International Review of Economics and Finance 37 (2015) 324–339

Contents lists available at ScienceDirect

International Review of Economics and Finance

journal homepage: www.elsevier.com/locate/iref

A new method of measuring herding in stock market and its empirical results in Chinese A-share market

Tian Xie [a,*], Yi Xu [b], Xinsheng Zhang [c]

[a] Research Center for Management Science and Information Analytics, School of Information Management and Engineering, Shanghai University of Finance and Economics, China
[b] School of Mathematical Sciences, Fudan University, China
[c] Department of Statistics, School of Management, Fudan University, China

ARTICLE INFO

Article history:
Received 26 May 2014
Received in revised form 28 November 2014
Accepted 6 December 2014
Available online 13 December 2014

Jel classification:
C12
C21
G02
G14

Keywords:
Herding
Chinese A-share market
APT
Fama–French Three-Factor Model
Cross-sectional dispersion

ABSTRACT

We propose a new method based on Arbitrage Pricing Theory to test herding pattern. With the innovative WCSV model, we theoretically prove that this method can test strong herding patterns while filtering out weaker ones, showing better discriminating power than previous methods; also, it can detect time points that affect the overall herding level. Empirically, we apply this method to Chinese A-share market and conclude that the market turmoil of Year 2007–2008 caused long-lasting herding with a decaying trend. Fama–French Three-Factor Model is proved to be an applicable underlying APT model in WCSV model with good model fitting and significant improvement compared to univariate CAPM. Also, we suggest that herding is a relative rather than absolute concept, which depends highly on the chosen benchmark.

1. Introduction

Herding is no longer a new topic in behavioral finance field with its research history for over 20 years. Since herding is possibly related to many psychological factors, it is always very difficult to be quantified and measured. Many scholars have built theoretical models to quantitatively measure herding, such as Lakonishok, Shleifer, & Vishny (1992), Christie & Huang (1995), Chang, Cheng, & Khorana (2000) and Hwang & Salmon (2004). Their designs are all enlightening and they initiate a totally new area of quantitative research in this topic.

Mainstream research methodologies on herding can be classified into two types according to their research objects. Methodologies of the first type focus on investment funds and test herding by analyzing their transactions, see Lakonishok, Shleifer, & Vishny (1992), Grinblatt, Titman, & Wermers (1995) and Wermers (1999). Methodologies of the second type look closely at individual shares and study herding with public market data such as return rates, as represented by Christie & Huang (1995) and Chang, Cheng, & Khorana (2000). In China, funds are required by supervising regulations to disclose quarterly position data. Nonetheless, it is unpersuasive and controversial to test herding with such data frequency. According to Christie & Huang (1995), herding is a short-term

* Corresponding author at: School of Information Management and Engineering, Shanghai University of Finance and Economics, 777 Guoding Rd., Shanghai 200433, China. Tel.: +86 188 0173 4045.
E-mail addresses: txie10@fudan.edu.cn (T. Xie), yixu10@fudan.edu.cn (Y. Xu), xszhang@fudan.edu.cn (X. Zhang).

http://dx.doi.org/10.1016/j.iref.2014.12.004

phenomenon and its significance will decrease sharply for data with frequency lower than monthly. Therefore, we mainly focus on the second type and use accessible daily public data to improve testing accuracy with increased data frequency.

However, existing models all contain theoretical defects, some of which have already been discovered by other researchers, and we also reveal more in our following analyses. Flaws of these models may lead to results of low reliability. Generally, these models make too many assumptions about markets to simplify calculation. Further, mainstream models recognize CAPM equilibrium as the sign of no-herding condition. This is poorly grounded because the classical CAPM lacks explanatory power, so such models cannot ensure the credibility of their results. In this paper, we adopt the fundamental idea of Christie & Huang (1995) and Hwang & Salmon (2004) while integrating Arbitrage Pricing Theory (APT) to design a new model (WCSV model). In the empirical part, we prove that this model fits better with Chinese stock market.

Comparing with previous methods, our new method has following features: (1) the WCSV model we proposed integrates multi-factor APT, which improves the model's suitability and robustness. Empirically, it also has significantly better fitting than previous ones. Essentially, WCSV model is a direct deduction of APT, which suggests that it is a reliable proof method with good properties. (2) Previous methods mostly focus on testing the existence of significant herding. We take a different view by thinking that complete rationality cannot be achieved in real-life stock markets, especially in emerging markets such as China. Thus we turn our focus to spotting those time points with extremely significant herding. Herding time points detected by this method can be proved to show a strong-enough herding pattern. Chang, Cheng, & Khorana (2000) and Hwang & Salmon (2004) built innovative models to test herding, which provide us much insight. In this paper, we start by thoroughly discussing their models. With an intention to establish an improved methodology, we dig into model defects and possible ways to mend them. Following the discussion, we propose a new method by constructing WCSV statistic and WCSV model, and set the criterion of herding detection. Our method is original and innovative in defining and separating strong herding points and strong influential points to measure herding. After theoretically proven its feasibility, we turn to the empirical test part of Chinese A-share market data, using CCK model and WCSV model respectively, to show its superiority. Data are analyzed both in the long term (Year 2006 to 2013) and in short-to-medium terms (annually) with different conclusions. Comparison between APT, which is the basis of WCSV model, and CAPM, which is the basis of most existing models, is also made to further support our methodology.

The remainder of this paper is organized as follows. Section 2 introduces two most fundamental methodologies of testing herding, and follows it by analyzing their defects. Section 3 throughly describes our new method and WCSV model which is based on modifications of previous methods. We proceed step by step to introduce the main model structure, its indication of herding and testing procedures to detect strong herding points and strong influential points. Section 4 describes the data. Section 5 discusses the empirical results of WCSV model applied to Chinese A-share market. We also testify the reasonability of Fama–French Three-Factor Model as the underlying APT model and compare it with univariate CAPM to show the superiority. Section 6 concludes the paper.

2. Reviews of two fundamental methodologies

2.1. The return dispersion model by Chang, Cheng, & Khorana (2000)

Christie & Huang (1995) (hereafter CH) point out that investors' investment decision process depends highly on the overall market situation and higher absolute value of market return generates bigger return dispersion. On an extreme market trend, individual investors will suppress their own willings and act in convergence with group behavior, which means that returns of individual stocks will not deviate too much from the whole market. Following this thought, they propose a model to test herding phenomenon by measuring the impact of extreme market trend on return dispersion. In their model, return dispersion is measured by Cross-Sectional Standard Deviation (CSSD) and Cross-Sectional Absolute Deviation (CSAD), which are calculated as

$$\mathrm{CSSD}_t \triangleq \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(r_{it}-r_{mt})^2}, \qquad \mathrm{CSAD}_t \triangleq \frac{1}{N}\sum_{i=1}^{N}|r_{it}-r_{mt}|$$

where N is the number of companies in the market portfolio, r_{it} is the daily return of company i's stock at time t, r_{mt} is the daily market return at time t. They suggest that during normal periods, the rational asset pricing model works; under extreme market pressure, returns of individual stock tend to cluster, causing a significant decrease in return dispersion. Their study on stock market of the U.S. shows no significant herding. Demirer & Kutan (2006) use CH's approach to study Chinese market by distinct levels of individual firm, individual industry and the whole market, which shows no evidence of herding in Shanghai and Shenzhen A-share market.

Chang, Cheng, & Khorana (2000) (hereafter CCK) improve CH's work by modifying their model and proposing a new nonlinear regression model. They obtain a corollary from CAPM in market equilibrium, stating that in normal times, ECASD should be linearly related with market return. In significant herding period, returns of individual stocks move in the same direction toward a consensus and the linear assumption is no longer valid.

They define Expected Cross-Sectional Absolute Deviation (ECSAD) for an equal-weighted market portfolio as

$$\mathrm{ECSAD}_t = \frac{1}{N}|\beta_i-\beta_m|\mathrm{E}\left(r_{mt}-r_{ft}\right),$$

where r_{ft} is the return rate of a risk-free portfolio, $\beta_m = \frac{1}{N}\sum_{i=1}^{N}\beta_i$ is the beta of the equal-weighted market portfolio. They claim that the ECSAD should be in a positive relationship with the expectation r_m under CAPM equilibrium.

CCK use observed value $CSAD_t$ and r_{mt} to estimate unobservable $ECSAD_t$ and $E(r_{mt})$. Basing on the positive linear relationship, CCK put forward a non-linear regression model:

$$CSAD_t = \alpha + \gamma_1 |r_{mt}| + \gamma_2 r_{mt}^2 + \varepsilon_t. \tag{1}$$

CCK believe that market is in equilibrium when CAPM holds, and CSAD, as the measure of return dispersion, should be linearly related to average market return r_{mt}. That is to say, γ_2 equals 0. However, when significant herding exists, CAPM will be invalid and thus CSAD is no longer linearly related to r_{mt}. The quadratic term $\gamma_2 r_{mt}^2$ can be a signal of such trend: when significant herding exists, CSAD tends to decrease and γ_2 is notably negative.

Their empirical studies find no significant herding in developed markets such as the U.S. and Hong Kong, in contrast with significant herding in emerging markets such as Korea and Taiwan. In recent years, many scholars have applied and improved CH and CCK's models with some representative results. Tan, Chiang, Mason, & Nelling (2008) adopt CCK's method to test Chinese market with daily trade data of 87 dual-listed companies. They conclude that herding exists in both A-share and B-share markets of Shanghai and Shenzhen, which is different from Demirer & Kutan (2006). For other related applications, see Chiang & Zheng (2010), Yao, Ma, & He (2014), Demirer, Kutan, & Zhang (2014), Gebka & Wohar (2013) and Balcilar, Demirer, & Hammoudeh (2013).

CCK method has a simple and practicable model structure but also some flaws that cannot be neglected.

1. It may be inappropriate to use $CSAD_t$ and r_{mt} to estimate the unobservable $ECSAD_t$ and $E(r_{mt})$. To compare $CSAD_t$ with $ECSAD_t$, we calculate the expectation of CSAD at time t by CAPM,

$$\begin{aligned} E(CSAD_t) &= \frac{1}{N}\sum_{i=1}^{N} E|r_{it} - r_{mt}| \geq \frac{1}{N}\sum_{i=1}^{N} |E(r_{it}) - E(r_{mt})| = \frac{1}{N}\sum_{i=1}^{N}\left| r_{ft} + \beta_i E\left(r_{mt} - r_{ft}\right) - E(r_{mt})\right| = \frac{1}{|\beta_m| N}\sum_{i=1}^{N} |\beta_i - \beta_m| \left| E(r_{mt}) - r_{ft}\right| \\ &\geq \frac{1}{|\beta_m|}\frac{1}{N}\sum_{i=1}^{N} |\beta_i - \beta_m| E\left(r_{mt} - r_{ft}\right) = \frac{1}{|\beta_m|} ECSAD_t. \end{aligned}$$

   In an equal-weighted portfolio, β_m hardly equals 1. The above inequality actually shows that ECSAD can deviate greatly from the expectation of CSAD, possibly causing wrong judgements about herding, which means the model is quite coarse per se. This is the most inevitable drawback of CCK method.
2. Since it is widely acknowledged that beta of an individual stock varies continuously, using constant β_is to represent time-variant patterns is too unrealistic to maintain the credibility of testing results.
3. Both financial practices and researches have shown that CAPM doesn't appear to be a very suitable model for real markets.

2.2. The state-space model by Hwang and Salmon (2004)

Hwang & Salmon (2004) (hereafter HS) believe that herding results from imitating others and suppressing personal information. They assume that CAPM estimates are biased and construct a state-space model to quantify the deviation between the real market parameter and the equilibrium market parameter. Compared with previous studies, this model tests the existence of herding through Cross-Sectional Standard Deviance (CSSD) of time-variant beta. CSSD is defined as

$$Std_c(\beta_{imt}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\beta_{imt} - E_c(\beta_{imt}))^2}.$$

The asset beta has low long-term volatility so the cross-sectional beta has low volatility as well. HS therefore assume it as the combination of an average term and an equal-variance stochastic disturbance term,

$$\log Std_c(\beta_{imt}) = \mu_m + \nu_{mt},$$

where $\mu_m = E \log Std_c(\beta_{imt})$, $\nu_{mt} \sim$ i. i. d. $(0, \sigma_{m\nu}^2)$.

HS claim that during herding time periods, $Std_c(\beta_{imt})$ should differ from the biased actual value $Std_c(\beta_{imt}^b)$. Further, they define $h_{mt} \in [0, 1]$ as the herding factor, which suffices $Std_c(\beta_{imt}^b) = Std_c(\beta_{imt})(1 - h_{mt})$. They recognize $h_{mt} = 0$ as a sign of no herding and $h_{mt} = 1$ as that of complete herding, while generally $0 < h_{mt} < 1$, indicating some herding patterns. They propose that

$$\log Std_c\left(\beta_{imt}^b\right) = \log Std_c(\beta_{imt}) + \log(1 - h_{mt}).$$

Let $H_{mt} \triangleq \log(1 - h_{mt})$, so a significantly negative H_{mt} provides evidence of herding. H_{mt} is assumed to follow a stationary AR(1), which changes the basic model to

$$\begin{aligned} \log Std_c\left(\beta_{imt}^b\right) &= \mu_m + H_{mt} + \nu_{mt} \\ H_{mt} &= \phi_n H_{m(t-1)} + \eta_{mt} \end{aligned} \tag{2}$$

where $\eta_{mt} \sim$ i. i. d. $(0, \sigma_{m\eta}^2)$.

They conduct fitting procedures for this classic state-space model, whose parameters can be estimated using Kalman Filter. A significant $\sigma^2_{m\eta}$ can be seen as the evidence of herding, while a significant $\phi > 0$ supports the first-order autoregressive structure in the model. $\sigma^2_{m\eta} = 0$ equivalently means $H_{mt} = 0$, which implies no herding existence.

HS model is widely acknowledged as well. Demirer, Kutan, & Chen (2010) synthetically incorporate models proposed by Christie & Huang (1995), Chang, Cheng, & Khorana (2000), and Hwang & Salmon (2004) to study Taiwan market. Their test with CH model shows no evidence of herding while CCK and HS models both manifest strong evidence. They therefore declare the insufficient sensitivity of CH model.

Among all measuring methods of herding, HS model is the first mature model that takes the time-variant pattern of beta into consideration, which brings us some instructive insight. Under normal circumstances, cross-sectional variance of beta is assumed to fluctuate around the mean value within a small range; oppositely, during herding periods, cross-sectional variance of beta decreases significantly, and a component H_{mt} can be obtained to represent herding. However, this model also has some implicit problems, making it hard to be applied in practice.

1. H_{mt} is assumed to follow a stationary process AR(1), which is a feasible guess rather than a fact. Such assumption can simplify calculations, but without analysis of higher order effects, it may not satisfy real markets and even bring wrong information.
2. The dependent variable in this model is cross-sectional variance of time-variant beta. However, there is still no consensus or an effective method to estimate time-variant beta. HS obtain beta by regressions using monthly data, which undoubtedly undermines data usage efficiency. Estimating beta by other methods may also be inappropriate and cause expanded errors after repetitions.
3. Empirically, $\sigma^2_{m\eta}$ is rarely insignificant. This may imply that herding almost always exists, or that HS model lacks discriminating power.

3. New methodology

In this section, we present a new measuring method to sensitively measure the relative level of herding. This method requires fewer preliminary assumptions and suits Chinese stock market better. Empirical studies also show its good quality.

3.1. WCSV statistic and WCSV model

Traditional return dispersion measures CSSD and CSAD are defined by Christie & Huang (1995). We propose a new statistic, namely Weighted Cross-Sectional Variance (WCSV), which is calculated as

$$\mathrm{WCSV}_t \triangleq \sum_{i=1}^{n} w_{it}(r_{it} - r_{mt})^2, \tag{3}$$

where r_{it} is the return of stock i on day t; r_{mt} is the market average return weighted by market value on day t; w_{it} is the market value weight of stock i on day t, $\sum_i w_{it} = 1$. This statistic determines weight factor w_{it} by market value, which treats the overall market as a market portfolio to consider its return dispersion.

Define cross-sectional expectation, variance and covariance as

$$\begin{aligned}
\mathrm{E}_c(x_{it}) &= \sum_{i=1}^{n} w_{it}x_{it}, \\
\mathrm{Var}_c(x_{it}) &= \sum_{i=1}^{n} w_{it}(x_{it} - \mathrm{E}_c(x_{it}))^2, \\
\mathrm{Cov}_c(x_{it}, y_{it}) &= \sum_{i=1}^{n} w_{it}(x_{it} - \mathrm{E}_c(x_{it}))(y_{it} - \mathrm{E}_c(y_{it})).
\end{aligned}$$

Then WCSV can be also expressed as

$$\mathrm{WCSV}_t = \mathrm{Var}_c(r_{it}).$$

Assume that during normal periods, each stock return always satisfies a certain Arbitrage Pricing Theory (APT) model (Ross, 1976) whose first factor is the risk premium in CAPM. In our paper, we assume that these APT models are all well-designed. That is to say,

1. Return is sufficiently explained by factors in APT model;
2. β_{imt} and all β_{ikt} are pairwise cross-sectional uncorrelated. Note that Hwang & Salmon (2001) prove that β_{imt} has cross-sectional mean of 1 and all β_{ikt} has cross-sectional mean of 0. This requirement means:

$$\begin{aligned}
&\sum_{i=1}^{n} w_{it}\beta_{ik't}\beta_{ik''t} = \mathrm{Cov}_c\left(\beta_{ik't}, \beta_{ik''t}\right)\begin{cases} >0 & k' = k'' \\ =0 & k' \neq k'' \end{cases}, \qquad k', k'' \in \{1, 2, \ldots, K\}, \\
&\sum_{i=1}^{n} w_{it}\beta_{ikt}(\beta_{imt} - 1) = \mathrm{Cov}_c(\beta_{ikt}, \beta_{imt}) = 0, \qquad k \in \{1, 2, \ldots, K\}.
\end{aligned}$$

If the APT equation is sufficient, it can be written as

$$r_{it}-r_{ft}=\beta_{imt}\left(r_{mt}-r_{ft}\right)+\sum_{k=1}^{K}\beta_{ikt}F_{kt}.$$

Deduct $(r_{mt}-r_{ft})$ on both sides,

$$r_{it}-r_{mt}=(\beta_{imt}-1)\left(r_{mt}-r_{ft}\right)+\sum_{k=1}^{K}\beta_{ikt}F_{kt}.$$

Take the square of both sides,

$$(r_{it}-r_{mt})^2=(\beta_{imt}-1)^2\left(r_{mt}-r_{ft}\right)^2+\sum_{k=1}^{K}\beta_{ikt}^2F_{kt}^2+C_{it},$$
$$C_{it}=\sum_{k=1}^{K}(\beta_{imt}-1)\beta_{ikt}\left(r_{mt}-r_{ft}\right)F_{kt}+\sum_{\substack{k'\neq k''\\ k',k''=1}}^{K}\beta_{ik't}\beta_{ik''t}F_{k't}F_{k''t}.$$

Sum these equations with weight w_{it},

$$\sum_{i=1}^{n}w_{it}(r_{it}-r_{mt})^2=\mathrm{Var}_c(\beta_{imt})\left(r_{mt}-r_{ft}\right)^2+\sum_{k=1}^{K}\mathrm{Var}_c(\beta_{ikt})F_{kt}^2. \tag{4}$$

Note that the weighted sum of C_{it} is zero. By using observed values to replace expected values, a sample version can be derived as

$$\mathrm{WCSV}_t=\sum_{i=1}^{n}w_{it}(r_{it}-r_{mt})^2=\mathrm{Var}_c(\beta_{imt})\left(r_{mt}-r_{ft}\right)^2+\sum_{k=1}^{K}\mathrm{Var}_c(\beta_{ikt})F_{kt}^2+\varepsilon_t. \tag{5}$$

There are two major assumptions in WCSV model.

Assumption 1. $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$ are constants when the stock market structure is stable.

This assumption originates from Hwang & Salmon (2004) model and we strengthen it further. HS assume that cross-sectional variance can be represented by mean and disturbance when the stock market structure does not undergo huge changes. Since our model has an embedded error term, we merge the disturbance part of their assumption into the error term, leaving the constant mean only. This assumption is basically equivalent to HS assumption, with even better analytical properties. Remark that this characteristic only applies to short-to-medium-term stock market data.

Assumption 2. In stock market, herding always exists. Stock market herding conditions of different time points differ in the level rather than the existence.

In existing models that test herding existence, explanations about weak or extremely weak herding are still quite vague. Actually, all investors can never be rational enough to completely eliminate the effect of herding at any time point. Thus we recognize herding as ubiquitous. Under such a strong assumption, those time points classified by previous studies as "insignificant herding" are now classified as "weak herding". Further, we can obtain the benchmark of identifying strong herding based on weak herding. In other words, we recognize herding as a relative rather than absolute concept, which is reasonable.

With these two assumptions, we rewrite WCSV model as

$$\mathrm{WCSV}_t=\gamma_m\left(r_{mt}-r_{ft}\right)^2+\sum_{k=1}^{K}\gamma_kF_{kt}^2+\varepsilon_t. \tag{6}$$

By Assumption 1, we view $(r_{mt}-r_{ft})^2$ and F_{kt}^2 as the independent variables, $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$ as the parameters to be estimated. In this way, we can obtain a linear regression model without intercept. Similar to CCK model, WCSV is used to detect herding rather than measure herding level.

Unlike CCK model, which intends to check whether linearity holds by adding a quadratic term, in WCSV model, the equation should hold, or at least approximately hold, under normal situations. Comparatively, our model has more advantages and looser preliminary assumptions of stock market operation process. Considering the based pricing model, APT is also more precise than CAPM.

3.2. WCSV model and herding

As discussed in the previous section, theoretically, in normal stock market with no herding, WCSV model is well descriptive of stock market. However, herding always exists in reality. In this section, we will study how model properties deteriorate while herding arises, and then use these characteristics to find and describe herding.

On the one hand, according to Christie & Huang (1995) and Chang, Cheng, & Khorana (2000), herding can cause a decline of return dispersion of stock market. WCSV statistic in our model is also a measure of return dispersion. Under Assumption 2, herding exists more or less at any time. This means that though WCSV is not a herding measure per se (because it is affected by daily market factors), its value calculated from actual data should always be lower than that of the ideal no-herding condition. Meanwhile, $(r_{mt} - r_{ft})^2$ and F_{kt}^2 are both positive, so the dependent variable WCSV and regression coefficients change in the same direction. That is to say, the estimated $\mathrm{Var}_c(\beta_{imt}^H)$ and $\mathrm{Var}_c(\beta_{ikt}^H)$ in the previous section (H means "with historical data") are actually lower than the due value with no herding.

On the other hand, APT claims that the price of an individual stock is determined by several market factors and corresponding sensitivities of these factors. We follow Hwang & Salmon (2004) by believing that individual stock's sensitivities of market factors are lower than the due levels with no herding. In this way, one single market factor has more consistent effect on all stock prices in terms of degree and direction than normal, which means investors no longer base their judgment on the idiosyncratic characteristics of an individual stock. In our model, this phenomenon can be reflected by the decrease of $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$.

Considering the two opinions above, we find that both the herding judgment based on return dispersion by Christie & Huang (1995) and Chang, Cheng, & Khorana (2000) and that based on beta dispersion by Hwang & Salmon (2004) can reach a uniformity in WCSV model. In fact, both methods now translate into comparing $\mathrm{Var}_c(\beta_{imt}^H)$ and $\mathrm{Var}_c(\beta_{ikt}^H)$ obtained from real data with $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$, the ideal values with no herding. If the former values are significantly lower, herding existence can be concluded. Following this thought, the problem of testing herding now transforms into whether $\mathrm{Var}_c(\beta_{imt}^H)$ and $\mathrm{Var}_c(\beta_{ikt}^H)$ significantly differ from $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$.

However, since we have accepted the assumption that herding always exists, testing the existence of herding seems to be meaningless. Theoretically, it cuts off our way to obtain $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$: Strictly, since the level of herding is not clear, it is unfeasible to figure out $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$ from $\mathrm{Var}_c(\beta_{imt}^H)$ and $\mathrm{Var}_c(\beta_{ikt}^H)$.

Seeking another approach, we can compare the herding level of a certain time point and the average herding level obtained from historical data. If the former is significantly stronger (or not significantly weaker) than the latter, we define this time point as of strong herding. Denote the daily actual $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$ as $\mathrm{Var}_c(\beta_{imt}^b)$ and $\mathrm{Var}_c(\beta_{ikt}^b)$. Then in the model, we compare $\mathrm{Var}_c(\beta_{imt}^b)$ and $\mathrm{Var}_c(\beta_{ikt}^b)$ with $\mathrm{Var}_c(\beta_{imt}^H)$ and $\mathrm{Var}_c(\beta_{ikt}^H)$, respectively.

Our focus is now shifted to answering these two questions: (1) Which time points (strong herding point) are of strong herding? (2) Since $\mathrm{Var}_c(\beta_{imt}^H)$ and $\mathrm{Var}_c(\beta_{ikt}^H)$ are significantly smaller than $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$, which time points (strong influential point) contribute most? Remark that "strong influential point" should be stronger than "strong herding point" in terms of herding level.

Overall, if significantly more strong herding points are detected during a certain time period, this time period is judged as strong herding period.

3.3. Testing for strong herding points

First we consider a special case when $K = 0$, and the model is degenerated to a single-factor case. Under normal circumstances,

$$\mathrm{WCSV}_t = \mathrm{Var}_c(\beta_{imt})\left(r_{mt} - r_{ft}\right)^2 + \varepsilon_t.$$

Define $\widehat{\gamma}_{mt}$ as the unbiased OLS estimator of $\mathrm{Var}_c(\beta_{imt}^H)$ and σ_γ^2 as its variance. Here, $\mathrm{Var}_c(\beta_{imt}^H)$ is the cross-sectional variance of β_{imt} using historical data and is the indicator of the historical average herding level. To compare the actual daily herding level with historical average herding level, we need an expression about $\mathrm{Var}_c(\beta_{imt}^b)$. Let's assume that in terms of expectation,

$$\mathrm{WCSV}_t = \mathrm{Var}_c\left(\beta_{imt}^b\right)\left(r_{mt} - r_{ft}\right)^2 \Rightarrow \mathrm{Var}_c\left(\beta_{imt}^b\right) = \frac{\mathrm{WCSV}_t}{\left(r_{mt} - r_{ft}\right)^2}.$$

Then we compare this value with $\mathrm{Var}_c(\beta_{imt}^H)$, and design an one-sided hypothesis test.

$$H_0 : \mathrm{Var}_c\left(\beta_{imt}^b\right) = \mathrm{Var}_c\left(\beta_{imt}^H\right) \qquad \text{v.s.} \qquad H_1 : \mathrm{Var}_c\left(\beta_{imt}^b\right) < \mathrm{Var}_c\left(\beta_{imt}^H\right).$$

If the null hypothesis H_0 is rejected, we can recognize time point t as a strong herding point.

For multivariate models that $K \geq 1$, since there does not exist a uniform measure describing the cross-sectional variance tuples, the method above can no longer be applied directly. Fortunately, in univariate model, comparing $\mathrm{Var}_c(\beta_{imt}^b)$ and $\mathrm{Var}_c(\beta_{imt}^H)$ is equivalent to comparing WCSV_t and $\widehat{y}_t \triangleq \mathrm{Var}_c(\beta_{imt}^H)(r_{mt} - r_{ft})^2$ by multiplying $(r_{mt} - r_{ft})^2$ on both sides. The original hypothesis can be written as

$$\textbf{Test 1} : H_0 : \mathrm{WCSV}_t = \widehat{y}_t \qquad \text{v.s.} \qquad H_1 : \mathrm{WCSV}_t < \widehat{y}_t.$$

Strong herding points can be detected by rejecting H_0. We can easily generate it to the multivariate case, where $\widehat{y}_t$ is also the OLS predictor, namely

$$\widehat{y}_t \triangleq \mathrm{Var}_c\left(\beta_{imt}^H\right)\left(r_{mt}-r_{ft}\right)^2+\sum_{k=1}^{K}\mathrm{Var}_c\left(\beta_{ikt}^H\right)F_{kt}^2 \tag{7}$$

with its variance

$$\mathrm{Var}\left(\widehat{y}_t\right)=\sigma^2\mathbf{x}_t^{'}\left(\mathbf{X}^{'}\mathbf{X}\right)^{-1}\mathbf{x}_t \tag{8}$$

where σ^2 is the common variance of residuals; $\mathbf{X}$ is the design matrix; $\mathbf{x}_t^{'}$ is the t-th row of $\mathbf{X}$. Remark that the hypothesis test for every time point is independent, and $\widehat{y}_t$ is not identically distributed because the market factors are time-variant.

Although the exact distribution of an individual stock's return is unknown, the method still works. Notice that WCSV_t is a weighted average of daily stock price data from a large sample of individual stocks, by Central Limit Theorem, WCSV_t converges to a normal distribution asymptotically, so does $(\mathrm{WCSV}_t-\widehat{y}_t)$.

By Assumption 2, we can infer that the average herding level obtained from historical data can also be recognized as herding. If we further relax the judging criterion by recognizing a certain herding level that is not significantly weaker than the historical average herding level as herding, we can construct a new hypothesis test:

Test 2 : $H_0:\mathrm{WCSV}_t=\widehat{y}_t$ v.s. $H_1:\mathrm{WCSV}_t>\widehat{y}_t$.

Strong herding points can be detected by not rejecting H_0. Note that Test 2 makes slightly less strict herding judgment than Test 1. Comparing two tests, we have the following conclusions:

1 If the null hypothesis of Test 1 is rejected, this time point can be judged as herding in the strong sense;
2 If both null hypotheses are not rejected, this time point can be judged as herding in the weaker sense;
3 If the null hypothesis of Test 2 is rejected, there is inadequate evidence to judge this point as herding.

Since the limiting distribution of $(\mathrm{WCSV}_t-\widehat{y}_t)$ is normal, under H_0,

$$\frac{\mathrm{WCSV}_t-\widehat{y}_t}{\sqrt{\sigma^2\mathbf{x}_t^{'}\left(\mathbf{X}^{'}\mathbf{X}\right)^{-1}\mathbf{x}_t}}\xrightarrow{D}N(0,1), \tag{9}$$

and the sample version is

$$Z_t\triangleq\frac{\mathrm{WCSV}_t-\widehat{y}_t}{\sqrt{s^2\mathbf{x}_t^{'}\left(\mathbf{X}^{'}\mathbf{X}\right)^{-1}\mathbf{x}_t}}\xrightarrow{D}t_{n-K-1}, \tag{10}$$

where Z_t represents the standardized value (a.k.a. z-score), and the unbiased estimator of σ^2 is

$$s^2=\frac{1}{n-K-1}\sum_{t=1}^{n}\left(\mathrm{WCSV}_t-\widehat{y}_t\right)^2.$$

Note that Test 1 and Test 2 are with the same null hypothesis but different inequality directions in the alternative hypothesis. Under H_0, Z_t is centralized at 0. In Test 1, $Z_t<t_{n-K-1}(\alpha)\approx\Phi^{-1}(\alpha)$ is sufficient to reject H_0 with significant level α ($\alpha=0.05$ in this paper), where $t_k(\alpha)$ is the lower α quantile of t distribution with degree of freedom k, and $\Phi^{-1}(\alpha)$ is the inverse of the cumulative density function of standard normal distribution. With the same significant level, in Test 2, $Z_t>t_{n-K-1}(1-\alpha)\approx\Phi^{-1}(1-\alpha)$ is sufficient to reject H_0 (implying inadequate evidence of herding).

In the view of p-value, p-value (Test 1) $=t_{n-K-1}^{-1}(Z_t)<\alpha$ will reject H_0 in Test 1 to show strong herding, and $p-$ value (Test 2) $=t_{n-K-1}^{-1}(-Z_t)<\alpha$ implies inadequate evidence of herding. Since t distribution is symmetric about 0, p-values of an individual point in Test 1 and Test 2 sum up to 1. That is to say, $1-p$-value (Test 1) $=p$-value (Test 2) is an indicator of herding and a higher value of $1-p$-value (Test 1) suggests a higher herding level. A summary of this test is shown in Table 1.

3.4. Testing for strong influential points

In the previous section, we propose a new hypothesis to detect strong herding. Now, we come back to the other question. By Assumption 2, the average herding level $\mathrm{Var}_c(\beta_{imt}^H)$ and $\mathrm{Var}_c(\beta_{ikt}^H)$ obtained from historical data should be lower than the no-herding market due level $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$. This difference is mainly contributed by two parts: One is the prevalent herding level of most time points in the market, which we refer as systematic part; the other is from few strong influential points that pull down the average value much more than other points, and we refer the excess as idiosyncratic part.

Based on this idea, we focus on the idiosyncratic part in hope of finding strong influential points. Since these strong influential points always come under extreme market conditions, finding them will help us detect time periods with high market stress. A common outlier detection method is introduced below.

We use DFFITS statistics (Belsley, Kuh, & Welsch, 1980). DFFITS statistic examines how the OLS predictor $\widehat{y}_t$ changes by deleting the corresponding observation to detect whether it is an outlier observation. The expression is

$$\mathrm{DFFITS}_t = \frac{\widehat{y}_t - \widehat{y}_{(t)}}{\sqrt{s_{(t)}^2 \mathbf{x}_t' \left(\mathbf{X}_{(t)}' \mathbf{X}_{(t)}\right)^{-1} \mathbf{x}_t}} \tag{11}$$

where $\widehat{y}_t$ is the OLS predictor of WCSV_t; $\widehat{y}_{(t)}$ is the OLS predictor of WCSV_t after deleting time point t; $s_{(t)}^2$ is the unbiased estimator of residual variance; and $\mathbf{X}_{(t)}$ is the matrix $\mathbf{X}$ after deleting the t-th row.

It is not hard to see that if time point t is in normal condition, $\widehat{y}_t$ and $\widehat{y}_{(t)}$ should not deviate much; otherwise, $\widehat{y}_t$ is biased and $\widehat{y}_{(t)}$ should be closer to the real value because the high leverage time point is deleted, and time t is judged as an outlier.

According to Christie & Huang (1995), return dispersion tends to decrease with herding stress, which means $\widehat{y}_t$ will be lower than the values under no herding. That is to say, the adjusted predictor $\widehat{y}_{(i)}$ produced by deleting this observation should be significantly higher than $\widehat{y}_i$ with herding. The one-sided hypothesis test for strong influential point at time t can be written as

$$H_0 : \widehat{y}_t = \widehat{y}_{(t)} \qquad \text{v.s.} \qquad H_1 : \widehat{y}_t < \widehat{y}_{(t)}.$$

Rejecting H_0 can imply severe underestimate and lack of fit for the model at time t, which suggests time t is a strong influential point.

From this perspective, when DFFITS is significantly negative, market stress can be detected. Belsley, Kuh, & Welsch (1980) suggest that the critical values for judging the outliers are $\pm 2\sqrt{p/n}$, where $p = K + 1$ is the number of factors in the model. In our model, we only concern about influential points that pull up the average herding level, so we set our criterion as that when $\mathrm{DFFITS}_t < -2\sqrt{(K+1)/n}$, time point t is recognized as a strong influential point.

3.5. Summary of WCSV model

In this part, we combine the ideas of testing herding existence by return dispersion and beta dispersion, and propose a new WCSV model to measure herding. According to the model, we design a strong herding point test and a strong influential point test. The former aims to detect trading days with strong herding, while the latter proceeds further by aiming to detect trading days that significantly increase the overall herding level. To summarize, the former test studies the phenomenon while the latter test focuses on the reason.

As analyzed above, CCK model is theoretically deficient and HS model empirically lacks discrimination. Compared with CCK model, WCSV model has no severe systematic bias between expectations and estimates and considers the time-variant pattern of beta, with a relatively reliable theory. Compared with HS model, WCSV model takes a totally different view by treating cross-sectional variance of beta as a parameter to lower actual practice difficulty, and intuitively demonstrates the herding patterns with critical values by the asymptotic distribution to ensure the model's sensitivity and discrimination.

WCSV model requires fewer preliminary assumptions as well. This model sets strict standards for APT structure, but in our empirical study, Fama–French Three-Factor Model can meet requirements quite well. This is natural considering that herding is originally a relative phenomenon and people cannot recognize a period when herding totally vanishes. Without huge market turmoils, $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$ should also remain stable and work as the benchmark of testing herding.

Certainly, these are some remaining problems, but we undoubtedly make some progress of detecting abnormal market behaviors based on the model's lack-of-fit. Eventually, the combination in WCSV model of return dispersion model and beta dispersion model is also of great value.

4. Data

4.1. Data description

In this paper, we examine the constituents from CSI 300 Index (a.k.a. Shanghai–Shenzhen 300 Index). The CSI 300 Index is a capitalization-weighted stock market index compiled by China Securities Index Company, Ltd. This index is designed to reflect the

Table 1
A summarization of strong herding point testing, $\alpha = 0.05$.

Z_t	$1 - p$-Value (Test 1)	Explanation of time t
$(-\infty, t_{n-K-1}(\alpha))$	$(1-\alpha, 1]$	Herding point in the strong sense.
$[t_{n-K-1}(\alpha), t_{n-K-1}(1-\alpha))$	$(\alpha, 1-\alpha]$	Herding point in the weaker sense.
$[t_{n-K-1}(1-\alpha), +\infty)$	$[0, \alpha]$	Inadequate evidence to judge this point as herding.

overall trend of Chinese A-share market, balancing Shanghai Stock Exchange (SHSE) and Shenzhen Stock Exchange (SZSE). The 300 stocks traded in Shanghai and Shenzhen stock exchanges cover more than 60% of total market value and are picked with strict criteria on a rolling basis, which makes this index convincingly representative. Since the CSI 300 Index has been calculated since April 8, 2005, considering the year completeness, we choose January 4, 2006 to December 31, 2013 as our data period.

Daily return is calculated as $R_t = \frac{P_t}{P_{t-1}} - 1$. The risk-free return we use is the daily compounded value of 1-year fixed bank interest rate.

Fig. 1 is the trend chart of CSI 300 Index from the very start to recent time. There are 1941 valid observation points in our research period. We can calculate the daily value of Fama–French Three-Factor Model from market data and corporate finance data, e.g. Table 2. Fig. 2 is the broken line graph of calculated WCSV.

4.2. Short description of Fama–French Three-Factor Model

In terms of the underlying APT model, we use the Three-Factor Model introduced by Fama & French (1993). Fama–French Three-Factor Model is an extension of CAPM with its market risk premium ($r_m - r_f$) as one of the factors, while market value factor (Small Minus Big, SMB) and book to market ratio factor (High Minus Low, HML) as the other two. The model is

$$r_{it} - r_{ft} = \beta_{imt}\left(r_{mt} - r_{ft}\right) + \beta_{i1t}\mathrm{SMB}_t + \beta_{i2t}\mathrm{HML}_t. \tag{12}$$

In our research, we take the total market value of a stock as its market value and the latest disclosure of owner's equity as its book value. Following generally-accepted rules, we divide stocks into two groups—Big (50%) and Small (50%)—by their market value; three groups—Low (30%), Medium (40%) and High (30%)—by their book to market ratio. Intersection of two dividing rules produces six groups: B/L, B/M, B/H, S/L, S/M and S/H. For each group, a portfolio can be obtained by weight of market value, and

$$\begin{aligned}\mathrm{SMB}_t &= \frac{1}{3}[r_t(\mathrm{S/L}) + r_t(\mathrm{S/M}) + r_t(\mathrm{S/H}) + r_t(\mathrm{B/L}) + r_t(\mathrm{B/M}) + r_t(\mathrm{B/H})],\\ \mathrm{HML}_t &= \frac{1}{2}[r_t(\mathrm{S/H}) + r_t(\mathrm{B/H}) - r_t(\mathrm{S/L}) - r_t(\mathrm{B/L})].\end{aligned}$$

With Fama–French Three-Factor Model as the underlying APT model, WCSV model can be written as

$$\mathrm{WCSV}_t = \gamma_m\left(r_{mt} - r_{ft}\right)^2 + \gamma_S\mathrm{SMB}_t^2 + \gamma_H\mathrm{HML}_t^2 + \varepsilon_t. \tag{13}$$

5. Empirical results

In this section, we will apply CCK model and WCSV model to Chinese stock data respectively. We will firstly discuss CCK model's fitting result and its empirical disadvantages. Then we will use WCSV model to solve the two questions proposed in the previous section. In addition to the whole time period of Year 2006–2013, each individual year is also investigated to study the yearly herding sensitivity. By comparing model fitting of different underlying APT models in WCSV model, we will also show that the multi-factor model we use is applicable and superior to single-factor CAPM.

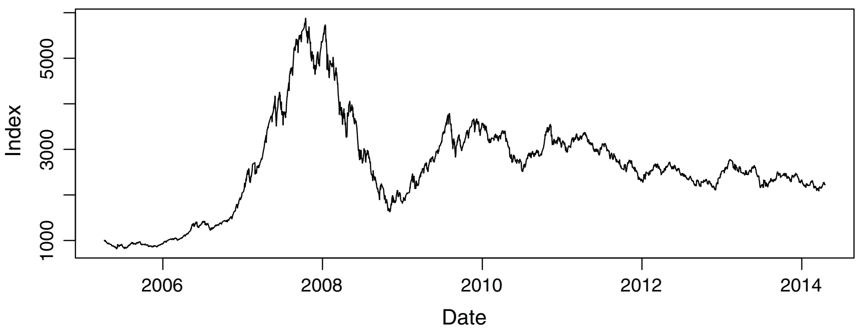

Fig. 1. CSI 300 Index. This figure describes the variation trend of CSI 300 Index between 8 Apr. 2005 and 18 Apr. 2014 with 2192 effective sampling days. This reflects the overall situation of CSI 300 Index from its establishment to recent period.

Table 2
Descriptive statistics. This table summarizes descriptive statistics of useful variables, covering 1941 effective trading days from the first trading day of 2006 (4 Jan. 2006) to the last trading day of 2013 (31 Dec. 2013). $r_m - r_f$, SMB, HML are the independent variables in Fama–French Three-Factor Model. WCSV is the dependent variable in our model.

Year	2006	2007	2008	2009	2010	2011	2012	2013	Overall
# observations	241	242	247	244	242	244	243	238	1941
Panel A: $r_m - r_f$									
Min	−0.059162	−0.0991673	−0.0751759	−0.0620745	−0.0503126	−0.0386784	−0.0368213	−0.0601705	−0.0991673
Max	0.0483344	0.052546	0.0729974	0.0545205	0.0363653	0.0288501	0.0355581	0.0275556	0.0729974
Mean	−0.0019412	−0.005027	−0.0118524	−0.0029255	−0.005811	−0.0084471	−0.0079624	−0.0076175	−0.0064611
Variance	0.000214749	0.00046138	0.000706298	0.000330791	0.000198042	0.000119284	0.00010774	0.000120731	0.000291673
Skewness	0.002678	−0.6909182	0.1860113	−0.3506033	−0.2876751	0.151075	0.8531116	−0.0263597	−0.2381546
Kurtosis	1.7482665	2.0637016	0.3366912	0.7001557	1.1767322	0.9017046	2.2899141	2.4650245	2.476634
Panel B: SMB									
Min	−0.0349256	−0.049138	−0.0526267	−0.0403673	−0.0250372	−0.0164611	−0.0193225	−0.0210525	−0.0526267
Max	0.0252316	0.0447583	0.0327501	0.0193756	0.0225689	0.0078694	0.0176401	0.0129735	0.0447583
Mean	−0.0010879	−0.000108632	−0.000347344	0.000443878	0.000231529	−0.000585745	−0.000729455	−0.00062706	−0.000349996
Variance	0.000089209	0.000201969	0.00019038	0.000079703	0.000046354	0.000021912	0.000031988	0.000028514	0.000086458
Skewness	−0.5705375	−0.2192579	−0.5991037	−1.0697475	−0.502608	−0.7544126	−0.4140672	−0.8032402	−0.5775905
Kurtosis	1.1247698	0.8102555	0.7837189	3.0756631	1.5424938	0.5705104	1.0826468	1.7691644	3.3256605
Panel C: HML									
Min	−0.0335337	−0.032214	−0.0261204	−0.0232113	−0.0443855	−0.0203543	−0.0216249	−0.0255417	−0.0443855
Max	0.0290166	0.0461393	0.0179687	0.0254845	0.027292	0.0248871	0.0297607	0.0245815	0.0461393
Mean	−0.0029469	−0.0013795	−0.0024645	−0.001905	−0.0025106	−0.000371856	−0.0011039	−0.0022244	−0.0018617
Variance	0.000070829	0.000119247	0.0000516	0.000068989	0.000094414	0.000051324	0.000048697	0.000072252	0.000072461
Skewness	0.1351796	0.4738846	−0.2266403	0.4200036	−0.0727129	0.3682822	0.3735391	0.1966389	0.2182664
Kurtosis	1.5591123	1.4135774	0.4459452	0.6512603	1.5662077	0.8165835	1.7452147	0.3715548	1.5751272
Panel D: WCSV									
Min	0.000100712	0.000137221	0.0000368	0.000059147	0.000055377	0.000064768	0.000058022	0.00009595	0.0000368
Max	0.0018634	0.0023149	0.0015138	0.000931297	0.0010419	0.000512119	0.000661604	0.000786657	0.0023149
Mean	0.000456873	0.000648833	0.00048759	0.000335582	0.000240936	0.000183981	0.000174655	0.000255997	0.000348278
Variance	8.67E−08	1.13E−07	6.65E−08	2.63E−08	2.64E−08	7.53E−09	8.15E−09	1.35E−08	6.82E−08
Skewness	1.9697482	1.4748926	1.4977302	0.9587959	2.1145431	1.1703115	2.0229257	1.5082751	2.1312375
Kurtosis	4.8680976	3.2153039	2.8715172	0.9111671	5.6591093	1.3978683	5.3101106	2.8617364	6.6137191

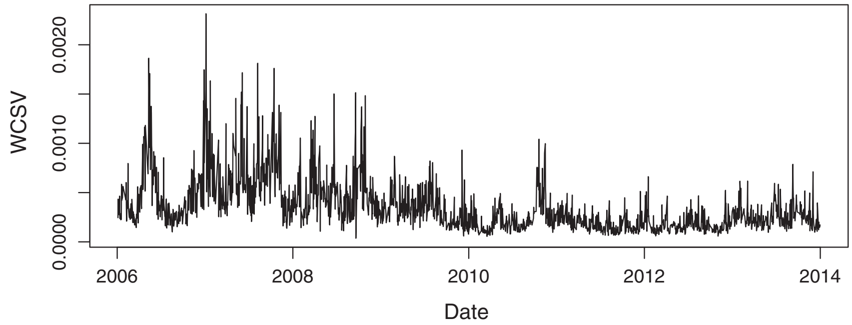

Fig. 2. Plot of WCSV time series. This figure describes the variation trend of the dependent variable WCSV in our model between 4 Jan. 2006 and 31 Dec. 2013 with 1941 effective sampling days. $WCSV_t$ is calculated as $\sum_i w_{it}(r_{it} - r_{mt})^2$, where weight w_{it} is the market value weight of stock i on day t; r_{it} is the return of stock i on day t; and r_{mt} is the market average return on day t.

5.1. Results of Chang, Cheng, & Khorana (2000) model

Firstly we carry out empirical test with CCK model on data of Year 2006–2013. The result is shown in Table 3(a). Adjusted R^2 for the model with an intercept is calculated as

$$\text{Adjusted } R^2 = 1 - \frac{(n-1)\left(1-R^2\right)}{n-p},$$

As shown in Table 3(a), CCK model concludes that there is a relatively higher level of herding in Chinese A-share market in Year 2008, in contrast with no significant herding existence in other years. By testing the whole dataset of Year 2006–2013, CCK model can still detect significant herding in the stock market.

In the previous section, we stated several theoretical defects of CCK model. Further, as can be seen from the empirical results, adjusted R^2 is too low, showing that the model is at most a trend detector rather than a good fit. However, this result can still provide useful information for our reference.

These results give rise to the following possibilities:

1. There exists systematic bias by using CSAD as an estimate for ECSAD, which can cause model dysfunction. Our theoretical derivation shows that if the underlying APT in WCSV model is properly chosen, such bias can vanish.

Table 3

OLS estimators of (a) CCK model (b) WCSV model based on Fama–French Three-Factor Model (c) WCSV model based on CAPM for 2006–2013 Chinese A-share market. This table lists parameter estimates and Adjusted R^2 of OLS regression under three model with Year 2006–2013 data analyzed yearly and integrally. t-Values are shown in brackets.

	(a) CCK model				(b) WCSV model (based on Fama–French Three-Factor Model)				(c) WCSV model (based on CAPM)	
Year	$\hat{\alpha}$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	Adjusted R^2	$\hat{\gamma}_m$	$\hat{\gamma}_S$	$\hat{\gamma}_H$	Adjusted R^2	$\hat{\gamma}_m$	Adjusted R^2
2006	0.01500***	0.09895*	0.57262	0.0837	0.30366***	1.07545***	1.27594***	0.6321	0.66358***	0.3242
	(28.73)	(1.69)	(0.47)		(5.68)	(7.45)	(8.39)		(10.80)	
2007	0.02067***	0.02678	0.17810	0.0085	0.11452***	0.80020***	1.27299***	0.5904	0.24137***	0.1341
	(31.60)	(0.52)	(0.25)		(4.10)	(9.48)	(9.58)		(6.20)	
2008	0.01655***	0.14343***	−1.25596**	0.0979	0.12595***	0.62683***	0.22242***	0.6288	0.23088***	0.3631
	(31.37)	(3.72)	(−2.32)		(7.47)	(9.79)	(6.55)		(11.91)	
2009	0.01448***	0.14356***	−1.32177	0.0896	0.17464***	0.52689***	1.25174***	0.5997	0.29397***	0.2702
	(30.94)	(3.02)	(−1.44)		(7.19)	(5.80)	(9.07)		(9.56)	
2010	0.01277***	0.03767	0.97801	0.0453	0.16496***	0.91722***	0.73487***	0.6934	0.33945***	0.2977
	(27.72)	(0.66)	(0.77)		(6.27)	(7.04)	(14.19)		(10.18)	
2011	0.01044***	0.09430*	1.54830	0.1321	0.20518***	1.25979***	0.94962***	0.6635	0.39687***	0.3930
	(32.96)	(1.84)	(0.22)		(7.22)	(6.07)	(11.46)		(12.61)	
2012	0.00990***	0.10751***	0.62037	0.2674	0.30290***	0.73023***	0.72014***	0.6809	0.49318***	0.4695
	(39.61)	(3.21)	(0.76)		(9.47)	(5.35)	(9.53)		(14.70)	
2013	0.01260***	0.02854	1.48810	0.0703	0.25280***	1.27524***	0.98656***	0.6589	0.43024***	0.2880
	(42.99)	(0.72)	(1.44)		(7.78)	(6.84)	(11.45)		(9.86)	
Overall	0.01294***	0.18572***	−0.90698***	0.1475	0.14800***	0.77581***	1.12858***	0.6007	0.27920***	0.2341
	(66.77)	(9.79)	(−2.81)		(16.67)	(24.67)	(26.49)		(24.38)	

* Significant at 10% level.
** Significant at 5% level.
*** Significant at 1% level.

2. During normal time periods, asset pricing does not satisfy CAPM in Chinese A-share market, showing the lack of explanatory power of CAPM. Our model is strengthened by applying a multi-factor APT, which can effectively improve model explanatory power.
3. Systematic herding always exists in Chinese A-share market. This result is reasonable, and our model further utilizes this characteristic to find strong herding points and strong influential points;
4. The time-variant pattern of individual stock beta is too strong to be ignored in Chinese A-share market. This feature prompts us to relax the constraint of constant beta to time-variant beta, and detect the existence of herding through the change of beta dispersion.

WCSV model modifies CCK model by mending its theory deficiency and therefore our idea can also provide reference value to other markets.

5.2. Results of WCSV model over Year 2006–2013

Firstly, we use ordinary least square regression for each years data respectively. The result is shown in Table 3(b). Adjusted R^2 for the no-intercept model is calculated as

$$\text{Adjusted } R^2 = 1 - \frac{n\left(1-R^2\right)}{n-p}.$$

Judging from Adjusted R^2, WCSV model significantly fits better than CCK model. Adjusted R^2 reaches an average of over 0.6 while the average is around 0.1 for CCK model. This may result from the superiority of our model design or the stronger explanatory power.

Next, we apply WCSV model to test strong herding points and strong influential points. Empirical results of testing strong herding points are shown in Fig. 3, with standardized deviance values (a.k.a. z-scores) Z_t of daily data shown in the chart. When conducting influential point test, we calculate all DFFITS statistics and plot them in Fig. 4. In Fig. 3, points below the lower dashed line are deemed as with herding in stronger sense; points between two dashed lines are deemed as with herding in weaker sense; and the other points, which constitute the majority, cannot be judged in terms of herding. In Fig. 4, points below the lower dashed line can be considered as strong influential points that cause the estimator of beta dispersion to decrease.

We can see from Fig. 3, most observed WCSV_t are far bigger than $\hat{y}_t$. This fact shows that for Chinese stock market, $\text{Var}_c(\beta^H_{imt})$ and $\text{Var}_c(\beta^H_{ikt})$ are indeed underestimated and high-leverage points have extremely high effect on the overall estimate.

Due to the huge number of data points, needles in the two figures are dense and the differences among time periods in terms of herding can be hardly shown. To solve this problem, we separate yearly data and count herding points in stronger sense, herding points in stronger and weaker sense and strong influential points for each year. The summary is in Table 4(a). From the table, we can easily spot strong abnormality pattern from observations during Year 2007–2010, while the frequency density is particularly high in Year 2008 and 2010. Strong influential points mainly occur in Year 2007 and 2008. From Fig. 4, we can find obviously high absolute values as well as high volatility of DFFITS statistics during Year 2007–2008, while the values are stable and approximately 0 both before and after that period. Such results suggest that herding over Year 2006–2013 mainly results from market abnormality during Year 2007–2008, which still remains even after considerably long time but does alleviate gradually.

Factual supports can also be found. The period of Year 2007–2008 was an unforgettable time for Chinese stock market, when it underwent tremendous ups and downs. From Fig. 1, we can find that the whole Chinese A-share market suffered from a high volatility of market index from Year 2007 to 2008. This trend can be explained by the substantial market stress during that period, which shows the model can fit real market to some degree.

The story began in Year 2006 when Shanghai Index opened at 1161.91 points at year start and closed at 2675.47 points at year end, multiplied more than twice. Year 2007 continued the crazy upward trend, pulling Shanghai Index to its historical peak of 6124.04 points. This marked the two-year stock index increase percentage as 427%, an uncommon ascent both

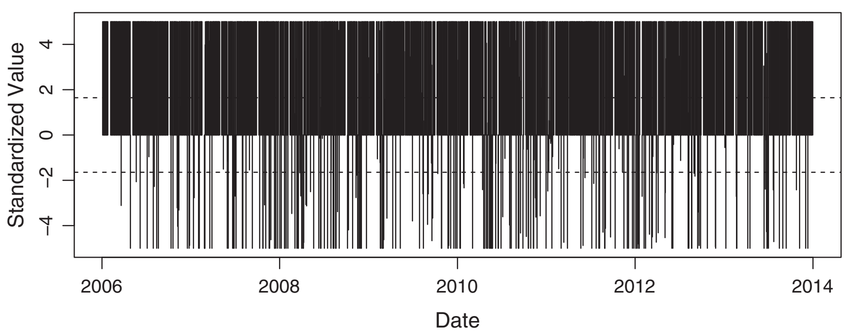

Fig. 3. Results of strong herding point test over data from 2006 to 2013. This figure depicts the z-score of every observation WCSV_t for estimator $\widehat{y}_t$ by WCSV model (with Fama–French Three-Factor Model as the underlying APT) with the integral data of Year 2006–2013. z-score is calculated as $Z_t = \frac{\text{WCSV}_t - \widehat{y}_t}{\sqrt{s^2 \mathbf{x}_t' (\mathbf{X}'\mathbf{X})^{-1} \mathbf{x}_t}}$. Critical values are approximately $\pm$ 1.64.

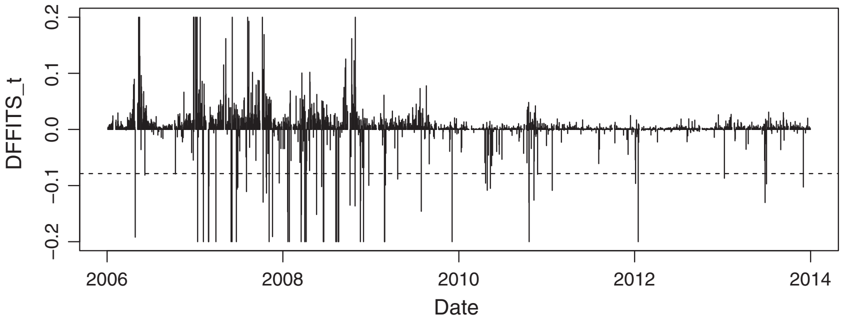

Fig. 4. Results of strong influential point test over data from 2006 to 2013. This figure depicts DFFITS statistics by WCSV model (with Fama–French Three-Factor Model as the underlying APT) for Year 2006–2013 to illustrate the effect on the predicted value of deleting an observation, which shows whether the observation contributes a high leverage. DFFITS_t is calculated as $\frac{\widehat{y}_t-\widehat{y}_{(t)}}{\sqrt{s^2_{(t)}\mathbf{x}'_t\left(\mathbf{X}'_{(t)}\mathbf{X}_{(t)}\right)^{-1}\mathbf{x}_t}}$. Critical values are approximately $-2\sqrt{3/n}\approx-0.079$.

domestically and globally. There were mainly two underlying reasons: constant strong capital support and favorable stock investment policy environment. China achieved a GDP increase rate of 16.97% in 2005 and 22.88% in 2006, among the highest of all recent years and such unprecedented economical growth contributed greatly to stock market. Also, an influential reform – the reform of the shareholder structure in listed companies – took place in China to strengthen the securities market and to correct the long-term distorted pricing mechanism. In addition, the government broadened the resources of stock market capital and institutional investors got developed, such as open-ended funds, insurance funds and QFIIs. By the end of Year 2007, the number of institutional investors rose to 4.7 million, 38% more than that of Year 2005. All these investors paid great effort to market-making and created the historical index peak.

After discussing the reasons for the upsurge of Chinese stock market in Year 2007, the plunge in Year 2008 can also be partly explained. The reform of the shareholder structure in listed companies, which intended to solve the long-lasted dual-class structure problem, finally began to realize the full marketability of restricted shares. However, at that time, no one retained the accepting ability or willingness to such bubbles. Besides, Year 2008 witnessed the worldwide financial crisis, which affected China both directly and indirectly. Under such circumstances, no doubt Shanghai Index fell to 1664 points by the year end.

5.3. Results of WCSV model applied to individual year data

As mentioned above, WCSV model uses estimators $\mathrm{Var}_c(\beta^H_{imt})$ and $\mathrm{Var}_c(\beta^H_{ikt})$ obtained from historical data as reference values of $\mathrm{Var}_c(\beta_{imt})$ and $\mathrm{Var}_c(\beta_{ikt})$ to construct a linear predictor to test herding of a single day. As demonstrated in Table 4(a), estimators from Chinese A-share market data show big yearly changes. Regarding this phenomenon, we think different time periods (different

Table 4
Summary of strong herding points and strong influential points in each year over data from 2006 to 2013. This table summarizes the strong herding points based on WCSV model (with Fama–French Three-Factor Model as the underlying APT) as well as the number of trading days (a) with integral data (b) with single-year data of Year 2006–2013.

		Only stronger		Stronger + weaker		Strongly influential	
Year	Total	Frequency	Proportion	Frequency	Proportion	Frequency	Proportion
(a) With integral data of Year 2006–2013							
2006	241	18	0.074689	24	0.099585	3	0.012448
2007	242	33	0.136364	39	0.161157	19	0.078512
2008	247	47	0.190283	54	0.218623	22	0.089069
2009	244	28	0.114754	40	0.163934	6	0.024590
2010	242	44	0.181818	53	0.219008	7	0.028926
2011	244	24	0.098361	37	0.151639	1	0.004098
2012	243	32	0.131687	39	0.160494	2	0.008230
2013	238	22	0.092437	27	0.113445	4	0.016807
Overall	1941	248	0.127769	313	0.161257	64	0.032973
(b) With single-year data in Year 2006–2013							
2006	241	27	0.112033	46	0.190871	8	0.033195
2007	242	26	0.107438	44	0.181818	9	0.037190
2008	247	31	0.125506	56	0.226721	8	0.032389
2009	244	23	0.094262	49	0.200820	9	0.036885
2010	242	23	0.095041	48	0.198347	9	0.037190
2011	244	27	0.110656	56	0.229508	11	0.045082
2012	243	25	0.102881	52	0.213992	10	0.041152
2013	238	22	0.092437	52	0.218487	9	0.037815

years in this paper) may feature different levels of systematic herding. Now we further our research by studying yearly differences of intra-year relative herding levels over Year 2006–2013.

Table 4(b) is a summary of strong herding points and strong influential points estimated from each individual year data. From this table, we can see that among years, frequencies of yearly strong herding points do not differ much. This implies similar yearly herding volatility on the basis of systematic herding. Even by comparing Year 2013, the relatively normal year, with the obviously abnormal and strong-herding Year 2008, we cannot find notable differences.

In terms of each individual year, a small and similar number of strongly influential points exist. This illustrates that no strong lack-of-fit phenomenon occurs for WCSV model during different short-to-medium-term time periods in Chinese stock market and yearly fitting degrees are close. That is to say, WCSV model is naturally appropriate for modeling Chinese stock market. This result also shows model robustness in short-to-medium terms, which can empirically support our hypothesis that the variation of cross-sectional variance of beta is small in short-to-medium terms.

Note that the herding frequencies we get from individual year data in this section are different from those in last section, where we study multi-year integral data. Frequencies of strong herding points differ mainly because the criteria of herding (yearly average and total average) are different. Frequencies of strong influential points differ mainly because the sensitivity of a single point decreases when the number of points increases. This phenomenon also demonstrates the strong leverage property of WCSV model, which means the detection of abnormal points must be based on the existence of large quantities of normal points. The best function of WCSV model is thus confirmed as inspecting the strongest herding period when long-term herding exists and fluctuates. From another perspective, herding is only a relative phenomenon without an absolute criterion.

5.4. Comparison of Fama–French Three-Factor Model and CAPM

In this part, we focus on the underlying APT model. Since we use Fama–French Three-Factor Model as the underlying APT model in our WCSV model, it is natural to check whether a simpler model can also work well. Univariate CAPM is implemented as the underlying APT of WCSV model and we use ordinary least square regression for each individual year data and overall data respectively. Results are shown in Table 3(c).

We can see a much lower Adjusted R^2 estimated from the CAPM-based model than the previous one. This also verifies better suitability of Fama–French Three-Factor Model than CAPM in WCSV model. By adding two factors, the model fits much better, showing that CAPM does lack explanatory power. CCK model is based on CAPM, which may be the main reason that it is lack of fit. Thus, it is reasonable to improve test accuracy by involving more factors in WCSV model.

Referring to the fact that the rational pricing model has obviously better fit since Year 2010 as shown in Fig. 4, we tend to believe that Fama–French Three-Factor Model is suitable for Chinese stock market, but the entire market situation was abnormal over Year 2007–2008 under some certain conditions such as herding.

At last, we test the multicollinearity of independent variables (squares of Fama–French three factors). We employ principal component analysis and list the result in Table 5. Without extremely small eigenvalues, the result can empirically support Fama–French Three-Factor Model to be a feasible choice to reflect Chinese stock market.

5.5. Analysis of covariance items based on Fama–French Three-Factor Model

In the methodology part, we employ Fama–French Three-Factor Model as the underlying APT model, which is

$$r_{it}-r_{ft}=\beta_{imt}\left(r_{mt}-r_{ft}\right)+\beta_{i1t}\mathrm{SMB}_t+\beta_{i2t}\mathrm{HML}_t.$$

WCSV model can be written as

$$\begin{aligned}\mathrm{WCSV}_t=&\ \mathrm{Var}_c(\beta_{imt})\left(r_{mt}-r_{ft}\right)^2+\mathrm{Var}_c(\beta_{iSt})\mathrm{SMB}_t^2+\mathrm{Var}_c(\beta_{iHt})\mathrm{HML}_t^2\\&+\mathrm{Cov}_c(\beta_{imt},\beta_{iSt})\left(r_{mt}-r_{ft}\right)\mathrm{SMB}_t+\mathrm{Cov}_c(\beta_{imt},\beta_{iHt})\left(r_{mt}-r_{ft}\right)\mathrm{HML}_t\\&+\mathrm{Cov}_c(\beta_{iSt},\beta_{iHt})\mathrm{SMB}_t\mathrm{HML}_t+\varepsilon_t.\end{aligned}\tag{14}$$

Table 5
Principal component analysis. The table shows the result of principal component analysis of independent variables. PC1–PC3 are three principal components of (**X'X**) with corresponding eigenvalues, eigenvalue proportions and orthonormal eigenvectors listed in this table. We recognize strong collinearity when the biggest eigenvalue exceeds the smallest eigenvalue by 100 times and severe collinearity when the biggest eigenvalue exceeds the smallest eigenvalue by 1000 times.

	PC1	PC2	PC3
Eigenvalue	1.27096378	0.97212845	0.75690778
Proportion	0.4237	0.3240	0.2523
$(r_m - r_f)^2$	0.551678	−0.634478	0.541376
SMB^2	0.687529	−0.021524	−0.725838
HML^2	0.472181	0.772641	0.424348

Table 6
OLS results of the overfitting model. This table lists parameter estimates and Adjusted R^2 of OLS regression under our WCSV model (with Fama–French Three-Factor Model as the underlying APT model) with covariance terms with Year 2006–2013 data analyzed yearly and integrally. *t*-Values are shown in brackets.

Year	$\hat{\gamma}_{mm}$	$\hat{\gamma}_{SS}$	$\hat{\gamma}_{HH}$	$\hat{\gamma}_{mS}$	$\hat{\gamma}_{mH}$	$\hat{\gamma}_{SH}$	Adjusted R^2
2006	0.42917***	1.03367***	1.33351***	−0.40118***	0.58785***	−0.15829	0.6571
	(7.19)	(7.05)	(8.64)	(−3.51)	(3.74)	(−0.80)	
2007	0.11840***	0.96158***	1.52730***	0.14727	−0.21560	−0.92982***	0.6259
	(4.14)	(10.90)	(11.14)	(1.48)	(−1.46)	(−4.94)	
2008	0.17658***	0.82354***	1.52646***	−0.33827***	−0.31387***	0.31756	0.6740
	(9.86)	(11.12)	(6.99)	(−5.30)	(−3.29)	(1.64)	
2009	0.21131***	0.80550***	1.74963***	−0.40233***	−0.07715	1.09380***	0.6522
	(8.32)	(8.16)	(10.81)	(−3.66)	(−0.73)	(5.05)	
2010	0.19385***	0.79737***	0.72467***	−0.15114*	−0.04179	−0.22036	0.6973
	(6.65)	(5.27)	(13.53)	(−1.92)	(−0.55)	(−1.42)	
2011	0.26876***	2.04927***	0.90730***	−0.70562***	0.01695	−0.04015	0.6968
	(8.92)	(8.11)	(10.59)	(−5.37)	(0.20)	(−0.16)	
2012	0.38159***	1.61474***	0.84998***	−0.62728***	0.02115	0.67254***	0.7138
	(9.87)	(7.23)	(9.44)	(−4.13)	(0.21)	(2.69)	
2013	0.37860***	1.84075***	0.98821***	−0.85284***	−0.22995**	0.81846***	0.7048
	(9.11)	(9.01)	(11.33)	(−5.24)	(−2.31)	(4.00)	
Overall	0.17349***	0.80872***	1.10973***	−0.19206***	−0.02715	−0.09917*	0.6095
	(18.02)	(25.63)	(26.22)	(−6.52)	(−0.71)	(−1.66)	

* Significant at 10% level.
** Significant at 5% level.
*** Significant at 1% level.

Our preliminary assumption is that all betas are uncorrelated and all covariance terms are 0. But in our empirical study, this assumption should be further investigated into. To determine the suitability of this assumption, we use OLS to estimate the following overfitting model.

$$\begin{aligned}\mathrm{WCSV}_t = {} & \gamma_{mm}\left(r_{mt}-r_{ft}\right)^2 + \gamma_{SS}\mathrm{SMB}_t^2 + \gamma_{HH}\mathrm{HML}_t^2 \\ & + \gamma_{mS}\left(r_{mt}-r_{ft}\right)\mathrm{SMB}_t + \gamma_{mH}\left(r_{mt}-r_{ft}\right)\mathrm{HML}_t + \gamma_{SH}\mathrm{SMB}_t\mathrm{HML}_t + \varepsilon_t.\end{aligned} \tag{15}$$

If the last three coefficients are insignificant compared to the first three ones, the original model can still be deemed as representative. Results are shown in Table 6.

We can find that the model fits a little better after adding cross terms. However, the cost of cross terms is too high. Compared with the original model, the new model has three more factors but only an insignificant increase of Adjusted R^2. Recall the modification from CAPM to Fama–French Three-Factor Model: by adding only two factors, a strong model improvement can be achieved. Deleting cross terms from WCSV model based on Fama–French Three-Factor Model is empirically suggested.

On the contrary, the advantage of ignoring cross terms is obvious. In our original model, all variables and coefficients are non-negative, while cross terms don't necessarily have such property. With non-negative variables and coefficients, we can compare cross-sectional variances simply by comparing the predictors. This property offers a convenient way to test herding.

6. Conclusion

In this paper, we discuss the theoretical defects and empirical problems of existing herding-measuring models. On this basis, we propose an improved method to detect herding in stock market. With this method, we introduce effective tests to find (1) which time points suffer from herding; (2) which time points pull up the average herding level. We also apply this method to real data of Chinese A-share market over Year 2006–2013.

Compared with existing models, the new method has several improved properties. Firstly, despite its strict requirements for APT model structure, WCSV model has objective and reasonable assumptions about market. If basic assumptions of our model are well satisfied, WCSV model does hold under rational pricing mechanism. On the contrary, existing methods cannot ensure it. Secondly, WCSV model effectively utilizes data and makes the result more reliable by integrating more market information into the method. In addition, while tested empirically, our model can show a detailed result of herding days. Herding days identified by WCSV model can be proved to be sufficiently strong compared with the overall average herding level during the whole research period.

Empirically, Fama–French Three-Factor Model, as the underlying APT of WCSV model, basically satisfies preliminary assumptions and presents good model fitting results when applied to Chinese A-share market data. It also features significant improvement compared to univariate CAPM. On the one hand, this verifies the necessity of improving model fitting by considering several factors; on the other hand, Fama–French Three-Factor Model shows great feasibility even under such strict requirements of APT models, so it is practical.

Our empirical test about Chinese A-share market data over Year 2006–2013 has several presentable results. Since Year 2007, higher frequency of strong herding points can be easily detected, which reaches the peak in Year 2008 and 2010 with a downtrend since then. Meanwhile, strong influential points mainly cluster in Year 2007–2008. In previous sections, we have briefly described the general situation during these years. We reach the following convincible conclusion: Huge market turmoil in Year 2007–2008 causes the model to be severely lack of fit and decreases market stability. The abnormal market fluctuation has long-lasting influences with a decaying trend. Meanwhile, for every individual year, relative intra-year herding does not differ much from each other. Therefore, we suggest that herding is a relative concept. With different benchmarks and overall sample sensitivities, different results may be obtained.

Overall, though WCSV model is not very complete, our new method does avoid some design defects of existing methods and reflect market characteristics better. WCSV model can improve accuracy and efficiency of herding detection.

Acknowledgment

The authors thank the editor Carl R. Chen and three referees for their invaluable help. All remaining errors are our responsibility.

References

Balcilar, M., Demirer, R., & Hammoudeh, S. (2013). Investor herds and regime-switching: Evidence from Gulf Arab stock markets. *Journal of International Financial Markets, Institutions and Money*, *23*, 295–321.

Belsley, D.A., Kuh, E., & Welsch, R.E. (1980). *Regression diagnostics: Identifying influential data and sources of collinearity*. John Wiley & Sons.

Chang, E.C., Cheng, J.W., & Khorana, A. (2000). An examination of herd behavior in equity markets: An international perspective. *Journal of Banking & Finance*, *24*, 1651–1679.

Chiang, T.C., & Zheng, D. (2010). An empirical analysis of herd behavior in global stock markets. *Journal of Banking & Finance*, *34*, 1911–1921.

Christie, W.G., & Huang, R.D. (1995). Following the pied piper: Do individual returns herd around the market? *Financial Analysts Journal*, *31–37*.

Demirer, R., & Kutan, A.M. (2006). Does herding exist in Chinese stock markets? *Journal of International Financial Markets, Institutions and Money*, *16*, 123–142.

Demirer, R., Kutan, A.M., & Chen, C.D. (2010). Do investors herd in emerging stock markets?: Evidence from the Taiwanese market. *Journal of Economic Behavior & Organization*, *76*, 283–295.

Demirer, R., Kutan, A.M., & Zhang, H. (2014). Do ADR investors herd?: Evidence from advanced and emerging markets. *International Review of Economics & Finance*, *30*, 138–148.

Fama, E.F., & French, K.R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, *33*, 3–56.

Gebka, B., & Wohar, M.E. (2013). International herding: Does it differ across sectors? *Journal of International Financial Markets, Institutions and Money*, *23*, 55–84.

Grinblatt, M., Titman, S., & Wermers, R. (1995). Momentum investment strategies, portfolio performance, and herding: A study of mutual fund behavior. *American Economic Review*, *85*, 1088–1105.

Hwang, S., & Salmon, M. (2001). *A new measure of herding and empirical evidence*. Warwick Business School, Financial Econometrics Research Centre.

Hwang, S., & Salmon, M. (2004). Market stress and herding. *Journal of Empirical Finance*, *11*, 585–616.

Lakonishok, J., Shleifer, A., & Vishny, R.W. (1992). The impact of institutional trading on stock prices. *Journal of Financial Economics*, *32*, 23–43.

Ross, S.A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, *13*, 341–360.

Tan, L., Chiang, T.C., Mason, J.R., & Nelling, E. (2008). Herding in Chinese stock markets: An examination of A and B shares. *Pacific-Basin Finance Journal*, *16*, 61–77.

Wermers, R. (1999). Mutual fund herding and the impact on stock prices. *Journal of Finance*, *54*, 581–622.

Yao, J., Ma, C., & He, W.P. (2014). Investor herding behaviour of Chinese stock market. *International Review of Economics & Finance*, *29*, 12–29.