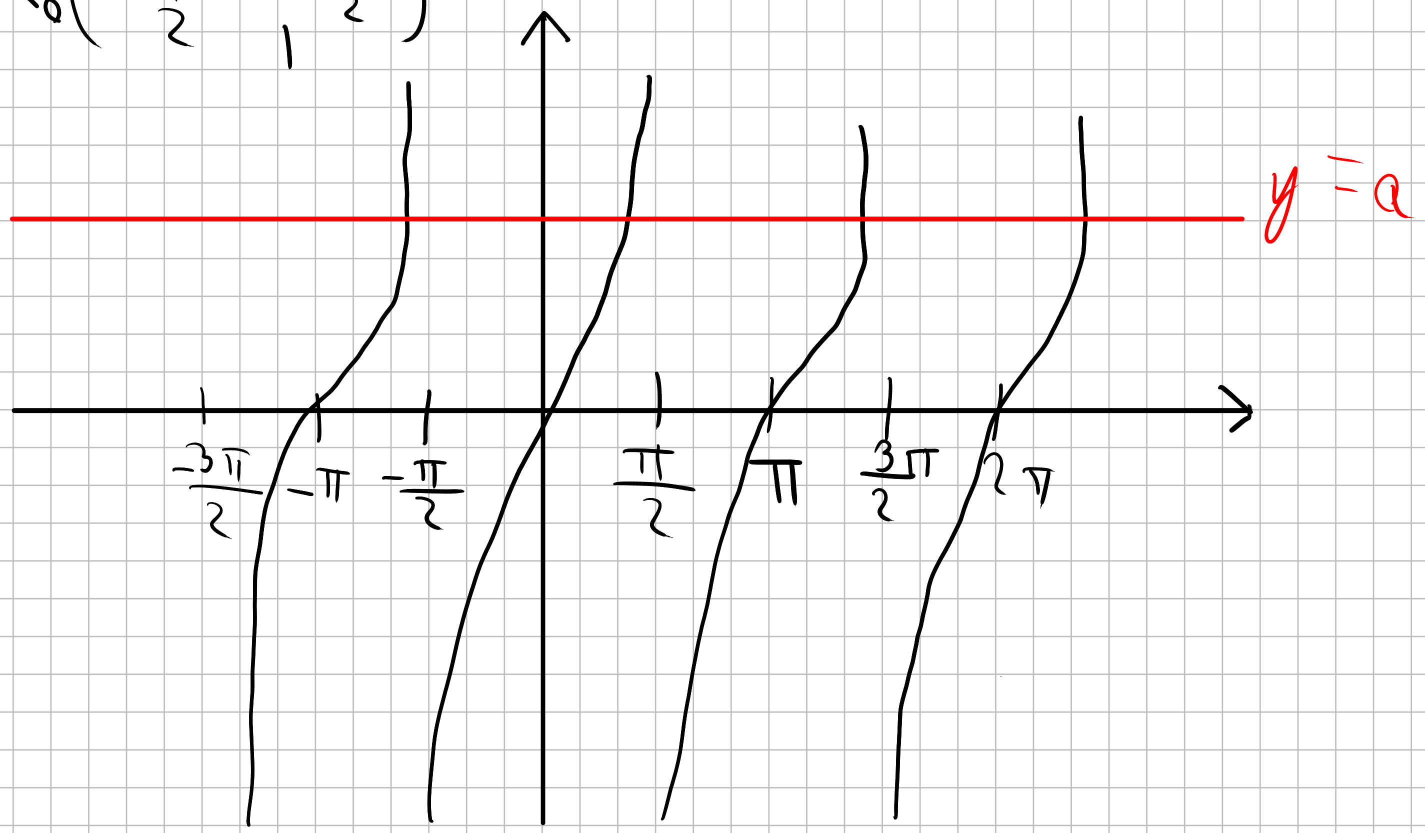
Wstęp do matematyki

$\operatorname{tg} x = a \quad a \in R$

$x = x_0 + k\pi$

$x_0 \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



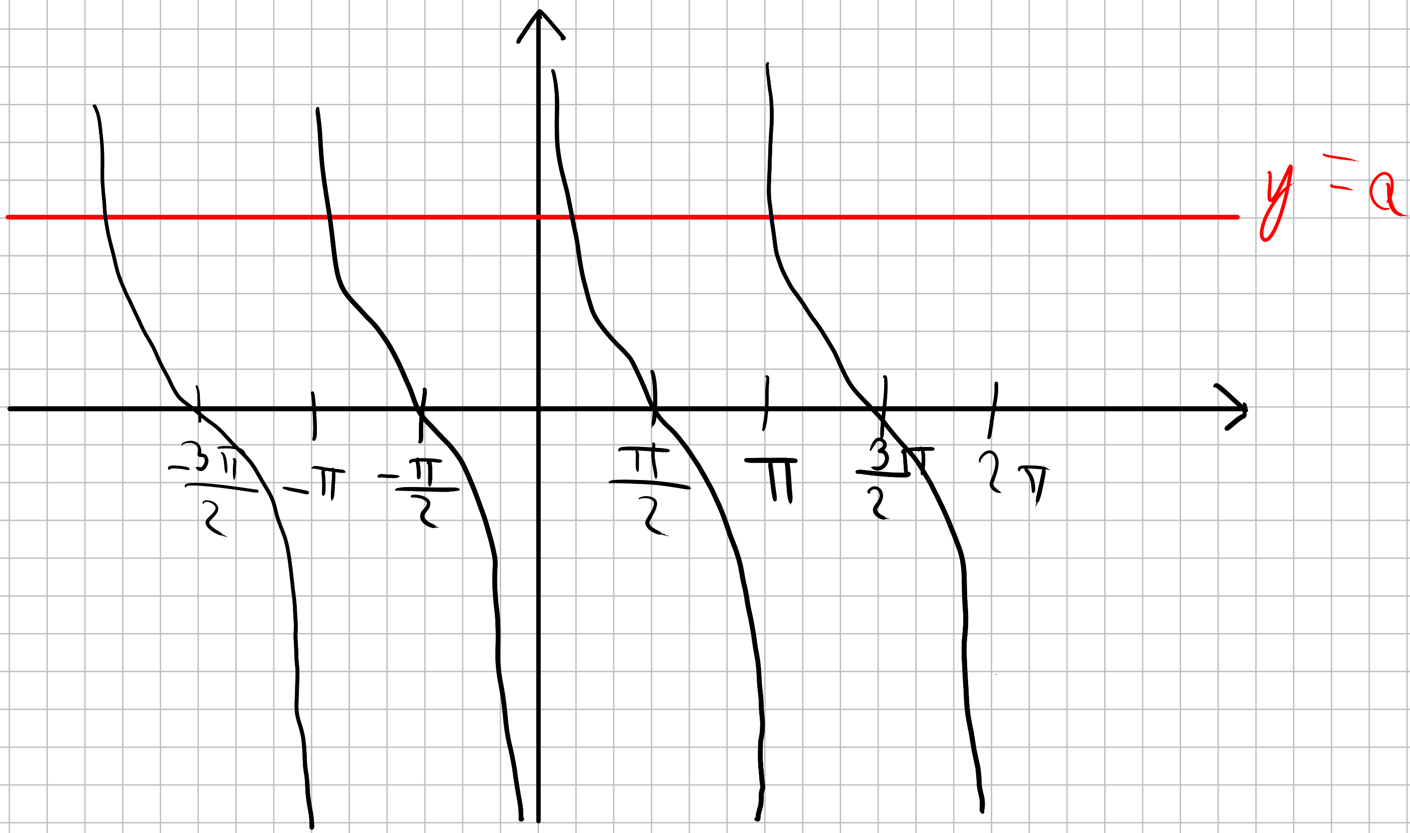
$\operatorname{tg} x = a \wedge a \in R \Rightarrow x = x_0 + k\pi \wedge k \in Z$
$\wedge x_0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \qquad \operatorname{tg} x_0 = a$

$D = R - \left\{\frac{\pi}{2} + k\pi : k \in Z\right\}$

$ctg\, x = a \quad a \in R$

$x = x_0 + k\pi$

$k \in Z \quad ctg\, x_0$



$ctg\, x = a \wedge a \in R \iff x_0 + k\pi \wedge k \in Z \wedge x_0 \in (0, \pi)$

$D = R - \{k\pi : k \in Z\}$

$3 \sin x = 2 \cos^2 x$

$3 \sin x = 2(1 - \sin^2 x)$

$2 \sin^2 x - 3 \sin x - 2 = 0$

$\Delta = 9 - 4 \cdot 2 \cdot 2 = 25$

$t_1 = \frac{-3-5}{4} = -2 \vee t_2 = \frac{-3+5}{4} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6} + 2k\pi \quad \vee \quad \cancel{\frac{\pi}{6} = \frac{1}{2}}$

$\sin^2 2x = 1 - \sin^2 x$

$(2 \sin x \cos x)^2 = \cos^2 x$

$4 \sin^2 x \cos^2 x - \cos^2 x = 0$

$\cos^2 x (4 \sin^2 x - 1) = 0$

$\cos^2 x = 0 \vee 4 \sin^2 x = 1$

$\cos x = 0 \vee \sin x = \frac{1}{2} \vee \sin x = -\frac{1}{2}$

$\cos x = 0$

$(x = \frac{\pi}{2} + k\pi \wedge k \in \mathbb{Z}) \vee (x = \frac{\pi}{6} + 2k\pi \wedge k \in \mathbb{Z}) \vee (x = \frac{5}{6}\pi + 2k\pi)$

$\left[(x = \frac{\pi}{2} + k\pi) \vee (x = -\frac{\pi}{6} + 2k\pi) \vee (x = \frac{5}{6}\pi + 2k\pi) \vee (x = -\frac{\pi}{6} + 2k\pi) \vee (x = \frac{7}{6}\pi + 2k\pi)\right] \wedge k \in \mathbb{Z} \wedge x \in$

$\frac{7}{6}\pi = \frac{\pi}{6} + \pi$

$-\frac{\pi}{6} = \frac{5}{6} - \pi$

$\{x: x=\frac{\pi}{6}+k\pi\} \cup \{x: x=\frac{5}{6}+2k\pi\}$

$\{x: x=-\frac{\pi}{6}+k\pi\}$

$x \in \{-\frac{\pi}{6}+k\pi, \frac{\pi}{6}+k\pi, \frac{\pi}{2}+k\pi\} \quad k\in Z$

nierówności

$\sin x > -\frac{\sqrt{3}}{2}$

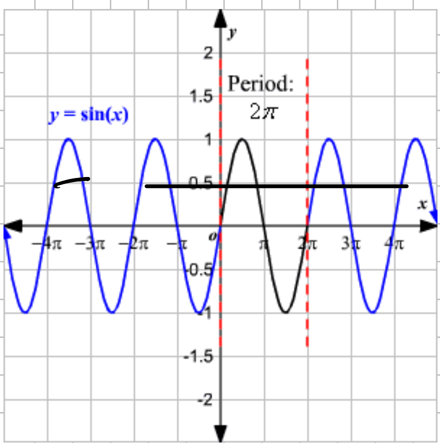
a) $x \in \langle -\pi, 2\pi \rangle$ b) R

a)

$\sin x = \frac{\sqrt{3}}{2}$

$(x=-\frac{\pi}{3}+2k\pi) \vee (x=\frac{4}{3}\pi+2k\pi) \wedge x \in \langle -\pi, 2\pi \rangle$

$(x=-\frac{\pi}{3} \vee x=\frac{5}{3}\pi \vee x=\frac{4}{3}\pi \vee x=-\frac{2}{3}\pi) \wedge x \in \langle -\pi, 2\pi \rangle$



$\sin x > -\frac{\sqrt{3}}{2}$

$x \in \langle -\pi, -\frac{2}{3}\pi) \cup (-\frac{\pi}{3}, \frac{4}{3}\pi) \cup (\frac{5}{3}\pi, 2\pi \rangle$

R $\quad \sin x = -\frac{\sqrt{3}}{2} \wedge x \in R$

$\left(-\frac{\pi}{3} + 2k\pi, \frac{4}{3}\pi + 2k\pi\right) \quad k \in Z$

$\sin x > -\frac{\sqrt{3}}{2} \wedge x \in R \Longleftrightarrow x \in \bigcup_{k \in Z} \left(-\frac{\pi}{3} + 2k\pi, \frac{4}{3}\pi + 2k\pi\right)$

$\left[\frac{\cos x}{1-\cos x} \geq 0 \quad \wedge x \in D\right]$

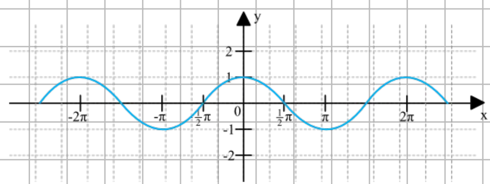
$-\frac{2\cos x - 1}{1-\cos x} \geq 0 \wedge x \in D$

D

$\cos x \neq 1$

$x \neq 1 + k\pi$

$\cos x \geq \frac{1}{2}$



$\cos x = \frac{1}{2}$

$\cos \frac{\pi}{3} = \frac{1}{2}$

$x \in \left(\bigcup_{x \in Z} \left\langle -\frac{\pi}{3} + 2k\pi, \frac{\pi}{2} + 2k\pi \right\rangle \wedge\right.$

$\left. x \in D\right)$

$x \in \left(\bigcup_{x \in Z} \left\langle -\frac{\pi}{3} + 2k\pi, 2k\pi\right)\right) \cup (2k\pi,$

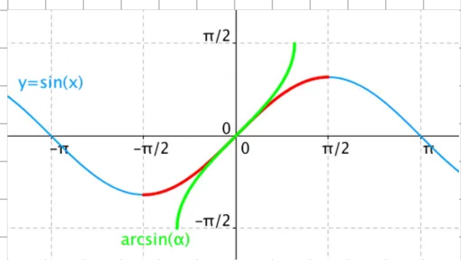
Funkcje Cyklometryczne

odwrotne do funkcji trygonometrycznych ograniczonych do pewnych przedziałów.

arcus sinus

$y = \arcsin x \iff x = \sin y, \quad x \in \langle -1, 1 \rangle$

$y \in \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$



$y = \arccos x$

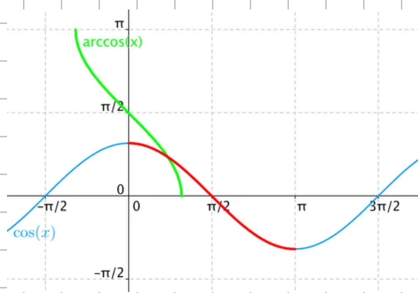
$\iff x = \cos y \quad x \in \langle -1, 1 \rangle$

$y \in \langle 0, \pi \rangle$

$D = \langle -1, 1 \rangle$

$ZW \langle 0, \pi \rangle$

ściśle malejąca



$\operatorname{tg} x \quad (-\frac{\pi}{2}, \frac{\pi}{2})$

$\bigwedge_{x \in \mathbb{R}} \operatorname{tg}(\operatorname{arctg} x) = x$

$$\bigwedge_{x\in(-\frac{\pi}{2},\frac{\pi}{2})} \operatorname{arctg}(\operatorname{tg}(x)) = x$$

daj pan spokój

$$\arcsin\frac{\sqrt{2}}{2}=\alpha \equiv \sin\alpha\frac{\sqrt{2}}{2} \wedge \alpha\in\langle-\frac{\pi}{2},\frac{\pi}{2}\rangle$$

$$\Rightarrow \alpha=\frac{\pi}{4}$$

$$\operatorname{arcctg}(-1)=\alpha$$

2) Wyznaczyć dziedzinę naturalną

$$f(x)=\arcsin(\log_{\frac{1}{3}}x)$$

$$D=\{x\in\mathbb{R}: x>0 \wedge \log_{\frac{1}{3}}3 \leq \log_{\frac{1}{3}}x \leq \log_{\frac{1}{3}}\frac{1}{3}$$

$$D=\{x\in\mathbb{R}: x>0 \wedge x\in\langle\frac{1}{3},3\rangle\} = \langle\frac{1}{3},3\rangle$$

3) Rozwiąż nierówność

$$\arccos x \leq \frac{5}{6}\pi \qquad \bigwedge_{x\in\langle 0,\pi\rangle} \arccos(\cos x) = x$$

$$D=\langle -1,1\rangle$$

z własności funkcji

$$\frac{5}{6}\pi = \arccos\left(\cos\frac{5\pi}{6}\right)$$

$$\arccos x \leq \frac{5}{6}\pi \wedge x \in \langle -1, 1 \rangle$$

$$\arccos x \leq \arccos\left(\cos\frac{5}{6}\pi\right)$$

$$x \geq \cos\frac{5}{6}\pi$$ — bo malejąca ($\arccos x$)

$$x - \cos\frac{5}{6}\pi \wedge x \in \langle -1, 1 \rangle$$

$$x \geq \overset{\text{redukcja}}{-\cos\frac{\pi}{6}} \quad \text{— " —}$$

$$x \geq -\frac{\sqrt{3}}{2} \quad \text{— " —}$$

$$x \in \left\langle -\frac{\sqrt{3}}{2}, 1 \right\rangle$$

$$\arcsin(-x) = -\arcsin x$$

$$\operatorname{arctg}(-x) = -\operatorname{arctg} x$$

$$\arccos(-x) = \pi - \cos x$$

$$\operatorname{arcctg}(-x) = \pi - \operatorname{ctg} x$$

$$2 \mid n \Rightarrow n^2 + \left\lfloor \frac{n+1}{2} \right\rfloor \cdot n + n$$

$$\sim(2 \mid n) \Rightarrow n^2 + \left\lfloor \frac{n}{2} \right\rfloor \cdot (n+1) + n$$

$$2 \mid n \Rightarrow n\left(n + \left\lfloor \frac{n+1}{2} \right\rfloor + 1\right)$$

$$1024 = n^2 + \frac{n+1}{2} \cdot n + n$$

$$2n^2 + n^2 + n + n - 1024 = 0$$

$$3n^2 + 2n - 1024 = 0$$

$$\Delta = 4 - 4 \cdot 3 \cdot -1024$$

$$\sqrt{\Delta} = 110{,}87$$

$$x_1 = \frac{-2 + 110{,}87}{6} = \underline{\underline{18}}$$

$$\cancel{x_2 = \frac{-2 - 110{,}87}{6} = -19}$$

$$(0{,}125)^x \cdot (\sqrt{2})^{x+1}$$

$$\left(\frac{1}{8}\right)^x \cdot \left(2^{\frac{1}{2}}\right)^{x+1}$$

$$2^{-3x} \cdot 2^{\frac{x+1}{2}} = 2^{-3x + \frac{x+1}{2}}$$

b) $(0.125)^x \cdot (\sqrt{2})^{x+1} = \left(\frac{4}{\sqrt[3]{2}}\right)^{3x}$,

$$\left(2^2 : 2^{\frac{1}{3}}\right)^{3x} = 2^{\frac{5}{3} \cdot 3x}$$

$$2^{-3x + \frac{x+1}{2}} = \left(\frac{4}{\sqrt[3]{2}}\right)^{3x}$$

$$2^{-3x + \frac{x+1}{2}} = 2^{5x}$$

różnowartościowości

$-3x + \frac{x+1}{2} = 5x$

24 kolokwium

21.

$\frac{x+1}{2} = 8x$

c) $21 \cdot 3^x - 25 \cdot 5^x = 81 \cdot 3^x - 125 \cdot 5^x$,

$x + 1 = 16x$

$x = \frac{1}{15}$

$7 \cdot 3 \cdot 3^x - 5^2 \cdot 5^x = 3^4 \cdot 3^x - 5^3 \cdot 5^x$

$7 \cdot 3^{x+1} - 5^{x+2} = 3^{x+4} - 5^{x+3}$

$7 \cdot 3^{x+1} - 3^{x+4} = 5^{x+2} - 5^{x+3}$

$7 \cdot 3^{x+1} - 27 \cdot 3^{x+1}$

$-20 \cdot 3^{x+1} = 5^{x+2} - 5 \cdot 5^{x+2}$

$-20 \cdot 3^{x+1} = -4 \cdot 5^{x+2}$

$5 \cdot 3^{x+1} = 5^{x+2}$

$5 \cdot 3^{x+1} = 5 \cdot 5^{x+1}$

$3^{x+1} = 5^{x+1}$

i) $\left(\frac{1}{2}\right)^{3x} + 64 < \left(\frac{1}{2}\right)^{x-2} \cdot (1 + 2^{2-x})$,

$\left(\frac{1}{2}\right)^{3x} + 64 < \left(\frac{1}{2}\right)^{x-2} \cdot (1 + 2^{2-x})$

$2^{-3x} + 2^6 < 2^{2-x} + 2^{2-x} \cdot 2^{2-x}$

$2^{-3x} + 2^6 < 2^{2-x} + 2^{4-2x}$

$(2^{-x})^3 + 64 < 4 \cdot 2^{-x} + 16 \cdot (2^{-x})^2$

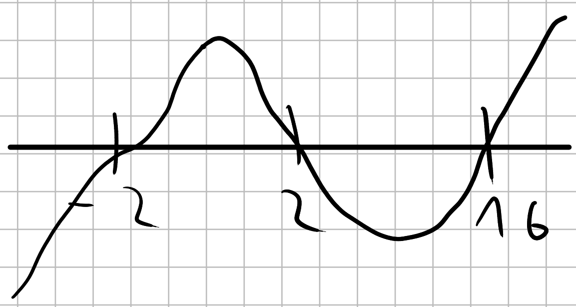
$(2^{-x})^3 - 4 \cdot 2^{-x} - 16 \cdot (2^{-x})^2 + 64 < 0$

$t^3 - 4t - 16t^2 + 64 < 0$

$t(t^2-4) - 16(t^2-4) < 0$

$(t-16)(t-4)(t+2)$ $\wedge\ t>0 \wedge t\in D$

$t=16 \vee t=2 \vee t=-2$



$t\in(-\infty,-2)\cup(2,16) \wedge t>0 \wedge t\in D$

$t\in(2,16)$

funkcja malejąca

$2^{-x}\in(2,16)$

$2<2^{-x}<16$

1
2
4
8
16

$2^1<2^{-x}$ $\qquad$ $2^{-x}<2^4$

$1<-x$ $\qquad$ $-x<4$

$x<-1$ $\qquad$ $x>-4$

$x\in(-4,-1)$

$\frac{1}{3^x-1} > \frac{9}{9-3^x}$ $\qquad$ $3^x\neq 1$ $\qquad$ $x\neq 0 \wedge x\neq 2$

$D_f = R-\{0,2\}$

$9\cdot 3^x - 9 > 9 - 3^x \wedge x<3$

$9-3^x$

$3^x\cdot 3^2 - 3^2 - 3^2 - 3^x > 0 \wedge x<3$

$3^x\cdot 3^2 - 2\cdot 3^2 - 3^x > 0 \wedge x\in D_f$

$(3^x-2)\cdot 3^2 - 3^3 > 0 \wedge x\in D_f$

$3^x<3^1$

$x<1$

$3^2(3^x-2-1)>0$

$3^2(3^x-3)>0 \quad // \cdot 3^2$

$3^x > +3^1$

$x>1$ $\qquad x^5 > x^2 \wedge 5>2$ rosnąca

$\underline{x \in (1,2) \cup (2,3) \cup (0,1)}$

5/ *

7/ *

8 *

9

10

11

12

ZADANIE 10. Obliczyć

a) $\left(\sqrt[3]{125}\right)^{2(1-\log_5 4)}$, $\quad$ *b)* $16^{-\log_2 \sqrt{2}} \cdot 3^{-\log_{\frac{1}{3}} 5} \cdot 4^{\log_2 \sqrt{8}} \cdot \frac{1}{2}$, $\quad$ *c)* $\log_2 5 \cdot \log_5 6 \cdot \log_6 7 \cdot \log_7 8$.

$\sqrt[3]{125}^{\,2(1-\log_5 4)}$

$5^{2-\log_5 16}$ $\qquad 5^x=16$

$\frac{5^2}{16} = \frac{25}{16} = 1,5625$ $\qquad 5^{5^x}=16$

b) $16^{-\log_2 \sqrt{2}} \cdot 3^{-\log_{\frac{1}{3}} 5} \cdot 4^{\log_2 \sqrt{8}} \cdot \frac{1}{2}$,

1
2
4
8
16

$16^{-\log_2\sqrt{2}} \cdot 3^{-\log_{\frac{1}{3}} 5} \cdot 4^{\log_2 \sqrt{8}} \cdot \frac{1}{2}$

$\cancel{2^{-\log_2 4\sqrt{2}}} \cdot \frac{1}{3}^{\log_{\frac{1}{3}} 5} \cdot \cancel{2^{\log_2 4\sqrt{2}}} \cdot \frac{1}{2}$

$\qquad 1 \qquad\qquad\qquad\qquad 1$

$5:2=2,5$

8	2
4	2
2	2
1	

11)

$\log_a b \cdot \log_b c = \log_a c$

$2^{\log_3 5} = 5^{\log_5 2 \cdot \log_3 5}$

12)

ZADANIE 12. Obliczyć $\log_{a^2}\frac{1}{b}$ wiedząc, że $\log_a b = \sqrt{2}$, gdzie a,b są liczbami dodatnimi i $a \neq 1$.

$\log_{a^2}\frac{1}{b}$ $\log_a b = \sqrt{2}$

$\log_{a^2}\frac{1}{b} = \log_a \sqrt{\frac{1}{b}}$

$\log_a b^{-2} = -2\log_a b = -2\sqrt{2}$

ZADANIE 13. Określić znak liczby:

a) $\log_a b$, jeżeli $a > 1$ i $b > 1$,

b) $\log_a b$, jeżeli $a \in (0,1)$ i $b \in (0,1)$,

c) $\log_a b$, jeżeli $a \in (0,1)$ i $b > 1$.

a) $\log_a b$ $a > 1 \wedge b > 1$

$+$

$\log_2 2 = 1$

b)

$\log_{\frac{1}{2}}\frac{1}{4} = 2$ $+$

c) $\log_{\frac{1}{2}} 2 = -1$ $-$

$f(x) = \log_x (x^2 - 4)$

$x > 0 \wedge x \neq 1$

$\log_{x^2-1} 2x+1-\frac{1}{x}$

$$\underline{\underline{5-15}}$$

ech...