

공 일 수학 과제 #1

2015170305

김태희

$$\frac{dV}{dt} = g - \frac{c_d}{m} V^2$$

$$(g = 9.81 \text{ m/s}^2, c_d, m \neq 0)$$

$$\int \frac{1}{a^2 - x^2} dx = \frac{1}{a} \tanh^{-1} \frac{x}{a}$$

$$(1) \int \frac{1}{g - \frac{c_d}{m} V^2} dV = \int 1 dt$$

$$= \frac{m}{c_d} \int \frac{1}{\frac{mg}{c_d} - V^2} dV = \frac{\frac{m}{c_d}}{\sqrt{\frac{mg}{c_d}}} \tanh^{-1} \frac{V}{\sqrt{\frac{mg}{c_d}}} = \int 1 dt = t + C$$

$$\tanh^{-1} \frac{V}{\sqrt{\frac{mg}{c_d}}} = \sqrt{\frac{c_d g}{m}} (t + C)$$

$$V = \sqrt{\frac{mg}{c_d}} \tanh \left(\sqrt{\frac{c_d g}{m}} (t + C) \right)$$

$$\text{at } t=0, V=0.$$

$$0 = \sqrt{\frac{mg}{c_d}} \times \frac{e^{\sqrt{\frac{c_d g}{m}}} - e^{-\sqrt{\frac{c_d g}{m}}}}{e^{\sqrt{\frac{c_d g}{m}}} + e^{-\sqrt{\frac{c_d g}{m}}}}$$

$$e^{\sqrt{\frac{c_d g}{m}}} = e^{-\sqrt{\frac{c_d g}{m}}}$$

So $C=0$ then

$$\therefore V(t) = \sqrt{\frac{mg}{c_d}} \tanh \left(\sqrt{\frac{c_d g}{m}} t \right)$$

(2). at the given conditions

$$V(t) = 51.69 \tanh(0.190 t)$$

$V(t)$ has asymptotic line

$$y = 51.69$$

terminal velocity = 51.69 m/s

time at terminal velocity = $t \rightarrow \infty$

$$(3) V(t_{i+1}) = V(t_i) + \left[g - \frac{C_d}{m} V(t_i)^2 \right] (t_{i+1} - t_i), \Delta t = 2$$

(3) plot is always bigger when $t > 0$ than (2) plot. Because

$\left. \frac{dV}{dt} \right|_{t=t_i}$ is always bigger than $\left. \frac{dV}{dt} \right|_{t=t_m}$ ($i < m \leq i+1$) which means

$\frac{d^2V}{dt^2} < 0$. Analytic $V(t)$ can be represented as $V(t_{i+1}) = V(t_i) + \int_{t_i}^{t_{i+1}} \frac{dV}{dt} dt$.

$$\text{And } \int_{t_i}^{t_{i+1}} \frac{dV}{dt} dt < \int_{t_i}^{t_{i+1}} \left. \frac{dV}{dt} \right|_{t=t_i} dt = \left. \frac{dV}{dt} \right|_{t=t_i} \times (t_{i+1} - t_i)$$

So (3) plot is bigger than (2) plot at $t > 0$.

(4) $\Delta t = 1$, (4) plot is more near to (2) plot than (3) plot.

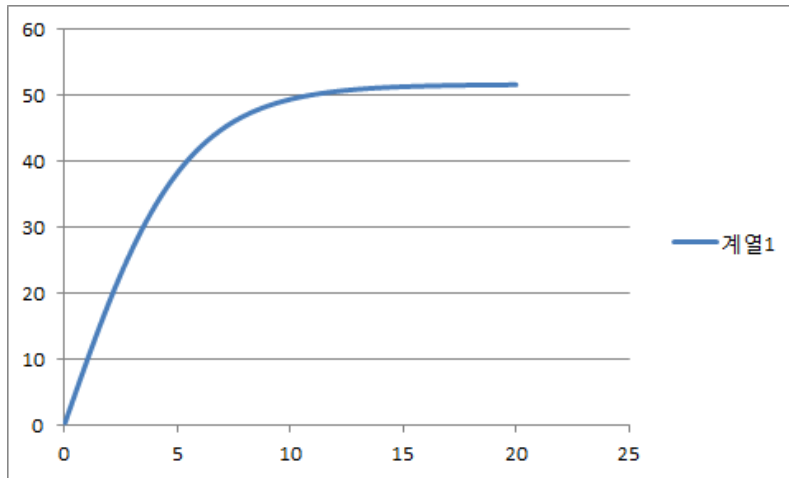
$$(5) m = 95$$

$$V(t) = 61.06 \tanh(0.161 t)$$

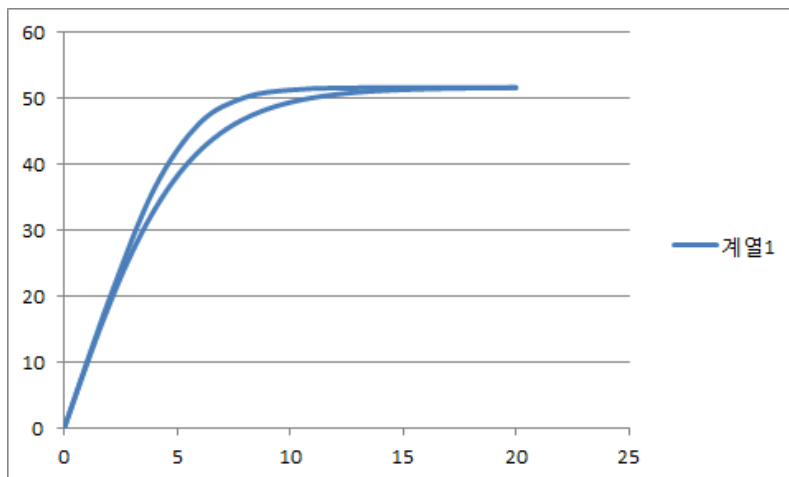
terminal velocity is 61.06 m/s.

at (5) plot asymptotic line becomes $y = 61.06$ and the time when (5) plots closely approach asymptotic line becomes later than (2)(3)(4) plots.

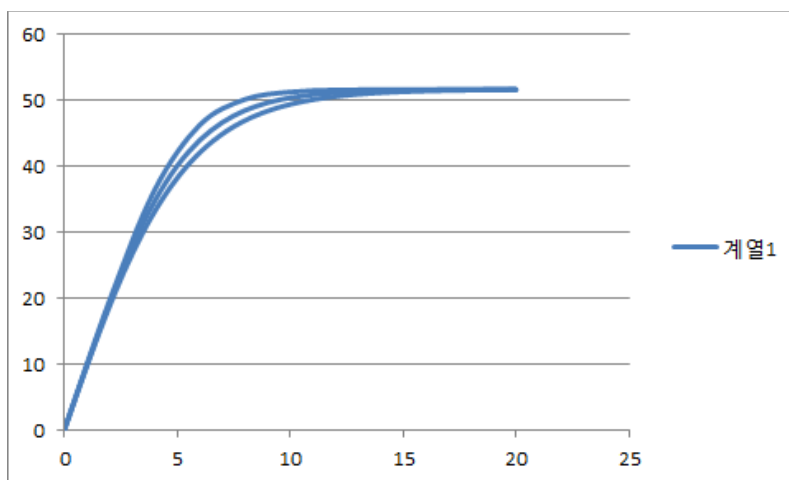
과제 2번



과제 3번



과제 4번



과제 5번

