Professur für Thermofluiddynamik

ANNULiGhT

Representation of flame dynamics in terms of phasors and flame impulse response

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1 From UIR to frequency response

The goal of this exercise is to play around with different UIR and understand their impact on FTF features.

From the Github repository, download the 5 UIR, named with the convention *UIR_caseX.mat*. Each file contains a unique variable h, which is an array filled with all the coefficients of the UIR.

$$h = [h_0 \quad h_1 \; \ldots \; h_N]$$

The UIR is sampled was a time step $dt = 2$ms. For each case,

- Plot the UIR $h(t)$.
- Evaluate the FTF in the frequency range of interest [0–1500 Hz].
- Plot the Nyquist diagram.
- Plot the Bode diagram of the frequency response.

Additionally, you can create your own UIR by changing the coefficients h_k.

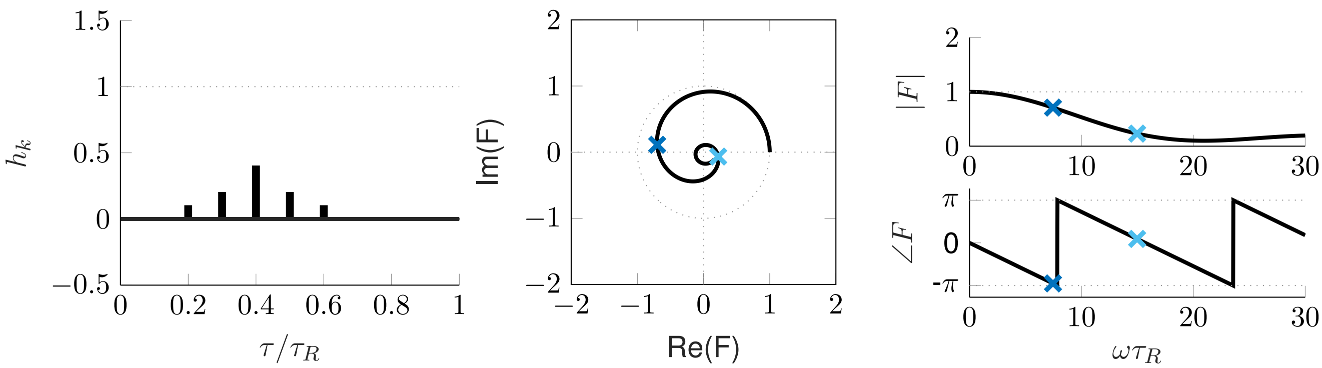

Figure 1: UIR, Nyquist plot and Bode diagram of the frequency response

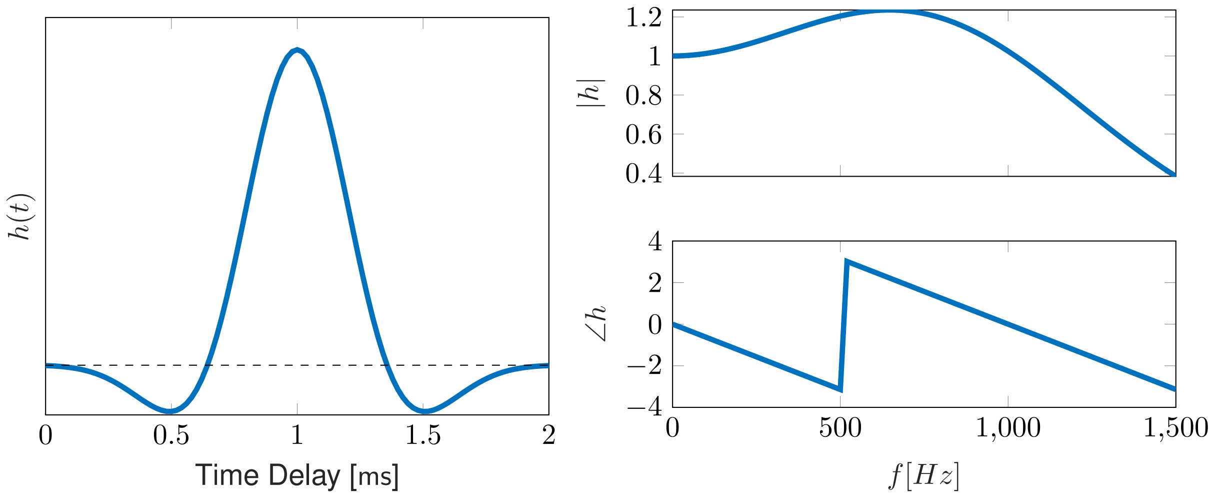

Figure 2: UIR, Nyquist plot and Bode diagram of the frequency response

After analyzing the different cases, what conclusion can we draw? What features must the UIR have for the frequency response to exhibit a gain higher than one? How is the low frequency limit of the frequency response $|F(\omega = 0)|$ satisfied?

2 Identify a presumed UIR from experimental data

The following is widely based on AEsoy et al. (CNF 2020). We are grateful to NTNU for providing experimental data and scripts. The goal of this exercise is to find a DTD model that best fit experimental data.

The UIR is assumed to be described by the equation

$$h(t) = \frac{g}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(t-\tau)^2}{2\sigma^2}\right) \cos(\beta(t-\tau)) \tag{1}$$

The first term is the Gaussian distribution with a mean value τ and a standard deviation σ (see Polifke et al. (AIAA 2001)). The model is extended with the amplitude g and the modulating term with frequency β.

The associated transfer function is:

$$\hat{h}(\omega) = \frac{g}{2} \exp\left(-\frac{1}{2}(\omega+\beta)^2\sigma^2 - j\omega\tau\right) + \frac{g}{2} \exp\left(-\frac{1}{2}(\omega-\beta)^2\sigma^2 - j\omega\tau\right) \tag{2}$$

The UIR and the transfer function are illustrated in Fig. 2.

2.1 Experimental data

From Github, download the experimental data. Plot the Bode diagram of the frequency response.

2.2 First model

Find the model that best fit the data, i.e. identify the parameters g, β, σ, τ.

- Use the phase of the frequency response to determine τ.
- Write a script to find, through a optimization loop, the other 3 parameters. Note that for a perfectly premixed flame, mass and energy conservation in the low frequency limit give $|F(\omega \approx 0)| = 1$. This translates to $g = \exp\left(\frac{1}{2}\beta^2\sigma^2\right)$. This constraint allows to remove g from the fitting procedure. It is reconstructed afterwards from β and σ.

Comment on the model (see Fig. 3). Is it acceptable?

2.3 Additional model

The model obtained in the previous step, DTD_1 is not fully satisfying. Indeed, the "oscillations" observed for the gain are not captured. This is typical of a phenomena that has a different time-scale. We therefore decide to combine this first model with an additional model, DTD_2. The global model is

$$DTD_{tot}(\omega) = DTD_1 + DTD_2 \tag{3}$$

To determine the parameters of this additional model:

- Subtract the model DTD_1 from the experimental data. For example, for the gain $\Delta G = G_{exp} - G_{DTD_1}$.
- Identify the frequency band where the modulation in gain is observed
- Fit a sine wave to the data and extract the wavelength λ between 2 consecutive peaks. This value will be used for the initialization of the optimization. $\tau_2 = \tau_1 + 2\pi/\lambda$.
- Write an optimization loop to find the parameters g_2, β_2, σ_2, τ_2.

2.4 Global model

Write an optimization loop to identify the parameters for the global model. Use the models DTD_1 and DTD_2 obtained from the previous steps as a starting point for the loop.

Note: for perfectly premixed flame, mass and energy conservation in the low frequency limit give $|F(\omega \approx 0)| = 1$. This translates to

$$1 = g_1 \exp\left(-\frac{1}{2}\beta_1^2\sigma_1^2\right) + g_2 \exp\left(-\frac{1}{2}\beta_2^2\sigma_2^2\right) \tag{4}$$

For numerical reasons, it is interesting to define one gain as a function of the other. For example:

$$g_1 = \exp\left(\frac{1}{2}\beta_1^2\sigma_1^2\right)\left(1 - g_2 \exp\left(-\frac{1}{2}\beta_2^2\sigma_2^2\right)\right) \tag{5}$$

This constraint allows us to remove g_1 from the fitting procedure.

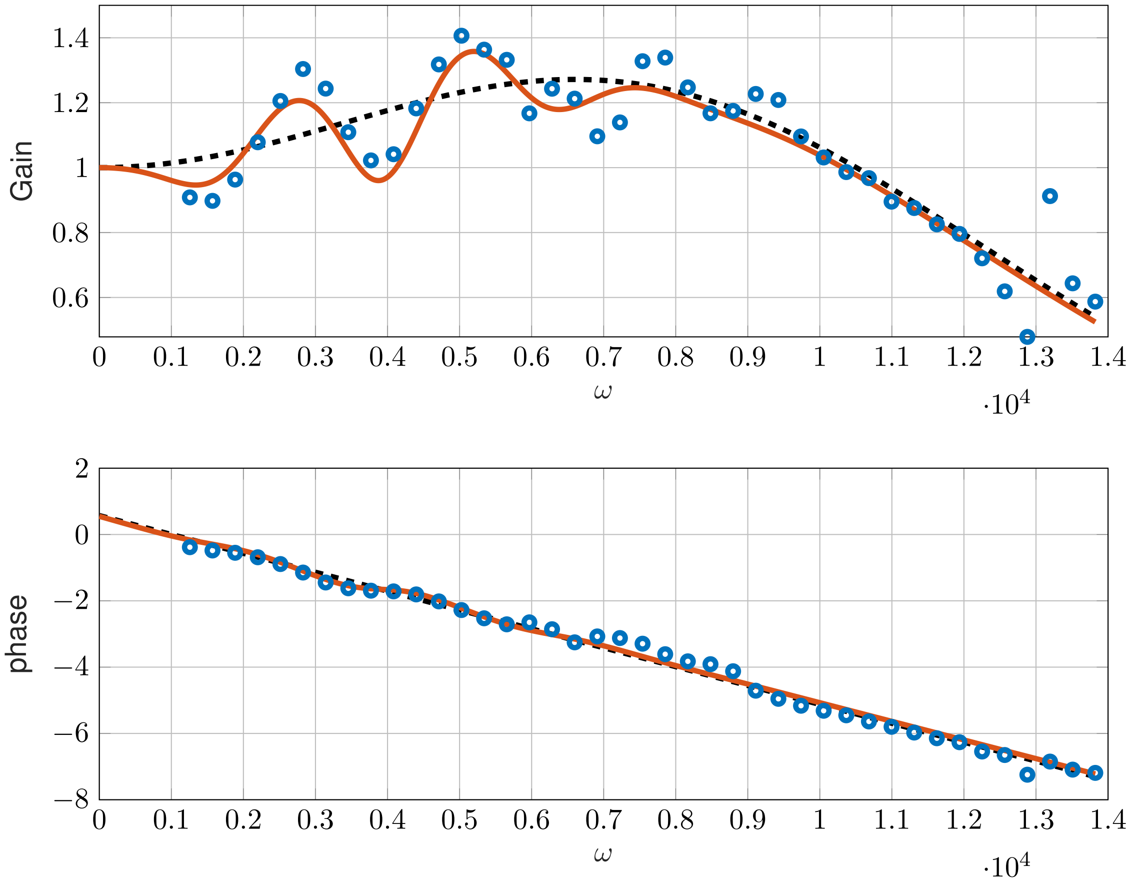

Figure 3: Gain and phase of the frequency response for the experimental data (dots), DTD_1 (black dashed line) and the global model (full red line)