A Dynamic Analysis of Fixed- and Adjustable-Rate Mortgage Terminations*

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Abstract

This paper provides a side-by-side comparison of loan-level statistical models for fixed- and adjustable-rate mortgages. Multinomial logit models for quarterly conditional probabilities of default and prepayment are estimated. We find that the estimated impacts of embedded option values for prepayment and default are generally quite similar across both FRM and ARM loans, providing additional empirical support for the basic predictions of the options theory. We also find that differences in estimates of conditional probabilities of prepayment and default associated with mortgage age, origination period, original LTV, and relative loan size, indicate the continued significance of these other economic and demographic factors for empirical models of mortgage terminations.

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Introduction

This paper adds to the growing body of empirical research into the causes and correlates of single-family residential mortgage terminations based on the options theory of prepayment and default by presenting a side-by-side comparison of statistical models for fixed- and adjustable-rate mortgages. The empirical analysis is based on a discrete-time / discrete-choice framework that utilizes data on the event histories of individual mortgage loans. A multinomial logit specification is used to account for the discrete-choice nature of default and prepayment decisions and to account for the competing-risks of default and prepayment.[1]

We report estimates of multinomial logit models for quarterly conditional probabilities of prepayment and default for conventional, conforming, 30-year, single-family mortgages. The statistical estimates were obtained using data on nearly 1.3 million mortgages originated over the period 1979-93.[2] Contemporaneous measures of mortgage premium values and borrower equity distributions were used to reconstruct individual mortgage event histories for analysis in a dynamic discrete-time logit model. We were also able to augment the basic loan-level data with detailed information on the contractual terms that determined the periodic changes in the coupon rates on adjustable-rate mortgages, and to directly compare the performance of fixed-rate and adjustable-rate loans within the same econometric model.

Options Theory and Mortgage Prepayment and Default

Options theory has been the dominant paradigm for research on residential mortgage prepayment and default in recent years and holds that mortgage borrowers will exercise embedded call (prepayment) or put (default) options when either of these

[1] A number of recent studies analyze mortgage prepayment and default in a competing risks hazard-modeling framework. See, for example, Deng, Quigley and Van Order (1996, 2000), and Deng (1997). Further discussion of the relationship of the statistical method to alternative approaches such as proportional hazards models is provided later in the paper.

[2] Conventional loans are not government insured. Conforming loans meet underwriting and maximum loan amount requirements for eligibility for purchase by U.S. government-sponsored housing enterprises Fannie Mae and Freddie Mac. The data used in the empirical analysis were from a historical mortgage database compiled by OFHEO to support a broad range of research and analysis activities.

alternatives becomes financially attractive -- that is, when they are “in the money.”[3] Many empirical studies have applied financial options theory to the analysis of prepayment and default probabilities for single-family fixed-rate mortgages (FRM).[4] Options-based empirical models of adjustable-rate mortgage (ARM) performance are less prevalent, owing to the greater complexity of the underlying index and coupon dynamics, and the relative paucity of publicly available loan-level data. This has complicated the application of options-based empirical models to ARM loans, and limited direct statistical comparisons of FRM and ARM performance within a common empirical framework.[5] A primary objective of the present paper was to compare FRM and ARM performance when the options values of prepayment and default are computed in a consistent manner by accounting for the unique features of ARM contracts. Evidence that FRM and ARM borrowers respond in similar ways to comparable measures of the options values of prepayment and default would provide additional empirical support for the options theory.[6]

For the research reported in this paper, we used loan-level data on conventional, conforming, single-family residential mortgages to study FRM and ARM prepayment and default behavior in an options-based empirical model. The availability of loan-level data allowed us to account for the dynamics of mortgage coupon rates to reflect the specific characteristics of ARM contracts (e.g., interest rate index, periodic and lifetime adjustment caps, interest rate margins, rate adjustment look-back periods, frequency of interest rate adjustments) and to approximate current coupon rates and monthly payment

[3] See Dunn and McConnell (1981), Foster and Van Order (1984, 1985), Buser and Hendershott (1984), Brennan and Schwartz (1985), Kau, Keenan, Muller, and Epperson (1985, 1990), and Hendershott and Van Order (1987),

[4] See, for example, Green and Shoven (1986), Schwartz and Torous (1989), Quigley and Van Order (1990, 1995), Archer, Ling and McGill (1996), Deng, Quigley and Van Order (1996, 2000), and Deng (1997).

[5] Previous empirical studies have tended to either: (1) model ARM prepayment and default rates as functions of variables observed at mortgage origination; (2) study ARM prepayment and default at the aggregate level; or (3) use prepayment and default rates on FRMs as a benchmark to estimate the relative termination rates of ARMs using empirical multiples. For example, Sa-Aadu (1988) used only variables observed at the time of origination to analyze ARM defaults but did not address prepayment. Lea and Zorn (1986) analyzed ARM termination experience in response to changes in macro-level variables. Cunningham and Capone (1990) studied relative termination experience of ARM to FRM in a small sample of loans from Houston, Texas.

[6] This does not imply that FRM and ARM loans will have similar observed rates of prepayment and default over the same economic cycle, as changes in ARM coupons will result in different realized values for the prepayment and default options.

amounts on ARM mortgages at each period over the lives of the loans. Loan-level data on approximately 630,000 ARM loans originated between 1982 and 1993, and 650,000 FRM loans originated between 1979 and 1993, were used in the empirical analysis.[7] Mortgage performance was measured through the end of 1995.[8]

Statistical Methods

Mortgage prepayment and default are highly duration-dependent, exhibiting characteristic age-profiles that increase during the first years following origination, peak sometime between the fourth and seventh years, and decline over the remaining years. Like other duration- or age-dependent processes, mortgage terminations are highly amenable to analysis using a variety of statistical survival-time models, including parametric hazard models, semi-parametric or proportional hazard models, and discrete-time models.[9] A discrete-time / discrete-choice model is applied in the present paper. In what follows we briefly summarize the different approaches and explain why a discrete-time / discrete-choice approach is appropriate for analyzing mortgage prepayment and default risks.

Parametric Models

Parametric models provide a complete parameterization of the probability distribution of survival times. Examples include the exponential, Weibull, gamma, log-logistic, log-normal, and various mixtures of these and other parametric distributions. A key advantage of parametric models is parsimony, since relatively few parameters are needed to completely describe the distribution of survival times. Once one has estimates of these parameters, the distribution of survival times is completely determined, thereby

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[7] Data on FRM loans were limited to conventional, conforming, 30-year (greater than 240 months original term to maturity) mortgages on single-family, single-unit properties. The FRM sample is a 10-percent random sample from a much larger database of historical mortgage originations.

[8] Default and prepayment events correspond to the occurrence of last-paid-installments that are subsequently identified as defaults or prepayments. In the case of default events, there may exist delays of a few months to as long as two years following a borrower's decision to cease payment on a loan and the culmination of foreclosure proceedings that result in a loan being classified as a default. Thus, it is necessary to limit the sample to cohorts with at least two years actual experience.

[9] Standard references in statistics and economics include Kalbfleish and Prentice (1980) and Lancaster (1990).

facilitating hypothesis testing, forecasting, or simulation. However, parametric models also impose greater restrictions on the specification and therefore are less likely to provide a good fit to empirical data for processes for which there is no theoretical justification for a given functional form. While one may specify the survival distribution in terms of an arbitrary mixture of different parametric models until the desired goodness-of-fit is attained, it is not clear that this conveys any particular advantage over more flexible specifications that are also easier to implement, such as proportional hazards or discrete-time models.

Proportional Hazards Models

One popular statistical survival-time model that has been applied to analyzing mortgage prepayment and default is the proportional hazard model.[10] The proportional hazard model is an appropriate method when one is willing to assume that the conditional termination rate or hazard function (defined as the ratio of the density to the survivorship proportion, $\lambda(t) = f(t)/S(t)$) can be written as the product of an exponential function of covariates and a baseline hazard that captures the underlying pattern of the hazard function over time which is identical across all subjects in the sample, $\lambda(t) = \lambda_0(t) \cdot e^{-X\beta}$. The reference to "proportionality" derives from the constant proportional impact of the covariates in the exponential component on the overall hazard rate.[11]

Estimation of the covariate parameter vector β can be achieved using a variety of canned routines for computing Cox (1975) partial-likelihood estimates that do not require specification and estimation of the baseline hazard component given by $\lambda_0(t)$. The Cox proportional hazard specification is particularly convenient when one wishes to test various hypotheses about the possible impact of a covariate on overall survivorship without having to specify a complete distribution of survival times. The proportional

[10] See, for example, Deng, Quigley and Van Order (1996, 2000), and Deng (1997).
[11] Covariate vector X may be functions of time, in which case they are referred to as "time-varying" covariates.

hazard model is often referred to as a "semi-parametric" model, due to the combination of the partial-likelihood approach with non-parametric estimation of the baseline hazard.[12]

Discrete-Time Models

Discrete-time models are usually specified in terms of the conditional probabilities of termination within specified time intervals. As such, discrete-time models often conform more closely to the way events are actually measured in empirical data, even when the underlying decision-making process may be defined in continuous time. This is almost always the case in terms of mortgage performance data.[13] Even if borrowers are assumed to decide to exercise embedded default (put) options in continuous time, this exercise may not be revealed until the next payment date when the lender fails to receive the scheduled payment from the borrower.

Whereas the hazard function in continuous time is defined as the instantaneous rate of failure conditional on survival to a given point in time, the discrete-time hazard is the probability that an event occurs at time *t*, or more precisely, in the interval from *t* to $t+1$, given that an event has not already occurred prior to *t*. Specification and estimation of a discrete-time model may be fully consistent with specification of parametric or semi-parametric models in continuous-time, from which it possible to derive the corresponding conditional probabilities of failure within discrete intervals of time by integrating the hazard function.[14] However, in cases where the events are observed only at discrete time points, as is almost always the case with mortgage performance data, a discrete-time model is appropriate. A grouped-duration model can be specified to estimate the discrete time behavior of mortgage prepayment and default. However, the Cox Partial Likelihood estimation approach, which is designed for estimating a continuous proportional hazard model, is inappropriate for estimating a grouped-duration model if there exist more than

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[12] Han and Hausman (1990) have proposed a method by which a flexible baseline function can be estimated non-parametrically together with the proportional hazard parameters given by β. A preliminary effort to apply the Han and Hausman (1990) approach to default and prepayment hazards is described in Calhoun (1998).

[13] Allison (1995), in introducing discrete-time methods, specifically mentions mortgage payment behavior as a case where "a discrete-time model is the natural way to go."

one failure observations in each discrete time interval.[15] Though unbiased estimates for a grouped-duration model may be obtained by using a Maximum Likelihood approach,[16] the approach typically requires significant amount of computational resources, particularly when the grouped-duration model is specified with competing risks and time-varying covariates.

Discrete Choice Theory and Competing Risks

The period-by-period decisions by borrowers to prepay, default, or continue payment on a loan conform to a situation of discrete choice for which the multinomial logit model is widely recognized as an appropriate methodology.[17] Given the inherent discreteness of the historical loan-level mortgage performance data, a discrete-time model is a more natural choice than a continuous-time specification. Mortgage prepayment and default may also be viewed as competing risks, since they are mutually exclusive and the occurrence of one event precludes or "censors" the chance of ever observing the waiting time until the other event.[18]

The multinomial logit specification provides a joint model for the probabilities of prepayment and default and accounts for the competing risks nature of these events. A joint model for the conditional probabilities of prepayment and default is also implied by the fact that both types of decisions are likely to depend on common unobserved factors so that there is some inherent simultaneity in the decisions of borrowers. Options theory predicts, for example, that borrowers may delay exercising either one of their embedded

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[14] As discussed in Allison (1982), in some cases the parameters estimates obtained from a discrete time model will be identical to those in the continuous time specification, such as when the hazards can be specified in terms of a log-linear model and all the explanatory variables are categorical.
[15] See Kalbfleish and Prentice (1980) for a discussion.
[16] See, for example, Deng, Quigley and Van Order (2000).
[17] See, for example, D. McFadden (1978).
[18] In the case of parametric and semi-parametric models, the assumption of independent competing risks can simplify the estimation of model parameters. For example, a standard approach is to view different types of terminations as arising from the existence of latent or potential failure times for each termination type. If these random failure times are independently distributed than we have an independent competing risks model, and the derivation of marginal distributions and accounting for censoring is greatly simplified. Whether or not the form of the dependence can be identified in the case where the underlying competing risks are not independent depends on whether we have information in addition to the proportions surviving and the type of failure. In the absence of additional information any dependent competing risks model will be observationally equivalent to an independent competing risks model. These points follow the discussion of competing risks and identification issues in Lancaster (1990), pp. 154-155.

options if doing so increases the value of future exercise, making an assumption of independent competing risks questionable.

The Empirical Model

We have estimated the competing risks of mortgage prepayment and default using a discrete-time multinomial logit model that accounts for the event histories of individual mortgage loans.[19] The data used in the present analysis allowed us to reconstruct individual loan event histories for quarterly transitions by combining mortgage origination information with contemporaneous values of time-dependent factors. In the process of creating quarterly event histories, each loan contributes an additional observed "transition" for every quarter from origination up to and including the period of mortgage termination.[20] All of the explanatory variables except mortgage age (measured in quarters) were recoded to categorical values.[21] The use of categorical outcomes for the explanatory variables is an inherently flexible approach to model specification, allowing the data to reveal any underlying non-linearities. The use of categorical explanatory variables also allowed us to "reduce" the data to a smaller number of physical loan records, each representing a unique combination of the categorical variables, to which a frequency count was assigned and applied as a sampling weight in subsequent statistical analyses.[22] Given sufficient computer resources, this approach avoids the need to

[19] Several empirical studies have applied static form of the logit or other qualitative response models to analyze mortgage prepayment and default behavior. Examples of previous applications of the multinomial logit model are Campbell and Dietrich (1983), Zorn and Lea (1989), and Cunningham and Capone (1990).

[20] The term "transition" is used here to refer to periods in which loans remain active, or in which default or prepayment is observed to occur.

[21] The categorical outcomes were subsequently treated as dummy variables in the logit estimation. This has the additional benefit of avoiding over-parameterization of the model through the use of specific functional forms (i.e., linear, quadratic, logarithmic). The only variable entered directly using a specific functional form was the age variable, which was entered as a quadratic. The implications of this specification for the empirical results are discussed in greater detail below in summarizing the model estimates.

[22] The original ARM sample of approximately 630,000 mortgage originations produced a weighted event-history sample of 3.8 million records representing nearly 10.6 million quarterly transitions. The FRM sample of approximately 650,000 mortgage originations produced a weighted event-history sample of 1.64 million records representing 9.9 million quarterly transitions. The smaller number of weighted event-history records for FRMs is consistent with the fact that each loan contributes fewer unique combinations of the explanatory variables because the coupon rates on these loans are fixed at origination and do not change over the life of the loan.

undertake choice-based sampling in order to assure that sufficient numbers of rare events like mortgage defaults are obtained.[23]

The empirical model is specified in terms of the age-specific conditional probabilities of prepayment and default given by:

$$\pi_P(t) = f_P\left(t, MP(t), EQ(t), X(t)\right) \tag{1}$$

$$\pi_D(t) = f_D\left(t, MP(t), EQ(t), X(t)\right) \tag{2}$$

where *t* is mortgage age. The two key options-theoretic predictors of mortgage prepayment and default are the mortgage premium value at age *t*, given by *MP(t)*; and the equity position of the borrower at age *t*, given by *EQ(t)*. The remaining explanatory variables are represented by *X(t),* comprising a collection of exogenous fixed-effects and time-varying factors related to the conditional probabilities of prepayment and default. The direct dependence of the conditional probabilities on mortgage age recognizes the existence of other borrower processes and unobserved heterogeneity that induce duration dependence in the conditional rates of termination and help to explain the typical age patterns of default and prepayment.[24]

Mortgage Premium Value

The options theory views mortgages as long-term bonds issued by borrowers who retain embedded call (prepayment) and put (default) options. The call option value of the mortgage is a function of the difference between the present value of the "anticipated" future stream of mortgage payments discounted at the current market rate of interest, *R(t)*, and the present value of the mortgage evaluated at the current note rate, *C(t)*.[25] Following

[23] It has been demonstrated for static logit models that choice-based sampling results in biased estimates of the coefficients of the logit constant terms, for which relatively simple corrections are available, based on the sample versus population distributions of the explanatory variables across groups defined by dependent variable outcomes (Costlett, 1981).

[24] Other borrower processes include residential mobility, employment mobility, involuntary unemployment, and demographic events related to household formation and dissolution, mortality, and fertility. Ideally, given suitable household-level data, these other processes would be modeled jointly with mortgage terminations. See Lancaster (1990) for a discussion of the impact of unobserved heterogeneity on estimates of duration dependence in econometric models of transition probabilities.

[25] The precise value of the mortgage premium is unknown, and depends on the uncertain future time path of mortgage payments and the associated future probabilities of prepayment and default. In this paper, we have elected to approximate the mortgage premium using empirical values known, or at least knowable, by both borrowers and observers of borrower behavior. To facilitate the comparison of FRM and ARM performance, we have also imposed the additional simplification of using the current payment amount to represent current and future payment amounts, although for ARM loans these payments will vary along with the levels of future interest rates. The dependence of current premium values on the uncertain exercise of financial options has been one motivation for the development of "option-adjusted spreads" in valuing the payment streams associated with financial assets (for example, see Fabozzi and Richard, 1995). For a

Deng, Quigley and Van Order (1996), we have elected to approximate the call option value using the relative *mortgage premium* given by:

$$MP(t)=\frac{\sum_{\tau=1}^{T-t} PAY(t)\cdot\left\{\frac{1}{1+R(t)}\right\}^{\tau}-\sum_{\tau=1}^{T-t} PAY(t)\cdot\left\{\frac{1}{1+C(t)}\right\}^{\tau}}{\sum_{\tau=1}^{T-t} PAY(t)\cdot\left\{\frac{1}{1+R(t)}\right\}^{\tau}} \tag{3}$$

where T is the original term to maturity for a mortgage originated at $t=0$ (first payment due at $t=1$), and *PAY(t)* is the current monthly payment that will fully amortize the outstanding mortgage balance over remaining term $T-t$ at the current coupon rate *C(t).* This expression can be rewritten as:

$$MP(t)=\frac{\dfrac{1-\left(\dfrac{1}{1+R(t)}\right)^{T-t}}{R(t)}-\dfrac{1-\left(\dfrac{1}{1+C(t)}\right)^{T-t}}{C(t)}}{\dfrac{1-\left(\dfrac{1}{1+R(t)}\right)^{T-t}}{R(t)}} \tag{4}$$

For moderately large values of the remaining term, $T-t$, the terms in powers of $T-t$ are close to zero, so that mortgage premium can be approximated by the following expression for the relative spread between the current coupon rate on the mortgage and the market rate of interest:

$$MP(t)=\left\{\frac{C(t)-R(t)}{C(t)}\right\} \tag{5}$$

The relative mortgage premium values for ARMs and FRMs were derived in exactly the same manner, except that the current coupon is always equal to the coupon at origination for FRMs.[26]

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discussion of the implications of stochastic term structure for the construction of mortgage premium values in default models, see Deng (1997).

[26] Note that negative spreads between the current coupon and the market rate of interest are associated with positive values of the mortgage premium and *vice versa.*

Borrower Equity

The equity position of the borrow is determined by the difference between the market value of the property securing the loan, $P(t)$, and the unpaid mortgage balance, $UPB(t)$:

$$EQ(t) = P(t) - UPB(t) \tag{6}$$

Consistent with the assumptions of the financial options theory, which emphasizes the total value of the financial instrument, the definition of borrower equity is often expanded to include the current mortgage value, *MV(t)*, given by:

$$MV(t) = \sum_{\tau=1}^{T-t} PAY(t) \cdot \left\{ \frac{1}{1+C(t)} \right\}^{\tau} - \sum_{\tau=1}^{T-t} PAY(t) \cdot \left\{ \frac{1}{1+R(t)} \right\}^{\tau} \tag{7}$$

when this value is positive. Thus, an expanded definition of borrower equity is given by:

$$EQ(t) = P(t) + \max[0, MV(t)] - UPB(t) \tag{8}$$

The expression for borrower equity in (8) assumes that when borrowers assess the current value of the default option, they consider the financial benefit of having a below market rate of interest on their current loan, under the assumption that once they have defaulted they will immediately take out another loan to purchase another property.[27]

Ideally, periodic observations on the values of individual properties would be used to update individual house values and borrower equity at the same frequency (monthly) at which the decision to prepay or default can be exercised. In fact, the lack of continuous updating of individual housing values introduces significant asymmetries in the information sets of borrowers and lenders, and complicates the implementation and testing of the options theory of prepayment and default. In light of the measurement difficulties associated with borrower equity at the loan level, researchers have resorted to various means of simulating the distribution of borrower equity.[28]

We have characterized the equity positions of individual borrowers using *ex ante*

[27] See Foster and Van Order (1984). This ignores the potential implications for a borrower's credit standing and their subsequent ability to qualify for another mortgage.

[28] The most familiar example is Foster and Van Order (1984, 1985). They used a Monte Carlo simulation of a synthetic mortgage pool in conjunction with a house price diffusion process and actual default and prepayment rates to reconstruct a time-series for the number of borrowers in a negative equity position. Under additional restrictions on the model (i.e., that only borrowers with negative equity default, and only

probabilities of negative equity. The probability of negative equity is a function of the current loan balance and the probability of individual house price outcomes that lie below this value. Distributions of individual housing values relative to the value at mortgage origination were determined by applying estimates of house price drift and volatility obtained from independent estimates based on the OFHEO House Price Index (Calhoun, 1996). The House Price Index (HPI) is based on a modified version of the weighted-repeat-sales (WRS) methodology (Case and Shiller, 1987, 1989), and is consistent with the assumption that housing values are generated by a log-normal diffusion process.[29]

Individual house prices are assumed to obey a non-stationary log-normal diffusion process in which individual house price appreciation since mortgage origination is normally distributed with variance $\sigma^2(t)$ around the expected rate of appreciation from the HPI given by $\beta(t)$.[30] For the individual borrower with original house price *P(0)*, the probability of negative equity is given by:

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\begin{aligned}
PNEQ(t) &= \Pr\left\{EQ(t) < 0\right\} \\
&= \Phi\left\{\frac{\ln(UPB(t)) - \ln(P(0)e^{\beta(t)} + \max[0, MV(t)])}{\sigma(t)}\right\}
\end{aligned}
\tag{9}
$$

where $\Phi(x)$ is the standard normal cumulative distribution function evaluated at *x*, and where we have included the mortgage value as a component of borrower equity when this value is positive. The final expression for the probability of negative equity shows that the equity position of the borrower depends on the drift and volatility of house price appreciation rates, the expected current LTV, and loan amortization. Rather than representing a point estimate for the expectation of the borrower regarding his current equity status, this variable characterizes the entire distribution of equity values underlying these expectations.

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borrowers with positive equity prepay), the time-series for the number of borrowers with negative equity (various levels) was used in regressions for conditional default and prepayment probabilities.

[29] Estimates of the parameters of the log-normal diffusion process are produced as by-products of the application of the WRS methodology. Deng, Quigley, and Van Order (1996) applied a similar approach using WRS indexes for 26 metropolitan areas estimated using Freddie Mac data.

[30] Estimates of expected appreciation or drift in house prices are obtained directly from the estimated values of the HPI for each of the nine U.S. Census divisions. Estimates of diffusion volatility $\sigma^2(t)$ are computed using the estimated parameters for the error variance of individual log-differences in housing prices that are

ARM Coupon Rate Dynamics

To represent the current financial value of the prepayment option, we needed to trace the path of the coupon rate over the active life of individual ARM loans. The coupon rate resets periodically to a new level that depends on the underlying index, plus a fixed margin, subject to periodic and lifetime caps and floors that specify the maximum and minimum amounts by which the coupon can change on any one adjustment and over the life of the loan.[31] Accordingly, the current ARM coupon rate $C(t)$ can be specified as follows:

$$\begin{aligned} C(t) = \max\{ \min [\ & Index(t-S) + \text{Margin}, \\ & C(t-1) + A(t)\cdot Period\_UpCap, C(0) + A(t)\cdot Life\_UpCap\ ], \\ & C(t-1) - A(t)\cdot Period\_DownCap(t), C(0) - A(t)\cdot Life\_DownCap\ \} \end{aligned} \tag{10}$$

where $Index(t)$ is the underlying index value at time *t*, *S* is the "lookback" period, and *Margin* is the amount added to $Index(t-S)$ to obtain the "fully-indexed" coupon rate. The periodic adjustment caps are given by $Period\_UpCap$ and $Period\_DownCap$, and are multiplied by dummy variable $A(t)$ which equals 0 except during scheduled adjustment periods when it equals 1. The maximum lifetime adjustments are determined by $Life\_UpCap$ and $Life\_Down\_Cap$.

Additional Interest Rate Variables

Recent studies of mortgage terminations have emphasized the importance of path dependence in interest rates for distinguishing among borrowers more or less likely to exercise the prepayment option when the opportunity arises.[32] The tendency for the most responsive borrowers to prepay first, so that the remaining sample of borrowers are those with lower conditional probabilities of prepayment, contributes to the observed seasoning

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obtained from the second-stage of the WRS method for each Census division. See Calhoun (1996) for additional details.

[31] Detail on specific ARM contracts was obtained in some cases from loan-level information, and in other cases was obtained using plan-level detail for loans in certain product categories.

[32] For example, see the discussions of borrower heterogeneity and path dependence in Bartholomew, Berk, and Roll (1988), and the discussion of burnout in Richard and Roll (1989).

or “burnout” of mortgage pools. We have included a dummy variable (BURNOUT) that indicates whether the borrower has missed a previous refinance opportunity.[33]

Expectations about future interest rates and differences in short-term and long-term borrowing rates associated with the slope of the Treasury yield curve influence the choice between FRM and ARM loans and the timing of refinancing. We have used the ratio of the 10-year Constant Maturity Treasury yield (CMT10) to the 1-year Constant Maturity Treasury yield (CMT1) to define the slope of yield curve. A high value for the slope of the yield curve indicates relatively favorable short-term rates, increasing the likelihood that a borrower refinances to an ARM to take advantage of the lower initial coupons.

Mortgage Age

Conditional probabilities of mortgage default and prepayment exhibit characteristic age-profiles that increase during the first years following origination, peak sometime between the fourth and seventh years, and decline thereafter. Under a pure options model, the typical age patterns of conditional default and prepayment rates might be attributed entirely to the diffusion of housing values and the introduction of unobserved differences (heterogeneity) in the equity positions of individual borrowers, resulting in differences in the rates of default and prepayment among particular subsets of

[33] The dummy variable equals one if the spread between the note rate on the mortgage and the average market rate of interest has been 200 basis points or greater during any two of the past 8 quarters. We experimented with a number of alternative burnout specifications such as the sum of all past negative spreads. One practical problem with utilizing cumulative measures like the sum of past negative spreads arises when one intends to apply the model to project mortgage cash flows under the extreme interest rate assumptions of a stress test. For example, the risk-based capital stress test being develop by OFHEO for Fannie Mae and Freddie Mac assumes that interest rates increase by up to 600 basis points and then remain at historically high levels for up to 10 years. In this case the burnout variable specified in terms of the sum of past negative spreads takes on unrealistically high values and drives prepayment rates to unrealistically low levels. As an alternative we experimented with specifications that would be much more limited in terms of the range of outcomes they may describe, but which are also more constrained in terms of the values they can take on under extreme interest rate scenarios. The particular specification reported here was selected from among several alternatives that used different numbers of quarters and different threshold levels for interest rates changes. The estimation results were generally quite similar across the different combinations and we selected the combination of looking back 8 quarters for at least 2 quarters of interest rate spreads of at least 200 basis points as a reasonable compromise, and one that corresponds to conventional wisdom about the magnitude of changes that have been required historically to motivate borrowers to prepay. We found that the hurdle of 200 basis points worked well in comparison with the tested alternatives, including 100 and 300 basis point hurdles.

individual borrowers. As these differences emerge following mortgage origination, the observed average conditional default and prepayment rates will initially increase. Eventually, as “high risk” borrowers depart the sample or mortgage pool, the average conditional rates of default and prepayment will decline. The existence of other demographic and economic processes that may “trigger” mortgage default, and the inability to measure the diffusion of house prices and the distribution of borrower equity precisely, imply the need to account directly for age-specific differences in conditional rates of default and prepayment. For this reason, mortgage age is included as an additional explanatory variable in the empirical model.

Other Variables

Dummy variables for origination years were included to account for differences in the performance of specific mortgage origination cohorts due to excluded factors such as regional income growth and unemployment, or changes in mortgage underwriting standards.[34] Dummy variables for the original loan-to-value (LTV) ratio were also included. The LTV ratio serves as an indicator of the income and net worth of the borrower at mortgage origination, and directly determines the initial equity position of the borrower. Higher values of LTV at origination increase the probability of default and lower the probability of prepayment because there is a greater likelihood that the borrower will be in a negative equity position early in the life of the loan. High LTV borrowers are also more likely to have fewer economic resources to finance the transactions costs of prepayment or endure spells of unemployment or other trigger events that might otherwise cause them to exercise the default option in a sub-optimal manner. Finally, high LTV borrowers have demonstrated a willingness to “leverage” the financing of the home purchase, which may portend a greater sophistication or “ruthlessness” in the exercise of the default option.

Dummy variables were also included to account for the current season (quarter) of the calendar year, in recognition of the potential impact of weather, school schedules, and

[34] The sample includes ARM loans for origination years 1982-93 and FRM loans for origination years 1979-93. As noted by Vandell (1993), significant changes in underwriting guidelines for Fannie Mae and Freddie Mac occurred in 1985 and 1988, respectively.

seasonal employment patterns on residential mobility. A dummy variable for the occupancy status of the property was included to distinguish mortgages on owner-occupied units from investor loans. Owner occupants should be less likely than investors to exercise the default option given the direct benefits they receive from the consumption of housing services. Owner occupants should be more likely to prepay than investors for non-financial reasons such as residential mobility.

The ability to bear the transactions costs of refinancing, and the ability to weather economic stress and avoid default will be correlated with the income level of the household. Given the lack of data on household income at origination, we elected to use a measure of relative loan size as a proxy for the relative income level of the household. Relative loan size was defined as the ratio of the original loan amount relative to the average-sized loan originated in the same state during the same origination year.[35] Loan size and borrower income may be assumed to be highly correlated given the existence of loan qualification requirements that limit monthly payments, and therefore loan size, to a multiple of borrower income, and the fact that housing services (and therefore demand for mortgage debt) are a normal good for most mortgage borrowers.

For the estimation results reported in this paper, all the explanatory variables were entered into both the default and prepayment functions. While our stated justifications for including particular variables have emphasized their specific relevance to either prepayment or default, the estimated models represent a "reduced-form" specification in which all of the explanatory variables are included in the logit "regression" functions for both prepayment and default. In the vernacular of survival analysis, mortgage prepayment and default are "competing risks," each having direct actuarial impacts on the probability of occurrence of the other event. In addition, the underlying hazards for prepayment and default may be causally related, although we lack sufficient information to identify such a model.[36]

[35] Price Waterhouse (1990) reported significant differences in claim rates for FHA mortgages stratified by loan size. Smaller loans were observed to fail at significantly higher rates than other loans.
[36] For an example of simultaneous equations hazards models, see Lillard (1993).

Empirical Results

Table 1 reports multinomial logit coefficient estimates for quarterly conditional probabilities of mortgage prepayment and default. Separate estimates are reported based on separate applications of the model to FRM and ARM loan data. In what follows we briefly review the estimates for the mortgage age effects before moving on to discuss the other variables.

Selected sets of coefficient estimates (all except seasonality) are also shown graphically in Exhibits 1 and 2. Some explanation of the exhibits may be helpful in interpreting the results. For all variables except mortgage age, the vertical axis represents the contribution of the explanatory variable to the logit "regression" function, $X_k \cdot \beta_k$, for explanatory variable *k*, where the value of X_k is equal to 1 when the loan falls into a particular category and 0 otherwise, and the values of the β_k coefficients are the same as those reported in Tables 1 and 2 for the logit prepayment and default functions, respectively. Because all of the explanatory variables (except mortgage age) are 0/1 dummies, the same scale applies to every panel in Exhibits 1 and 2. For the panels in Exhibits 1 and 2 showing the results for mortgage age, the vertical axis reports the sum of the corresponding logit intercept coefficient and the quadratic age function. Thus, the intercepts reported in the first panel of each of the exhibits correspond to the overall intercept of the respective logit regression functions. Although these panels are drawn to different scales than the others, the units are the same as for the other panels and correspond directly to the coefficient values reported in Tables 1 and 2.

Mortgage Age Effects

Exhibit 1 shows that the underlying expected rates of prepayment by age are very similar for FRM and ARM loans. This has particular significance for assessing the validity of the options-based empirical model. As discussed above, mortgage age plays a dual role in capturing both "structural" duration-dependence in prepayment rates, as well as the "residual" impact of unobserved heterogeneity among borrowers that contributes to "spurious" duration dependence in prepayment rates.[37] The fact that the underlying age patterns for FRM and ARM loans are so similar suggests that most systematic differences have been explained by the other explanatory variables. The fact that the most influential variable is the mortgage premium representing the call option value of prepayment represents strong empirical support for the options model, and confirms the basic consistency of FRM and ARM borrower behavior. We will return to the discussion of the impact of the mortgage premium variables below.

Exhibit 2 shows that the underlying average rates of default are initially higher for ARMs, but they subsequent rise less rapidly with age and eventually decline more rapidly than the corresponding estimates for FRMs. The more rapid decline in the underlying age-specific conditional probabilities of default for ARM borrowers may indicate a greater level of unobserved heterogeneity in the determinants of mortgage default than exists for FRM borrowers, or additional differences related to the mortgage selection process.

The selection of an ARM contract over a FRM loan may signal a number of differences among borrowers not fully measured by the included explanatory variables. For example, ARM borrowers may have more limited time horizons for residential mobility, so they are less risk averse regarding future changes in interest rates. Alternatively, the lower initial coupons and monthly payments on ARM loans make it easier for lower income borrowers to qualify, and they may be more likely to default earlier in the life of the mortgage. The resulting age patterns of mortgage default indicate higher early defaults, and more rapid improvement in the underlying risk profile of the sample of ARM borrowers than occurs among FRM borrowers.

[37] The use of a quadratic function for age conveniently reduces the number of discrete parameters that must be estimated. We also experimented with estimating models that included separate categorical variables for each age, so that the age function was essentially unrestricted. The results of this analysis confirm that a quadratic age function fits almost perfectly for default, and somewhat less so for prepayment. In the case of prepayment, the unrestricted age function showed that prepayment probabilities would initially be closer to zero in the first year, then rise more rapidly over years 1 to 3, peak slightly sooner and higher around year 4, and then decline more or less linearly after year 5. Due to the small number of loans surviving to years 9 to 12, the coefficient estimates become highly variable at these outer years. The quadratic is generally consistent with the overall pattern of prepayment intensities and does not allow for erratic estimates at the higher ages, so that it smoothes the data in the way one would like. In future research we plan to experiment with alternatives such as combinations of cubic and linear splines that will allow more flexibility in the earlier ages and still smooth the estimates at the later ages.

The Impact of Option Values

The estimates for the option-related variables for FRMs and ARMs indicate that borrower behavior on these loans is consistent with the predictions of the options theory of prepayment and default. Conditional probabilities of prepayment are negatively related to the mortgage premium value (positively related to relative spreads), and conditional probabilities of default are positively related to the probability of negative equity. Conditional default rates are also negatively related to the mortgage premium values. Although the mortgage value was included as a component of borrower equity, we find that the mortgage premium value also has a direct impact on conditional default rates.[38] In addition, conditional prepayment rates are negatively related to the probability of negative equity, which is consistent with the expectation that borrowers will be less likely to refinance or sell their properties in declining markets.

The overall consistency of the independent estimates for FRM and ARM termination models provides strong empirical support for the options-based model. While the patterns of estimates across product types are quite similar, the multinomial logit estimates do indicate some differences in the responsiveness of FRM and ARM borrowers to comparable option value outcomes. For example, ARM prepayment and default probabilities appear to be more responsive to the likelihood of extreme outcomes for borrower equity than those of FRM borrowers, given the larger difference in the range of borrower equity coefficient estimates for ARM borrowers shown Table 1 and Exhibits 1 and 2. ARM prepayment and default probabilities are generally somewhat less sensitive to differences in the mortgage premium value, with the exception of the coefficient estimate for the probability of prepayment for loans in the most negative spread category (i.e., largest mortgage premium category). Again, this finding seems consistent with the hypothesis that borrowers with more limited time horizons for remaining in the home are more like to choose ARM contracts and will be less responsive to the value of the mortgage premium.

Other Interest Rate Variable Effects

There are some interesting differences in the impact of the yield curve slope and burnout variables on FRM and ARM prepayment and default probabilities. These variables have relatively smaller impacts on conditional probabilities of prepayment and default for ARM borrowers. This is also consistent with what one would expect from an options-theoretic viewpoint. For example, when confronted with an upward sloping yield curve, an FRM borrower has a financial incentive to switch to an ARM mortgage to take advantage of lower short-term rates offered by lenders. ARM borrowers, many of whom have coupon rates indexed to relatively short-term Treasury yields or other cost-of-funds indexes, already enjoy this advantage, and would be less likely to refinance. In addition, if borrowers select ARM mortgages because they have shorter time horizons for residential mobility, they have no incentive to incur the transactions costs associated with refinancing that would be required to take advantage of a steeply sloped yield curve.[39]

There appears to be no impact of path dependence in interest rates on ARM prepayment and default probabilities. The burnout variable is used primarily to distinguish borrowers who have failed to prepay when the prepayment option was previously in-the-money. ARM borrowers should be in this situation less frequently than FRM borrowers, due to the periodic adjustment of ARM coupons. In addition, as discussed above, even when market rates decline enough to produce significant spreads on ARM mortgages, the original selection of an ARM contract may indicate a limited time horizon, reducing the potential benefits of exercising the prepayment option on lifetime wealth, and making it less likely that ARM borrowers respond to prepayment opportunities.

Origination Year Effects

Both the FRM and ARM models produced a wide range of default coefficient estimates for the origination year dummies. This indicates that there are important factors associated with specific mortgage cohorts that are not completely captured by the two key

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[38] The finding of a direct effect of the mortgage premium on conditional default rates may be explained in part by the fact that we cannot measure borrower equity directly at the loan level.

options related variables. The range of origination year coefficient estimates for conditional default probabilities is greatest for ARMs, but mainly in the early years when this product was a relatively new innovation, and prior to standardization of ARM underwriting guidelines after 1985.

Origination year effects are generally smaller for prepayment, but still indicate that there is a great deal of variation in prepayment probabilities that is not explained by the options value of prepayment. One source of difference in the pattern of ARM prepayment origination year effects could be the timing of the introduction of different types of ARM contracts into the historical sample. Although information on different ARM products was used to provide accurate updating of ARM coupon rates, no attempt was made to account for the changing composition of the sample as different ARM product types were introduced to the market and began to appear in the sample.[40]

Original Loan-to-Value Ratios

The impact of LTV at origination is quite similar for the FRM and ARM models, showing a strong positive relationship between LTV and default, and very little impact on prepayment. This is consistent will numerous other studies and confirms the continued significant of this key underwriting factor on subsequent mortgage performance. The default coefficient estimates for LTV indicate a slight deterioration in performance in the 70-75 LTV category, which is consistent with less stringent screening of loans for which borrowers have 25-30 percent equity at origination.[41]

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[39] ARM borrowers have already revealed themselves as unwilling to pay higher fixed coupon rates to acquire the full benefit of the embedded prepayment option on a fixed-rate mortgage.

[40] Although the FRM and ARM samples cover different time periods, we do not believe this presents any problems given the inclusion of separate origination year dummies and additional contemporaneous explanatory variables specific to time periods in which loans are at risk of prepayment or default (e.g., equity, mortgage premium, burnout, seasonality, yield curve slope). We experimented with more flexible functional forms, such as estimating separate models for each origination cohort and interacting origination years with other explanatory variables. We found that these specifications tended to produce more erratic coefficient patterns and to highlight historical anomalies rather than identifying consistent patterns.

[41] A similar finding is reported in a recent analysis by MacDonald and Holloway (1996) comparing the relative early payment performance of balloon and FRM loans. The authors suggest that this result may be due to the presense of low-documentation loan products in the 71-75 percent LTV category.

Other Variables

Conditional probabilities of default vary significantly for loans secured by owner-occupied properties versus those for investor loans. The impact on ARM default rates is substantially larger than that estimated for FRMs. The impact of occupancy status on both FRM and ARM prepayments is quite small, but does indicate a tendency for owner-occupants to prepay at higher rates than investors.

Relative loan size appears to be much more important for explaining differences in prepayment rates than it is for default. Smaller loans are less likely to prepay regardless of product type, although the range of coefficient estimates is slightly greater for FRM loans. Surprisingly, relative loan size has only negligible impact on the probability of default for both FRM and ARM loans. Combined with the results for prepayment, this seems to suggest that, to the extent that loan size is a proxy for borrower income, it is more likely to be associated with differences in non-financial prepayments, such as those due to residential mobility, rather than an indicator of the ability of the household to avoid default.[42]

Both FRM and ARM loans exhibit seasonal variation in conditional probabilities of prepayment and default, although the range of coefficient estimates is greater for fixed-rate loans. If ARM borrowers were generally more likely to relocate, that being one reason for selecting an ARM rather than a FRM, then one would expect seasonal factors to be less important. If an ARM were selected in part because a move was anticipated, then this would be consistent with the attenuation of seasonal effects for ARM borrowers.

Summary and Conclusions

The empirical analysis used individual mortgage event histories to estimate a discrete-time hazards model for FRM and ARM performance based on a multinomial-logit specification for quarterly conditional probabilities of default and prepayment. The explanatory variables for both ARMs and FRMs comprised a variety of fixed-effects and time-varying covariates, including contemporaneous indicators of the

[42] This view is supported by recent research on securitization of low-income loans meeting the requirements of the Community Reinvestment Act (CRA). The relatively more stable prepayment speeds of CRA-eligible loans is cited as a feature increasing their value as a collateral for securitization. See Brown

current values of the embedded default and prepayment options. Information on individual ARM contract terms was used to reconstruct the dynamics of ARM coupons. Using this increased level of detail, the empirical analysis explored the relationship between FRM and ARM termination rates and the dynamics of the underlying options-related determinants of prepayment and default.

The estimated impacts of the options values on conditional probabilities of prepayment and default are generally quite similar across both FRM and ARM borrowers. We believe that this provides additional support for the basic predictions of the options theory, as it implies similar behavior by FRM and ARM borrowers when faced with the same financial incentives. When FRM and ARM behavior differs, it is mainly with respect to other fixed-effects. In many cases, these differences can be related to the alternative motivations of FRM and ARM borrowers in the mortgage selection process.

What are the potential implications of these findings for the pricing of FRM and ARM securities? One implication of the similarities between the FRM and ARM estimates is that in the absence of historical data on ARM loan performance one could apply a model of FRM performance combined with realistic modeling of changes in ARM coupons under different interest rate scenarios. Our estimates confirm that the underlying options-related incentives to default and prepay are very similar, so that one would expect very little difference in valuations based on either set of model estimates. This does not imply that ARM and FRM securities would receive similar prices under the same assumptions about future changes in housing values and interest rates, only that the underlying propensities to default or prepay will be similar when individual borrowers are faced with the same financial incentives.

We have attempted to assess the significance of options-related determinants of prepayment and default outcomes in the context of an empirical model that must accommodate the existence of other economic and demographic events and observed and unobserved heterogeneity among individual borrowers (i.e., violations of the assumptions of a pure options model). While the results support the basic predictions of the options theory, differences in estimates of conditional probabilities of prepayment and default

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and Westhoff (1998).

associated with mortgage age, year of origination, original LTV, and relative loan size, indicate the continued significance of these other processes for empirical models of mortgage terminations.

This paper has attempted to advance the state of empirical research on mortgage terminations by modeling FRM and ARM loans with comparable levels of precision. We report empirical estimates based on a large, temporally and geographically diverse sample of historical mortgage experience, supplemented by detailed characteristics on the dynamics of ARM coupons rates. Like many other empirical studies of mortgage terminations, the analysis was subject to limitations in our ability to measure the option values of prepayment and default. These limitations notwithstanding, the results support the application of similar options-based empirical models to the analysis of FRM and ARM loan performance.

Table 1
Comparison of Multinomial Logit Parameter Estimates for Quarterly Conditional Prepayment and Default Probabilities

	30-Year FRM		ARM	
Explanatory Variables	Prepayment	Default	Prepayment	Default
Constant	-4.795	-6.873	-4.932	-5.916
	(0.00)	(0.00)	(0.00)	(0.00)
Age	0.094	0.113	0.090	0.065
	(0.00)	(0.00)	(0.00)	(0.00)
Age Squared	-0.002	-0.002	-0.002	-0.002
	(0.00)	(0.00)	(0.00)	(0.00)
1979	-0.201	0.273		
	(0.00)	(0.00)		
1980	-0.161	0.402		
	(0.00)	(0.00)		
1981	0.238	0.970		
	(0.00)	(0.00)		
1982	0.347	0.853	0.531	1.393
	(0.00)	(0.00)	(0.00)	(0.00)
1983	0.030	0.394	0.079	1.213
	(0.02)	(0.00)	(0.00)	(0.00)
1984	0.134	0.202	-0.412	0.705
	(0.00)	(0.00)	(0.00)	(0.00)
1985	0.026	-0.103	-0.130	0.390
	(0.00)	(0.03)	(0.00)	(0.00)
1986	-0.221	-0.496	-0.908	-0.995
	(0.00)	(0.00)	(0.00)	(0.00)
1987	-0.245	-0.559	-0.044	-0.285
	(0.00)	(0.00)	(0.00)	(0.00)
1988	-0.067	-0.437	-0.036	-0.114
	(0.00)	(0.00)	(0.00)	(0.00)
1989	0.033	0.148	-0.114	-0.170
	(0.00)	(0.00)	(0.00)	(0.00)
1990	0.089	0.250	0.340	0.369
	(0.00)	(0.00)	(0.00)	(0.00)
1991	0.225	-0.051*	0.472	0.062*
	(0.00)	(0.27)	(0.00)	(0.13)
1992	0.025	-0.584	0.241	-0.707
	(0.00)	(0.00)	(0.00)	(0.00)
1993	-0.251	-1.263	-0.009	-1.977

(continued on following page)

Table 1 (continued)
Comparison of Multinomial Logit Parameter Estimates for
Quarterly Conditional Prepayment and Default Probabilities

Explanatory Variables	30-Year FRM Prepayment	30-Year FRM Default	ARM Prepayment	ARM Default
LTV ≤ 60	0.134	-1.348	0.094	-1.310
	(0.00)	(0.00)	(0.00)	(0.00)
60 < LTV ≤ 70	0.024	-0.090	-0.017	-0.260
	(0.00)	(0.03)	(0.00)	(0.00)
70 < LTV ≤ 75	-0.077	0.416	-0.100	0.257
	(0.00)	(0.00)	(0.00)	(0.00)
75 < LTV ≤ 80	-0.000*	0.197	-0.059	0.209
	(0.90)	(0.00)	(0.00)	(0.00)
80 < LTV ≤ 90	-0.017	0.309	0.082	0.448
	(0.00)	(0.00)	(0.00)	(0.00)
90 < LTV ≤ 100	-0.063	0.516	0.000	0.656
PNEQ ≤ 0.05	0.412	-1.147	0.700	-1.052
	(0.00)	(0.00)	(0.00)	(0.00)
0.05 < PNEQ ≤ 0.10	0.200	-0.222	0.264	-0.317
	(0.00)	(0.00)	(0.00)	(0.00)
0.10 < PNEQ ≤ 0.15	0.149	-0.056*	0.060	-0.182
	(0.00)	(0.11)	(0.00)	(0.00)
0.15 < PNEQ ≤ 0.20	0.086	-0.031*	0.046	-0.036*
	(0.00)	(0.47)	(0.00)	(0.21)
0.20 < PNEQ ≤ 0.25	-0.078	0.191	-0.010*	0.165
	(0.00)	(0.00)	(0.53)	(0.00)
0.25 < PNEQ ≤ 0.30	-0.143	0.414	-0.158	0.314
	(0.00)	(0.00)	(0.00)	(0.00)
0.30 < PNEQ ≤ 0.35	-0.150	0.376	-0.289	0.451
	(0.00)	(0.00)	(0.00)	(0.00)
0.35 < PNEQ	-0.475	0.475	-0.613	0.658
MP ≤ -0.20	-1.101	-0.326	-1.335	-0.233
	(0.00)	(0.00)	(0.00)	(0.00)
-0.20 < MP ≤ -0.10	-0.704	-0.147	-0.506	0.028*
	(0.00)	(0.00)	(0.00)	(0.18)
-0.10 < MP ≤ 0.0	-0.605	-0.377	-0.279	-0.029*
	(0.00)	(0.00)	(0.00)	(0.11)
0.0 < MP ≤ 0.10	-0.432	-0.283	-0.031	-0.169
	(0.00)	(0.00)	(0.00)	(0.00)
0.10 < MP ≤ 0.20	0.585	0.077	0.362	-0.051
	(0.00)	(0.00)	(0.00)	(0.00)
0.20 < MP ≤ 0.30	1.093	0.451	0.837	0.150
	(0.00)	(0.00)	(0.00)	(0.00)
0.30 < MP	1.163	0.605	0.901	0.304

(continued on following page)

Table 1(continued)
Comparison of Multinomial Logit Parameter Estimates for Quarterly Conditional Prepayment and Default Probabilities

Explanatory Variables	30-Year FRM Prepayment	30-Year FRM Default	ARM Prepayment	ARM Default
CMT10/CMT1 < 1.0	-0.224	0.239	0.134	0.001*
	(0.00)	(0.00)	(0.00)	(0.98)
1.0 ≤ CMT10/CMT1 <1.2	-0.261	0.261	-0.182	0.083
	(0.00)	(0.00)	(0.00)	(0.00)
1.2 ≤ CMT10/CMT1 < 1.5	0.038	-0.083	-0.068	-0.002*
	(0.00)	(0.00)	(0.00)	(0.90)
1.5 ≤ CMT10/CMT1	0.447	-0.517	0.116	-0.082
Burnout (No Chance to Refi)	0.143	-0.272	0.071	-0.026*
	(0.00)	(0.00)	(0.00)	(0.05)
Burnout (Missed Chance to Refi)	-0.143	0.272	-0.071	0.026
Winter	-0.152	-0.128	-0.104	-0.056
	(0.00)	(0.00)	(0.00)	(0.00)
Spring	0.148	0.023*	0.044	0.035
	(0.00)	(0.26)	(0.00)	(0.01)
Summer	-0.008	-0.044	-0.009	0.023*
	(0.01)	(0.03)	(0.01)	(0.08)
Fall	0.012	0.145	0.069	-0.001
Investor	-0.126	0.272	-0.121	0.647
	(0.00)	(0.00)	(0.00)	(0.00)
Owner-Occupant	0.126	-0.272	0.121	-0.647
Loan Size ≤ 0.40	-0.616	0.017*	-0.389	-0.134*
	(0.00)	(0.76)	(0.00)	(0.10)
0.40 < Loan Size ≤ 0.60	-0.339	-0.035*	-0.290	0.074
	(0.00)	(0.28)	(0.00)	(0.01)
0.60 < Loan Size ≤ 0.75	-0.119	-0.078	-0.139	0.051
	(0.00)	(0.01)	(0.00)	(0.03)
0.75 < Loan Size ≤ 1.00	0.066	-0.079	0.005*	0.007*
	(0.00)	(0.00)	(0.29)	(0.73)
1.00 < Loan Size ≤ 1.25	0.221	-0.040*	0.171	0.027*
	(0.00)	(0.10)	(0.00)	(0.17)
1.25 < Loan Size ≤ 1.50	0.334	0.078	0.279	0.021*
	(0.00)	(0.01)	(0.00)	(0.35)
1.50 < Loan Size	0.452	0.137	0.363	-0.046

Note: All models were estimated by the method of maximum likelihood using the SAS® CATMOD procedure. Empirical p-values are shown in parentheses. An asterisk indicates that the coefficient is not statistically significant from zero at the 5-percent level for an asymptotic-normal test. CATMOD treats the last category for each variable grouping as the omitted category and computes the coefficient estimate for this category as the negative sum of the other coefficients for that variable group, and no standard error estimate is reported for this category.

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Exhibit 1
Multinomial Logit Prepayment Parameter Estimates

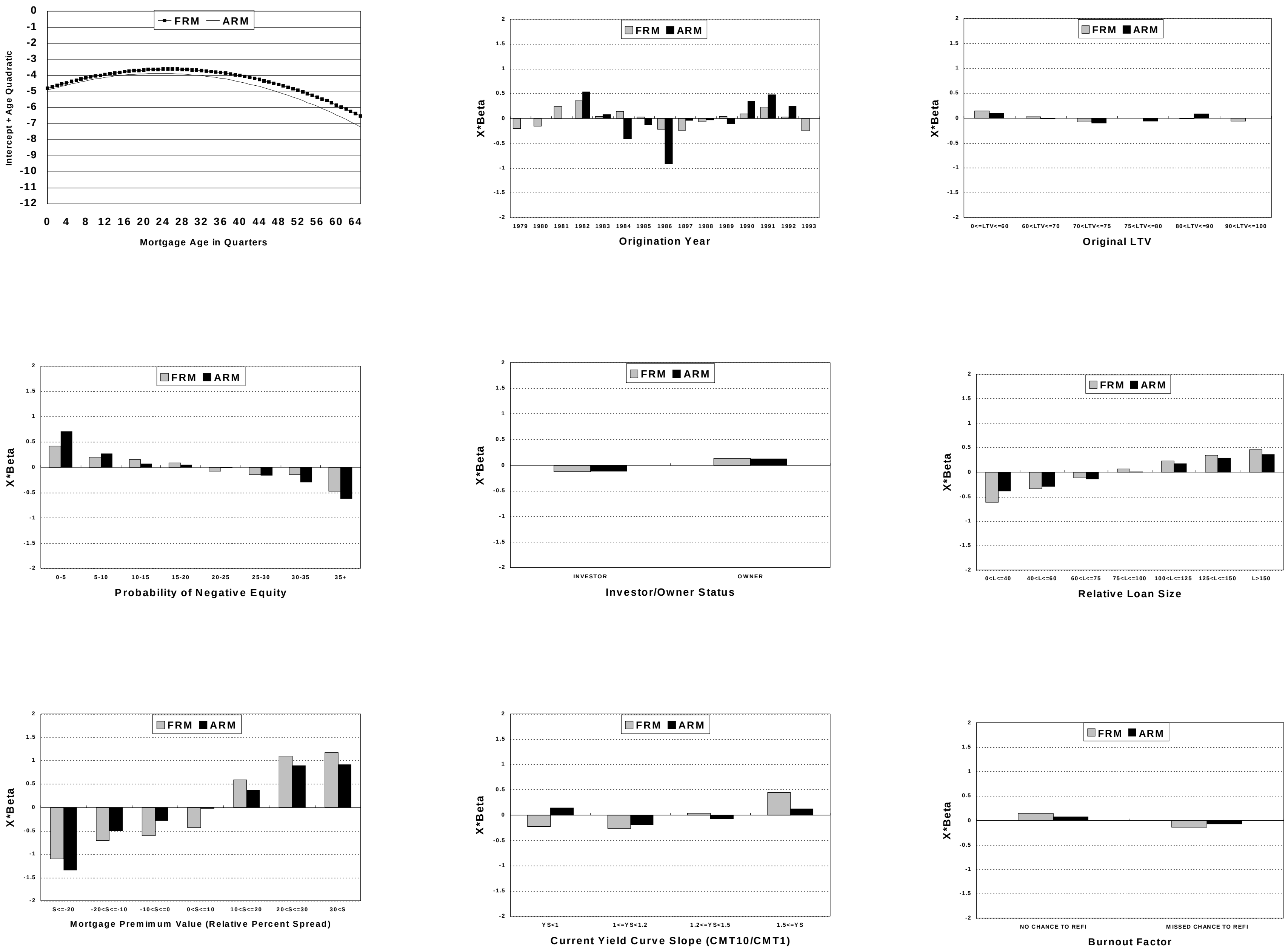

Exhibit 2
Multinomial Logit Default Parameter Estimates