Notes on Advanced Macroeconomics, Edition for Final Exam

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1 jian da ti, 15'x2

1.1 RBC model (6 eqs) vs classical model (7 eqs), match similar eqs and differ

古典模型	RBC模型	差别
AS	**AS**	
$Y = A \cdot F(K, N)$	$Y_t = A_t N_t^{1-\alpha}$	生产函数本质相同，但RBC中技术可以包含技术冲击$A_t = A_{t-1}^{\rho a} e^{\epsilon_t^a}$
$\max \pi \Rightarrow \frac{W}{P} = F_N(K, N)$	$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$	劳动力需求方程本质相同，由厂商PMP得到
$N = N\left(\frac{W}{P}\right), N' > 0$	$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$	古典模型直接用ad hoc假设劳动力市场出清；RBC里劳动市场的出清由供需双方决定，这一均衡由消费者的UMP得到的劳动力供给方程刻画
AD	**AD**	
$Y = C + I + G + \delta K$	$Y_t = C_t + I_t$	会计等式本质相同
$C = C(Y, T - \pi)$	$Q_t = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-a} \frac{P_t}{P_{t+1}}$	古典模型认为消费是税后收入的函数；而RBC模型的微观基础认为跨期消费是由欧拉方程刻画的
$I = I(q-1), I' > 0, q \equiv \frac{F_k - (r + \delta - \pi)}{r - \pi}$		古典模型认为投资取决于之前时期的产出水平；而RBC中UMP的欧拉方程决定消费后也相应决定了投资，所以RBC比古典模型少了一个方程
$\frac{M}{P} = m(Y, r), m_1 > 0, m_2 < 0$	$\frac{M_t}{P_t} = \frac{Y_t}{Q_t^{-n}}$	古典模型对于流动性需求没有微观基础；RBC模型还引入了债券价格，可以使模型表达李嘉图等价等概念

In conclusion, classical model assumes a world where market always clears based on simple aggregated relationships; RBC model, using micro foundation and incorporating technology shocks, still portraits a perfectly competitive world but with mechanism rendering market-unclearing possible. The key difference is that RBC model, originated from Ramsey model, internalizes consumption and saving decisions. Therefore RBC consists of six equation, where as classical consists of seven.

1.2 ?

2 jisuan tuidao ti, 20'x2

2.1 second way to deduct NKPC, three equations, acceptable price setting eq

Now sticky price price-adjusting equation [1]

$$\hat{p}_t^{\triangle} = \underbrace{(1-\theta\beta)\sum_{k=0}^{\infty}(\theta\beta)^k E_t P_{t+k}^*}_{\text{weighted average}} = (1-\theta\beta)\hat{p}_t^* + \theta\beta E_t \hat{p}_{t+1}^{\triangle} \tag{1}$$

The θ is the opposite of the one in previous PAE, here rather is to stay put. price adjusting based on acceptable price setting eq in future periods, β mean subjective discount.
With the expression of total price being

$$\Rightarrow \hat{p}_t = (1-\theta)\sum_{k=0}^{\infty}\theta^k \hat{p}_{t-k}^{\triangle} \tag{2}$$

$$\Rightarrow \hat{p}_t = (1-\theta)\hat{p}_t^{\triangle} + \theta\hat{p}_{t-1} \text{ (expression of total price based on APSE and PAE)} \tag{3}$$

From the first equation $\hat{p}_t = (1-\theta)\sum_{k=0}^{\infty}\theta^k \hat{p}_{t-k}^{\triangle}$ we have

$$\hat{p}_t = (1-\theta)P_t^{\triangle} + \theta P_{t-1} \tag{4}$$

$$\Rightarrow (1-\theta)P_t^{\triangle} = \hat{p}_t - \theta\hat{p}_{t-1} \tag{5}$$

$$\Rightarrow (1-\theta)E_t P_{t+1}^{\triangle} = E_t\hat{p}_{t+1} - \theta\hat{p}_t \tag{6}$$

$$\Rightarrow (1-\theta)\beta E_t \hat{P}_{t+1}^{\triangle} = \beta E_t\hat{p}_{t+1} - \theta\beta\hat{p}_t \tag{7}$$

From the second equation $\hat{p}_t = (1-\theta)\hat{p}_t^{\triangle} + \theta\hat{p}_{t-1}$, first use sticky price price-adjusting equation $\hat{p}_t^{\triangle} = (1-\theta\beta)\hat{p}_t^* + \theta\beta E_t\hat{p}_{t+1}^{\triangle}$ to substitute

$$\Rightarrow \hat{p}_t - \theta\hat{p}_{t-1} = (1-\theta)\hat{p}_t^{\triangle} = (1-\theta)(1-\theta\beta)\hat{p}_t^* + (1-\theta)\theta\beta E_t P_{t+1}^{\triangle} \tag{8}$$

then use APSE $\hat{p}_t^* = \hat{p}_t + \gamma\tilde{y}_t$ to substitute

$$\Rightarrow \hat{p}_t - \theta\hat{p}_{t-1} = (1-\theta)(1-\theta\beta)(\hat{p}_t + \gamma\tilde{y}_t) + (1-\theta)\theta\beta E_t P_{t+1}^{\triangle} \tag{9}$$

use the conclusion $(1-\theta)E_t P_{t+1}^{\triangle} = E_t\hat{p}_{t+1} - \theta\hat{p}_t$ above

$$\Rightarrow \hat{p}_t - \theta\hat{p}_{t-1} = (1-\theta)(1-\theta\beta)(\hat{p}_t + \gamma\tilde{y}_t) + \theta\beta E_t\hat{p}_{t+1} - \theta^2\beta\hat{p}_t \tag{10}$$

$$\Rightarrow [1+\theta^2\beta - (1-\theta)(1-\theta\beta)]\hat{p}_t - \theta P_{t-1} = (1-\theta)(1-\theta\beta)\gamma\tilde{y}_t + \theta\beta E_t \text{ (break APSE)} \tag{11}$$

$$\Rightarrow [1+\theta^2\beta - (1-\theta)(1-\theta\beta)]\hat{p}_t - \theta P_{t-1} - \theta\beta\hat{p}_t = (1-\theta)(1-\theta\beta)\gamma\tilde{y}_t + \theta\beta(E_t\hat{p}_{t+1} - \hat{p}_t) \tag{12}$$

$$\Rightarrow \theta(\hat{p}_t - \hat{p}_{t-1}) = (1-\theta)(1-\theta\beta)\gamma\tilde{y}_t + \theta\beta(E_t\hat{P}_{t+1} - \hat{p}_t) \text{ (match terms so there is inflation)} \tag{13}$$

$$\Rightarrow \theta\hat{\pi}_t = (1-\theta)(1-\theta\beta)\gamma\tilde{y}_t + \theta\beta E_t\hat{\pi}_{t+1} \text{ (here comes inflation)} \tag{14}$$

$$\Rightarrow \hat{\pi}_t = k\hat{y}_t + \beta E_t\hat{\pi}_{t+1} \text{ (NKPC)} \tag{15}$$

[1] See more in Calvo, 1983

2.2 sticky price and sticky wage to labor demand eq, opt problem, with hints

- sticky price
  - UMP of household

Household face two stages of optimization. Since goods are heterogenous, UMP begins by choosing items $i \in I$. The first stage is sufficient to acquire good demand equation and expression of total price .
We go by method of minimization, as in this case Lagrangian multiplier could be shadow price.

$$\begin{aligned} &\min \int_0^1 P_{it} C_{it} di \\ &s.t. (\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}} \geqslant C_t \end{aligned} \tag{16}$$

Form a Lagrangian, we would have

$$\underset{\{C_{it}\}}{\mathscr{L}} = \int_0^1 P_{it} C_{it} di + P_t [(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}} - C_t] \tag{17}$$

$$\Rightarrow P_{it} = P_t \frac{\epsilon}{\epsilon - 1} ((\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1} - 1}) \frac{\epsilon - 1}{\epsilon} C_{it}^{\frac{\epsilon-1}{\epsilon} - 1} \tag{18}$$

$$\Rightarrow P_{it} = P_t [(\int_0^1 C_{it}^{\frac{\epsilon-1}{\epsilon}} di)^{\frac{\epsilon}{\epsilon-1}})^{\frac{1}{\epsilon}}] C_{it}^{\frac{-1}{\epsilon}} \tag{19}$$

$$\Rightarrow \frac{P_{it}}{P_t} = (\frac{C_{it}}{C_t})^{\frac{-1}{\epsilon}} \tag{20}$$

$$\Rightarrow C_{it} = (\frac{P_{it}}{P_t})^{-\epsilon} C_t \text{ (the demand curve)} \tag{21}$$

From $\int_0^1 P_{it} C_{it} di = P_t C_t$ we have

$$\Rightarrow \int_0^1 P_{it} (\frac{P_{it}}{P_t})^{-\epsilon} C_t di = P_t C_t \tag{22}$$

$$\Rightarrow \int_0^1 P_{it} (\frac{P_{it}}{P_t})^{-\epsilon} di = P_t \tag{23}$$

$$\Rightarrow P_t = (\int_0^1 P_{it}^{1-\epsilon} di)^{\frac{1}{1-\epsilon}} \text{ (the expression of total price level)} \tag{24}$$

- sticky wage
  - PMP

With labor market being monopolistic comptitive and sticky wage, we can find labor supply equation and expression of total wage in PMP.
For a representative firm

$$\begin{aligned} &\min \int_0^1 W_{jt} N_{ijt} dj \\ &s.t. A_t [(\int_0^1 N_{ijt}^{\frac{\epsilon_W - 1}{\epsilon_W}} dj)^{\frac{\epsilon_W}{\epsilon_W - 1}}]^{1-\alpha} \geqslant A_t N_{it}^{1-\alpha} \end{aligned} \tag{25}$$

where i indexing firm, j indexing labor, t is time
Form a Lagrangian

$$\mathscr{L} = \int_0^1 W_{jt} N_{ijt} dj + W_t \{ A_t [(\int_0^1 N_{ijt}^{\frac{\epsilon_w - 1}{\epsilon_w}} dj)^{\frac{\epsilon_W}{\epsilon_W - 1}}]^{1-\alpha} - A_t N_{it}^{1-\alpha} \} \tag{26}$$

where Lagrangian multiplier is total wage level
Solving the optimization problem results

$$\Rightarrow \begin{cases} N_{ijt} = (\frac{W_{jt}}{W_t})^{-\epsilon_W} N_{it} \text{ (labor supply equation)} \\ W_t = (\int_0^1 W_{jt}^{1-\epsilon_W} dj)^{\frac{1}{1-\epsilon_W}} \text{ (expression of total wage level)} \end{cases} \tag{27}$$

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3.1 dynamic inefficiency: solow to ramsey to olg

Solow
The model has key equations

$$\begin{cases} Y = F(K, L) \\ I = S = sY \\ L_{t+1} = (1+n)L_t \\ k_{t+1} = \frac{(1-\delta)k_t + sf(k_t)}{(1+n)} \iff \dot{k} = sf(k) - (n+\delta)k \end{cases} \tag{28}$$

Put $k_{t+1} = \frac{(1-\delta)k_t + sf(k_t)}{1+n}$ at steady state

$$\Rightarrow (1+n)k^* = (1-\delta)k^* + sf(k^*) \tag{29}$$
$$-(n+\delta)k^* = -sf(k^*) \tag{30}$$
$$f(k^*) - (n+\delta)k^* = (1-s)f(k^*) = c^* \tag{31}$$
$$\Rightarrow c^* = f(k^*) - (n+\delta)k^* \tag{32}$$
$$\text{To maximize welfare at steady state} \iff \max c* \iff FOC : f'(k_g^*) = n + \delta \tag{33}$$

But for a certain production $y = k^a$ at steady state, using $\dot{k} = sf(k) - (n+\delta)k$

$$s(k^*)^\alpha - (n+\delta)k^* = 0 \tag{34}$$
$$\Rightarrow k^* = (\frac{s}{n+\delta})^{\frac{1}{1-\alpha}} \tag{35}$$
$$\Rightarrow f'(k^*) = \frac{\alpha}{s}(n+\delta) \tag{36}$$

comparing to $f'(k_g^*) = n + \delta$, for certain α and s it could be

$$\Rightarrow k^* < k_g^* \tag{37}$$

meaning there is dynamic inefficiency in Solow model if for certain α and s.
Ramsey
Ramsey has no dynamic inefficiency, here's why
The model has three key equations

$$\begin{cases} L_t = (1+n)L_{t-1} \\ k_{t+1} - (1-\delta)k_t = f(k_t) - c_t \\ \frac{u'(c_t)}{u'(c_{t+1})} = \beta[f'(k_{t+1}) + (1-\delta)] \end{cases} \tag{38}$$

Put $k_{t+1} - (1-\delta)k_t = f(k_t) - c_t$ at steady state

$$c^* = f(k^*) - k^* + (1-\delta)k^* = f(k^*) - \delta k^* \tag{39}$$
$$\text{To maxmize welfare at steady state} \Rightarrow \text{FOC: } f'(k_G^*) = \delta \tag{40}$$

Put $\frac{u'(c_t)}{u'(c_{t+1})} = \beta[f'(k_{t+1}) + (1-\delta)]$ at steady state resulting

$$\Rightarrow 1 = \beta[f'(k^*) + (1-\delta)] = \frac{1}{1+\rho}[f'(k^*) + (1-\delta)] \tag{41}$$
$$\Rightarrow 1+\rho = f'(k^*) + 1 - \delta \tag{42}$$
$$\Rightarrow f'(k^*) = \rho + \delta \tag{43}$$
$$\Rightarrow k_G^* > k^* \tag{44}$$

Capital per capita at steady state is lower than k_{gold}, meaning it is dynamic efficient.
OLG
OLG in central planner form:

$$\begin{aligned} &\max \sum_{t=0}^{\infty} (\beta_s)^t L_t[u(c_{1t}) + \beta u(c_{2,t+1})] \\ &s.t.\ c_{1t} + \frac{c_{2t}}{1+n} + (1+n)k_{t+1} = f(k_t) \end{aligned} \tag{45}$$

Equation of motion of capital per capita at the steady states

$$c^* \equiv c_1^* + \frac{c_2^*}{1+n} = f(k^*) - (1+n)k^* \tag{46}$$

Since we are maximizing welfare at steady state, let $\frac{\partial c^*}{\partial k^*} = 0$ to acquire first order condition

$$\Rightarrow f'(k_G^*) = 1+n \tag{47}$$

using production function at equilibrium

$$f'(k^*) = \alpha(k^*)^{\alpha-1} = \alpha\{[\frac{\beta(1-\alpha)}{(1+\beta)(1+n)}]^{\frac{1}{1-\alpha}}\}^{\alpha-1} \tag{48}$$
$$= \frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}(1+n) \tag{49}$$

for certain value of α

$$f'(k^*) < 1+n \tag{50}$$
$$\Rightarrow f'(k^*) < f'(k_G^*) \tag{51}$$
$$\Rightarrow k^* > k_G^* \tag{52}$$

Meaning there will be over-accumulation of capital $\iff$ existence of dynamic inefficiency in OLG model.
Conclusion

- Due to ad hoc given s, Solow model is unable to assure dynamic efficiency.
- Due to endogenous saving decision and discount factor β, Ramsey model is able to achieve dynamic efficiency.
- Due to limited life span of individuals, OLG model fails to achieve dynamic efficiency.