공열 수학 과제#1

2015170305

김태희

$$\frac{dV}{dt} = g - \frac{C_d}{m}V^2 \quad , \quad \int \frac{1}{a^2 - x^2}\,dx = \frac{1}{a}\tanh^{-1}\frac{x}{a}$$

($g = 9.81\ m/s^2$, $C_d, m \neq 0$)

(1) $$\int \frac{1}{g - \frac{C_d}{m}V^2}\,dV = \int 1\,dt$$

$$= \frac{m}{C_d}\int \frac{1}{\frac{mg}{C_d} - V^2}\,dV = \frac{\frac{m}{C_d}}{\sqrt{\frac{mg}{C_d}}}\tanh^{-1}\frac{V}{\sqrt{\frac{mg}{C_d}}} = \int 1\,dt = t + C$$

$$\tanh^{-1}\frac{V}{\sqrt{\frac{mg}{C_d}}} = \sqrt{\frac{C_d g}{m}}\,(t + C)$$

$$V = \sqrt{\frac{mg}{C_d}}\tanh\left(\sqrt{\frac{C_d g}{m}}\,(t + C)\right)$$

at $t = 0$, $V = 0$.

$$0 = \sqrt{\frac{m}{C_d}} \times \frac{e^{\sqrt{\frac{C_d g}{m}}} - e^{-\sqrt{\frac{C_d g}{m}}}}{e^{\sqrt{\frac{C_d g}{m}}} + e^{-\sqrt{\frac{C_d g}{m}}}}$$

$$e^{\sqrt{\frac{C_d g}{m}}} = e^{-\sqrt{\frac{C_d g}{m}}}$$

So $C = 0$ then

$$\therefore V(t) = \sqrt{\frac{mg}{C_d}}\tanh\left(\sqrt{\frac{C_d g}{m}}\,t\right)$$

(2). at the given conditions

$V(t) = 51.69 \tanh(0.190\, t)$.

V(t) has asymtotic line $y = 51.69$.

terminal velocity $= 51.69\ m/s$

time at terminal velocity $= +\infty$

(3) $V(t_{i+1}) = V(t_i) + \left[g - \frac{C_d}{m} V(t_i)^2\right](t_{i+1} - t_i)$, $\Delta t = 2$

(3) plot is always bigger when $t > 0$ than (2) plot. Because $\left.\frac{dV}{dt}\right|_{t=t_i}$ is always bigger than $\left.\frac{dV}{dt}\right|_{t=t_m}$ ($i < m \le i+1$) which means $\frac{d^2V}{dt^2} < 0$. Analytic $V(t_{i+1})$ can be represented as $V(t_{i+1}) = V(t_i) + \int_{t_i}^{t_{i+1}} \frac{dV}{dt}\, dt$.

And $\int_{t_i}^{t_{i+1}} \frac{dV}{dt}\, dt < \int_{t_i}^{t_{i+1}} \left.\frac{dV}{dt}\right|_{t=t_i} dt = \left.\frac{dV}{dt}\right|_{t=t_i} \times (t_{i+1} - t_i)$

So (3) plot is bigger than (2) plot at $t > 0$.

(4) $\Delta t = 1$, (4) plot is more near to (2) plot than (3) plot.

(5) $m = 95$

$V(t) = 61.06 \tanh(0.161\, t)$

terminal velocity is $61.06\ m/s$.

at (5) plot asymtotic line becomes $y = 61.06$ and the time when (5) plots closely approach asymtotic line becomes later than (2), (3), (4) plots.