Weak Theories and Parameter Instability: Using Flexible Least Squares to Take Time Varying Relationships Seriously

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A common assumption by time-series analysts is that the estimated coefficients remain fixed through time. Yet this strong assumption often has little grounds in substantive theory or empirical tests. If the true coefficients change over time, but are estimated with fixed-coefficient methods such as Ordinary Least Squares (OLS) or its many offshoots, then this can lead to significant information loss, as well as errors of inference. This article demonstrates a method, Flexible Least Squares (FLS), for exploring the relative stability of time-series coefficients. FLS is superior to other such methods in that it enables the analyst to diagnose the magnitude of coefficient variation and detect which particular coefficients are changing. FLS also provides an estimated vector of time-varying coefficients for exploratory or descriptive purposes. FLS properties are demonstrated through simulation analysis and an evaluation of the time-varying characteristics of explanations of presidential approval from 1978 to 1997.

As with many statistical methods used by political scientists, regression analysis was borrowed from the physical sciences where relationships are often invariant across both experimental observations and through time.[1] For example, if we drop an object from an altitude of 100 feet, we can easily calculate the velocity with which the object will hit the ground, and with great accuracy. Using regression analysis and multiple experiments, we can also discover the effect of wind speed, friction, and other factors on this velocity. We know with some degree of certainty that controlling for these other factors, the velocity will be the same across all similar experiments, next year, in ten years, or in a hundred years. Many such invariant relationships exist in the physical sciences (e.g., the rate of decay of a radioactive isotope, the relationship between pressure and temperature in contained gases, the rate of growth of biological populations, the change in crop yields as a function of fertilizer, etc.). However, one is hard pressed to find such invariant relationships in the science of politics.

Political science theories are weak in that they identify few such steadfast relationships, yet political scientists who use regression analysis and its many offshoots commonly assume parameter invariance when they restrict the coefficients to a single $K \times 1$ vector. Political scientists rarely give the parameter invariance assumption any serious consideration either through substantive theory or their choice of estimation methods. The purpose of this article is to demonstrate that this is a mistake and to illustrate a relatively new method for evaluating the parameter-invariance assumption. In particular we shall be concerned with the parameter-invariance assumption with respect to time.

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[1]Ordinary Least Squares regression was developed between 1795 and 1801 by the German physicist and mathematician Karl Friedrich Gauss. Sir Francis Galton, the renowned nineteenth-century British biologist, perhaps more than any other person was responsible for launching regression as a tool of statistical analysis.

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Flexible Least Squares (Kalaba and Tesfatsion 1989) provides a three-tiered method of exploring parameter variation. At the most general level, the analyst examines whether the OLS solution provides a good description of the observations by plotting the costs of assuming time-invariant parameters along a residual efficiency frontier. We shall define what constitutes a residual efficiency frontier more precisely below. Second, the analyst examines descriptive statistics for the time paths traced out by FLS time-varying coefficients. The standard deviation of the time-varying coefficients yields a measure of the magnitude of parameter variation. Additionally, the average of the FLS coefficients can be compared directly to the OLS solution to determine whether or not the OLS estimates give a good characterization of the data-generation process. Finally, the time paths traced out by the FLS estimates can be plotted and examined for evidence of systematic movement in the coefficients. Descriptive evidence from these coefficient time paths may also be useful for determining whether coefficient variation is consistent with substantive theory and provides detailed evidence about *how* the coefficients vary.

The next section defines FLS and provides a brief comparison with other methods. The subsequent three sections lay out the statistical rationale behind FLS, the tools for analysis, and how to obtain the time-varying coefficients. After that, examples are provided using simulation data and an analysis of the determinants of presidential approval from 1978 to 1997. Finally, we conclude with a discussion of the limitations of the method and implications for political science research.

Definition and Comparison of Flexible Least Squares

FLS is a multi-criterion estimator that seeks to discover the *particular coefficient vector that obtained at each time t, considering all time T.* In contrast, OLS seeks to find an *average coefficient vector for all time t, without taking into account possible coefficient variation.* Like OLS, FLS minimizes an objective function, but unlike OLS, the FLS objective function considers two different types of specification error—residual-measurement error due to specifying an incomplete set of independent variables, and residual-dynamic error due to possible coefficient variation for the included variables.[2] In minimizing the multi-criterion objective function, FLS is more flexible in that it allows temporal variation in the coefficients. OLS is just a special case of FLS in that a restriction is imposed that fixes the potentially time-varying coefficients to constant values. Absent true coefficient variation, the two methods yield identical results, but if the restriction is invalid then the two methods can yield different results.

Of course, the coefficient variation problem has attracted considerable attention from methodologists across various disciplines for many years. Early work included efforts to estimate the location of a shift in regression-parameters from one regime to another (Quandt 1958) and to test the null hypothesis that such a shift had occurred (Chow 1960). The familiar Chow test assumes a discrete shift in parameters and that the analyst knows the precise timing of the shift. It also assumes that parameters are fixed in the different subperiods. Of course, a discrete shift in parameters is only one type of parameter instability, and the analyst may not know the precise timing of a shift even after it has occurred.

Later work relied on recursive residuals to provide a test of parameter stability using the CUSUM and CUSUMSQ plots developed by Brown, Durbin, and Evans (1975) and formalized in a test statistic by Harvey and Collier (1977).[3] The standard version of the CUSUM test is available in some statistical packages, but is rarely reported by political scientists. Greene (2000, 294) criticizes this test as having low statistical power. These tests also reveal nothing about which particular coefficients vary, how much they vary, or whether variation is systematic. Rather, they are tests of *global* coefficient stability that apply to entire regression specifications.

Still other work has examined the application of the Kalman filter to linear regression models with various types of parameter variation assumed for the coefficients (Kalman 1960; Kalman and Bucy 1961; Beck 1983; Engle and Watson 1985). Typically, the analyst using the Kalman filter assumes a particular stochastic structure for the time-evolving coefficients and that the disturbances follow a specific distribution. The most commonly assumed stochastic structure for the time-evolving coefficients is first-order autoregressive, with the disturbances assumed to follow a normal distribution. The problem is that we are seldom in a position to know beforehand the stochas-

[2] The terms "measurement error" and "dynamic error" come from the literature on state-space modeling. See Newbold and Bos (1985) for a simple introduction; see Harvey (1990a) for a more comprehensive treatment.

[3] Greene (2000, 293–297) provides a description and example of application of the CUSUM and CUSUMSQ tests. The test is computed as an option in many statistical packages, including LIMDEP and SHAZAM. The RATS econometric package also provides an example procedure called "RECRESID.SRC" which computes recursive residuals for these tests and other purposes. The two tests are based on the cumulative sum of recursive residuals and squared sum of recursive residuals. Recursive residuals are residuals from the recursive estimates described in note 4 below.

tic process that moves the coefficients and may have little confidence that the disturbances are normal.

In contrast to these techniques, FLS can be applied where the expected coefficient instability is sudden or evolving slowly through time. It provides a global test for coefficient stability, evidence concerning which coefficients vary, how much they vary, and whether those variations follow some systematic pattern. FLS also requires no *ad hoc* prior assumptions about the structure driving the coefficient variation or about the distribution of disturbances from the estimated model. And, unlike methods relying on recursive estimates (Ploberger, Kramer, and Kontrus 1989; Hendry and Ericsson 1991), FLS uses true time-varying estimation and the full data set in computing the time paths of the coefficients.[4]

The Rationale of Flexible Least Squares

The general approach taken by FLS in exploring coefficient-time variation is to investigate the relative costs for fixed versus time-varying coefficient assumptions. These costs are gauged in terms of residual measurement and dynamic error. More precisely, suppose one's prior theoretical beliefs are of the following form.

Measurement Specification:

$$y_t - \mathbf{x}_t^{'}\boldsymbol{\beta}_t \approx 0,\ \mathrm{t} = 1,\ ..,\ \mathrm{T} \tag{1}$$

Dynamic Specification:

$$\boldsymbol{\beta}_{t+1} - \boldsymbol{\beta}_t \approx 0,\ \mathrm{t} = 1,..,\ \mathrm{T}-1 \tag{2}$$

where:

$$\mathbf{x}_t^{'} = (x_{t1},\ldots,x_{tk})$$
$$= 1 \times \mathrm{K} \text{ row vector of regressors}$$
$$\boldsymbol{\beta}_\mathrm{t} = (\beta_{t1},\ldots,\beta_{tk})'$$
$$= \mathrm{K} \times 1 \text{ column vector of unknown coefficients}$$

These prior beliefs posit simply that (1) the model is linear and well specified, and (2) the degree of coefficient variation through time is very small. These, of course, are the assumptions made when choosing the fixed-coefficient specification.

Now, in truth, there are a very large number of possible coefficient sequences that could emerge for each variable. Some of these sequences posit a relatively fixed $\boldsymbol{\beta}_t$ (the OLS solution); others posit a time-varying $\boldsymbol{\beta}_t$. For any one of these sequences, $\boldsymbol{\beta}_t$ could fail to satisfy (1) the prior measurement specification, or (2) the prior dynamic specification. More directly, we could fail to minimize prediction error due to improper model specification, or we could fail to minimize prediction error because there is parameter variation. If we assign a cost to each of these two different types of specification error, then it is possible to map the cost of violating either prior, given a particular set of observations, along a residual efficiency frontier (REF).

This mapping along the REF is similar to an evaluation by consumers of the resource maximizing choice between two commodities along a Pareto efficiency frontier. The researcher may choose to allow no parameter variation, as is usually done in social science research. The researcher may also allow different amounts of parameter variation as determined by a smoothing parameter to be discussed below. In this sense, FLS tells the user what are the feasible efficient trade-offs between dynamic and measurement-specification errors, efficient in the sense that there is no way to obtain smaller dynamic errors without an increase in measurement-specification error, and vice versa. However, the optimal choice can only be made on the basis of a researcher's utility for different amounts of parameter variation. FLS makes explicit the costs of fixed versus time-varying assumptions in terms of residual-measurement error. As such, it provides the tools required for analysts to make reasonable choices between the two options.

Now, let us define the costs associated with the two types of specification error more precisely. The cost of violating the prior measurement specification is

$$r_M^2(\boldsymbol{\beta},T) = \sum_{t=1}^{T}(y_t - \mathbf{x}_t^{'}\boldsymbol{\beta}_t)^2. \tag{3}$$

The cost of violating the prior dynamic specification is

$$r_D^2(\boldsymbol{\beta},T) = \sum_{t=1}^{T-1}(\beta_{t+1} - \boldsymbol{\beta}_t)'(\boldsymbol{\beta}_{t+1} - \boldsymbol{\beta}_t). \tag{4}$$

Then the cost function $\mathrm{C}(\boldsymbol{\beta};\delta,\mathrm{T})$ that attains the REF for all possible choices is formed by taking the weighted sum of these two types of specification error as follows.

[4]See Harvey (1990b, 52–56) for a discussion of recursive least squares. Recursive coefficient estimates are just OLS estimates based on the first n observations for n = k…N. For forward recursions, the first k observations are always omitted so that estimation is possible. Additionally, each successive estimate is based on a sequentially larger sample so that the k+1st estimate is based on k+1 observations and the N$_{th}$ estimate is based on N-k observations. In practice, multicollinearity often results in the start point for the recursions occurring much later than the k+1st observation. In either case, none of the N-k coefficient estimates is based on the full sample.

$$C(\boldsymbol{\beta};\ \delta,T) = \frac{\delta}{1-\delta}\sum_{t=1}^{T-1}(\boldsymbol{\beta}_{t+1}-\boldsymbol{\beta}_t)'\,\mathbf{D}\,(\boldsymbol{\beta}_{t+1}-\boldsymbol{\beta}_t)+\sum_{t=1}^{T}(y_t-\mathbf{x}_t'\boldsymbol{\beta}_t)^2$$
$$0<\delta<1 \tag{5}$$

The FLS solution is the set of all coefficient sequence estimates, $\boldsymbol{\beta}_t$, that minimize the cost of *both* residual measurement and dynamic error in Equation 5. We minimize $C(\boldsymbol{\beta};\delta,T)$ with respect to $\boldsymbol{\beta}$ in order to obtain the sequence of FLS estimates. **D** is a suitably chosen scaling matrix that makes the cost function essentially invariant to the choice of units for the regressor variables. **D** is best specified as the diagonal matrix whose i^{th} diagonal term d_{ii} consists of the averaged sum of squared values for the i^{th} regressor variable over times t = 1…T $\left(\text{i.e., } d_{ii}=\left[\left(x_{1i}^2+\ldots+x_{ti}^2\right)\right],\ i=1\ldots k\right)$. The term containing δ imposes a weight on the prior dynamic errors that facilitate evaluating the relative cost of each type of specification error along a residual-efficiency frontier.

The term δ is a *smoothness parameter* that forces the potential time-varying vector, $\boldsymbol{\beta}_t$, toward or away from the fixed-coefficient OLS solution along the REF. With δ set very near 1, near total weight is given to minimizing $r_D^2(\boldsymbol{\beta},T)$ in Equations 4 and 5 with respect to $\boldsymbol{\beta}$. Of course, the minimum for $r_D^2(\boldsymbol{\beta},T)$ occurs where there is no coefficient variation. This is the "smoothest" solution, resulting in coefficients near the OLS estimate. As we move δ away from 1, greater priority is given to minimizing $r_M^2(\boldsymbol{\beta},T)$ in Equations 3 and 5 with respect to $\boldsymbol{\beta}$. Movement away from $\delta = 1$ allows coefficient variation in the minimization of $r_M^2(\boldsymbol{\beta},T)$, but only if the coefficient vector is time varying.

As should be obvious from the preceding discussion, there is a relationship between OLS and FLS estimates.[5] As δ moves to 1, then the FLS estimate converges to the fixed-coefficient OLS solution. In other words, the OLS solution lies on one end of the REF, so it is just a limiting case of FLS in which absolute priority is given to the dynamic prior over the measurement prior. As noted above, OLS is a simplification of FLS that restricts the coefficient vector to constancy. As a result of this simplification, the researcher using OLS may pay a considerable cost in terms of residual-measurement error in order to adopt the fixed-coefficient solution. Furthermore, the OLS solution is a weighted average of FLS estimates, but the particular weighting scheme may result in the average FLS estimate not being equal to the fixed-coefficient OLS estimate. This means that OLS estimates can be wrong due to time variation in the true coefficient vector.

[5] There is also a relationship between FLS and a time-varying Kalman filter representation. Lutkepohl (1993) shows that under certain assumptions FLS can be considered as a special Kalman filter.

The Tools of Flexible Least Squares

This section explains how the analyst uses FLS to evaluate global coefficient stability, determine which coefficients vary and by how much, and explore the patterns of coefficient variation.

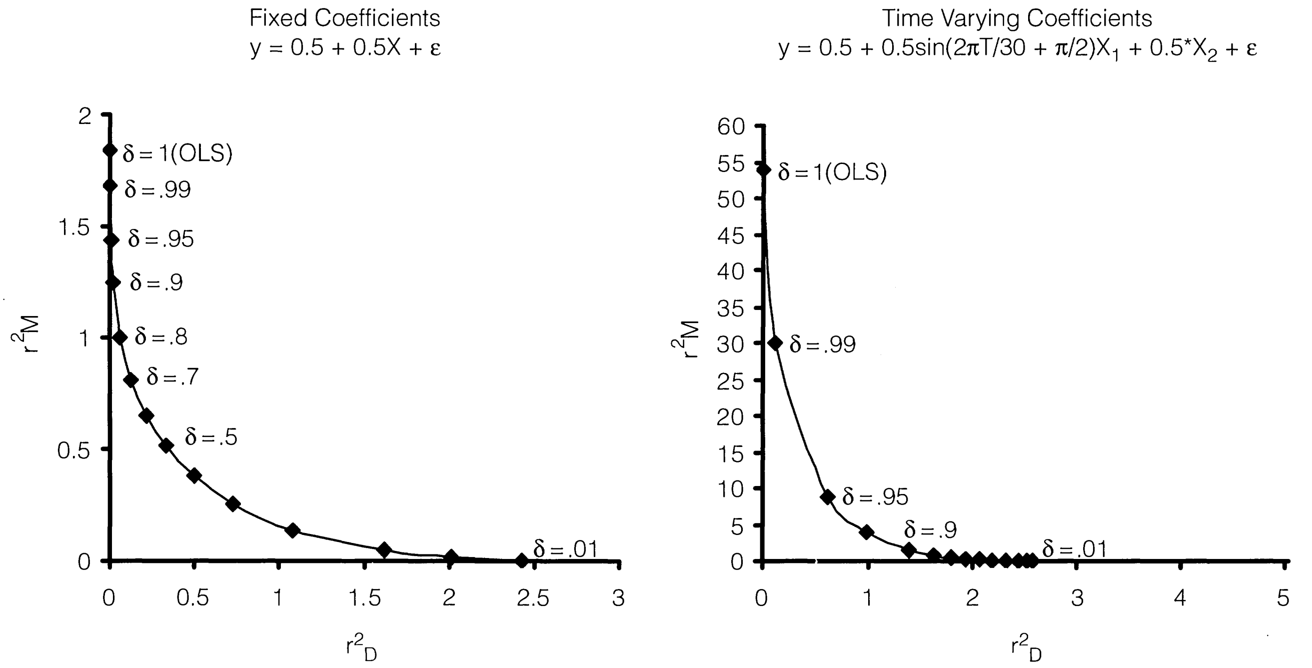
Evaluating Global Coefficient Stability

In evaluating global coefficient stability, the analyst traces out a residual efficiency frontier (REF) by changing the value of δ in the cost function across the range from 1 to 0. For purposes of discussion Figure 1 provides a schematic illustration of the REFs for fixed and time-varying coefficient models using simulated data. Residual-measurement error is on the vertical axis, while residual-dynamic error is on the horizontal axis. The downward sloping curve is the set of all pair combinations of r_M^2, r_D^2 which attain the residual efficiency frontier conditional on values of δ from 1 to 0.

Suppose we set δ near 1 and obtain the corresponding FLS coefficient-sequence estimate. Then the cost function, $C(\boldsymbol{\beta};\delta,T)$, places most of the weight on the dynamic-specification errors, forcing r_D^2 to be near zero. The left endpoint of the frontier gives the minimum possible value of r_M^2 subject to $r_D^2 = 0$. Hence, this endpoint reveals the cost in terms of residual-measurement error that must be paid for the analyst to choose the fixed-coefficient solution. This is called the OLS extreme point. At this point the FLS coefficient sequence is very near the fixed OLS estimate. Now suppose we set δ near 0 and obtain the corresponding FLS coefficient-sequence estimate. The cost function, $C(\boldsymbol{\beta};\delta,T)$, places most of the weight on the measurement-specification errors, forcing r_M^2 to be near zero. The right endpoint of the frontier gives the minimum possible value of r_D^2 subject to $r_M^2 = 0$. Hence, this endpoint reveals the minimum amount of time variation in the coefficients that must be allowed in order to have no residual-measurement error (i.e., a perfect fit for the regression).

Suppose now that for a given time-series experiment we hypothesize that the specifications in (1) and (2) are true. If the model truly has time-invariant coefficients, then starting from the OLS extreme point, the REF will indicate (as we move δ toward zero) only small decreases in measurement error are possible for large increases in

FIGURE 1 Residual Efficiency Frontier, Simulated Data



dynamic error. Thus, the REF should decline slowly with changing δ. In the left side of Figure 1, note that only small changes in residual-measurement error occurred as a result of allowing parameter variation by moving δ from 1 to 0.99. In contrast, if the true model has time-varying coefficients, then starting from the OLS extreme point large decreases in measurement error will be possible for allowing small increases in parameter variation. The REF should slope downward more steeply from the OLS extreme point. For example, note in the right side of Figure 1 that a change of δ from 1 to 0.99 produced a large decline in residual-measurement error. The REF will always have some downward slope due to the presence of ε in the measurement specification. The global test of whether coefficients are time varying is whether the decline in the REF is steeper than might occur for a specification containing only random disturbances impinging on the dependent variable.

Evaluating Which Coefficients Vary and By How Much

A second level of analysis is to observe the standard deviations and averages of the estimated sequence of time-varying coefficients, **β**, at different values of δ. The reason for doing so is to gather evidence concerning *which* particular coefficients exhibit the most time variation. The standard deviation of a time-varying coefficient sequence provides direct evidence concerning the existence and magnitude of the time variation. Because δ is a smoothing coefficient, the standard deviation of time-varying coefficients may change substantially when moving δ away from the OLS extreme point. However, caution is required in interpreting the standard deviations, since coefficients may show variability simply because a variable does not belong in the model and is drifting around near zero.

The average of a time-varying coefficient sequence can be compared directly with the OLS coefficient to check for large differences over the range of the REF. The OLS estimates may or may not provide a good summary of the time-varying coefficients, depending on the nature of the time variation and the value of δ. The average of a time-varying coefficient sequence can change substantially the further one moves δ away from 1. The average for a fixed-coefficient sequence should remain the same for all values of δ.

Evaluating the Pattern of Coefficient Variation

A third level of analysis is to plot the actual coefficient-vector sequences to observe the nature of the time variation. FLS generates an estimated time path for each regression coefficient, conditional on δ. The value of δ should be chosen so as not to arbitrarily restrict the coefficients to constancy. Typically, there is a threshold $\delta < 1$, below which the means, standard deviations, and residual-measurement errors change very little. Below this threshold, the choice of δ is arbitrary, since the qualitative patterns exhibited by the FLS time paths occur at all points along the REF, and the scale of the variations remains similar below the threshold.

Plots of the time-varying coefficients can be used to evaluate whether coefficient-time variation is consistent with substantive theory positing some breakpoint or a gradually changing process. Alternatively, the analyst may suspect that a breakpoint or gradual movement occurred, but the timing is unknown. In this case the plots can be used simply for exploratory purposes to determine whether the substantive model missed some unmodeled change that distorts the fixed-coefficient results.

Obtaining Flexible Least-Squares Estimates

In practice, how does one obtain the FLS coefficients for the three-tiered analysis described above? This section gives explicit procedures for choosing $\boldsymbol{\beta}$ that minimizes the FLS cost function in Equation 5.

First, define the following scalar, vectors, and matrices.

$\mu = \dfrac{\delta}{1-\delta} =$ scalar weight formed from the smoothing parameter δ.

$\boldsymbol{\beta} = (\boldsymbol{\beta}_1^{'},\ldots,\boldsymbol{\beta}_t^{'})' = TK \times 1$ column vector of coefficients.

$\mathbf{y} = (y_1,\ldots,y_t)' = T \times 1$ column vector of observations on y.

$\mathbf{X}' = (\mathbf{x}_1,\ldots,\mathbf{x}_t) = K \times T$ matrix of regressors.

$\mathbf{I} = K \times K$ identity matrix.

From these create the following design matrices.

$$\mathbf{G} = \begin{bmatrix} \mathbf{x}_1 & 0 & . & . & 0 \\ 0 & \mathbf{x}_2 & 0 & . & . \\ . & 0 & . & . & . \\ . & . & . & . & 0 \\ 0 & . & . & 0 & \mathbf{x}_t \end{bmatrix}$$

$$\mathbf{A}_t = \begin{cases} \mathbf{x}_1\mathbf{x}_1^{'} + \mu\mathbf{I} & \text{if } t = 1 \\ \mathbf{x}_t\mathbf{x}_t^{'} + 2\mu\mathbf{I} & \text{if } t \neq 1, T \\ \mathbf{x}_T\mathbf{x}_T^{'} + \mu\mathbf{I} & \text{if } t = T \end{cases}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & -\mu\mathbf{I} & \mathbf{0} & . & . & 0 \\ -\mu\mathbf{I} & \mathbf{A}_2 & -\mu\mathbf{I} & . & . & . \\ 0 & -\mu\mathbf{I} & . & . & . & . \\ . & . & . & . & . & 0 \\ . & . & . & . & . & -\mu\mathbf{I} \\ 0 & . & . & 0 & -\mu\mathbf{I} & \mathbf{A}_T \end{bmatrix}$$

The cost function in Equation 5 can then be expressed in matrix form as follows.

$$C(\boldsymbol{\beta};\delta;T) = \boldsymbol{\beta}'\mathbf{A}\boldsymbol{\beta} - 2\boldsymbol{\beta}'\mathbf{G}\mathbf{y} + \mathbf{y}'\mathbf{y} \qquad (6)$$

The first order necessary condition for minimization of the cost function, $C(\boldsymbol{\beta};\delta;T)$, with respect to the vector $\boldsymbol{\beta}$ in Equation 6 is as follows.

$$\mathbf{A}\boldsymbol{\beta} - \mathbf{G}\mathbf{y} = 0 \qquad (7)$$

The matrix $\mathbf{A}$ is positive definite for every $\mu > 0$ when the $T \times K$ regressor matrix $\mathbf{X}$ is of full rank. Therefore, the cost function is strictly convex and a function of $\boldsymbol{\beta}$. The minimum for Equation 6 is, therefore, the $TK \times 1$ column vector given by the following.

$$\boldsymbol{\beta} = \mathbf{A}^{-1}\mathbf{G}\mathbf{y} \qquad (8)$$

In order to obtain the FLS solution in Equation 8 it is necessary to invert the matrix $\mathbf{A}$, which has dimensions $TK \times TK$. This may be accomplished directly, but requires a lot of computational power given the large dimension of the matrix.

In lieu of direct inversion, a lower dimensional sequential updating procedure can be used. The sequential updating procedure takes the $T \times K$ dimensional problem of minimizing $C(\boldsymbol{\beta};\delta,T)$ with respect to $\boldsymbol{\beta}$ and decomposes it into a sequence of T linear-quadratic cost-minimization problems, each of dimension K. The first period estimate that is used to initialize the sequential updating procedure is obtained

$$\boldsymbol{\beta}_1 = \left[\mathbf{Q}_0 + \mathbf{x}_1\mathbf{x}_1^{'}\right]^{-1}\left[\mathbf{p}_0 + \mathbf{x}_1\mathbf{y}_1\right], \qquad (9)$$

where

$$\mathbf{Q}_0 = 0_{K\times K}$$
$$\mathbf{p}_0 = 0_{K\times 1}\,.$$

Having obtained the initialization coefficients, the procedure relies on a recurrence relation to obtain successive coefficient estimates while continuously updating prior coefficient estimates. The FLS coefficient for time t is given

$$\boldsymbol{\beta}_t = \left[\mathbf{Q}_{t-1} + \mathbf{x}_t\mathbf{x}_t'\right]^{-1}\left[\mathbf{p}_{t-1} + \mathbf{x}_t\mathbf{y}_t\right], \qquad (10)$$

where

$$\mathbf{Q}_t = \mu\left[\mathbf{Q}_{t-1} + \mu\mathbf{I} + \mathbf{x}_t\mathbf{x}_t'\right]^{-1}\left[\mathbf{Q}_{t-1} + \mathbf{x}_t\mathbf{x}_t'\right]$$
$$\mathbf{p}_t = \mu\left[\mathbf{Q}_{t-1} + \mu\mathbf{I} + \mathbf{x}_t\mathbf{x}_t'\right]^{-1}\left[\mathbf{p}_{t-1} + \mathbf{x}_t\mathbf{y}_t\right].$$

There is a FORTRAN program in Kalaba and Tesfatsion 1989 (see also 1990) that can be used to implement this sequential-updating procedure. The code is also available online at Tesfatsion's website at http://www.econ.iastate.edu/tesfatsi/vita.htm. The procedure can be run as such or incorporated into software packages such as S-plus that recognize FORTRAN language.

More conveniently, there are currently two software packages that contain FLS as a "canned" procedure. Version 8.0 of SHAZAM has a procedure to compute and plot FLS estimates and descriptive statistics; it also allows one with some additional effort to plot the REF. The most recent release of GAUSS's TSM 1.2 also provides FLS and GFLS, including estimates, descriptive statistics, and automated plots of the REF.[6] Given the usefulness of this approach, there is little question that other statistical packages should be incorporating FLS in the near future.

Example Analyses

In this section we apply FLS to some simulated data, as well as to actual empirical data on the determinants of presidential approval from 1978 to 1997. The purpose is to demonstrate to readers how to implement the method in detecting and exploring time-varying coefficients.

A Simulation Analysis

Simulated time-series data were generated with the following model.

$$y_t = 0.5 + \boldsymbol{\beta}_t' X_1 + 0.5X_2 + \varepsilon$$
$$\boldsymbol{\beta}_t = 0.5\sin\left(\frac{2\pi\, T}{30} + \frac{\pi}{2}\right) \qquad (11)$$

Notice that we have two fixed coefficients and one time-varying coefficient generating these data. The independent variables X_1 and X_2 are centered random-normal numbers with standard deviation set to unity. The residual is centered random-normal numbers with standard deviation set to 0.1. OLS estimates for one realization of the relationship between y_t, X_1, and X_2 are reported in Table 1. The two fixed coefficients are strongly significant, as they should be with such simulated data. However, notice that the time-varying relationship is not shown by the OLS estimate. The fixed coefficient for X_1 is small and nonsignificant, even though there is a strong systematic relationship between X_1 and y_t. Also, the regression diagnostics suggest some weak high-order autocorrelation in the residuals, undoubtedly due to the time-varying coefficient for X_1.

Now we shall apply the tools of FLS to explore the existence and nature of the coefficient-time variation. We first compute FLS estimates across a range of δ and plot the REF that is shown in the right side of Figure 1. This graph plots the squared error from the measurement equation (1) against the squared error from the dynamic equation (2) in the cost function for distinct values of δ ranging from 1 to 0. Beginning from the OLS extreme point in the upper left corner, the squared residual-measurement error declines rapidly to near zero. At δ = 0.9, virtually all of the squared error from the measurement equation disappears. Thus, allowing even small amounts of coefficient-time variation produces large reductions in explained variance from the measurement equation. This is evidence for global-coefficient instability, but tells nothing about which particular coefficient is changing.

TABLE 1 OLS Estimates for Simulated Data with One Time-Varying Parameter Changing in a Sinusoidal Pattern

Independent Variable	Coefficient
X_1	0.01
	(0.07)
X_2	0.39
	(6.07)
Intercept	0.48
	(6.51)
R^2	0.28
σ	0.73
Durbin-Watson d	2.35
Box-Ljung Q $\chi^2(24)$	43.65
N	100

Note: The numbers in parentheses are t-statistics.

[6]GFLS is the acronym for Generalized Flexible Least Squares. GFLS is a generalization of FLS that enables a more flexible specification than assumptions of a random-walk coefficient progression. GFLS can be viewed as an alternative to the Kalman filter for state-space models. See Kalaba and Tesfatsion (1990, 1996) for discussion, as well as citations to recent applications of FLS and GFLS. TSM 1.2 for GAUSS is a Microsoft WINDOWS-based program. For users who do not use Microsoft WINDOWS, the author can provide GAUSS code for DOS or UNIX-based versions for FLS and GFLS.

Table 2 Flexible Least Squares Summary Statistics for Simulated Data over the Residual Efficiency Frontier

δ	X_1	X_2	Intercept
.999	0.02 (0.04)	0.39 (0.03)	0.48 (0.01)
0.99	0.02 (0.17)	0.40 (0.07)	0.47 (0.03)
0.95	0.01 (0.37)	0.42 (0.08)	0.48 (0.04)
0.90	0.01 (0.45)	0.44 (0.08)	0.48 (0.05)
0.80	0.01 (0.52)	0.45 (0.08)	0.49 (0.05)
0.70	0.01 (0.56)	0.45 (0.07)	0.49 (0.06)
0.60	0.02 (0.58)	0.46 (0.07)	0.49 (0.06)
0.50	0.02 (0.60)	0.46 (0.07)	0.49 (0.06)
0.40	0.02 (0.61)	0.46 (0.07)	0.49 (0.06)
0.30	0.02 (0.62)	0.46 (0.07)	0.49 (0.06)
0.20	0.02 (0.63)	0.47 (0.07)	0.50 (0.06)
0.10	0.02 (0.63)	0.47 (0.07)	0.50 (0.06)
0.05	0.02 (0.64)	0.47 (0.07)	0.50 (0.06)
0.01	0.02 (0.64)	0.47 (0.07)	0.50 (0.06)

Note: The numbers in the table are time-varying coefficient averages at each specified δ. The numbers in parentheses are time-varying coefficient standard deviations at each specified δ.

Table 2 gives the means and standard deviations of the FLS coefficient sequences for values of δ along the REF. Note first that the FLS estimates for $\delta = 0.999$ (near the OLS extreme point of 1.0) are very close to the fixed-coefficient estimates in Table 1. This will *always* be the case, though there may be variations in how close δ must approach 1 to arrive at the OLS extreme point. Evidence concerning variation in specific coefficients comes from change in the means and standard deviations as we move away from the OLS extreme point. The coefficient means will vary if the OLS weighting scheme produces a bias; the coefficient standard deviations will increase monotonically if there is coefficient variation.

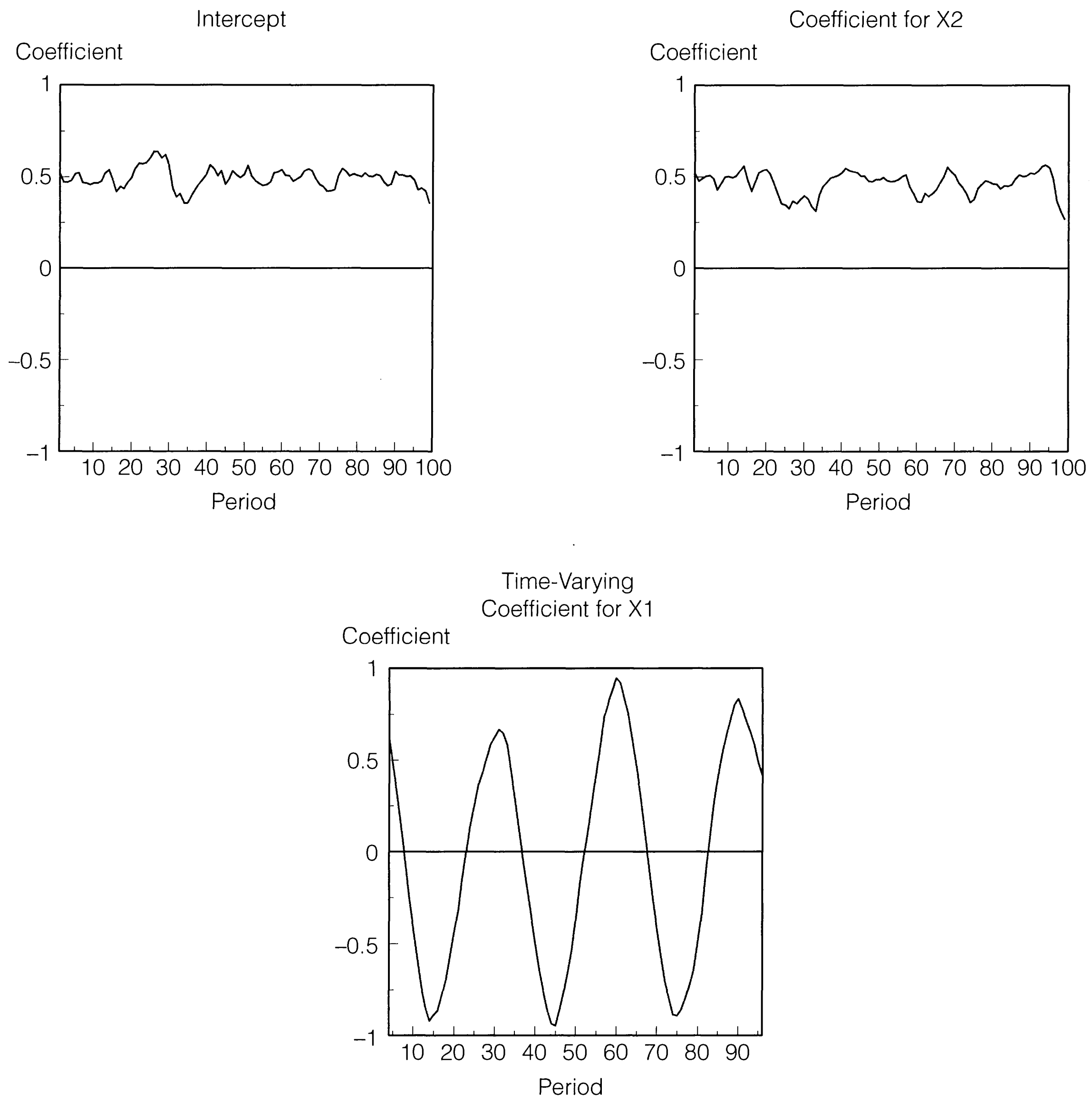
In this simulation, the average for the intercept term changes by only about 4 percent as we move away from the OLS extreme to the other end of the REF. The average for the fixed coefficient for X_2 changes by about 20 percent, which is within a reasonable range given the error in Equation 6. The average for the time-varying coefficient for X_1 shows no change at all. The absence of change in the mean for X_1 shows that the OLS weighting scheme produces no systematic bias even though the true coefficient varies through time. This is because the sinusoidal coefficient in Equation 6 is centered on zero, and the variations average out.

More evidence on time variation is in the standard deviations for these coefficients. As we move away from the OLS extreme point, the standard deviations for the intercept and fixed coefficient of X_2 remain quite small across the entire range of the REF. This implies no time variation in these two coefficients. However, the standard deviation for the time-varying coefficient for X_1 moves very quickly to large values away from the OLS extreme point. At the extreme right end of the REF, the coefficient standard deviation for X_1 is thirty-two times larger than at $\delta = 0.999$! This is an extreme example and very strong evidence for time variation in the coefficient for X_1. Still, the nature of the time variation in X_1 remains obscure due to aggregation of the coefficient sequence into summary statistics.

Thus, it is also useful to plot the coefficient sequences at some particular value of δ. As long as we are off the OLS extreme point, the particular value of δ is unimportant since only the scale of the coefficient variation changes as we move away from this point. The standard deviations change little for $\delta < 0.5$, so we arbitrarily set δ at 0.5. Figure 2 contains the coefficient sequences for the intercept, the fixed coefficient for X_2, and the time-varying coefficient for X_1. The jagged appearance of the fixed coefficients in the upper part of the Figure 2 reflects the error process for Equation 11. The fixed coefficients are near their respective true means as indicated in the preceding summary statistics, with standard deviations about the same as the simulated disturbances. The graph for the time-varying coefficient for X_1 reveals the true sinusoidal change occurring with respect to time, also showing the greater relative variation.

Comparing the results from this three-tiered analysis to the OLS results in Table 1 suggests why the coefficient for X_1 was not significantly different than zero, as well as the inadequacy of an analysis that ignores possible variation in coefficients through time. The true average coefficient for X_1 was indeed near zero, implying that there was no systematic relationship between X_1 and the dependent variable. However, FLS allowed us to uncover the true

FIGURE 2 Flexible Least Squares Coefficients, Simulated Data



time-varying relationship without making strong assumptions about the underlying data generation structure or about the distribution of disturbances.

A Time-Varying Parameter Model of Presidential Approval

Beginning with Mueller's (1970) classic study, a virtual cottage industry developed examining the effect of the economy on aggregate public approval of the president's job performance. While early work often concluded that the economy is important, the particular mechanisms for these effects were also in doubt. Mueller's (1970) correlational analysis found a strong relationship between unemployment and presidential approval, but Hibbs' (1974) reanalysis of Mueller's data found no statistically significant relation between unemployment and presidential approval after accounting for autocorrelation effects. Kernell (1978) found that unemployment rates are not a key determinant of presidential popularity, while Kenski (1977) concluded that inflation is a strong determinant. The consensus that emerged from this early work was that inflation is probably more important than unemployment in determining presidential approval, but that unemployment could also be important (Monroe 1978, 1979; MacKuen 1983).

Following the lead of Key (1968), most early scholars also concluded that it is current or past economic performance that affects public evaluations of the president. The public was usually deemed as psychologically retrospective, rather than prospective to the future state of the economy. Challenging this perspective, MacKuen, Erikson, and Stimson concluded, "controlling for business expectations, no other measure of economic sentiment directly affects approval. Economic conditions affect presidential popularity only to the extent that economic conditions alter expectations of the economic future" (1992, 603). In their path-breaking analysis, MacKuen, Erikson, and Stimson (1992) showed that the effects of unemployment and inflation on presidential approval disappear once a measure for citizens' expectations about the future state of the economy is included in the analysis. This was a serious challenge to conventional wisdom that current and past economic conditions matter and suggested an incredibly rational and forward-looking citizenry able to ignore all conditions except future economic prospects.

Various scholars have questioned MacKuen, Erikson, and Stimson's (1992) results, suggesting that it is implausible that the electorate is actually this rational or insulated. Others suggested that their statistical analysis was flawed, because it failed to consider the stationarity properties of the approval time series (e.g., Clark and Stewart 1994). Indeed, there has been considerable debate over whether the presidential approval time series is stationary or integrated (e.g., Beck 1993; Williams 1993; Ostrom and Smith 1993; Smith 1993; Alvarez and Katz 1996; DeBoef and Granato 1997; Box-Steffensmeier and Smith 1998). A stationary time series should be modeled in levels, while an integrated time series should be modeled in differences (Granger and Newbold 1974). Those who see the approval time series as stationary tend to find the economy as important, while those who see it as integrated tend to find the economy as unimportant. Clark and Stewart (1994) attempt to strike a middle ground by estimating an error-correction model that treats presidential approval as integrated and requiring differencing, but in a long-term equilibrium relationship with economic evaluations. Using this approach, they find that presidential approval responds to both retrospective and prospective economic evaluations. However, their analysis depends on presidential approval being nonstationary and in long-term equilibrium with economic evaluations.

Of course, all of these analyses assume a time-invariant relationship between economic conditions and citizens' approval of the president's job performance. Yet there is reason to believe that these relationships are not temporally fixed. Edwards, Mitchell, and Welch (1995) showed that issue salience is extremely important in determining issue effects on presidential approval. As issues become more salient, their impact on presidential approval becomes increasingly large; as they become less salient, their impact becomes increasingly small. Thus, it makes sense that more pronounced economic conditions should produce larger effects on citizen evaluations of the president. Additionally, Hetherington (1996) showed that the media is important in the formation of national economic evaluations from the actual state of the economy. A media that is exuberant in reporting economic conditions may actually alter citizen perceptions of economic conditions as occurred in the 1992 presidential election (Hetherington 1996).

FLS enables exploring the time-varying properties of the economy-approval relationship. As a measure of presidential approval we use monthly Gallup survey results from January 1978 through December 1997. We model presidential approval as a function of lagged presidential approval, quarterly change in unemployment, inflation, and citizen evaluations of business conditions for the next twelve months. We shall call this last variable Prospective Evaluations. This measure is taken from the Survey of Consumer Attitudes and Behavior conducted by the University of Michigan's Survey Research Center. Prospective Evaluations is the same variable that MacKuen, Erikson, and Stimson (1992) find fully explains the economic component of presidential approval.[7]

Results for the fixed-coefficient model are presented in Table 3 and correspond closely with the results reported in MacKuen, Erikson, and Stimson (1992). Approval appears highly inertial and approaching unit-root behavior with a dynamic coefficient of 0.88. Prospective Evaluations is strongly significant, but the coefficients for inflation and change in unemployment are both small and nonsignificant. The fixed-coefficient model seems to support the claims of MacKuen, Erikson, and Stimson (1992) that citizens are incredibly forward looking and able to shut out past and present economic conditions in their evaluations of presidential job performance.

Is there global evidence of time variation in the coefficients for this model? The model was reestimated with Flexible Least Squares sweeping across the entire range of

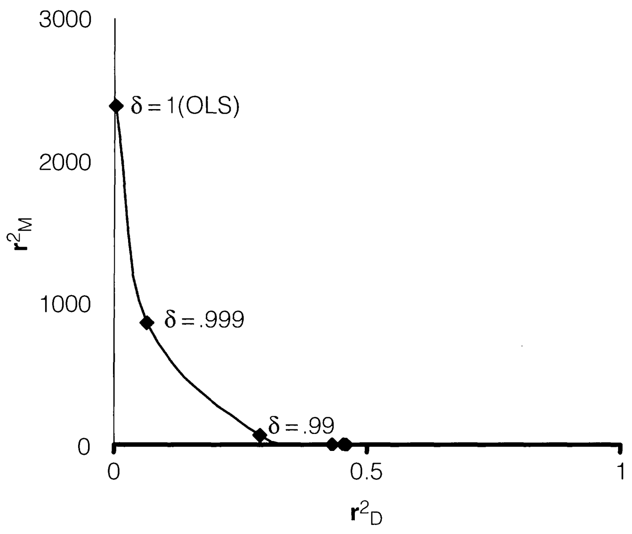
[7] The model also includes dummy variables for the start of each presidential administration, along with a modest events series which includes the Iran hostage crisis, the Reagan assassination attempt, the Beirut bombing, the Iran-Contra scandal, the U.S. invasions of Grenada, Panama, and Haiti, and the Persian Gulf War. Specifically, the events series is coded 1 for December 1979, January 1980, April 1981, January 1990, August 1990 through February 1991, and October 1994. It is coded –1 for February and March of 1980, June 1981, November 1983, and December 1986.

TABLE 3 Fixed Coefficient Estimates (OLS) for Approval Equation

Independent Variable	Coefficient
Approval_{t-1}	0.88 (33.10)
Prospective Evaluations	0.04 (2.85)
ΔUnemployment	−0.22 (−0.21)
Inflation	−0.13 (−0.16)
Intercept	2.29 (1.31)
R^2	0.88
σ	3.89
Durbin-Watson d	1.90
Box-Ljung Q $\chi^2(23)$	27.59
N	236

Note: The dependent variable is presidential approval. The numbers in parentheses are t-statistics.

FIGURE 3 Residual Efficiency Frontier Presidential Approval



δ and plotting the REF reported in Figure 3. At δ = 0.99, virtually all of the squared error from the measurement equation disappears. Thus, allowing even small amounts of coefficient-time variation produces extremely large reductions in explained variance from the measurement equation. This sharply declining REF provides *very* strong evidence that at least some of the coefficients are changing through time.[8]

Which particular coefficients are changing and by how much? Table 4 reports the coefficient averages and standard deviations at each value of δ along the REF. Notice first that the coefficients with δ very near 1 are almost precisely the same as those reported in Table 3.[9] As we change δ by a small amount to 0.999, the coefficient averages shift substantially, as do the standard deviations. The coefficient average for Prospective Evaluations triples in magnitude relative to the OLS result. Equally interesting, the magnitude of the coefficient average for inflation becomes almost twice as large. The average coefficient for change in unemployment is over eight times larger! As we move δ toward zero, the coefficient averages stabilize at about δ = 0.95, suggesting the extreme coefficient instability that was also evident from the plot of the REF in Figure 3.

The evidence in Table 4 also shows temporal variation in the coefficient for the lagged-dependent variable. The results for the lagged-dependent variable are instructive of an important side benefit of FLS in evaluating stationarity problems. The usual procedure in modeling a nonstationary time series is to take first differences; most test statistics for evaluating stationarity also rely on first differences and ignore possible time variation. Now, suppose that we model a suspected unit-root time series in levels with a lagged-dependent variable as was done with the approval series above. First differencing to achieve stationarity assumes that the coefficient on this lagged-dependent variable is *precisely* 1 and remains fixed through time. This implies a time-varying coefficient average of 1, with a standard deviation of exactly zero.[10] Tesfatsion and Veitch (1990) and Lutkepohl (1993) provide empirical examples where allowing time variation resolves unit-root problems. We see yet another example through analysis of the approval series.

The changes for lagged approval in the first column of Table 4 allow us to evaluate whether the approval series is stationary. An augmented Dickey-Fuller test performed on the univariate approval series shows that we are unable to reject the null hypothesis of a unit root at

[8]Note also that a CUSUM test shows that the cumulative sum of recursive residuals exceeds the CUSUM bounds at the thirteenth observation and remains outside the bounds for the remainder of the series.

[9]In this case "very near 1" means δ = 0.999999.

[10]Deviations above 1 imply an explosive process and would obviously be problematic if the vector were time varying. Harvey (1990b, Section 6.5) discusses the implications of time variation in parameters of ARIMA noise models in more detail, including the difference operator.

TABLE 4 Summary Statistics for Flexible Least Squares Estimates over the Residual Efficiency Frontier, Approval Equation

δ	$Approval_{t-1}$	Prospective Evaluations	ΔUnemployment	Inflation
≈1	0.88	0.04	0.22	−0.13
	(0.00)	(0.00)	− (0.00)	(0.00)
0.999	0.22	0.12	−1.79	−0.21
	(0.13)	(0.08)	(0.01)	(0.06)
0.99	0.17	0.12	−1.40	−0.23
	(0.13)	(0.08)	(0.02)	(0.07)
0.95	0.17	0.12	−1.30	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.90	0.17	0.12	−1.30	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.80	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.70	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.60	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.50	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.40	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.30	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.20	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.10	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.05	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)
0.01	0.17	0.12	−1.29	−0.24
	(0.13)	(0.08)	(0.02)	(0.08)

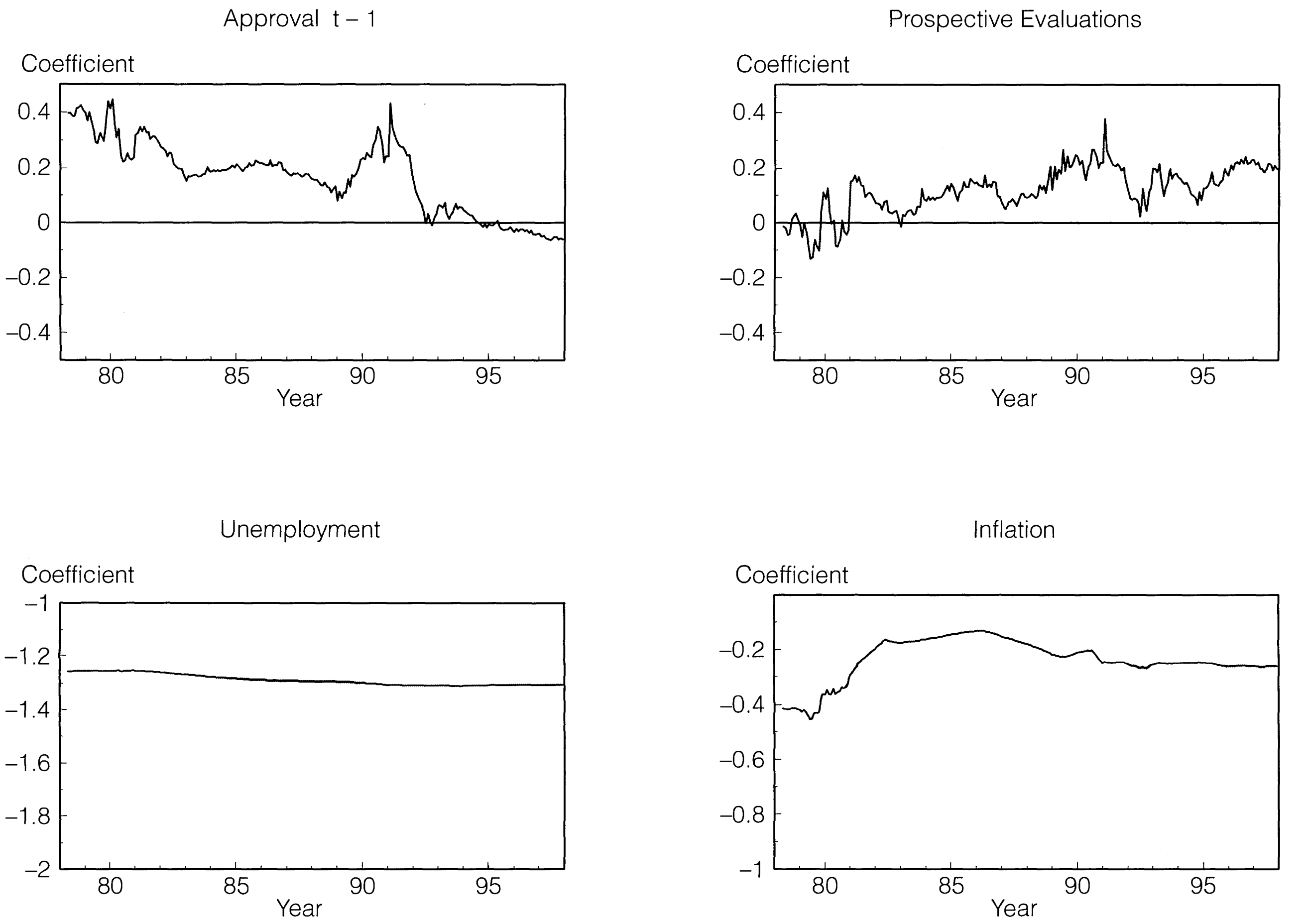
Note: The numbers in the table are time-varying coefficient averages at each specified δ. The numbers in parentheses are time-varying coefficient standard deviations at each specified δ.

the conventional 0.05 level.[11] Yet, as we change δ from near 1 to 0.999 the dynamic coefficient changes drastically from 0.88 to about 0.22. At $\delta = 0.95$ the dynamic coefficient stabilizes to a coefficient average of about 0.17. Thus, allowing only small amounts of coefficient-time variation resolves unit-root problems for approval. This implies that past analyses that have treated approval as nonstationary or used error-correction technologies may be inappropriate. Rather, the structural time series components of approval should be treated as time varying (e.g., see Harvey 1990b, Section 6.5).

What interesting patterns exist in the time-varying coefficients? Figure 4 plots the coefficient sequences for lagged approval and the three economic variables with δ set arbitrarily to 0.5. Note that the evidence in Table 4 shows that we could have set δ to any value less than 0.95, since the coefficient averages and standard deviations stabilized after this threshold. The coefficient for unemployment is steady, with each 1 percent increase in unemployment producing about a 1.29 percent decline in presidential approval. This is a much larger effect than for

[11]The augmented Dickey-Fuller test statistic is −2.76 with a 0.05 critical value of −3.41.

FIGURE 4 Flexibile Least Squares Coefficients, Approval Equation



either inflation or Prospective Evaluations, suggesting that current unemployment conditions are *very* important to evaluations of presidential job performance. This result runs contrary to findings by early scholars who found unemployment to be less important than other economic variables. It is also inconsistent with the strong position taken by MacKuen, Erikson, and Stimson (1992) that Prospective Evaluations are all that matter. Using an approach that recognizes the time-varying nature of the economy-approval relationship shows that the public is markedly aware of surrounding economic conditions and reward or punish the president accordingly.

The effects of inflation and Prospective Evaluations on presidential approval are smaller and exhibit significant fluctuations through time. The coefficient for inflation has been centered on about –0.25 since the 1982–83 recession but has been declining marginally through time. Inflation was more important, however, during the hyper-inflationary era of the late 1970s and early 1980s. This may suggest the importance of issue salience to public evaluations of the president's job performance. The effect of Prospective Evaluations on presidential approval has been much more volatile from month to month, but has averaged about 0.12 over the period of the analysis.

The coefficient for lagged approval shows that approval's inertia never exceeded about 0.46 over the entire period of the analysis, which is again inconsistent with the unit-root hypothesis. Approval's inertia is time varying with a secular decline through the Carter and Reagan administrations, a resurgence and subsequent decline during the Bush administration, and almost no inertia through the first Clinton administration. These declines suggest that presidential approval has increasingly rested on shaky ground. Modern presidents are less likely to receive the "benefit of the doubt," and approval can turn sharply with changing events, economic fortunes, and political drama.

Sampling Variability and Flexible Least Squares

A natural next step is to seek confidence bands around the estimated vector of coefficients. Recall, however, that FLS makes no assumptions about the distribution of disturbances associated with each of the T-point estimates. In the absence of such assumptions, there is no analytical way to obtain such confidence bands. An alternative approach to gauging the sampling variability of the coefficients would be to apply the bootstrap (Efron 1979). Caution is required, however, when using this approach with FLS. Standard errors generated in this fashion are *conditional* standard errors. Observing the REF for any of the preceding analyses shows that residual-measurement error (and therefore the FLS residual) changes dramatically between the OLS extreme point down to a threshold determined by the amount of time variation in the coefficients. Given this limitation, we should view FLS as a diagnostic or exploratory tool for evaluating the basic compatibility of data with theories.

When using FLS to diagnose the appropriateness of conventional estimators, evidence of time-varying relationships may suggest a need for model *respecification.* Not until a model has been obtained for which the coefficients stay robustly constant over time can the traditional machinery for statistical inference be usefully applied, because otherwise the model is seriously misspecified. FLS is a simple practical diagnostic tool for moving along the path to a well-specified model, at which point standard statistical techniques can be applied. FLS is thus a complement to other statistical tools and not in any sense a replacement for them.[12]

Even with a fully specified model, there may be time variation in the vector of estimated parameters due to time variation in the true data-generation process. Under this circumstance, an appropriate method for inference would be the time-varying Kalman filter. In order to use the time-varying Kalman filter it is necessary to make strong assumptions about the distribution of disturbances associated with each of the T-point estimates, as well as the data-generation process driving the coefficients. This approach also requires the analyst to have good initialization values for both the parameters and their variances. Otherwise, estimates will often be nonsensical. Under the usual circumstance, initialization values are difficult to obtain, but if an analysis has already been done with FLS, then these are readily available.

Conclusions

Political science theories are weak in the sense that they rarely specify the types of invariant relationships that are common in the physical sciences. Furthermore, empirical work in political science commonly relies on data that were not generated through processes that remain steadfast across space or through time. Under these circumstances, naïve belief in the primacy of OLS and other fixed-coefficient methods will often lead to errors of inference. As the preceding examples show, true relationships are sometimes missed, because OLS inappropriately weights the time-varying coefficients. False relationships sometimes appear for the same reason. For both reasons, political scientists should pay closer attention to the spatial and temporal stability of modeled relationships.

In this study we have been concerned with the temporal stability of modeled relationships. A beginning point in assessing temporal stability should always be substantive theory. The researcher should ask whether past work or intuition suggests fixed or time-varying relationships. If relationships are likely to be changing, then Flexible Least Squares and its three-tiered mode of analysis provides an arsenal of tools for exploring the nature of the changes.

Relative to other methods, FLS requires fewer assumptions about the underlying data generation process and disturbances. FLS relies on all of the data to generate coefficient sequences and employs true varying coefficient estimation techniques. It provides information about global coefficient stability, as well as about the particular coefficients that vary and the relative magnitude of their variation. FLS also generates a time-varying coefficient sequence that can be used to evaluate the consistency of substantive theory with actual data.

[12] Time variation in the vector of estimated parameters can provide evidence of several different types of model misspecification. For example, suppose the true relation is $y_t = \beta_0 + \beta_1 X_t + \varepsilon_t$ with dynamic equation $\beta_1 = \alpha_0 + \alpha_1 Z_t + v_t$. The researcher models only the first equation. Then, the true relationship is interactive between X_t and Z_t. Temporal variation in the omitted variable, Z_t, will show up through β_t. Obviously there is a myriad of possibilities for the omitted dynamic equation (e.g., see Hastie and Tibshirani 1993, 761–762 for a summary). Another possibility is that the variances of the disturbances are changing through time and feeding back to the mean of the data generation process. Such a mechanism has been described by Engle, Lilien, and Robins (1987) and can be modeled as an ARCH-M process using nonlinear methods. A third possibility is that there are unmodeled regime changes. This is an omitted-variable problem that again manifests itself through time variation in the estimated parameters. If the regime changes are endogenous to the model, then regime-switching estimators with shifts captured through the dynamic equation may be appropriate (e.g., see Hamilton 1994, 677–699).

FLS also has limitations as discussed above. Classical hypothesis testing using the estimated vector of coefficients is not straightforward. More generally, all time-varying parameter methods produce T-point estimates for each effect, rather than a single point estimate. We could, of course, use the mean of the time-varying estimates as our point estimate and the standard error of the mean as a measure of precision. However, this ignores the idea that the estimated effect is a random variable with draws of vectors of outcomes rather than single outcomes. Relatedly, Beck (1983) observes that time-varying parameter models challenge the entire meaning of statistical significance, because coefficients can be nonsignificant due to changing signs over time, rather than a lack of true relations. We observed this with the sinusoidal simulation shown in Figure 2 above. Thus, the use of FLS and other time-varying parameter techniques suggests a limitation of the classical statistical model under which we currently develop scientific knowledge.[13]

FLS provides a nice arsenal of tools for exploring parameter time variation, but it is not the only tool. If FLS seems too technical or is unavailable due to software limitations, researchers might consider other alternatives. These include Chow tests for structural breaks, CUSUM and CUSUMSQ plots for global-coefficient stability, and recursive estimation for exploring which particular coefficients vary.[14] These methods are limited for reasons noted above, but at least provide a sense of whether there might be a problem. Whichever tool is used to explore time variation, the analyses presented in this study should have convinced political scientists to take time-varying relationships more seriously.

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[13]Achen (1982) suggests another reason why the entire notion of statistical significance should be challenged. In particular, he notes that insignificance due to a large error term does not have the same meaning as insignificance due to a "small" estimate.

[14]Additionally, Hastie and Tibshirani (1993) propose a class of time-varying estimators that enable exploring time variation in a broader class of models, including those with limited dependent variables and nonnormal disturbances.

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