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Abstract

Within a unified framework, the author conducts an empirical investigation of dynamic interrelationships among inflation, inflation uncertainty, relative price dispersion, and output growth. Focusing on the Canadian industrial sector, the author finds weak evidence that inflation uncertainty rises with the level of inflation, with short-run inflation uncertainty minimized at a trend inflation rate of approximately 3 per cent. With regard to the predictions of menu-cost and signal-extraction models, evidence that relative price dispersion rises with both trend inflation and inflation uncertainty is obtained. However, the significance of this evidence varies according to whether a weighted or unweighted measure of relative price dispersion is employed, in addition to the symmetry characteristics of the process governing the evolution of inflation uncertainty. The primary result is that across a variety of different model specifications, inflation uncertainty significantly lowers output growth, an effect of considerable size and duration.

JEL classification: E3; E31
Bank classification: Inflation and prices

Résumé

Dans un cadre empirique unifié, l'auteur étudie les liens dynamiques entre l'inflation, l'incertitude entourant celle-ci, la dispersion des prix relatifs et la croissance de la production, en mettant l'accent sur le secteur industriel canadien. D'après certains des résultats qu'il présente, la hausse du niveau de l'inflation ferait augmenter l'incertitude de l'inflation, l'incertitude touchant un creux en courte période lorsque l'inflation tendancielle avoisine les 3 %; cependant, les résultats en question sont peu significatifs. En ce qui concerne les prévisions des modèles à coûts d'étiquetage et des modèles à signaux brouillés, l'auteur constate que la dispersion des prix relatifs s'accroît à la fois avec l'inflation tendancielle et avec l'incertitude de l'inflation. Le degré de signification de ce résultat varie toutefois selon que l'on utilise une mesure pondérée ou non de la dispersion des prix relatifs; le caractère symétrique ou non du processus régissant l'évolution de l'incertitude de l'inflation tire aussi à conséquence. Le principal point à retenir est que, dans un vaste éventail de spécifications mises à l'essai, l'incertitude entourant l'inflation a un effet de réduction important, persistant et significatif sur la croissance de la production.

Classification JEL : E3; E31
Classification de la Banque : Inflation et prix

1. Introduction

In his Nobel address, Friedman (1977) argues that inflation uncertainty rises with the level of inflation, and that increased inflation uncertainty reduces economic efficiency via the distortion of price signals, at least during some transitional period of adaptation. Moreover, by confounding expectations of the present value of nominal cash flows, inflation uncertainty leads optimizing economic agents to make savings and investment decisions that are generally revealed to be suboptimal ex post. Clearly, such distortions may exert adverse effects on the efficiency of resource allocation and the level of real economic activity.

A closely related channel through which inflation may induce welfare-diminishing resource misallocation is its influence on the variability of relative prices. Menu-cost models generally imply that relative price dispersion rises with trend inflation. In contrast, signal-extraction models predict increased relative price dispersion in response to increased inflation uncertainty. To the extent that inflation uncertainty rises with the level of inflation, joint consideration of these closely interrelated effects is necessary in order to isolate their individual contributions to relative price dispersion.

The insight due to Friedman (1977) that inflation uncertainty rises with the level of inflation is formalized in the innovative model of Ball (1992), who derives a positive relationship between inflation uncertainty and the level of inflation driven by uncertainty concerning the monetary policy regime. Essentially, his model predicts that during periods of low inflation, the central bank will promote a constant inflation rate, leading to low inflation uncertainty. In contrast, during high inflation periods, the central bank may be reluctant to tighten monetary policy because of the associated temporary output and employment costs. Clearly, this potentially asymmetric monetary policy response implies a positive relationship between the level of inflation and inflation uncertainty.

Even if inflation is fully anticipated, the existence of fixed costs of changing prices implies discontinuous price setting. In the menu-cost models of Sheshinski and Weiss (1979, 1983), Ball and Romer (1993), Rotemberg (1983), Benabou (1992), and Diamond (1993), among others, firms maintain fixed nominal prices so long as real prices lie within nondegenerate intervals. Within this framework, variation across firms in the administrative and resource costs of implementing price changes relative to the costs of maintaining suboptimal prices implies asynchronization in price setting under non-zero trend inflation. As the resultant relative price changes reflect nominal rigidities as opposed to fundamental resource scarcity, resource misallocation is induced.

Strictly speaking, the price-adjustment rules underlying the positive relationship between trend inflation and relative price dispersion associated with typical menu-cost models are optimal only under a constant positive rate of inflation. However, Jaramillo (1999) suggests that the effect of trend inflation on relative price dispersion is inherently asymmetric. In particular, he argues that trend deflation increases relative price dispersion to a greater extent than does trend inflation of equal magnitude.

In an extension of the Lucas (1972) signal-extraction model of relative price deduction from imperfect information, Barro (1976) derives a positive relationship between the variance of unexpected inflation and relative price dispersion. Essentially, when the noise component of observed prices is highly variable, individual firms attribute changes in output prices primarily to changes in the aggregate price level. Consequently, expected changes in relative prices are biased downwards in magnitude, and firms adjust output less in response to all shocks, including idiosyncratic demand shocks. This reduced price elasticity of supply in response to increased inflation uncertainty implies an increase in the variability of relative prices. The consumer search signal-extraction model of Benabou and Gertner (1993), in which output markets are imperfectly competitive while information gathering is endogenous, also predicts increased relative price dispersion in response to increased inflation uncertainty.

Within a unified empirical framework, we investigate these closely interrelated hypotheses concerning inflation, inflation uncertainty, relative price dispersion and output growth. Our analysis is based on the generalized autoregressive conditional heteroscedasticity (GARCH) class of models introduced by Engle (1982) and Bollerslev (1986), which facilitate the joint estimation of parametric conditional mean and conditional covariance functions. In particular, we employ the multivariate GARCH in mean or GARCH–M model introduced by Bollerslev, Engle, and Wooldridge (1988), which allows for direct relationships linking conditional first and second moments. Focusing on the Canadian industrial sector, we find weak evidence that inflation uncertainty rises with the level of inflation, with short-run inflation uncertainty minimized at a trend inflation rate of approximately 3 per cent. With regard to the predictions of menu-cost and signal-extraction models, evidence that relative price dispersion rises with both trend inflation and inflation uncertainty is obtained. However, the significance of this evidence varies according to whether a weighted or unweighted measure of relative price dispersion is employed, in addition to the symmetry characteristics of the process governing the evolution of inflation uncertainty. Our primary result is that across a variety of different model specifications, inflation uncertainty significantly lowers output growth, an effect of considerable size and duration.

The organization of this paper is as follows. The next section discusses properties of the data and comments on the sources of outliers. Section 3 develops the empirical framework, presents estimation results, and describes the impulse-response analysis. Finally, section 4 offers conclusions and recommendations for further research.

2. Properties of the Data

In modelling dynamic interrelationships among inflation, inflation uncertainty, relative price dispersion, and output growth in Canada, we identify the foreign economy as the United States, an empirically reasonable first-order approximation. A relatively small open economy, Canada is inseparably linked to the United States, both economically and geographically. Indeed, bilateral trade with the United States accounts for a substantial majority of Canadian imports and exports, while macroeconomic fluctuations originating in the United States dramatically influence the Canadian business cycle.

Reflecting these empirical facts, the data set consists of several domestic and foreign macroeconomic variables observed with monthly frequency over the period 1961:01 through 2001:04 inclusive. In particular, the domestic price level, P, corresponds to the seasonally adjusted industrial producer price index for Canada, while the foreign price level, P^f, is represented by the seasonally adjusted industrial producer price index for the United States. Furthermore, real industrial production, Y, is seasonally adjusted and measured at factor cost, while the corresponding measure of potential output, Y^P, is calculated with the linear filter described in Hodrick and Prescott (1997). Finally, the price of oil, P^{OIL}, is measured by the foreign currency price of West Texas Intermediate crude oil expressed as a period average, while the nominal exchange rate, S, pertains to spot transactions and is expressed as the period average price of foreign currency in terms of domestic currency. All price and output indexes were retrieved from the CANSIM database maintained by Statistics Canada, the price of oil was obtained from the West Texas Research Group, and the nominal exchange rate was extracted from the International Financial Statistics database maintained by the International Monetary Fund.

We adopt as our measure of relative price dispersion the standard deviation of inflation rates across an exhaustive set of 139 industries. To assess the sensitivity of our empirical results to alternative variable definitions, both weighted and unweighted measures of relative price dispersion are constructed. Our weighted measure of relative price dispersion is defined as follows, where industry weights $w_{i,t} \equiv P_{i,t-1} Y_{i,t-1} / \sum_{j=1}^{N_t} P_{j,t-1} Y_{j,t-1}$ correspond to nominal output shares that sum to unity by construction:

$$WRPD_t \equiv \sqrt{\sum_{i=1}^{N_t} w_{i,t} \left(\Delta \ln P_{i,t} - \sum_{j=1}^{N_t} w_{j,t} \Delta \ln P_{j,t} \right)^2} . \tag{1}$$

Owing to data availability constraints, summations are taken over the maximum number of industries, N_t, for which observations are available, which is typically less than $N_T = 139$.[1] All price indexes are seasonally unadjusted, while all output indexes are seasonally adjusted.

Our unweighted measure of relative price dispersion imposes equal weights across industries, which vary with time to reflect time variation in the number of industries for which observations are available:

$$URPD_t \equiv \sqrt{\frac{1}{N_t} \sum_{i=1}^{N_t} \left(\Delta \ln P_{i,t} - \frac{1}{N_t} \sum_{j=1}^{N_t} \Delta \ln P_{j,t} \right)^2} . \tag{2}$$

Maximum disaggregation is pursued in order to effectively capture the variability of relative prices across industries. Brief descriptions of the specific industries under consideration are available on request.

Line graphs of our alternative measures of relative price dispersion versus contemporaneous trend inflation appear below. To filter out short-run volatility while preserving the underlying trend of inflation, the seasonal as opposed to first logarithmic difference of the aggregate price level is employed, the former being proportional to a one-year moving average of the latter. Visual inspection reveals that both our weighted and unweighted measures of relative price dispersion tend to rise with the level of inflation. Indeed, both measures of relative price dispersion exhibit pronounced and sustained increases during the mid 1970s and early 1980s, periods of general macroeconomic instability punctuated by high and volatile inflation. While these visual impressions are consistent with the predictions of both menu-cost and signal-extraction models,

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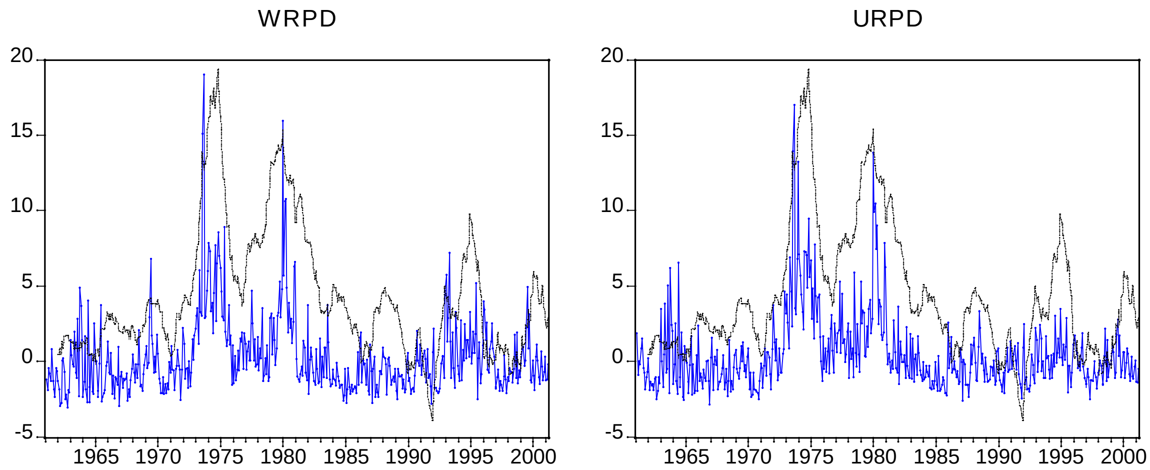
1. Allowing the industry coverage of our alternative relative price dispersion measures to vary with data availability has little quantitative effect. To elaborate, the correlation between weighted measures of relative price dispersion based on balanced and unbalanced panels of 50 and 139 industries is 0.94. Furthermore, the correlation between unweighted measures of relative price dispersion based on balanced and unbalanced panels of 73 and 139 industries is 0.95.

further analysis is required to determine whether trend inflation and inflation uncertainty cause relative price dispersion.

Figure 1: Relative Price Dispersion versus Trend Inflation



Note: Relative price dispersion is represented by solid lines, while dashed lines depict trend inflation.

3. Empirical Analysis

Menu-cost models predict that relative price dispersion rises with trend inflation, whereas signal-extraction models give precedence to the effect of inflation uncertainty. Indeed, in the basic signal-extraction model, trend inflation exerts no effect on the distribution of relative prices. To the extent that inflation uncertainty rises with the level of inflation, joint consideration of these closely interrelated effects is necessary in order to isolate their individual contributions to relative price dispersion. Yet the existing empirical literature on these issues either confuses or fails to distinguish between these distinct testable hypotheses of menu-cost and signal-extraction models. A notable exception is Grier and Perry (1996), who employ a bivariate GARCH-M framework to simultaneously test both hypotheses. On the basis of a variety of different model specifications and sample periods in the United States, they find that inflation uncertainty dominates trend inflation as a predictor of relative price dispersion.

The joint hypothesis advanced by Friedman (1977), that inflation uncertainty rises with the level of inflation and reduces the level of real economic activity, has been the subject of extensive empirical investigation. As discussed in a survey paper by Holland (1993), proxy measures of inflation uncertainty derived from survey data or time-varying parameter models tend to be positively correlated with the level of inflation and negatively correlated with output growth in the United States. Congruent results have recently been obtained with proxy measures of inflation uncertainty derived from fixed-parameter GARCH models. Indeed, in an empirical investigation of the hypothesis that inflation uncertainty rises with the level of inflation, Grier and Perry (1998) find unanimous support across the Group of Seven industrial nations. Furthermore, within a bivariate GARCH-M framework, Grier and Perry (2000) find that inflation uncertainty significantly lowers output growth across a number of different model specifications and sample periods in the United States.

In an extension of the analysis of Grier and Perry (1996, 1998, 2000), we analyze dynamic interrelationships among inflation, inflation uncertainty, relative price dispersion, and output growth in Canada. In the pursuit of efficiency gains in estimation and increased test power, the conditional means, variances and covariances of inflation, relative price dispersion, and output growth are simultaneously modelled within a trivariate GARCH-M framework. To determine whether our results are robust to the information set upon which our proxy measure of inflation uncertainty is conditional, both autoregressive and reduced-form Phillips curve specifications of the conditional mean of inflation are considered. Our statistical inference employs procedures that are robust to departures from conditional multivariate normality, while both symmetric and asymmetric multivariate GARCH specifications of the conditional covariance function are employed.

3.1 Specification of the conditional mean function

In estimating the conditional mean of inflation, we follow the majority of researchers in this area and model expectations formation as a fixed parameter autoregressive process:

$$\Delta \ln P_t = \mu_1 + \sum_{i=1}^{p_t} \phi_{1,i} \Delta \ln P_{t-i} + \varepsilon_{1,t}. \qquad (3)$$

Theoretical support for this univariate forecasting framework is provided by the decomposition theorem derived by Wold (1938), which states that any covariance stationary purely linearly indeterministic stochastic process has an infinite-order moving-average representation. As discussed in Hamilton (1994), any infinite-order moving-average process can be approximated to any required degree of accuracy by an autoregressive process of sufficient order. Typically, a relatively low-order autoregressive process is found to provide an empirically adequate representation.

As noted by Crawford and Kasumovich (1996), by excluding all information apart from the history of inflation, an autoregressive model may ignore additional information employed by economic agents in forming expectations of inflation. In recognition of this myopia, we also consider the following reduced-form Phillips curve model:

$$\Delta \ln P_t = \mu_t + \sum_{i=1}^{p_t} \phi_{1,i} \Delta \ln P_{t-i} + \theta_{1,1} \ln \frac{Y_{t-1}}{Y_{t-1}^P} + \theta_{1,2} \Delta \ln (S_{t-1} P_{t-1}^f) + \theta_{1,3} \Delta \ln \frac{P_{t-1}^{OIL}}{P_{t-1}^f} + \varepsilon_{1,t} . \quad (4)$$

As specified, demand-side pressures enter via the output gap, while supply-side pressures are captured by import price inflation and real oil price inflation. We note that the relative complexity of this model of the conditional mean of inflation, which implicitly conditions on information unavailable to economic agents at the forecast origin, may give rise to the underestimation of inflation uncertainty.

The conditional mean of relative price dispersion is modelled as an autoregressive process augmented by lagged absolute trend inflation in a potentially asymmetric manner

$$RPD_t = \mu_2 + \sum_{i=1}^{p_2} \phi_{2,i} RPD_{t-i} + \theta_{2,2} |\Delta_s \ln P_{t-1}| + \theta_{2,3} I_{t-1} |\Delta_s \ln P_{t-1}| + \varepsilon_{2,t} , \qquad (5)$$

where I_t is an indicator function that activates if and only if trend inflation is negative. Given that trend inflation and relative price dispersion are simultaneously determined, lagged as opposed to contemporaneous trend inflation is considered as a potential explanatory factor throughout in order to facilitate consistent parameter estimation while abstracting from the effects of transitory commodity price shocks.

Motivated by the signal-extraction model of Lucas (1972), the conditional mean of output growth is modelled as an autoregressive process augmented by unexpected inflation:

$$\Delta \ln Y_t = \mu_3 + \sum_{i=1}^{p_3} \phi_{3,i} \Delta \ln Y_{t-i} + \theta_{3,1} \varepsilon_{1,t-1} + \varepsilon_{3,t} . \qquad (6)$$

As specified, the effects of inflation shocks on output growth are restricted to be symmetric and proportional to their size. Given that unexpected inflation and inflation uncertainty are intimately related, it is important to condition on unexpected inflation in estimating the partial effect of inflation uncertainty on output growth.

Consider the following trivariate restricted vector autoregressive moving-average (VARMA) model, which nests both autoregressive and reduced-form Phillips curve specifications of the conditional mean of inflation:

$$\Delta \ln P_t = \mu_1 + \sum_{i=1}^{p_1} \phi_{1,i} \Delta \ln P_{t-i} + \theta_{1,1} \ln \frac{Y_{t-1}}{Y_{t-1}^P} + \theta_{1,2} \Delta \ln (S_{t-1} P_{t-1}^f) + \theta_{1,3} \Delta \ln \frac{P_{t-1}^{OIL}}{P_{t-1}^f} + \varepsilon_{1,t},$$

$$RPD_t = \mu_2 + \sum_{i=1}^{p_2} \phi_{2,i} RPD_{t-i} + \theta_{2,2} |\Delta_S \ln P_{t-1}| + \theta_{2,3} I_{t-1} |\Delta_S \ln P_{t-1}| + \varepsilon_{2,t},$$

$$\Delta \ln Y_t = \mu_3 + \sum_{i=1}^{p_3} \phi_{3,i} \Delta \ln Y_{t-i} + \theta_{3,1} \varepsilon_{1,t-1} + \varepsilon_{3,t}.$$

As specified, a positive and statistically significant estimate of $\theta_{2,2}$ would suggest that relative price dispersion rises with absolute trend inflation as predicted by menu-cost models, while a positive and statistically significant estimate of $\theta_{2,3}$ would suggest that negative trend inflation increases relative price dispersion to a greater extent than does positive trend inflation of equal magnitude.

We estimate our VARMA models of the conditional means of inflation, relative price dispersion, and output growth by full-information maximum likelihood assuming a conditional multivariate normal error distribution. To preclude residual autocorrelation, the autoregressive lag orders are set equal to the seasonal frequency throughout. The conditional loglikelihood function is maximized with the Marquardt algorithm employing two-sided numeric-first derivatives. This algorithm modifies the popular BHHH algorithm of Berndt, Hall, Hall, and Hausman (1974) with the addition of a correction matrix or ridge factor to the Hessian approximation. In addition to generally increasing the rate of convergence, this ridge correction mitigates occasional matrix inversion problems associated with near singularity of the outer product of the gradient. Robust standard errors are calculated with the heteroscedasticity-consistent coefficient covariance matrix estimator derived by Bollerslev and Wooldridge (1992). Estimation results appear in Tables A.1 through A.4 of the Appendix, with asymptotic *t*-ratios reported in parentheses.

Consistent with the predictions of menu-cost models, our estimation results suggest that trend inflation significantly raises relative price dispersion when considered in isolation. However, contrary to the hypothesis advanced by Jaramillo (1999), we find little or no evidence that trend deflation increases relative price dispersion to a greater extent than does trend inflation of equal magnitude. Indeed, to the extent that the relationship between trend inflation and relative price dispersion is asymmetric, our results suggest that trend inflation increases relative price dispersion, while trend deflation exerts no effect.

Under empirically adequate specifications, the ordinary residuals derived from our estimated VARMA models should exhibit no discernible autocorrelation or autoregressive conditional heteroscedasticity. To examine whether these properties are approximately satisfied, we subject the levels, squares, and cross products of the ordinary residuals to Ljung-Box (1978) tests for autocorrelation up to integer multiples of the seasonal frequency. These tests for autocorrelation in the squares and cross products of the ordinary residuals may be interpreted as tests for autoregressive conditional heteroscedasticity. Indeed, Granger and Teräsvirta (1993) show that the Ljung-Box (1978) test for autocorrelation in the squared ordinary residuals is asymptotically equivalent to the Lagrange multiplier test for autoregressive conditional heteroscedasticity derived by Engle (1982). We also examine whether there exist significant departures from normality with the test of Jarque and Bera (1980), which compares the third and fourth moments to those of the normal distribution.

We find little or no evidence of residual autocorrelation, suggesting that our estimated VARMA models adequately describe the dynamic evolution of the conditional means of the variables under consideration. However, there exists abundant evidence of autoregressive conditional heteroscedasticity, as evidenced by significant autocorrelation among the squares and cross products of the ordinary residuals. Moreover, some evidence of non-normality is found, in part attributable to the existence of residual leptokurtosis. Given that GARCH models can explain such leptokurtosis, our residual diagnostic test results suggest that the extension of our VARMA models to explicitly incorporate GARCH errors is warranted.

3.2 Specification of the conditional covariance function

To establish some preliminaries, under uncertainty, the probability density function associated with an event is subjective and conditional on a learning process. More precisely, uncertainty concerns the degree to which a variable is unpredictable, a concept related to conditional second and potentially higher moments.

There exists an extensive empirical literature concerned with the quantification of uncertainty, and a wide variety of alternative parametric and nonparametric measures have been proposed. Prior to the advent of the GARCH class of models, perhaps the two most commonly used measures were the cross-sectional dispersion of individual forecasts derived from surveys, and a rolling standard deviation of the variable under consideration.

Survey-based measures provide an indication of the heterogeneity of expectations across forecasters at a point in time. However, since such measures ignore the uncertainty associated with individual forecasts, they may fail to adequately approximate actual uncertainty. In addition, survey data pertaining to the variable under consideration may be limited or nonexistent.

A rolling standard deviation measure captures time variation in unconditional volatility. However, if volatility is to some extent predictable, then a rolling standard deviation measure will tend to overestimate uncertainty. Furthermore, this upward estimation bias may itself be time-varying, and the subsequent incorporation of uncertainty proxy measures so derived into econometric specifications may give rise to errors in variables issues. Finally, it is both logically inconsistent and statistically inefficient to employ a volatility measure that implicitly assumes constant volatility over some interval of time when the resultant series varies with time.

Contemporary research involving the quantification and analysis of uncertainty typically employs the GARCH class of models, which facilitate the joint estimation of parametric conditional mean and conditional covariance functions. Since the seminal works of Engle (1982) and Bollerslev (1986), a proliferation of alternative univariate and multivariate GARCH specifications has arisen. In addition to potentially offering arbitrarily large efficiency gains in estimation, such models facilitate the consistent estimation of correctly specified relationships linking conditional first and second moments. In contrast, Pagan and Ullah (1988) show that a two-stage procedure in which uncertainty proxy measures are generated outside of the model of interest typically results in inconsistent parameter estimation.

A general multivariate GARCH model for N dimensional innovation vector ε_t is given by $\varepsilon_t = H_t^{1/2} z_t$, where z_t is an N dimensional vector of independent standard normal random variables. It follows that $\varepsilon_t | I_{t-1} \sim N(0, H_t)$, where I_{t-1} denotes the information set available at time $t-1$. To complete the model, parameterization of the process governing the evolution of symmetric and positive definite conditional covariance matrix H_t is required. Since the sheer number of parameters associated with a general parameterization is overwhelming, all useful specifications must necessarily restrict the dimension of the parameter space. However, such restrictions may impose important untested characteristics on the process governing the evolution of the conditional covariance matrix.

The constant conditional correlations (CCORR) model of Bollerslev (1990), in which the conditional covariances between pairs of innovations are proportional to the product of the corresponding conditional standard deviations, while the individual conditional variances are usually constrained to follow univariate GARCH processes, is often employed as an empirically reasonable working hypothesis:

$$h_{ii,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1},$$

$$h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}}. \tag{7}$$

As specified, conditional variance forecasts are linear in current conditional variances, while squared innovations drive forecast revisions. The conditional covariance matrix is positive-definite if and only if the conditional correlations, ρ_{ij}, render a positive definite correlation matrix and the individual conditional variances are all positive. This requires $\omega_i > 0$, $\alpha_i \geq 0$ and $\beta_i \geq 0$ while identification of β_i requires $\alpha_i > 0$ for all $i = 1, \ldots, N$. The CCORR model is covariance stationary if and only if $\alpha_i + \beta_i < 1$ for all $i = 1, \ldots, N$.

Since the CCORR model restricts conditional variances to respond symmetrically to positive and negative innovations of equal magnitude, it may be inappropriate for purposes of quantifying inflation uncertainty, which is commonly thought to depend on perceptions of the monetary policy regime. In particular, Brunner and Hess (1993) and Joyce (1995) argue that in the presence of monetary policy regime uncertainty, positive unexpected inflation should increase inflation uncertainty to a greater extent than negative unexpected inflation of equal magnitude.

Intuitively, we expect inflation uncertainty to be minimized in the presence of zero unexpected inflation. The asymmetric CCORR or ACCORR model of Kroner and Ng (1998) allows for asymmetric uncertainty response while satisfying this property, where $\eta_{i,t} \equiv \max(0, \varepsilon_{i,t})$:

$$h_{ii,t} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \gamma_i \eta_{i,t-1}^2 + \beta_i h_{ii,t-1},$$

$$h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}}. \tag{8}$$

This asymmetric extension of the CCORR model is based on the threshold ARCH (TARCH) model introduced independently by Zakoïan (1994) and Glosten, Jagannathan and Runkle (1993). As specified, if $\gamma_1 > 0$, then positive unexpected inflation disproportionately raises inflation uncertainty. Positive definiteness of the conditional covariance matrix requires $\omega_i > 0, \alpha_i \geq 0$, $\alpha_i + \gamma_i \geq 0$, and $\beta_i \geq 0$ for all $i = 1, \ldots, N$. Under the assumption that the standardized innovations are symmetrically distributed, the ACCORR model is covariance stationary if and only if $\alpha_i + \gamma_i/2 + \beta_i < 1$ for all $i = 1, \ldots, N$.

The hypothesis due to Friedman (1977) and Ball (1992), that inflation uncertainty rises with the level of inflation, may be directly tested by augmenting the conditional covariance function with a predetermined function of inflation. Motivated by Brunner and Hess (1993), we consider the following state-dependent conditional variance model:

$$h_{11,t} = \omega_1 + \alpha_1\varepsilon^2_{1,t-1} + \gamma_1\eta^2_{1,t-1} + \beta_1 h_{11,t-1} + \delta_1(\Delta_S \ln P_{t-1} - \delta_2)^2. \tag{9}$$

As specified, if $\delta_1 > 0$, there exists a positive quadratic relationship between lagged trend inflation and inflation uncertainty, with δ_2 representing that level of trend inflation at which inflation uncertainty is minimized.

Consider the following trivariate GARCH-M model, which extends our VARMA specification of the conditional mean function to allow for the dependence of both relative price dispersion and output growth on a proxy measure of inflation uncertainty derived from a state-dependent ACCORR specification of the conditional covariance function:

$$\Delta\ln P_t = \mu_1 + \sum_{i=1}^{p_1}\phi_{1,i}\Delta\ln P_{t-i} + \theta_{1,1}\ln\frac{Y_{t-1}}{Y^P_{t-1}} + \theta_{1,2}\Delta\ln(S_{t-1}P^f_{t-1}) + \theta_{1,3}\Delta\ln\frac{P^{OIL}_{t-1}}{P^f_{t-1}} + \varepsilon_{1,t},$$

$$h_{11,t} = \omega_1 + \alpha_1\varepsilon^2_{1,t-1} + \gamma_1\eta^2_{1,t-1} + \beta_1 h_{11,t-1} + \delta_1(\Delta_S \ln P_{t-1} - \delta_2)^2,$$

$$RPD_t = \mu_2 + \sum_{i=1}^{p_2}\phi_{2,i}RPD_{t-i} + \theta_{2,1}h^{1/2}_{11,t} + \theta_{2,2}\left|\Delta_S\ln P_{t-1}\right| + \theta_{2,3}I_{t-1}\left|\Delta_S\ln P_{t-1}\right| + \varepsilon_{2,t},$$

$$h_{22,t} = \omega_2 + \alpha_2\varepsilon^2_{2,t-1} + \gamma_2\eta^2_{2,t-1} + \beta_2 h_{22,t-1},$$

$$\Delta\ln Y_t = \mu_3 + \sum_{i=1}^{p_3}\phi_{3,i}\Delta\ln Y_{t-i} + \theta_{3,1}\varepsilon_{1,t-1} + \theta_{3,2}h^{1/2}_{11,t} + \varepsilon_{3,t},$$

$$h_{33,t} = \omega_3 + \alpha_3\varepsilon^2_{3,t-1} + \gamma_3\eta^2_{3,t-1} + \beta_3 h_{33,t-1},$$

$$h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}}\,\forall i > j;\ i, j = 1, 2, 3.$$

This general empirical framework nests all of the specifications considered in this study. A positive and statistically significant estimate of $\theta_{2,1}$ would constitute evidence that relative price dispersion rises with inflation uncertainty as predicted by signal-extraction models. Furthermore, a positive and statistically significant estimate of $\theta_{2,2}$ would suggest that relative price dispersion rises with absolute trend inflation as predicted by menu-cost models, while a positive and statistically significant estimate of $\theta_{2,3}$ would constitute evidence that negative trend inflation increases relative price dispersion to a greater extent than does positive trend inflation of equal magnitude. Finally, a negative and statistically significant estimate of $\theta_{3,2}$ would suggest that inflation uncertainty reduces output growth.

We estimate nested variants of this GARCH-M model by full-information maximum likelihood assuming a conditional multivariate normal error distribution. In general, provided that certain regularity conditions are satisfied, exact maximum likelihood estimators are consistent, asymptotically normal, and asymptotically efficient. The conditional loglikelihood function is maximized with the Marquardt algorithm employing two-sided numeric-first derivatives. Robust standard errors are calculated using the heteroscedasticity-consistent coefficient covariance matrix estimator derived by Bollerslev and Wooldridge (1992), which remains consistent irrespective of the validity of the conditional multivariate normality assumption. The parameter estimates also remain consistent and asymptotically normal regardless of the validity of this assumption, provided, of course, that the conditional mean and conditional covariance functions are correctly specified. Estimation results appear in Tables A.5 through A.12 of the Appendix, with asymptotic *t*-ratios reported in parentheses.

Under empirically adequate specifications, the standardized residuals derived from our estimated multivariate GARCH-M models should exhibit no discernible autocorrelation or remaining autoregressive conditional heteroscedasticity. To examine whether these properties are approximately satisfied, we subject the levels, squares, and cross products of the standardized residuals to Ljung-Box (1978) tests for autocorrelation up to integer multiples of the seasonal frequency. We also examine whether there exist significant departures from normality with the test of Jarque and Bera (1980).

Most studies define standardized residuals as the ratio of ordinary residuals to conditional standard deviations, an approach that is valid only if all conditional covariances are identically zero. The remainder premultiply the ordinary residual vector by the inverse of the lower triangular Cholesky factor of the conditional covariance matrix, rendering the standardized residuals dependent on the ordering of the variables in the system. A more appropriate approach involves derivation of the inverse square root of the estimated conditional covariance matrix with a spectral decomposition as $\hat{H}_t^{-1/2} = X_t \Lambda_t^{-1/2} X_t^{\mathrm{T}}$, where X_t is a square matrix containing N distinct orthonormal eigenvectors, and Λ_t is a diagonal matrix containing the corresponding strictly positive eigenvalues. In the absence of model misspecification, the resultant standardized residuals, $\hat{z}_t = \hat{H}_t^{-1/2} \hat{\varepsilon}_t$, are consistent for their population counterparts, since both $\hat{H}_t$ and $\hat{\varepsilon}_t$ are consistent. Furthermore, these standardized residuals are by construction invariant to the ordering of the variables in the system.

We find little or no evidence of residual autocorrelation or autoregressive conditional heteroscedasticity, as evidenced by the absence of significant autocorrelation among the levels, squares, and cross products of the standardized residuals. Moreover, all estimated multivariate

GARCH-M models are covariance stationary and satisfy the sufficient conditions for positive definiteness of the conditional covariance matrix. While the models to some extent explain the pronounced leptokurtosis exhibited by inflation, relative price dispersion, and output growth, abundant evidence of non-normality remains. Although the implied highly significant departures from conditional multivariate normality render our parameter estimates inefficient, the combined results of our standardized residual diagnostic tests suggest that they remain consistent and asymptotically normal. Thus, statistical inference within the framework of our estimated multivariate GARCH-M models appears justified.

In agreement with the hypothesis of Friedman (1977) and Ball (1992), the estimated effect of trend inflation on inflation uncertainty is positive across all of the alternative model specifications under consideration, with inflation uncertainty minimized at a trend inflation rate of approximately 3 per cent. However, this effect is never statistically significant at conventional levels. In contrast, little or no empirical support is found for the hypothesis of Brunner and Hess (1993) and Joyce (1995), who argue that positive unexpected inflation should increase inflation uncertainty to a greater extent than negative unexpected inflation of equal magnitude. Indeed, the estimated differential impact of positive unexpected inflation on inflation uncertainty is often negative and is always statistically insignificant.

Consistent with the predictions of menu-cost and signal-extraction models, the estimated effects of trend inflation and inflation uncertainty on relative price dispersion are uniformly positive. However, the significance of these effects varies according to whether a weighted or unweighted measure of relative price dispersion is employed, in addition to the symmetry characteristics of the process governing the evolution of inflation uncertainty. To elaborate, inflation uncertainty dominates trend inflation as a predictor of relative price dispersion in models where a weighted measure of relative price dispersion is used. In contrast, trend inflation alone is a significant predictor of relative price dispersion in symmetric multivariate GARCH-M models employing an unweighted measure of relative price dispersion.

The hypothesis advanced by Friedman (1977), that inflation uncertainty adversely affects the level of real economic activity, receives robust empirical support. Indeed, the estimated effect of inflation uncertainty on output growth is uniformly negative and statistically significant at the 5 per cent level, irrespective of the conditioning information set used in the generation of our proxy measure of inflation uncertainty and the symmetry characteristics of the process governing its evolution.

It should be noted that several econometric issues hinder our statistical inference. Collinearity between trend inflation and inflation uncertainty confounds statistical attempts to estimate their partial effects on relative price dispersion. Furthermore, although quasi-maximum likelihood estimation of GARCH models in the presence of asymmetric departures from normality remains consistent, simulation studies such as Bollerslev and Wooldridge (1992) suggest that such estimation may be extremely inefficient. As the standardized residuals associated with inflation and relative price dispersion exhibit significant positive skewness, this limitation of quasi-maximum likelihood estimation is to some extent binding.

3.3 Impulse-response analysis

In a seminal paper, Sims (1980) introduced the impulse-response function as a tool to trace the propagation of a shock through a linear system of dynamically interrelated endogenous variables. Within this context, the impulse-response function for a vector-stochastic process, y_t, is defined as the difference between two realizations of y_{t+h}, conditional on identical histories, I_{t-1}:

$$IRF_y(s, \delta, J_{t-1}) = \mathrm{E}[y_{t+s} | \varepsilon_t = \delta, I_{t-1}] - \mathrm{E}[y_{t+s} | \varepsilon_t = 0, I_{t-1}]. \tag{10}$$

Under the shock profile, the process is perturbed by a shock, $\varepsilon_t = \delta$, at time *t*, while under the benchmark profile, no such shock occurs. Typically, δ corresponds to a specific column of a conformable identity matrix.

Within a linear framework, these impulse responses are symmetric and history-independent, while their interpretation is straightforward if the innovations are contemporaneously uncorrelated. In practice, however, the existence of contemporaneous correlations across innovations is typical, and such innovations may be interpreted as having a common component that cannot be assigned to a specific variable. To facilitate the interpretation of impulse responses under such circumstances, it is customary to apply an orthogonal transformation to the innovations.

Of particular interest within the context of multivariate GARCH models is the influence of shocks on future conditional variances and covariances. Lin (1997) defines the impulse-response function for symmetric multivariate GARCH models as

$$IRF_H(s, \delta) = \mathrm{E}[H_{t+s} | \varepsilon_t = \delta, H_t = H] - \mathrm{E}[H_{t+s} | \varepsilon_t = 0, H_t = H]_{,} \tag{11}$$

and shows that an asymptotic normal distribution applies. His analysis assumes zero conditional covariances, an uncomfortably strong assumption.

A natural solution to the problem of gaining interpretability of the impulse responses while acknowledging the existence of non-zero conditional covariances involves defining the impulse-response functions in terms of the standardized innovations, which are the fundamental sources of stochastic variation in GARCH models:

$$IRF_y(s, \delta, I_{t-1}) = \mathrm{E}[\mathrm{y}_{t+s}|z_t = \delta, I_{t-1}] - \mathrm{E}[y_{t+s}|z_t = 0, I_{t-1}],$$

$$IRF_H(s, \delta, I_{t-1}) = \mathrm{E}[\mathrm{H}_{t+s}|z_t = \delta, I_{t-1}] - \mathrm{E}[\mathrm{H}_{t+s}|z_t = 0, I_{t-1}]. \quad (12)$$

By definition, the standardized innovations are both conditionally and unconditionally contemporaneously uncorrelated. Furthermore, provided that the orthogonal transformation matrix corresponds to the inverse square root of the symmetric and positive-definite conditional covariance matrix, which may be derived with a spectral decomposition, these impulse responses are invariant to the ordering of the variables in the system.

Employing a factorization of the conditional covariance matrix to orthogonalize the ordinary innovations renders these impulse-response functions history-dependent. A solution to this problem is suggested by Koop, Pesaran, and Potter (1996), who emphasize that impulse-response functions are random variables for which various conditional versions may be defined. For instance, conditioning on or averaging across all possible histories renders the impulse responses history-independent:

$$IRF_y(s, \delta) = \mathrm{E}[y_{t+s}|z_t = \delta] - \mathrm{E}[y_{t+s}|z_t = 0],$$

$$IRF_H(s, \delta) = \mathrm{E}[H_{t+s}|z_t = \delta] - \mathrm{E}[H_{t+s}|z_t = 0]. \quad (13)$$

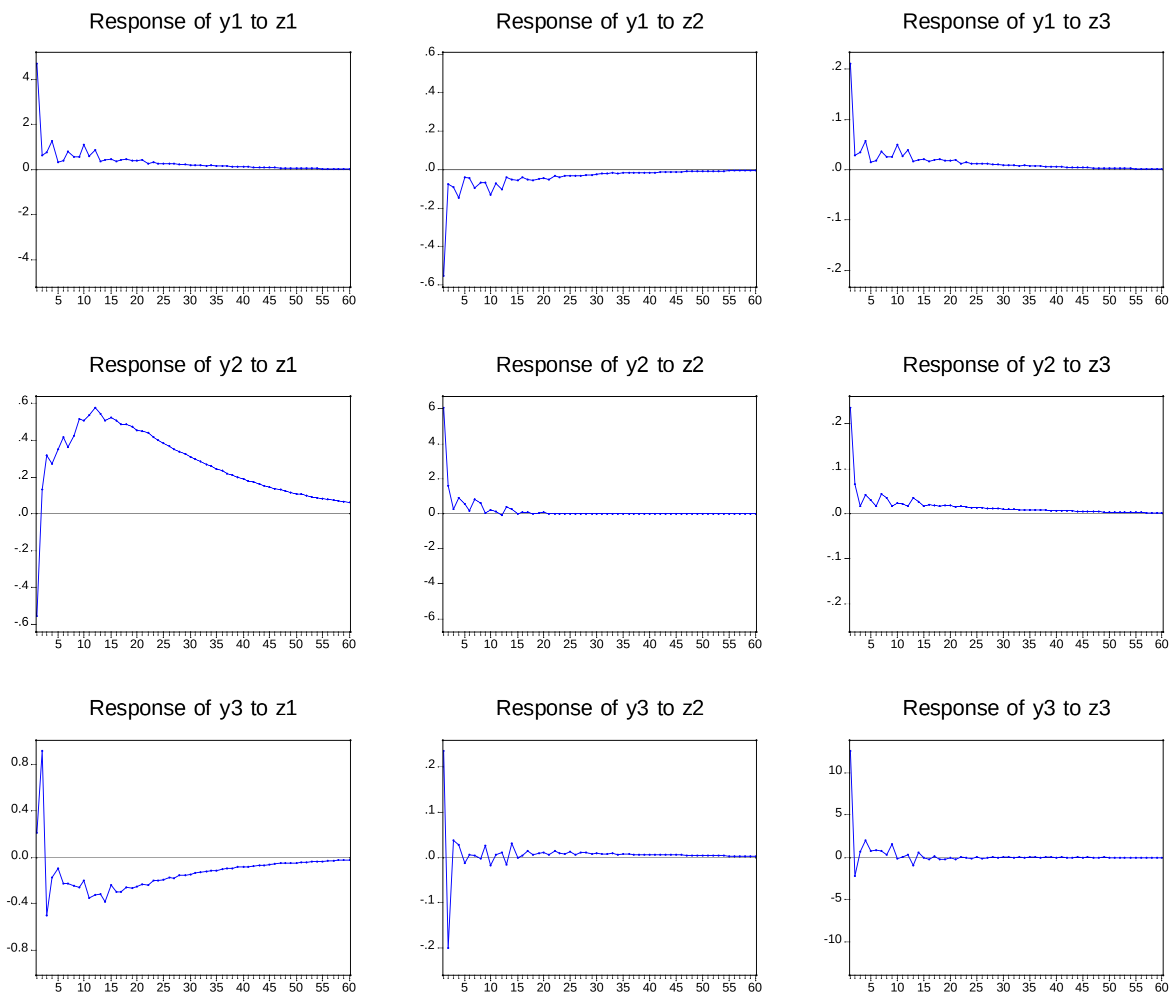
These impulse-response functions measure the extent to which optimal point forecasts of y_{t+s} and H_{t+s} at time $t-1$ are revised in response to a shock, $z_t = \delta$, at time *t*.

In general, the construction of dynamic forecasts within a non-linear framework requires application of the bootstrap, since analytic expressions are unavailable. While the multivariate GARCH-M model under consideration is highly non-linear, a recursive forecasting procedure provides a computationally tractable approximation. One-step-ahead point forecasts of the means, variances, and covariances of inflation, relative price dispersion, and output growth are treated as deterministic functions of observable quantities, while multiple-step-ahead forecasts are derived by replacing unobservable quantities with their conditional expectations. In particular, unobserved squared ordinary innovations are replaced by the corresponding conditional variances, while unobserved squared asymmetric innovations are replaced by appropriately weighted conditional variances.

Given that our estimated impulse responses differ imperceptibly across alternative model specifications, in Figures 2 and 3, we present those derived from our symmetric multivariate GARCH-M model, employing an autoregressive specification of the conditional mean of inflation and a weighted measure of relative price dispersion as representative. Visual inspection of our estimated mean and variance impulse responses reveals that the effects of unexpected inflation on inflation uncertainty, relative price dispersion, and output growth are all sizeable and persistent. To elaborate, a 4 per cent inflation shock is found to raise relative price dispersion through increased trend inflation and inflation uncertainty by approximately 0.6 per cent after one year. Furthermore, following a temporary direct stimulatory effect of approximately 0.8 per cent, an inflation shock of this magnitude is found to reduce output growth through increased inflation uncertainty by nearly 0.4 per cent after one year. That all of our estimated impulse responses approach zero as the forecast horizon tends towards infinity, confirms covariance stationarity of the conditional mean and conditional covariance functions.

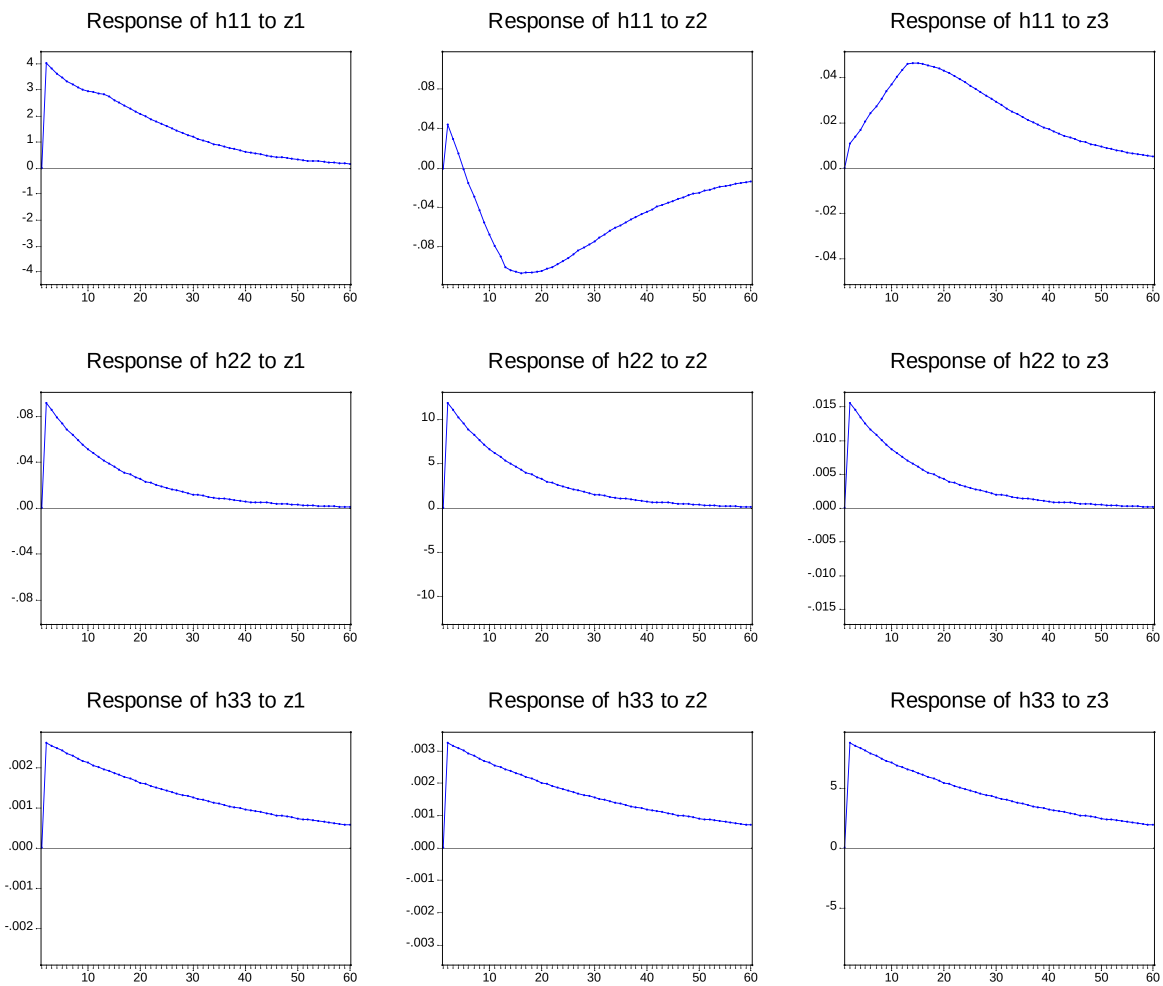
While our estimated impulse responses reveal that the effects of unexpected inflation on inflation uncertainty, relative price dispersion, and output growth are economically significant, assessing whether this evidence is statistically significant requires the derivation of confidence intervals. In principle, provided that an asymptotically pivotal limiting distribution exists, such confidence intervals may be calculated with a bootstrap simulation. In practice, however, the extreme computational intensity of such a procedure is prohibitive.

Figure 2: Estimated Mean Impulse Responses to Unit-Standardized Innovations



Note: The conditional means of inflation, relative price dispersion, and output growth are denoted by y_1, y_2, and y_3, respectively.

Figure 3: Estimated Variance Impulse Responses to Unit-Standardized Innovations



Note: The conditional variances of inflation, relative price dispersion, and output growth are denoted by h_{11}, h_{22}, and h_{33}, respectively.

4. Conclusion

An empirical investigation of dynamic interrelationships among inflation, inflation uncertainty, relative price dispersion, and output growth is conducted within a trivariate GARCH-M framework. Focusing on the Canadian industrial sector, weak evidence that inflation uncertainty rises with the level of inflation is found, with short-run inflation uncertainty minimized at a trend inflation rate of approximately 3 per cent. With regard to the predictions of menu-cost and signal-extraction models, evidence that relative price dispersion rises with both trend inflation and inflation uncertainty is obtained. However, the significance of this evidence varies according to whether a weighted or unweighted measure of relative price dispersion is employed, in addition to the symmetry characteristics of the process governing the evolution of inflation uncertainty. To the extent that any relationship between trend inflation and relative price dispersion is asymmetric, our results suggest that trend inflation increases relative price dispersion, while trend deflation exerts no effect. Our primary result is that across a variety of different model specifications, inflation uncertainty significantly lowers output growth, an effect of considerable size and duration.

We treat our results as preliminary and suggestive, as a number of substantive issues remain to be explored. First and foremost, we recommend the examination of whether our conclusions hold under alternative measures of inflation and relative price dispersion based on the consumer price index or the gross domestic product price deflator, both of which feature considerably broader sectoral coverage than does the industrial producer price index. Furthermore, although maximum likelihood estimation and inference as applied to time-dependent processes has only asymptotic justification, the consideration of lower observation frequencies has the potential to further elucidate dynamic interrelationships among inflation, inflation uncertainty, relative price dispersion, and output growth. Additional insights may be gained by decomposing inflation uncertainty into separate components attributable to parameter instability and heteroscedastic disturbances with the quasi-optimal Kalman filter advocated by Harvey, Ruiz, and Sentana (1992), which simultaneously accommodates time-varying parameters and GARCH errors. Finally, additional efforts to control for the effects of commodity price shocks on relative price dispersion may be warranted.

References

Ball, L. 1992. "Why Does High Inflation Raise Inflation Uncertainty?" *Journal of Monetary Economics* 29: 371–88.

Ball, L. and D. Romer. 1993. "Inflation and the Informativeness of Prices." NBER Working Paper No. 4267.

Barro, R. 1976. "Rational Expectations and the Role of Monetary Policy." *Journal of Monetary Economics* 2: 1–32.

Benabou, R. 1992. "Inflation and Efficiency in Search Markets." *Review of Economic Studies* 59: 299–329.

Benabou, R. and R. Gertner. 1993. "Search with Learning from Prices: Does Increased Inflation Uncertainty Lead to Higher Markups?" *Review of Economic Studies* 60: 69–94.

Berndt, E., B. Hall, R. Hall, and J. Hausman. 1974. "Estimation and Inference in Nonlinear Structural Models." *Annals of Economic and Social Measurement* 3: 653–65.

Bollerslev, T. 1986. "Generalized Autoregressive Conditional Heteroscedasticity." *Journal of Econometrics* 31: 307–27.

———. 1990. "Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Approach." *Review of Economics and Statistics* 72: 498–505.

Bollerslev, T., R. Engle, and J. Wooldridge. 1988. "A Capital Asset Pricing Model with Time-Varying Covariances." *Journal of Political Economy* 96: 116–31.

Bollerslev, T. and J. Wooldridge. 1992. "Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances." *Econometric Reviews* 11: 143–72.

Brunner, A. and G. Hess. 1993. "Are Higher Levels of Inflation Less Predictable? A State-Dependent Conditional Heteroscedasticity Approach." *Journal of Business and Economic Statistics* 11(2): 187–97.

Crawford, A. and M. Kasumovich. 1996. "Does Inflation Uncertainty Vary with the Level of Inflation?" Bank of Canada Working Paper No. 9.

Diamond, P. 1993. "Search, Sticky Prices and Inflation." *Review of Economic Studies* 60: 53–69.

Engle, R. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation." *Econometrica* 50: 987–1008.

Friedman, M. 1977. "Nobel Lecture: Inflation and Unemployment." *Journal of Political Economy* 85: 451–72.

Glosten, L., R. Jagannathan, and D. Runkle. 1993. "On the Relation Between the Expected Value and the Volatility of the Nominal Excess Return on Stocks." *Journal of Finance* 48: 1779–1801.

Granger, C. and T. Teräsvirta. 1993. "Modelling Nonlinear Economic Relationships." Oxford University Press.

Grier, K. and M. Perry. 1996. “Inflation, Inflation Uncertainty, and Relative Price Dispersion: Evidence from Bivariate GARCH-M Models.” *Journal of Monetary Economics* 38: 391–405.

———. 1998. “On Inflation and Inflation Uncertainty in the G7 Countries.” *Journal of International Money and Finance* 17: 671–89.

———. 2000. “The Effects of Real and Nominal Uncertainty on Inflation and Output Growth: Some GARCH-M Evidence.” *Journal of Applied Econometrics* 17: 45–58.

Hamilton, J. 1994. *Time Series Analysis*. Princeton University Press.

Harvey, A., E. Ruiz, and E. Sentana. 1992. “Unobserved Component Time Series Models with ARCH Disturbances.” *Journal of Econometrics* 52: 129–57.

Hodrick, R. and E. Prescott. 1997. “Postwar U.S. Business Cycles: An Empirical Investigation.” *Journal of Money, Credit and Banking* 29: 1–16.

Holland, S. 1993. “Comments on Inflation Regimes and the Sources of Inflation Uncertainty.” *Journal of Money, Credit and Banking* 25: 514–20.

Jaramillo, C. 1999. “Inflation and Relative Price Variability: Reinstating Parks’ Results.” *Journal of Money, Credit and Banking* 31 (3): 375–85.

Jarque, C. and A. Bera. 1980. “Efficient Tests for Normality, Homoskedasticity and Serial Dependence of Regression Residuals.” *Economics Letters* 6: 255–59.

Joyce, M. 1995. “Modelling U.K. Inflation Uncertainty: The Impact of News and the Relationship with Inflation.” Bank of England Working Paper No. 30.

Koop, G., M. Pesaran, and S. Potter. 1996. “Impulse Response Analysis in Nonlinear Multivariate Models.” *Journal of Econometrics* 74: 119–47.

Kroner, K. and V. Ng. 1998. “Modeling Asymmetric Comovements of Asset Returns.” *The Review of Financial Studies* 11: 817–44.

Lin, W. 1997. “Impulse Response Functions for Conditional Volatility in GARCH Models.” *Journal of Business and Economic Statistics* 15: 1–14.

Ljung, G. and G. Box. 1978. “On a Measure of Lack of Fit in Time Series Models.” *Biometrika* 65: 297–303.

Lucas, R. 1972. “Expectations and the Neutrality of Money.” *Journal of Economic Theory* 4: 103–24.

Pagan, A. and A. Ullah. 1988. “The Econometric Analysis of Models with Risk Terms.” *Journal of Applied Econometrics* 3: 87–105.

Rotemberg, J. 1983. “Aggregate Consequences of Fixed Costs of Price Adjustment.” *American Economic Review* 73: 433–36.

Sheshinski, E. and Y. Weiss. 1979. “Inflation and Costs of Price Adjustment.” *Review of Economic Studies* 54: 287–303.

Sheshinski, E. and Y. Weiss. 1983. “Optimum Pricing Policy Under Stochastic Inflation.” *Review of Economic Studies* 50: 513–29.

Sims, C. 1980. “Macroeconomics and Reality.” *Econometrica* 48: 1–48.

Wold, H. 1938. “A Study in the Analysis of Stationary Time Series.” Almqvist and Wiksell.

Zakoïan, J. 1994. “Threshold Heteroskedastic Models.” *Journal of Economic Dynamics and Control* 18: 931–44.

Appendix A

Table A.1: Estimation Results for Weighted VARMA Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$						
0.641	6.155	2.234	0.559	–0.497	0.214						
(1.774)	(5.609)	(2.996)	(4.502)	(–0.695)	(1.690)						
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.126	0.114	0.224	0.014	0.049	0.084	0.081	0.093	0.117	0.006	0.048	–0.094
(1.817)	(1.694)	(3.796)	(0.243)	(0.819)	(1.941)	(1.558)	(1.914)	(2.466)	(0.123)	(0.848)	(–1.744)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.367	–0.014	0.186	0.007	–0.038	0.120	–0.048	0.001	–0.038	–0.033	0.016	–0.031
(3.739)	(–0.181)	(3.195)	(0.131)	(–0.692)	(2.359)	(–0.927)	(0.010)	(–0.718)	(–0.666)	(0.267)	(–0.596)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.177	0.054	0.209	0.141	0.096	0.058	0.017	0.106	0.004	–0.031	–0.020	–0.117
(–3.625)	(1.102)	(4.557)	(2.847)	(1.969)	(1.214)	(0.356)	(2.211)	(0.091)	(–0.637)	(–0.401)	(–2.306)

	ε_1	ε_2	ε_3	$\varepsilon_2\varepsilon_1$	$\varepsilon_3\varepsilon_1$	$\varepsilon_3\varepsilon_2$
$Q(12)$	1.601	4.480	0.875	37.044***	30.655***	24.457**
$Q(24)$	19.247	28.296	20.982	39.057**	48.704***	33.986*
$Q^2(12)$	41.617***	66.316***	21.008*	–	–	–
$Q^2(24)$	53.534***	73.355***	41.474**	–	–	–
Skewness	0.537***	1.697***	–0.097	–	–	–
Kurtosis	7.150***	10.181***	3.403*	–	–	–
JB	359.928***	1235.556***	3.913	–	–	–
$\ln L = -4784.558$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.2: Estimation Results for Weighted VARMA Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$			
0.822	6.199	2.220	0.139	0.023	0.005	0.562	–0.512	0.263			
(2.292)	(5.632)	(2.973)	(2.529)	(0.999)	(1.890)	(4.526)	(–0.721)	(2.031)			
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.074	0.098	0.220	0.009	0.055	0.069	0.077	0.088	0.115	0.016	0.060	–0.085
(1.024)	(1.446)	(3.679)	(0.166)	(0.941)	(1.564)	(1.490)	(1.816)	(2.449)	(0.349)	(1.072)	(–1.580)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.367	–0.016	0.186	0.005	–0.034	0.120	–0.049	0.001	–0.039	–0.033	0.014	–0.033
(3.736)	(–0.194)	(3.182)	(0.097)	(–0.610)	(2.343)	(–0.955)	(0.014)	(–0.725)	(–0.661)	(0.246)	(–0.644)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.182	0.050	0.207	0.140	0.097	0.058	0.017	0.108	0.007	–0.028	–0.018	–0.113
(–3.731)	(1.028)	(4.506)	(2.845)	(1.988)	(1.216)	(0.357)	(2.240)	(0.165)	(–0.581)	(–0.348)	(–2.242)

	ε_1	ε_2	ε_3	$\varepsilon_2\varepsilon_1$	$\varepsilon_3\varepsilon_1$	$\varepsilon_3\varepsilon_2$
$Q(12)$	2.407	4.608	0.791	39.195***	30.447***	25.371**
$Q(24)$	17.176	28.306	21.099	40.869**	48.556***	34.945*
$Q^2(12)$	42.035***	66.363***	21.641**	–	–	–
$Q^2(24)$	53.202***	73.404***	42.659**	–	–	–
Skewness	0.490***	1.695***	–0.107	–	–	–
Kurtosis	7.101***	10.158***	3.401*	–	–	–
JB	348.127***	1228.423***	4.042	–	–	–
$\ln L = -4779.497$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.3: Estimation Results for Unweighted VARMA Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$						
0.638	6.005	2.226	0.569	–0.549	0.232						
(1.737)	(5.048)	(2.973)	(3.926)	(–0.713)	(1.889)						
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.118	0.117	0.227	0.021	0.035	0.084	0.077	0.097	0.123	0.008	0.047	–0.093
(1.693)	(1.754)	(3.970)	(0.392)	(0.631)	(1.975)	(1.472)	(1.921)	(2.591)	(0.176)	(0.808)	(–1.698)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.293	0.025	0.142	0.096	–0.067	0.099	–0.039	0.050	–0.085	–0.001	–0.015	0.017
(3.361)	(0.354)	(2.414)	(1.390)	(–0.974)	(1.953)	(–0.759)	(0.791)	(–1.358)	(–0.015)	(–0.258)	(0.299)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.175	0.051	0.207	0.145	0.091	0.062	0.016	0.106	0.003	–0.028	–0.022	–0.114
(–3.548)	(1.059)	(4.551)	(2.972)	(1.846)	(1.299)	(0.341)	(2.195)	(0.064)	(–0.579)	(–0.434)	(–2.244)

	ε_1	ε_2	ε_3	$\varepsilon_2\varepsilon_1$	$\varepsilon_3\varepsilon_1$	$\varepsilon_3\varepsilon_2$
Q(12)	1.712	6.484	0.951	54.164***	30.525***	28.727***
Q(24)	19.212	20.694	21.196	55.659***	48.489***	37.293**
Q^2(12)	42.223***	83.991***	21.515**	–	–	–
Q^2(24)	54.028***	90.069***	41.794**	–	–	–
Skewness	0.547***	1.811***	–0.098	–	–	–
Kurtosis	7.161***	9.660***	3.396*	–	–	–
JB	362.495***	1125.734***	3.834	–	–	–

ln L = –4759.440

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.4: Estimation Results for Unweighted VARMA Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$			
0.819	6.002	2.207	0.132	0.021	0.006	0.566	–0.573	0.278			
(2.234)	(5.020)	(2.943)	(2.433)	(0.955)	(2.175)	(3.905)	(–0.748)	(2.222)			
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.067	0.101	0.224	0.016	0.044	0.067	0.073	0.092	0.121	0.019	0.059	–0.085
(0.916)	(1.497)	(3.872)	(0.311)	(0.794)	(1.564)	(1.408)	(1.829)	(2.581)	(0.405)	(1.015)	(–1.548)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.295	0.025	0.142	0.095	–0.064	0.100	–0.041	0.051	–0.085	–0.000	–0.017	0.015
(3.379)	(0.353)	(2.413)	(1.364)	(–0.925)	(1.970)	(–0.795)	(0.809)	(–1.360)	(–0.003)	(–0.289)	(0.269)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.179	0.048	0.205	0.145	0.091	0.062	0.016	0.108	0.006	–0.025	–0.019	–0.110
(–3.646)	(0.990)	(4.503)	(2.977)	(1.860)	(1.303)	(0.339)	(2.222)	(0.136)	(–0.518)	(–0.378)	(–2.170)

	ε_1	ε_2	ε_3	$\varepsilon_2\varepsilon_1$	$\varepsilon_3\varepsilon_1$	$\varepsilon_3\varepsilon_2$
$Q(12)$	2.509	6.582	0.870	55.761***	30.332***	29.202***
$Q(24)$	17.114	20.772	21.337	57.162***	48.339***	38.358**
$Q^2(12)$	43.348***	84.099***	22.106**	–	–	–
$Q^2(24)$	54.378***	90.179***	42.844**	–	–	–
Skewness	0.502***	1.809***	–0.109	–	–	–
Kurtosis	7.131***	9.643***	3.396*	–	–	–
JB	353.838***	1120.548***	4.017	–	–	–
$\ln L = -4754.143$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.5: Estimation Results for Weighted Asymmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$				
0.734	4.117	7.434	0.514	0.160	–0.652	0.275	–1.028				
(2.047)	(4.410)	(3.471)	(2.720)	(1.101)	(–1.121)	(2.300)	(–2.436)				
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.129	0.152	0.222	–0.020	–0.006	0.105	0.063	0.053	0.129	0.005	0.066	–0.095
(2.590)	(3.004)	(4.474)	(–0.371)	(–0.123)	(2.172)	(1.272)	(1.202)	(2.881)	(0.103)	(1.296)	(–2.048)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.272	0.004	0.146	0.017	–0.011	0.081	0.039	–0.014	0.007	–0.022	–0.028	0.069
(5.173)	(0.062)	(2.540)	(0.361)	(–0.243)	(1.631)	(0.771)	(–0.290)	(0.137)	(–0.501)	(–0.530)	(1.423)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.181	0.021	0.164	0.110	0.082	0.056	0.024	0.099	–0.001	–0.029	–0.031	–0.106
(–3.621)	(0.435)	(3.456)	(2.416)	(1.671)	(1.165)	(0.487)	(2.115)	(–0.035)	(–0.625)	(–0.627)	(–2.308)
ω_1	ω_2	ω_3	α_1	α_2	α_3	γ_1	γ_2	γ_3	β_1	β_2	β_3
1.689	3.561	4.121	0.164	0.000	0.068	–0.023	0.330	–0.031	0.760	0.725	0.920
(1.651)	(3.011)	(1.037)	(2.832)	(0.000)	(2.086)	(–0.283)	(1.999)	(–0.893)	(8.320)	(10.577)	(20.370)
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.196	0.065	0.048	0.046	3.153							
(–2.940)	(1.370)	(0.963)	(0.674)	(1.547)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	11.263	7.067	1.247	10.263	21.108**	8.956
$Q(24)$	24.953	21.025	25.596	11.809	26.603	15.875
$Q^2(12)$	4.972	7.288	12.489	–	–	–
$Q^2(24)$	12.448	13.633	24.190	–	–	–
Skewness	0.602***	1.259***	–0.141	–	–	–
Kurtosis	5.689***	6.026***	3.207	–	–	–
JB	169.945***	303.441***	2.410	–	–	–
$\ln L = -4680.212$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.6: Estimation Results for Weighted Asymmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$	
0.770	3.991	7.000	0.044	0.037	0.010	0.441	0.146	–0.556	0.274	–0.965	
(2.216)	(4.275)	(3.294)	(0.857)	(1.731)	(3.536)	(2.304)	(1.002)	(–0.984)	(2.258)	(–2.252)	
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.051	0.147	0.240	–0.001	0.008	0.097	0.057	0.056	0.125	0.017	0.073	–0.110
(0.867)	(2.951)	(4.941)	(–0.026)	(0.161)	(1.985)	(1.179)	(1.328)	(2.777)	(0.369)	(1.350)	(–2.361)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.287	0.009	0.150	0.017	–0.006	0.083	0.039	–0.013	0.007	–0.019	–0.030	0.067
(5.517)	(0.148)	(2.618)	(0.353)	(–0.119)	(1.670)	(0.755)	(–0.254)	(0.142)	(–0.415)	(–0.559)	(1.382)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.181	0.023	0.168	0.113	0.084	0.056	0.020	0.098	–0.000	–0.026	–0.028	–0.099
(–3.642)	(0.483)	(3.537)	(2.486)	(1.721)	(1.162)	(0.422)	(2.096)	(–0.010)	(–0.556)	(–0.553)	(–2.163)
ω_1	ω_2	ω_3	α_1	α_2	α_3	γ_1	γ_2	γ_3	β_1	β_2	β_3
1.591	3.444	3.815	0.164	0.000	0.063	–0.031	0.324	–0.026	0.768	0.731	0.924
(1.435)	(3.027)	(0.972)	(2.836)	(0.000)	(2.010)	(–0.381)	(1.994)	(–0.772)	(7.792)	(10.879)	(20.748)
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.186	0.066	0.051	0.033	1.782							
(–2.758)	(1.398)	(1.016)	(0.494)	(0.438)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	9.910	6.475	1.221	10.472	19.993*	8.994
$Q(24)$	22.481	20.894	25.657	12.174	24.984	16.168
$Q^2(12)$	6.084	7.638	12.753	–	–	–
$Q^2(24)$	13.805	14.211	23.958	–	–	–
Skewness	0.579***	1.256***	–0.146	–	–	–
Kurtosis	5.568***	5.992***	3.230	–	–	–
JB	155.350***	298.943***	2.697	–	–	–
$\ln L = -4676.214$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.7: Estimation Results for Unweighted Asymmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$				
0.713	4.559	7.364	0.258	0.209	–0.555	0.287	–1.008				
(1.972)	(3.488)	(3.463)	(1.187)	(1.118)	(–0.935)	(2.406)	(–2.403)				
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.123	0.151	0.212	–0.003	–0.006	0.093	0.063	0.070	0.138	0.001	0.068	–0.096
(2.439)	(2.985)	(4.359)	(–0.047)	(–0.125)	(1.943)	(1.268)	(1.569)	(3.017)	(0.019)	(1.314)	(–2.039)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.233	0.013	0.120	0.072	0.010	0.097	–0.003	0.033	–0.033	–0.038	–0.036	0.135
(4.011)	(0.237)	(2.211)	(0.972)	(0.188)	(1.939)	(–0.053)	(0.513)	(–0.688)	(–0.703)	(–0.779)	(2.208)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.181	0.015	0.159	0.117	0.081	0.056	0.022	0.100	–0.004	–0.028	–0.036	–0.104
(–3.628)	(0.313)	(3.384)	(2.570)	(1.672)	(1.179)	(0.447)	(2.137)	(–0.108)	(–0.599)	(–0.725)	(–2.288)
ω_1	ω_2	ω_3	α_1	α_2	α_3	γ_1	γ_2	γ_3	β_1	β_2	β_3
1.676	1.446	5.026	0.144	0.000	0.081	–0.010	0.129	–0.038	0.767	0.870	0.906
(1.634)	(2.690)	(1.163)	(2.588)	(0.000)	(2.206)	(–0.121)	(1.177)	(–1.018)	(8.572)	(15.106)	(18.249)
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.186	0.068	0.075	0.052	3.316							
(–2.437)	(1.453)	(1.496)	(0.780)	(1.922)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	9.989	10.956	1.407	11.974	20.079*	10.936
$Q(24)$	23.520	19.734	26.214	13.768	26.294	17.523
$Q^2(12)$	4.831	5.015	12.554	–	–	–
$Q^2(24)$	12.177	11.843	24.257	–	–	–
Skewness	0.586***	1.525***	–0.154	–	–	–
Kurtosis	5.540***	8.357***	3.184	–	–	–
JB	153.170***	744.245***	2.534	–	–	–

$\ln L = -4655.120$

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.8: Estimation Results for Unweighted Asymmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$	
0.747	4.861	6.992	0.037	0.033	0.010	0.185	0.212	–0.513	0.293	–0.954	
(2.114)	(3.591)	(3.309)	(0.707)	(1.539)	(3.410)	(0.850)	(1.155)	(–0.893)	(2.412)	(–2.238)	
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.056	0.146	0.226	0.005	0.009	0.087	0.060	0.065	0.136	0.017	0.078	–0.109
(0.966)	(2.930)	(4.780)	(0.083)	(0.171)	(1.816)	(1.242)	(1.506)	(2.969)	(0.369)	(1.468)	(–2.356)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.236	0.011	0.120	0.073	0.015	0.098	–0.004	0.032	–0.033	–0.035	–0.040	0.134
(4.113)	(0.211)	(2.210)	(0.988)	(0.287)	(1.967)	(–0.072)	(0.490)	(–0.691)	(–0.653)	(–0.853)	(2.178)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.182	0.017	0.164	0.119	0.084	0.057	0.019	0.099	–0.004	–0.026	–0.032	–0.098
(–3.665)	(0.359)	(3.461)	(2.621)	(1.719)	(1.190)	(0.396)	(2.121)	(–0.106)	(–0.551)	(–0.643)	(–2.144)
ω_1	ω_2	ω_3	α_1	α_2	α_3	γ_1	γ_2	γ_3	β_1	β_2	β_3
1.774	1.441	4.626	0.133	0.000	0.074	0.007	0.131	–0.030	0.761	0.871	0.911
(1.680)	(2.667)	(1.087)	(2.400)	(0.000)	(2.095)	(0.077)	(1.186)	(–0.841)	(8.050)	(15.865)	(18.498)
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.185	0.070	0.075	0.051	3.158							
(–2.455)	(1.509)	(1.509)	(0.762)	(1.806)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	9.516	11.818	1.351	11.597	19.408*	10.683
$Q(24)$	22.270	20.664	26.242	13.612	24.887	17.292
$Q^2(12)$	5.874	5.201	12.696	–	–	–
$Q^2(24)$	13.579	12.001	23.855	–	–	–
Skewness	0.556***	1.512***	–0.155	–	–	–
Kurtosis	5.366***	8.216***	3.206	–	–	–
JB	133.839***	711.941***	2.722	–	–	–
$\ln L = -4649.620$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.9: Estimation Results for Weighted Symmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$				
0.760	3.728	7.537	0.574	0.179	–0.602	0.280	–1.034				
(2.052)	(3.384)	(3.526)	(2.540)	(1.113)	(–0.785)	(2.425)	(–2.433)				
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.136	0.148	0.222	–0.024	–0.002	0.094	0.066	0.043	0.123	0.011	0.060	–0.086
(2.754)	(2.922)	(4.448)	(–0.452)	(–0.034)	(1.940)	(1.338)	(0.966)	(2.797)	(0.237)	(1.169)	(–1.861)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.262	–0.021	0.143	0.021	–0.006	0.115	0.023	–0.030	0.010	–0.015	–0.016	0.056
(5.253)	(–0.300)	(2.621)	(0.412)	(–0.128)	(2.451)	(0.477)	(–0.631)	(0.195)	(–0.329)	(–0.292)	(1.102)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.179	0.018	0.169	0.113	0.087	0.058	0.019	0.094	–0.002	–0.028	–0.026	–0.101
(–3.647)	(0.382)	(3.592)	(2.527)	(1.770)	(1.199)	(0.385)	(1.993)	(–0.059)	(–0.596)	(–0.544)	(–2.227)
ω_1	ω_2	ω_3	α_1	α_2	α_3	β_1	β_2	β_3			
1.683	3.840	3.949	0.156	0.263	0.055	0.761	0.667	0.919			
(1.669)	(2.541)	(1.048)	(3.397)	(2.055)	(2.133)	(8.368)	(7.036)	(20.661)			
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.209	0.061	0.058	0.039	3.106							
(–2.918)	(1.301)	(1.112)	(0.595)	(1.257)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	12.738	7.103	1.034	9.584	20.551*	6.869
$Q(24)$	26.066	23.035	25.656	10.803	26.123	14.605
$Q^2(12)$	5.175	5.723	12.762	–	–	–
$Q^2(24)$	12.366	11.046	23.396	–	–	–
Skewness	0.643***	1.460***	–0.136	–	–	–
Kurtosis	5.872***	7.568***	3.184	–	–	–
JB	193.965***	575.771***	2.119	–	–	–
$\ln L = -4693.324$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.10: Estimation Results for Weighted Symmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$	
0.772	3.728	7.032	0.045	0.038	0.010	0.515	0.167	–0.527	0.282	–0.961	
(2.181)	(3.367)	(3.315)	(0.854)	(1.762)	(3.683)	(2.086)	(1.016)	(–0.714)	(2.388)	(–2.213)	
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.061	0.140	0.240	–0.009	0.016	0.086	0.063	0.043	0.124	0.024	0.071	–0.108
(1.038)	(2.819)	(4.931)	(–0.163)	(0.318)	(1.773)	(1.318)	(1.013)	(2.771)	(0.536)	(1.319)	(–2.342)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.272	–0.010	0.144	0.021	0.001	0.117	0.020	–0.027	0.007	–0.010	–0.020	0.052
(5.469)	(–0.143)	(2.616)	(0.408)	(0.012)	(2.464)	(0.411)	(–0.569)	(0.129)	(–0.220)	(–0.367)	(1.003)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.180	0.021	0.171	0.113	0.090	0.059	0.019	0.094	–0.003	–0.025	–0.023	–0.095
(–3.666)	(0.433)	(3.633)	(2.540)	(1.818)	(1.217)	(0.383)	(2.005)	(–0.072)	(–0.540)	(–0.465)	(–2.086)
ω_1	ω_2	ω_3	α_1	α_2	α_3	β_1	β_2	β_3			
1.664	3.777	3.831	0.145	0.255	0.054	0.767	0.675	0.921			
(1.539)	(2.491)	(1.018)	(3.040)	(2.003)	(2.080)	(7.712)	(7.073)	(20.56)			
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.198	0.067	0.056	0.033	2.233							
(–2.717)	(1.453)	(1.078)	(0.516)	(0.637)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	10.704	6.697	1.104	10.078	19.653*	7.068
$Q(24)$	23.297	23.049	25.947	11.415	24.699	14.851
$Q^2(12)$	6.238	5.914	12.803	–	–	–
$Q^2(24)$	13.797	11.274	23.123	–	–	–
Skewness	0.606***	1.475***	–0.139	–	–	–
Kurtosis	5.688***	7.640***	3.204	–	–	–
JB	170.273***	592.175***	2.335	–	–	–
$\ln L = -4689.325$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.11: Estimation Results for Unweighted Symmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$				
0.649	5.525	7.567	0.326	0.495	–0.418	0.288	–1.030				
(1.779)	(4.017)	(3.381)	(1.490)	(3.062)	(–0.567)	(2.506)	(–2.297)				
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.128	0.148	0.225	–0.000	–0.007	0.088	0.063	0.070	0.140	0.005	0.066	–0.087
(2.467)	(2.914)	(4.530)	(–0.001)	(–0.134)	(1.823)	(1.278)	(1.482)	(3.139)	(0.118)	(1.284)	(–1.804)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.213	0.024	0.109	0.067	0.005	0.084	–0.011	0.025	–0.052	–0.036	–0.042	0.075
(4.341)	(0.459)	(2.144)	(1.012)	(0.096)	(1.392)	(–0.203)	(0.352)	(–1.108)	(–0.643)	(–0.816)	(1.736)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.179	0.015	0.166	0.117	0.087	0.059	0.018	0.096	–0.007	–0.028	–0.029	–0.099
(–3.602)	(0.307)	(3.508)	(2.635)	(1.755)	(1.214)	(0.366)	(2.028)	(–0.181)	(–0.616)	(–0.602)	(–2.174)
ω_1	ω_2	ω_3	α_1	α_2	α_3	β_1	β_2	β_3			
1.503	1.107	3.948	0.142	0.112	0.055	0.788	0.857	0.919			
(1.445)	(1.580)	(1.040)	(3.117)	(1.838)	(2.108)	(8.648)	(12.647)	(20.453)			
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.193	0.060	0.069	0.025	3.123							
(–2.340)	(1.284)	(1.363)	(0.318)	(0.767)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	8.407	9.342	1.115	9.271	19.214*	12.280
$Q(24)$	21.828	18.765	25.454	10.732	25.461	17.867
$Q^2(12)$	4.773	6.072	13.258	–	–	–
$Q^2(24)$	11.544	10.864	23.656	–	–	–
Skewness	0.725***	1.632***	–0.142	–	–	–
Kurtosis	6.212***	9.312***	3.167	–	–	–
JB	243.231***	988.897***	2.121	–	–	–
$\ln L = -4662.398$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

Table A.12: Estimation Results for Unweighted Symmetric GARCH–M Model, 1962:03–2001:04

μ_1	μ_2	μ_3	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$	$\theta_{2,1}$	$\theta_{2,2}$	$\theta_{2,3}$	$\theta_{3,1}$	$\theta_{3,2}$	
0.699	5.541	7.097	0.049	0.037	0.009	0.223	0.482	–0.376	0.300	–0.973	
(2.015)	(3.988)	(3.235)	(0.920)	(1.692)	(3.097)	(0.980)	(3.069)	(–0.523)	(2.562)	(–2.157)	
$\phi_{1,1}$	$\phi_{1,2}$	$\phi_{1,3}$	$\phi_{1,4}$	$\phi_{1,5}$	$\phi_{1,6}$	$\phi_{1,7}$	$\phi_{1,8}$	$\phi_{1,9}$	$\phi_{1,10}$	$\phi_{1,11}$	$\phi_{1,12}$
0.056	0.143	0.237	0.009	0.008	0.082	0.059	0.066	0.138	0.020	0.076	–0.106
(0.940)	(2.872)	(4.953)	(0.164)	(0.158)	(1.706)	(1.242)	(1.499)	(3.063)	(0.440)	(1.423)	(–2.263)
$\phi_{2,1}$	$\phi_{2,2}$	$\phi_{2,3}$	$\phi_{2,4}$	$\phi_{2,5}$	$\phi_{2,6}$	$\phi_{2,7}$	$\phi_{2,8}$	$\phi_{2,9}$	$\phi_{2,10}$	$\phi_{2,11}$	$\phi_{2,12}$
0.223	0.029	0.112	0.074	0.008	0.084	–0.011	0.027	–0.051	–0.031	–0.045	0.075
(4.541)	(0.559)	(2.216)	(1.121)	(0.156)	(1.400)	(–0.208)	(0.371)	(–1.084)	(–0.547)	(–0.888)	(1.706)
$\phi_{3,1}$	$\phi_{3,2}$	$\phi_{3,3}$	$\phi_{3,4}$	$\phi_{3,5}$	$\phi_{3,6}$	$\phi_{3,7}$	$\phi_{3,8}$	$\phi_{3,9}$	$\phi_{3,10}$	$\phi_{3,11}$	$\phi_{3,12}$
–0.179	0.018	0.170	0.119	0.089	0.061	0.017	0.097	–0.008	–0.025	–0.025	–0.093
(–3.627)	(0.371)	(3.598)	(2.676)	(1.794)	(1.251)	(0.343)	(2.049)	(–0.193)	(–0.549)	(–0.516)	(–2.056)
ω_1	ω_2	ω_3	α_1	α_2	α_3	β_1	β_2	β_3			
1.648	1.097	3.883	0.143	0.109	0.054	0.774	0.860	0.921			
(1.500)	(1.587)	(1.021)	(2.972)	(1.820)	(2.060)	(7.814)	(13.097)	(20.320)			
ρ_{21}	ρ_{31}	ρ_{32}	δ_1	δ_2							
–0.190	0.066	0.070	0.029	2.536							
(–2.351)	(1.421)	(1.362)	(0.398)	(0.632)							

	z_1	z_2	z_3	z_2z_1	z_3z_1	z_3z_2
$Q(12)$	8.080	9.316	1.121	9.562	18.896*	12.431
$Q(24)$	20.780	18.823	25.736	11.166	24.329	17.801
$Q^2(12)$	5.673	6.341	13.402	–	–	–
$Q^2(24)$	12.820	11.027	23.695	–	–	–
Skewness	0.647***	1.623***	–0.143	–	–	–
Kurtosis	5.804***	9.168***	3.189	–	–	–
JB	186.833***	951.328***	2.302	–	–	–
$\ln L = –4655.965$						

Note: Rejection of the null hypothesis at the 1%, 5%, and 10% levels is indicated by ***, **, and *, respectively.

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