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Inflation, inflation uncertainty, and relative price dispersion: Evidence from bivariate GARCH-M models

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Abstract

One potential real effect of inflation is its influence on the dispersion of relative prices in the economy. Menu cost models generally imply that higher trend inflation will increase price dispersion. In contrast, signal extraction models predict that increased inflation uncertainty will raise relative price dispersion. Existing empirical studies do not distinguish between these separate hypotheses. We constuct a bivariate GARCH-M model of inflation and relative price dispersion to test these differing explanations in a single model and find that inflation uncertainty dominates trend inflation as a predictor of relative price dispersion.

Key words: Relative price dispersion; Inflation; Inflation uncertainty; GARCH

JEL classification: C32; E31

1. Introduction

Inflation is unpopular. Yet economists often have difficulty giving a convincing account of the real effects of inflation.[1] Recently, emphasis has been placed on inflation's effect on the distribution of relative prices in the economy. Menu

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[1] For recent surveys of research on the real effects of inflation see Cowen (1993) and Dowd (1993).

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cost models typically predict that higher average inflation will raise relative price dispersion while signal extraction models emphasize the effects of inflation uncertainty. All existing empirical work either confuses, or fails to distinguish between, these two separate effects.

Our contribution in this paper is to construct and estimate a bivariate GARCH-M model of inflation and relative price dispersion.[2] The GARCH-M model generates a time-series measure of inflation uncertainty and allows both the mean and the conditional variance of inflation to have effects on relative price dispersion. We find that the conditional variance of inflation is a consistently significant explanatory variable explaining price dispersion across a variety of sample periods, model specifications, and measures of relative price dispersion. In contrast, the level of inflation is only significant in models where inflation uncertainty is excluded. That is, inflation uncertainty dominates trend inflation in predicting the variability of relative prices. Our results imply greater empirical support for signal extraction than for menu cost models.

The paper is organized as follows. Section 2 briefly surveys the theoretical literature on inflation and relative price variability. Section 3 contains a review and critique of the empirical literature. In Sections 4 and 5 we describe the data and explain our statistical model. Section 6 reports the results of our tests, and we conclude in Section 7.

2. Inflation and relative price dispersion

Models with fixed costs of changing prices imply a positive link between trend inflation and relative price variability (e.g., Sheshinski and Weiss, 1977, 1983; Rotemberg, 1983; Benabou, 1992; Diamond, 1992; Ball and Romer, 1993). In these models, firms follow one-sided (**S**, **s**) pricing rules in the face of inflation. A firm's nominal price is held constant until its real price hits the lower boundary **s**. At this point, the nominal price is adjusted upward so that the real price is set at the upper boundary S. Either variations in fixed costs of price changes across firms or firm-specific shocks can create staggered price changes in the economy which implies positive relative price dispersion. Given the existence of staggered price setting, higher inflation increases the dispersion of relative prices.[3]

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[2] GARCH stands for generalized, autoregressive conditional heteroskedasticity. Based on the work of Engle (1982) these models assume that the conditional error variance can be described by a time-series model. GARCH-M means that the conditional variance is being included as a factor that influences the mean of some variable(s) in the system.

[3] Bordo (1980) and Carlton (1982) argue that price changes are more frequent for standardized commodities sold in auction-type markets than for more customized goods where long-term customer relationships are more important. Also, Ball and Cecchetti (1991) show that staggered wage setting can also imply a positive link between inflation and relative price dispersion.

Barro's (1976) signal extraction model (based on Lucas, 1972) derives a positive link between the variance of surprise inflation and relative price dispersion.[4] The logic of the argument proceeds as follows. As aggregate nominal shocks become more unpredictable, individual firms adjust output less in response to all shocks, including idiosyncratic real demand shocks. Prices must now move more in each market to equate quantity demanded with the now less variable quantity supplied. Firms' prices will be more widely dispersed the less firms respond to demand shocks with output changes, implying that increases in inflation uncertainty will raise relative price dispersion.[5]

In contrast to the menu cost models, trend inflation has no effect on the distribution of relative prices in the basic signal extraction model. As we show below, existing empirical studies have not presented tests capable of distinguishing between these competing hypotheses.

3. Existing evidence

A typical measure of relative price dispersion (RPD_t) in the literature is

$$RPD_t = 1/n \sum_{i=1}^{n} (\pi_{it} - \pi_t)^2 , \tag{1}$$

where π_t is the aggregate inflation rate and π_{it} is rate of price change in the ith individual commodity group.[6] The two most widely cited empirical papers providing evidence that inflation and relative price dispersion are positively related are Vining and Elwertowski (VE, 1976) and Parks (1978).[7] While VE argue that they show 'strong evidence', the only evidence presented is two pairs of aggregate inflation – relative price dispersion graphs. To make matters more complicated, VE refer to 'general price change instability' and seem to argue that there is a positive relationship between the variability of the inflation graph and

[4] See Barro (1976, pp. 13–14). Friedman (1977) makes a similar argument in his Nobel Address.

[5] Benabou and Gertner (1993) consider the welfare effects of increased inflation uncertainty under imperfect competition with endogenous information gathering. While increased inflation uncertainty raises relative price dispersion in their model, the welfare implications of the consumer search signal extraction model may differ from the traditional signal extraction case where inflation uncertainty lowers economic efficiency and welfare.

[6] Some studies weight the individual relative price changes by the size of the sector. Some use the standard deviation, rather than the variance, of price change dispersion. We do not have output data corresponding to the monthly price data we use, so our main empirical measure is unweighted. However, we do use a quarterly weighted price dispersion index similar to Parks, Fischer, and Bomberger and Makinen in the Appendix and show that our main results still obtain.

[7] A review of the Social Science Citation Index for 1990, 1991, and the first half of 1992 reveals that Vining and Elwertowski (1976) and Parks (1978) are cited 40 times.

the level of the relative price dispersion graph. They discuss the Lucas signal-extraction model and are definitely making an argument about inflation variability and price dispersion.

Parks (1978) does provide a statistical test of the relation between inflation and relative price dispersion. Parks estimates the equation:

$$RPD_t = \alpha + \beta \pi_t^2 + \varepsilon_t, \tag{2}$$

where π_t^2 is used as the regressor to 'account for episodes of both inflation and deflation'. Parks uses annual data from 1930–1975 on 12 sectors of Personal Consumption Expenditures and reports significant coefficients for β over the full sample and in pre-war and post-war subsamples. Parks also estimates several regressions adding $(\Delta\pi_t)^2$ to the right-hand side of the equation and reports significant effects for this variable as well.[8]

There are several cogent criticisms of these seminal works. For example, Driffil, Mizon, and Ulph (DMU, 1990) perform a statistical analysis that challenges Vining and Elwertowski's results. They use VE's data (see DMU, pp. 1050–1051) and show that the correlation coefficients between relative price dispersion and inflation rates for both pairs of VE's variables are 0.4435 and 0.2225 with corresponding t-statistics of 2.47 and 1.41, respectively. If the final observation for 1974 is omitted, the estimated simple correlation coefficients are 0.152 and -00.066 with t-statistics of 0.41 and -0.21.

DMU show that Vining and Elwertowski's data do not support the conclusion that trend inflation raises relative price dispersion. The DMU result is important because many papers cite VE as showing a significant inflation–price dispersion link. However, VE never claim the existence of an average inflation–relative price dispersion relationship. Rather, they argue that greater instability of inflation is associated with greater relative price dispersion. In actuality, DMU estimate Parks' model (Eq. (2) above) using the level, rather than the square of inflation, and do not find any significant relationship.[9]

Bomberger and Makinen (1993) replicate and extend Parks' basic model (Eq. (2) above) using annual Personal Consumption Expenditure data. They

[8] Parks derives a theoretical model which implies that unanticipated inflation (the level and the square) is the variable that should drive relative price dispersion. He uses the change in inflation as his measure of unanticipated inflation. Parks recognizes that VE argue that inflation instability promotes relative price dispersion and also argues that the significant of the square of unexpected inflation supports VE's contention.

[9] Thus VE and DMU disagree both about the significance of VE's findings and the hypothesis under consideration. VE say: 'There is strong statistical evidence that... as the general price level becomes less predictable relative to its trend value... the dispersion in relative prices increases.' DMU say: 'Vining and Elwertowski do not provide convincing evidence of a strong positive relationship between relative price variability and inflation in the United States.'

show that, if the oil shock years of 1974 and 1980 are omitted, or if energy price sectors are excluded from the price dispersion index, inflation has no significant effect on relative price dispersion. Their evidence supports Fischer (1981) who argues that the relation between inflation and relative price variability is dominated by food and energy shocks. Taylor (1981) also argues that the relationship between inflation and relative price dispersion occurs because energy shocks are driving both variables.[10]

Finally, most studies use squared inflation at time t to explain relative price variability at time t. Yet, inflation and price dispersion are jointly determined in some overall model where both of these variables are endogenous. Any contemporaneous correlation between the two variables is extremely difficult to interpret as causal.[11]

In our statistical work, we address each of these points. First, we use a bivariate GARCH model of inflation and relative price dispersion to estimate both the effect of trend inflation and conditional inflation variability on relative price dispersion. Second, we heed Fischer's, Taylor's, and Bomberger and Makinin's warnings and exclude energy prices from our relative price dispersion measure. Third, we use lagged rather than contemporaneous inflation in the regressions.

Our tests focus on separating the effects of trend inflation from inflation uncertainty as causes of relative price dispersion. Previous tests fail to make this distinction very clear. As noted, VE talk about general price change instability (i.e., inflation variance) but are frequently cited as showing an inflation level–relative price dispersion link. Similarly, Parks uses $(\Delta\pi_t)^2$ as a measure of unexpected inflation in later equations explaining relative price dispersion. He shows that this variable works better than does π_t^2. Taylor (1981) cites Parks and interprets $(\Delta\pi_t)^2$ as the variability of inflation. Strictly, $(\Delta\pi)^2$ is neither of these things. Further, neither the menu cost nor signal extraction models surveyed above imply that unanticipated inflation or the variability of inflation are the variables directly affecting relative price dispersion. Rather, the theories are concerned with (and we test for) the effects of average inflation vs. inflation uncertainty on relative price dispersion. We capture uncertainty, as opposed to variability, by using a GARCH estimate of the conditional variance of inflation.

[10] To eliminate the effects of supply shocks, Fischer creates measures of relative price dispersion that exclude the energy and food sectors. The inflation–relative price dispersion linkage is much weaker in these cases. In Bomberger and Makinen, the relationship disappears completely.

[11] We are indebted to our referee for this point. In this paper, we concentrate on testing the conflicting predictions of menu cost and signal extraction models about what moment of the inflation process influences relative price dispersion. All our equations use lagged variables on the right-hand side, including the GARCH equation for the conditional variance of inflation.

4. Data

We compute a monthly measure of relative price dispersion (RPD) as in (1) from the Producer Price Index over the 1948.01–1991.12 period. The overall rate of inflation is calculated from the Producer Price Index for all commodities and sectoral inflation rates are calculated from price indexes for fourteen commodity groups.[12] We begin this section by showing that our measure produces Parks-like results using his methods.

Eq. (3) is an OLS regression of relative price dispersion on squared inflation from 1948.01–1991.12 that typifies previous tests. The results here confirm a large positive correlation between the variance in relative prices and the squared aggregate inflation rate (in all equations reported in the text, the numbers in parentheses are t-statistics),

$$\underset{}{RPD_t} = \underset{(5.4)}{0.619} + \underset{(17.3)}{1.007}\pi_t^2 + v_t \qquad \text{(Log-likelihood} = -1235). \tag{3}$$

Eq. (4) replaces contemporaneous squared inflation with lagged squared inflation to reduce problems associated with the simultaneous determination of RPD_t and π_t^2 mentioned above. While the correlation between inflation and relative price dispersion is now lessened, there is still a positive and highly significant relation

$$RPD_t = \underset{(6.8)}{0.946} + \underset{(6.4)}{0.448}\pi_{t-1}^2 + v_t \qquad \text{(Log-likelihood} = -1334). \tag{4}$$

Finally, Eq. (5) adds one lag of price dispersion and a first-order moving average term to the model. Here all variables are significant at the 1% level.

$$RPD_t = \underset{(2.7)}{0.469} + \underset{(4.3)}{0.376}\pi_{t-1}^2 + \underset{(4.4)}{0.429}RPD_{t-1} - \underset{(4.9)}{0.384}v_{t-1} + v_t \tag{5}$$

$$\text{(Log-likelihood} = -1326).$$

A single-equation OLS framework does not easily allow tests of the hypothesis that inflation uncertainty raises relative price dispersion. In the next section, we develop a bivariate GARCH model to address this issue. We conclude this preliminary empirical section by specifying a baseline time-series model for inflation and then testing the null hypotheses that relative price dispersion and inflation each have a constant conditional variance. Our OLS inflation model

[12] The data come from CITIBASE. The groupings used are farm products, industrial commodities, fuels and power, chemicals, rubber and plastic products, lumber and wood products, pulp and paper, metal and metal products, machinery and equipment, furniture and household durables, mineral products, transportation, photographic equipment and supplies, and miscellaneous products.

appears in Eq. (6):

$$\underset{(3.6)}{\Pi_t = 0.114} + \underset{(5.0)}{0.214\Pi_{t-1}} + \underset{(5.1)}{0.213\Pi_{t-2}} + \underset{(3.5)}{0.139\Pi_{t-6}} + \underset{(4.1)}{0.181\varepsilon_{t-12}} + \varepsilon_t. \tag{6}$$

The inflation equation includes the first, second, and sixth lag of inflation, plus a twelfth-order moving average term. The residuals are white noise (Box–Pierce $Q(12) = 13.1$), but the squared residuals show significant, highly persistent conditional heteroskedasticity. The χ^2 statistics testing the serial independence of the squared residuals at 1, 4, and 8 lags are each above 100. In contrast the residuals from the RPD equation (Eq. (5) above) are both random and conditionally homoskedastic. The χ^2 statics testing the null hypothesis of no ARCH effects at 1, 4, and 8 lags are all less than 1. Inflation has a time-varying conditional variance, but RPD does not.

5. Bivariate GARCH-M model

In this section, we outline a bivariate GARCH-M model that allows joint estimation of the conditional means, variances, and covariances of inflation and relative price dispersion. GARCH-M is an extension of Engle, Lilien, and Robbins' (1987) ARCH-M model. These models allow the conditional variance of each endogenous variable to appear as a regressor in one or more of the conditional mean equations.

Since we show above that the conditional variance of relative prices is constant, we use the following simplified bivariate GARCH(1,1)-M model for inflation and relative price dispersion:

$$\pi_t = \beta_0 + \beta_1 \pi_{t-1} + \beta_2 \pi_{t-2} + \beta_3 \pi_{t-6} + \beta_4 \varepsilon_{t-12} + \varepsilon_t, \tag{7}$$

$$\sigma_{\varepsilon t}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \sigma_{\varepsilon t-1}^2, \tag{8}$$

$$RPD_t = \gamma_0 + \gamma_1 RPD_{t-1} + \gamma_2 v_{t-1} + \gamma_3 \pi_{t-1}^2 + \gamma_4 \sigma_{\varepsilon t}^2 + v_t, \tag{9}$$

$$\sigma_{vt}^2 = \bar{\sigma}_v^2, \quad \forall t, \tag{10}$$

$$COV_t = \rho \sigma_{\varepsilon t} \bar{\sigma}_v. \tag{11}$$

Eq. (7) describes the inflation rate as a function of the first, second, and sixth lags of inflation and a 12th-order moving average term. Eq. (8) is a GARCH (1,1) model of the conditional variance of inflation, where each of the α_i's are ≥ 0 and $\alpha_1 + \alpha_2 < 1$.[13] $\sigma_{\varepsilon t}^2$ is the variance of unpredictable inflation in period t. The GARCH (1,1) specification implies that the conditional variance of inflation at

[13] The first condition is necessary to ensure nonnegativity and the second is required for the stationarity of the conditional variance of inflation.

time t depends on the squared residual from Eq. (7) in $t-1$ and the conditional variance from $t-1$. We use this estimated conditional variance as our time series measure of inflation uncertainty.[14]

Eq. (9) describes the mean of relative price dispersion. The variables are the first lag of RPD, a first-order moving average term, the first lag of the squared inflation rate, and the conditional variance of inflation (Eq. (8)). If relative price dispersion is a function of trend inflation, then γ_3 will be positive and significant. If inflation uncertainty is the variable that drives price dispersion, then γ_4 will be positive and significant. We do not rule out the possibility that both variables are significant factors. Eq. (10) indicates the homoskedasticity of relative price dispersion. Finally, Eq. (11) is a simple constant correlation model of the covariance between ε and v.[15]

6. Results

Panel A of Table 1 reports maximum-likelihood estimates of Eqs. (7)–(11) with the effect of inflation uncertainty on *RPD* constrained to zero.[16] The sample is 528 monthly observations from 1948 through 1991. We calculate Box–Pierce Q statistics at 12 lags for the levels, squares, and cross-products of the residuals for all the GARCH systems estimated in the paper. In each case, these diagnostic tests indicate that the time-series models for the means and the GARCH model for the residual conditional variance–covariance adequately capture the joint distribution of the error terms. The estimated GARCH(1,1) process for the conditional variance of inflation is very significant and stable.[17]

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[14] Since $\sigma_{\varepsilon t}^2$ is the variance of inflation conditional on the model of the mean of inflation in Eq. (3), it does measure inflation uncertainty and not merely inflation variability. GARCH models of conditional variance are thus superior to moving unconditional standard deviations as a method of quantifying uncertainty, as opposed to quantifying variability. Also note that the conditional variance equation does not have its own random error term but is a deterministic relationship with unknown coefficients. This treatment is standard even in more common heteroskedasticity correction models.

[15] Constant correlations is a simplified covariance model attributable to Bollerslev (1990). We examine more complicated covariance models in our statistical work and the results do not change in any material way. See Fn. 21 below for more details.

[16] We assume that the two error terms, ε and v, are jointly normally distributed and estimate this system (Eqs. (3)–(7)) using a nonlinear maximum likelihood technique. Specifically we use a FORTRAN program called MGARCH which uses the Berndt, Hall, Hall, and Hausman (1974) algorithm. We thank Ken Kroner for supplying the software and troubleshooting advice.

[17] Both the moving average and the autoregressive components of the conditional variance are positive and significant at the 1% level. The coefficients sum to 0.937 indicating a persistent but stationary conditional variance process.

Table 1
Monthly inflation and relative price dispersion: 1948–1991

A. *GARCH (1,1) model – Constant conditional correlations*

$$\underset{(3.53)}{\Pi_t = 0.097} + \underset{(5.63)}{0.245\Pi_{t-1}} + \underset{(3.27)}{0.145\Pi_{t-2}} + \underset{(3.17)}{0.134\Pi_{t-6}} + \underset{(3.67)}{0.143\varepsilon_{t-12}} + \varepsilon_t$$

$$\sigma^2_{\varepsilon t} = \underset{(3.35)}{0.017} + \underset{(4.16)}{0.163\varepsilon^2_{t-1}} + \underset{(18.8)}{0.774\sigma^2_{\varepsilon t-1}}$$

$$RPD_t = \underset{(1.27)}{0.164} + \underset{(10.3)}{0.751 RPD_{t-1}} - \underset{(9.61)}{0.691 v_{t-1}} + \underset{(3.82)}{0.188\Pi^2_{t-1}} + v_t$$

$$COV_t = \underset{(7.10)}{0.197(\sigma_{\varepsilon t}\bar{\sigma}_v)} \qquad \text{Log-likelihood function} = -1716.5$$

B. *GARCH (1,1)-M model – Constant conditional correlations*

$$\underset{(3.45)}{\Pi_t = 0.097} + \underset{(5.69)}{0.232\Pi_{t-1}} + \underset{(3.68)}{0.156\Pi_{t-2}} + \underset{(2.92)}{0.115\Pi_{t-6}} + \underset{(3.60)}{0.133\varepsilon_{t-12}} + \varepsilon_t$$

$$\sigma^2_{\varepsilon t} = \underset{(4.24)}{0.023} + \underset{(5.17)}{0.211\varepsilon^2_{t-1}} + \underset{(21.7)}{0.713\sigma^2_{\varepsilon t-1}}$$

$$RPD_t = \underset{(1.73)}{-1.42} + \underset{(4.73)}{0.761 RPD_{t-1}} - \underset{(4.37)}{0.744 v_{t-1}} + \underset{(0.03)}{0.002\Pi^2_{t-1}} + \underset{(5.53)}{6.2\sigma^2_{\varepsilon t}} + v_t$$

$$COV_t = \underset{(4.96)}{0.197(\sigma_{\varepsilon t}\bar{\sigma}_v)} \qquad \text{Log-likelihood function} = -1699.6$$

Sample period is 1948.01–1991.12. T-statistics are given in parentheses.

The conditional correlation coefficient is positive with a t-statistic of 7.1, indicating a significant relationship between the residual covariances and the conditional variance of inflation. The *RPD* equation shows that π^2_{t-1} is still a positive and significant explanatory factor for relative price dispersion (t-statistic = 3.8).

Panel B in Table 1 adds the GARCH in mean term ($\sigma^2_{\varepsilon t}$) to the *RPD* equation.[18] The results here are dramatic. The coefficient value and significance level of trend inflation falls almost to zero, while inflation uncertainty is positive and significant with a t-statistic in excess of 5.5. *Inflation uncertainty, not trend inflation, drives relative price dispersion.*

Given that the error term in Eq. (9) above follows an MA(1) process, a single lag of inflation information may not be sufficient generate exogenous regressors. However, we can replicate all the results in the paper using Eq. (9′):

$$RPD_t = \gamma_0 + \gamma_1 RPD_{t-1} + \gamma_2 v_{t-1} + \gamma_3 \pi^2_{t-2} + \gamma_4 \sigma^2_{\varepsilon t-1} + v_t. \tag{9′}$$

[18] Note that this conditional variance is constructed using only lagged information about inflation innovations.

Table 2
Monthly inflation and relative price dispersion: 1948–1973

A. *GARCH (1,1) model – Constant conditional correlations*

$$\Pi_t = \underset{(1.96)}{0.086} + \underset{(2.36)}{0.134}\Pi_{t-1} + \underset{(2.79)}{0.156}\Pi_{t-2} + \underset{(1.64)}{0.101}\Pi_{t-6} + \underset{(1.58)}{0.095}\varepsilon_{t-12} + \varepsilon_t$$

$$\sigma_{\varepsilon t}^2 = \underset{(1.66)}{0.006} + \underset{(2.95)}{0.094}\varepsilon_{t-1}^2 + \underset{(23.1)}{0.888}\sigma_{\varepsilon t-1}^2$$

$$RPD_t = \underset{(1.01)}{0.163} + \underset{(4.71)}{0.643}RPD_{t-1} - \underset{(5.81)}{0.712}v_{t-1} + \underset{(2.30)}{0.286}\Pi_{t-1}^2 + v_t$$

$$COV_t = \underset{(3.89)}{0.333}(\sigma_{\varepsilon t}\bar{\sigma}_v) \qquad \text{Log-likelihood function} = -879.9$$

B. *GARCH (1,1)-M model – Constant conditional correlations*

$$\Pi_t = \underset{(1.90)}{0.065} + \underset{(3.92)}{0.175}\Pi_{t-1} + \underset{(2.21)}{0.105}\Pi_{t-2} + \underset{(2.39)}{0.112}\Pi_{t-6} + \underset{(2.91)}{0.135}\varepsilon_{t-12} + \varepsilon_t$$

$$\sigma_{\varepsilon t}^2 = \underset{(4.26)}{0.074} + \underset{(4.03)}{0.400}\varepsilon_{t-1}^2 + \underset{(3.83)}{0.348}\sigma_{\varepsilon t-1}^2$$

$$RPD_t = -\underset{(1.73)}{1.24} + \underset{(4.93)}{0.801}RPD_{t-1} - \underset{(3.89)}{0.676}v_{t-1} + \underset{(0.28)}{0.046}\Pi_{t-1}^2 + \underset{(4.95)}{5.1}\sigma_{\varepsilon t}^2 + v_t$$

$$COV_t = \underset{(2.90)}{0.246}(\sigma_{\varepsilon t}\bar{\sigma}_v) \qquad \text{Log-likelihood function} = -869.1$$

Sample period is 1948.01–1973.12. *T*-statistics are given in parentheses.

Here all information about inflation used in the *RPD* equation comes from period $t-2$ or earlier.[19] We can also replicate our results using a weighted, quarterly measure of relative price dispersion. See the Appendix for details on the index and the corresponding GARCH-M results.

In Table 2, we reestimate the models from Table 1 on a 1948–1973 subsample that conforms more closely to the periods studied by Vining and Elwertowski and Parks.[20] The results are almost identical to the full sample results in Table 1. When the effect of inflation uncertainty is constrained to zero in panel A, π_{t-1}^2 is a positive and significant influence on *RPD*, though the *t*-statistic is barely

[19] For example, when we use Eq. (9′) and repeat the experiment in Table 1, the coefficient on π_{t-2}^2 is positive and significant (coefficient = 0.216, *t*-statistic = 3.7) when $\sigma_{\varepsilon t-1}^2$ is excluded from the *RPD* equation . However when $\sigma_{\varepsilon t-1}^2$ is added, it is positive and significant at the 0.01 level (coefficient = 3.61, *t*-statistic = 3.15) and lagged inflation becomes completely insignificant (*t*-statistic falls to 0.7). All of our results survive using Eq. (9′) instead of (9). We are again indebted to our referee for this point.

[20] Vining and Elwertowski examine the 1948–1974 period. Parks examines the 1930–1975 period.

Table 3
Effects of alternative measures of trend inflation on relative price dispersion

	Bivariate GARCH model with σ^2 excluded from *RPD* equation		Bivariate GARCH-M model with σ^2 included in *RPD* equation		
Inflation variable	Inflation coefficient	Log-likelihood	Inflation coefficient	$\sigma_{\varepsilon t}^2$ coefficient	Log-likelihood
Π_{t-1}	0.299 (2.4)	− 1711	0.009 (0.1)	7.355 (7.4)	− 1700
$\lvert \Pi_{t-1} \rvert$	0.771 (3.2)	− 1713	− 0.175 (0.5)	7.111 (6.3)	− 1701
$(\Delta\Pi_{t-1})^2$	0.044 (3.1)	− 1719	− 0.059 (1.6)	6.672 (7.2)	− 1706

Sample period is 1948.01–1991.12. Numbers in parentheses are t-statistics.

Π_{t-1} is the first lag of the inflation rate.
$\lvert \Pi_{t-1} \rvert$ is the absolute value of the first lag of the inflation rate.
$(\Delta\Pi_{t-1})^2$ is square of the first lag of the change in the inflation rate.

The inflation coefficient in the left-hand column is from a bivariate GARCH model analagous to panel A of Table 1. The coefficients in the right-hand column are from a bivariate GARCH-M model analagous to panel B of Table 1. The other coefficients in the GARCH models are not reported here in the interest of space-saving.

above 2.0. In Table 2, panel B, the conditional variance of inflation is included in the *RPD* equation and once again inflation uncertainty dominates the trend inflation variable. The coefficient on inflation uncertainty is almost identical in the full and partial sample results (6.2 vs. 5.9). In a model where both variables are allowed to have effects, it is uncertainty about inflation, not trend inflation, which affects relative price dispersion.[21]

So far, we have used π_{t-1}^2 to measure trend inflation. While Parks and Bomberger and Makinen use this variable, there are other plausible candidates. In order to see whether the method of defining trend inflation matters, we estimate the GARCH and GARCH-M models over the full sample using alternative trend inflation variables. Table 3 reports the results of three such

[21] Our results also hold using the positive-definite functional form of the GARCH model. The residual covariance matrix (H_t) is given by: $H_t = C_0^T C_0 + C_1^T(\varepsilon_{t-1} v_{t-1}) C_1 + C_2^T H_{t-1} C_2$. Here H_t is the 2×2 conditional covariance matrix , C_0, C_1, and C_2 are all 2×2 coefficient matrices, with C_0 symmetric, and T indicates matrix transposition. The results again show that inflation uncertainty dominates the mean of inflation as the aggregate nominal factor influencing relative price dispersion. Lagged squared inflation is insignificant (t-statistic = 1.6) while the conditional variance of inflation is positive and significant with a t-statistic of 5.8.

experiments. To save space, only the coefficients on the inflation variables are reported. The left column of Table 3 shows the effect of the trend inflation variable on relative price dispersion when the conditional variance of inflation is not included in the price dispersion equation. The right column of the table shows the trend inflation coefficient and the inflation uncertainty coefficient when both variables are included in the relative price dispersion equation.

The specific inflation variables used are: 1) π_{t-1} (lagged inflation rate), 2) $|\pi_{t-1}|$ (absolute value of the lagged inflation rate), and 3) $(\Delta\pi_{t-1})^2$ (square of the lagged change in inflation). In each case, the inflation trend variables are positive and significant in GARCH systems where inflation uncertainty is constrained to have no effect. Yet in the full GARCH-M model, the inflation variable is always insignificant and the coefficient on the conditional variance of inflation is positive and very significant, with a t-statistic greater than 6.0 in each model. Regardless of the sample, covariance model, or trend inflation variable, inflation uncertainty and not average inflation is the relevant factor for explaining the dispersion of relative prices.

7. Discussion

Pioneering empirical papers by Vining and Elwertowski and Parks suggest some link between the inflation process and the dispersion of relative prices. Yet, the analyses in these and subsequent papers are not able to distinguish between average inflation and inflation uncertainty as the main factor influencing price dispersion. This distinction is important because it can help to discriminate between alternative macroeconomic models. Typical menu cost models predict that trend inflation matters, while signal extraction models emphasize the importance of inflation uncertainty. Both of these literatures cite the same empirical papers to support their conflicting positions.

We use bivariate GARCH-M models as a natural way to estimate the conditional variance of inflation and test its effect on relative price dispersion in a single system. Our results clearly show that inflation uncertainty, as measured by the conditional variance, dominates trend inflation as a predictor of relative price dispersion. The result holds over changes in the sample period, method of constructing the price dispersion index, covariance structure of the model, and definition of the trend inflation variable.

We find little support for menu cost models where trend inflation influences relative price dispersion. Rather, our results are consistent with signal extraction models where increased inflation uncertainty increases relative price dispersion. Our results also illustrate the utility of multivariate GARCH-M models for testing macroeconomic hypotheses involving uncertainty.

Appendix

Here we use an alternative index of relative price dispersion to check the robustness of our results in Table 1 in the text. We use quarterly data from 1949.02 through 1991.03 on Personal Consumption Expenditures in ten categories (again excluding the energy sectors). The sectors are motor vehicles, furniture, other durables, food, clothing, housing, household operations, transportation, medical care, and other services. These data are taken from the Citibase databank. Given that we have output data for these sectors we construct a weighted price dispersion index as follows:

$$WRPD_t = 1/10 \sum_{i=1}^{10} w_{it}(\pi_{it} - \pi_t)^2, \tag{A.1}$$

where w_{it} is the weight for the ith sector in time t. Preliminary testing of the

Table A.1
Quarterly inflation and weighted relative price dispersion: 1949–1991

A. *GARCH (1,1) model – Constant conditional correlations*

$$\underset{}{\Pi_t} = \underset{(2.03)}{0.602} + \underset{(4.09)}{0.292}\Pi_{t-1} + \underset{(4.05)}{0.344}\Pi_{t-2} + \underset{(2.63)}{0.225}\Pi_{t-3} + \varepsilon_t$$

$$\sigma^2_{\varepsilon t} = \underset{(1.90)}{0.154} + \underset{(2.27)}{0.146}\varepsilon^2_{t-1} + \underset{(11.4)}{0.807}\sigma^2_{\varepsilon t-1}$$

$$WRPD_t = \underset{(6.72)}{0.728} + \underset{(2.49)}{0.255}v_{t-4} - \underset{(1.45)}{0.140}v_{t-5} + \underset{(2.86)}{0.009}\Pi^2_{t-1} + v_t$$

$$\sigma^2_{vt} = \underset{(2.83)}{0.018} + \underset{(4.46)}{0.172}v^2_{t-1} + \underset{(28.9)}{0.837}\sigma^2_{vt-1}$$

$$COV_t = \underset{(3.74)}{0.238}\sigma_{\varepsilon t}\sigma_{vt} \qquad \text{Log-likelihood function} = -596.4$$

B. *GARCH (1,1)-M model – Constant conditional correlations*

$$\Pi_t = \underset{(2.11)}{0.616} + \underset{(4.79)}{0.285}\Pi_{t-1} + \underset{(5.07)}{0.361}\Pi_{t-2} + \underset{(3.06)}{0.216}\Pi_{t-3} + \varepsilon_t$$

$$\sigma^2_{\varepsilon t} = \underset{(2.11)}{0.118} + \underset{(3.00)}{0.073}\varepsilon^2_{t-1} + \underset{(22.5)}{0.871}\sigma^2_{\varepsilon t-1}$$

$$WRPD_t = \underset{(1.70)}{0.558} + \underset{(2.78)}{0.194}v_{t-4} - \underset{(1.40)}{0.135}v_{t-5} + \underset{(1.88)}{0.006}\Pi^2_{t-1} + \underset{(5.16)}{1.04}\sigma^2_{\varepsilon t} + v_t$$

$$\sigma^2_{vt} = \underset{(3.26)}{0.023} + \underset{(3.30)}{0.123}v^2_{t-1} + \underset{(29.9)}{0.847}\sigma^2_{vt-1}$$

$$COV_t = \underset{(4.52)}{0.268}\sigma_{\varepsilon t}\sigma_{vt} \qquad \text{Log-likelihood function} = -587.9$$

Sample is quarterly from 1949.02–1991.03. T-statistics are given in parentheses.

quarterly weighted price dispersion index (*WRPD*) shows that an MA(4), MA(5) time-series model fits best. Further, unlike the monthly unweighted index we find significant and persistent conditional heteroskedasticity in the squared residuals.

In Table A.1, we repeat the experiment shown in Table 1 in the text, with the new relative price dispersion measure and a GARCH(1,1) model of the conditional variance of *WRPD*. In panel A, with the effect of inflation uncertainty constrained to zero, lagged squared inflation is positive and significant in the *WRPD* equation. When inflation uncertainty is included (panel B), average inflation loses its significance and the conditional variance of inflation is positive and significant at the 0.01 level. Our results in the text hold up very well here using quarterly data, a different (and weighted) index, and a slightly different sample period.

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