha$	$2\pi - \alpha$
$\sin\beta$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$
$\cos\beta$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$
$\mathrm{tg}\,\beta$	$\mathrm{ctg}\,\alpha$	$-\mathrm{ctg}\,\alpha$	$-\mathrm{tg}\,\alpha$	$\mathrm{tg}\,\alpha$	$\mathrm{ctg}\,\alpha$	$-\mathrm{ctg}\,\alpha$	$-\mathrm{tg}\,\alpha$
$\mathrm{ctg}\,\beta$	$\mathrm{tg}\,\alpha$	$-\mathrm{tg}\,\alpha$	$-\mathrm{ctg}\,\alpha$	$\mathrm{ctg}\,\alpha$	$\mathrm{tg}\,\alpha$	-$\mathrm{tg}\,\alpha$	$-\mathrm{ctg}\,\alpha$

Na szczęście nie trzeba uczyć się na pamięć powyższej tabeli. Wystarczy zapamiętać poniższy schemat. Niech $\beta \in \left\langle \frac{\pi}{2}, 2\pi \right\rangle$. Kąt β przedstawiamy w postaci

$$\beta = n \cdot \frac{\pi}{2} \pm \alpha, \text{ gdzie } \alpha \in \left\langle 0, \frac{\pi}{2} \right) \wedge n \in \{1, 2, 3, 4\}.$$

Wówczas

$$f(\beta) = f(n \cdot \frac{\pi}{2} \pm \alpha) = \begin{pmatrix} \text{znak} \\ \text{Tabela nr. 1} \end{pmatrix} \cdot \begin{cases} f(\alpha), & \text{gdy } n - \text{parzyste} \\ cf(\alpha), & \text{gdy } n - \text{nieparzyste} \end{cases}$$

$f(\cdot)$ – funkcja trygonometryczna (sin, cos, tg, ctg). Zaś $cf(\cdot)$ – odpowiadająca funkcji f cofunkcja, wyznaczona według schematu

$$\sin \leftrightarrow \cos; \qquad \mathrm{tg} \leftrightarrow \mathrm{ctg}\,.$$

$\mathrm{tg}\left(\frac{3}{2}\pi + 2\right) \quad \alpha \in (0, \pi)$

$\frac{3}{2}\pi = 3 \cdot \frac{\pi}{2}$

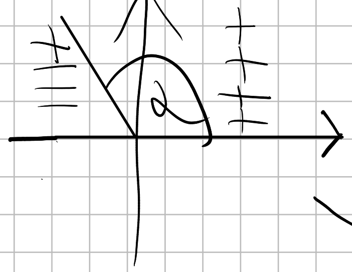
$\sin\frac{7}{4}\pi = \sin\left(2\pi - \frac{\pi}{4}\right) = \sin\left(4\frac{\pi}{2} - \frac{\pi}{4}\right)$

$-\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ Co my tu robimy?

$\mathrm{tg}\left(\frac{5}{6}\pi\right)$

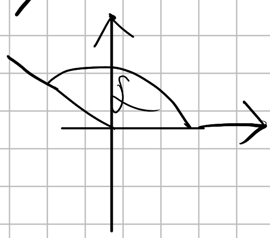
$\parallel$

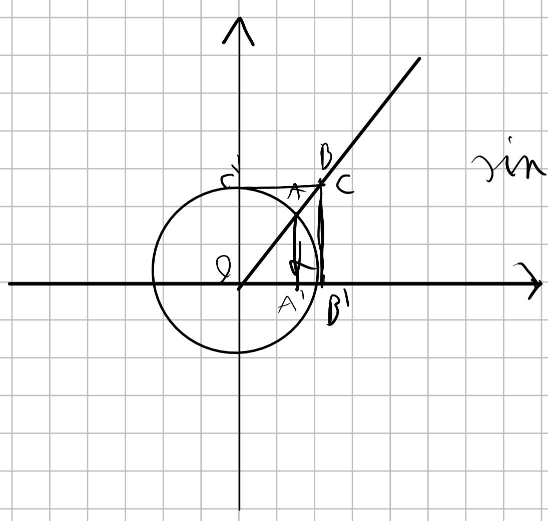
$-\frac{\sqrt{3}}{3}$



$\sin\left(-\frac{1742}{3}\pi\right) = -\sin\left(580\pi + \frac{2}{3}\pi\right)$

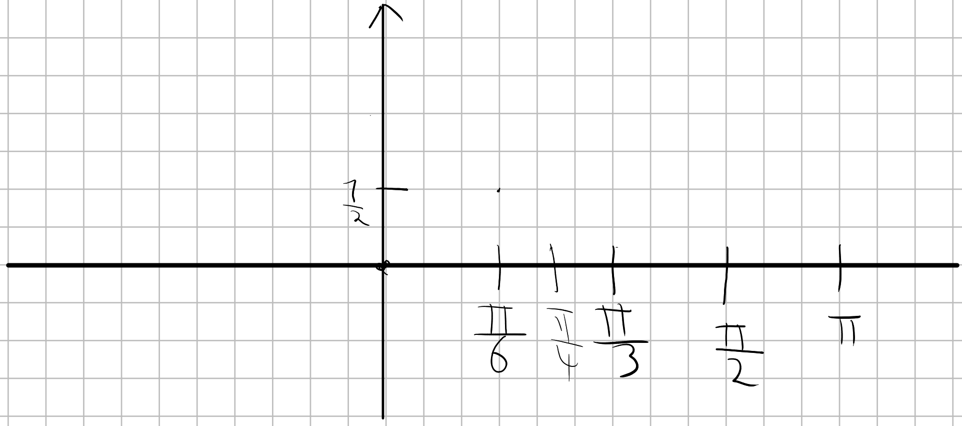
$= -\sin\left(\frac{2}{3}\pi\right) =$





$\sin\alpha = \frac{|AA'|}{|OA|} \qquad \cos\alpha = \frac{|OA'|}{|OA|}$

tg



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$\operatorname{tg}\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	x	0	x	0
$\operatorname{ctg}\alpha$	x	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	x	0	x