

NATIONAL SENIOR CERTIFICATE/ NASIONALE SENIORSERTIFIKAAT

GRADE/GRAAD 12

JUNE/JUNIE 2023

MATHEMATICS P1 MARKING GUIDELINE/ WISKUNDE V1 NASIENRIGLYN

MARKS/PUNTE: 150

This marking guideline consists of 15 pages./ *Hierdie nasienriglyn bestaan uit* 15 *bladsye*.

NOTE/LET OP:

- If a candidate answered a question TWICE, mark the FIRST attempt ONLY.

 Indien 'n kandidaat' n vraag TWEE keer beantwoord het, merk SLEGS die EERSTE poging.
- Consistent accuracy(CA) applies in ALL aspects of the memorandum. Volgehoue akkuraatheid geld deurgaans in ALLE aspekte van die memorandum.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossedout attempt.

Indien 'n kandidaat 'n poging vir 'n vraag deurgetrek het en nie die vraag weer beantwoord het nie, merk die poging wat deurgetrek is.

• The mark for substitution is awarded for substitution into the correct formula. *Die punt vir substitusie word vir substitusie in die korrekte formule toegeken.*

QUESTION 1/VRAAG 1

1.1.1
$$x^2 - 9 = 0$$
 OR/OF $x^2 - 9 = 0$ $(x+3)(x-3) = 0$ $x^2 = 9$ $x+3 = 0$ or/of $x-3 = 0$ $x = \pm \sqrt{9}$ OR/OF

 $x^{2}-9=0$ $x = \frac{-b \pm \sqrt{b^{2}-4ac}}{2a}$ $= \frac{-(0) \pm \sqrt{(0)^{2}-4(1)(-9)}}{2(1)}$ Answers only –
Full marks
Slegs antwoorde Volpunte

 $x^2 = 9$ \checkmark factors / faktore

 $x = \pm \sqrt{9}$ | \checkmark both answers / beide antwoorde

OR / OF

✓ correct substitution into correct formula / korrekte vervanging in korrekte formule

✓ both answers / beide antwoorde

(2)

x = -3 or / of x = 3

1.1.2	$x-5+\frac{2}{x}=0$ $x^2-5x+2=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Penalise 1 mark for incorrect rounding off. / Penaliseer 1 punt vir verkeerde afronding.	✓ standard form / standaardvorm
	$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$ $x = \frac{5 \pm \sqrt{17}}{2}$	✓ substitution / vervanging
	$\therefore x = 4,56 \text{ or/}of x = 0,44$	$\checkmark \checkmark x$ -values / waardes
1.1.3	$x = 1 + \sqrt{7 - x}$ $x - 1 = \sqrt{7 - x}$ $(x - 1)^2 = (\sqrt{7 - x})^2$ $x^2 - 2x + 1 = 7 - x$	(4) ✓ isolating surd / isoleer wortelvorm ✓ square both sides / kwadreer beide kante
	$x^{2}-x-6=0$ $(x+2)(x-3)=0$ $\therefore x \neq -2 \text{ or / of } x=3$	✓ standard form / standaardvorm ✓ factors / faktore ✓ selection / keuse (5)
1.1.4	$x^{2} + 2x - 15 \ge 0$ $(x+5)(x-3) \ge 0$ critical values/kritieke waardes $x = -5 \text{ or/of } x = 3$ $+ + - 5$ $-5 \qquad 3$	✓ critical values / kritieke waardes
	$x \le -5 \text{ or } / \text{ of } x \ge 3, x \in \mathbb{R}$ $\mathbf{OR} / \mathbf{OF}$	$ √√ x ≤ -5 or / of x ≥ 3, x ∈ \mathbf{R} $ (accuracy / akkuraatheid) $ OR/OF $
	$x \in (-\infty; -5] \text{ or } / \text{ of } x \in [3; \infty), x \in \mathbf{R}$	$x \in (-\infty; -5] \text{ or/of } x \in [3; \infty), x \in \mathbf{R}$ (3)

1.2
$$y + 2x = 3$$
(1)

$$y^2 - y = 3x^2 - 5x$$
(2)

From / Vanaf (1): y = -2x + 3(3)

(3) into/in (2):

$$(-2x+3)^2 - (-2x+3) = 3x^2 - 5x$$

$$4x^2 - 12x + 9 + 2x - 3 = 3x^2 - 5x$$

$$4x^2 - 12x + 9 + 2x - 3 - 3x^2 + 5x = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3)=0$$

$$x = 2$$
 or/of $x = 3$

$$y = -1 \text{ or/} of \quad y = -3$$

OR / OF

$$y + 2x = 3$$
(1)

$$y^2 - y = 3x^2 - 5x$$
(2)

$$x = \frac{3-y}{2}$$
(3)

Subst. / *Verv*. (3) into / *in* (2):

$$y^{2} - y = 3\left(\frac{3-y}{2}\right)^{2} - 5\left(\frac{3-y}{2}\right)$$

$$y^{2} - y = 3\left(\frac{9 - 6y + y^{2}}{4}\right) - 5\left(\frac{3 - y}{2}\right)$$

$$y^{2} - y = \frac{27 - 18y + 3y^{2}}{4} + \frac{-15 + 5y}{2}$$

$$4y^2 - 4y - 27 + 18y - 3y^2 + 30 - 10y = 0$$

$$y^2 + 4y + 3 = 0$$

$$(y+1)(y+3) = 0$$

$$y = -1 \text{ or } / \text{ of } y = -3$$

$$x = 2$$
 or $/of$ $x = 3$

 $\sqrt{y} = -2x + 3$

✓ substitution / *vervanging*

✓ standard form / standaardvorm

✓ factors / faktore

✓ *x*-values / *waardes*

✓ y-values / waardes

OR / OF

$$\checkmark x = \frac{3 - y}{2}$$

✓ substitution / *vervanging*

✓ standard form / standaardvorm

✓ factors / faktore

✓ y-values / waardes

✓ *x*-values / *waardes*

(6)

1.3	$\sqrt[n]{\frac{10^n + 2^{n+2}}{5^{2n} + 4(5^n)}}$	
	$= \left[\frac{2^n \times 5^n + 2^n \cdot 2^2}{5^n \cdot 5^n + 4(5^n)}\right]^{\frac{1}{n}}$	$\checkmark \frac{2^n \times 5^n + 2^n \cdot 2^2}{5^n \cdot 5^n + 4(5^n)}$
	$= \left[\frac{2^n(5^n+4)}{5^n(5^n+4)}\right]^{\frac{1}{n}}$	✓ factors / faktore
	$ = \left[\left(\frac{2}{5} \right)^n \right]^{\frac{1}{n}} $	✓ changing surd to exponent / verandering van wortel na eksponent
	$=\frac{2}{5}$	✓ answer / antwoord (4)
		[24]

QUESTION 2/VRAAG 2

2.1.1	$r = \frac{T_3}{T_2} = \frac{6x}{12}$	
	$=\frac{x}{2}$	✓ answer / antwoord (1)
2.1.2	-1 < <i>r</i> < 1	
	$-1 < \frac{x}{2} < 1$	✓ substitution / vervanging
	-2 < x < 2	✓ answer / antwoord (2)
2.1.3	$x=4 \Rightarrow a=6 \& r=2$	✓ values of a and r / waardes van a en r
	$S_{15} = \frac{a(r^n - 1)}{r - 1}$	
	$=\frac{6(2^{15}-1)}{2-1}$	✓ substitution / vervanging
	=196602	✓ answer / antwoord (3)
2.2	$T_1 = 6(2)^{-1} = 3$	
	$T_2 = 6(2)^{-2} = \frac{3}{2}$	✓ values of a and r / waardes van a en r
	$\therefore r = \frac{1}{2}$	
	$S_{\infty} = \frac{a}{1 - r}$	
	$=\frac{3}{1-\frac{1}{2}}$	✓ substitution / vervanging
	= 6	✓ answer / antwoord (3)

2.3.1	$S_{15} = -(15)^2 + 8(15)$	✓ substitution / vervanging
	=-105	✓ answer / antwoord
		(2)
2.3.2	$T_{15} = S_{15} - S_{14}$	✓ method / <i>metode</i>
	=-105-(-84)	
	=-21	✓ answer / antwoord
2.2.2		(2)
2.3.3	$T_1 = S_1 = 7 a + 14d = -21$	
	$S_2 = -(2)^2 + 8(2) = 12$ OR / OF $7 + 14d = -21$	
		✓ $T_2 = 5$ OR / OF $14d = -28$
	$\Rightarrow d = -2 \qquad \qquad d = -2$	✓ <i>d</i> = -2
	$a + (n-1)d = T_n$	
	7 + (n-1)(-2) = -169	✓ substitution / vervanging
	7 - 2n + 2 = -169	
	-2n = -178	
	n = 89	✓ answer / antwoord
	07.407	OR / OF
	OR / OF	OK / OI
	$S_n - S_{n-1} = T_n$	
	$-n^{2} + 8n - [-(n-1)^{2} + 8(n-1)] = -169$	✓ formula / formule
	$-n^2 + 8n - [-n^2 + 2n - 1 + 8n - 8] = -169$	✓ substitution / vervanging
	$-n^{2} + 8n + n^{2} - 2n + 1 - 8n + 8 = -169$ $-n^{2} + 8n + n^{2} - 2n + 1 - 8n + 8 = -169$	
		✓ simplification / vereenvoudiging
	-2n = -178	
	n = 89	✓ answer / antwoord
		(4)
		[17]

QUESTION 3/VRAAG 3

3.1	95; 72; y; 32;	
3.1	-23; $y-72$; $32-y$; (first diff./ eerste verskille)	
	y-49; $-2y+104$	
	y 12, 2y 1101	
	$\therefore y - 49 = -2y + 104$	((2nd 1:66 /
	3y = 153	✓ equating 2 nd differences / gelykstel van 2 ^{de} verskille
	y = 51	
3.2	y = 31	✓ answer / antwoord (2)
3.2	95 ; 72 ; 51 ; 32	
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	✓ 2 nd difference / 2 ^{de} verskil
	2 ; 2	2 difference / 2 verskti
	2a = 2 $3(1) + b = -23$ $1 - 26 + c = 95$	✓ <i>a</i> = 1
	$a = 1 \qquad \qquad b = -26 \qquad \qquad c = 120$	$\checkmark b = -26$
	$T_n = n^2 - 26n + 120$	$\checkmark c = 120 \tag{4}$
3.3	$T_{22} = (22)^2 - 26(22) + 120$	
	=32	✓ answer / antwoord
3.4		(1)
3.4	$n^2 - 26n + 120 = 1040$	✓ equating / gelykstel
	$n^2 - 26n - 920 = 0$	✓ standard form / standaardvorm
	(n-46)(n+20) = 0	✓ factors / faktore
	$n = 46 \text{ or / of } n \neq -20$	✓ answer / antwoord
	\mathbf{OR} / \mathbf{OF}	OR / OF
	$n^2 - 26n + 120 = 1040$	✓ equating / gelykstel
	$n^2 - 26n - 920 = 0$	✓ standard form / standaardvorm
	n 20n - 720 – 0	
	$-b \pm \sqrt{b^2 - 4ac}$	
	$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	$-(-26) \pm \sqrt{(-26)^2 - 4(1)(-920)}$	✓ substitution / vervanging
	$=\frac{-(-26)\pm\sqrt{(-26)^2-4(1)(-920)}}{2(1)}$	
	$\therefore n = 46 \text{ or / of } n \neq -20$	✓ answer / antwoord (4)
		[11]

QUESTION 4/VRAAG 4

4.1	S(0;3)	✓✓ answer / antwoord
		(2)
4.2.1	$x = -\frac{b}{2a} = -\frac{10}{2(-5)}$ $y = 5(-1)^2 + 10(-1) + 3$	✓ method /metode ✓ x-coordinate / x-koördinaat
	$x = -1 \qquad y = -2$	✓ substitution / <i>vervanging</i> ✓ <i>y</i> -coordinate / <i>y-koördinaat</i>
	T(-1;-2)	(4)
4.2.2	p=1 and $eq=-2$	✓ p=1
		$\checkmark q = -2 \tag{2}$
4.2.3	$\frac{5}{x+1} - 2 = 0$	✓ equating to 0 / stel gelyk aan 0
	$\therefore x = \frac{3}{2}$	
	\Rightarrow OR = 1,5 units / eenhede	✓ answer / antwoord
	→ OK = 1,3 umts/ centieue	(2)
4.2.4	$y \ge -2$; $y \in \square$	✓✓ answer / antwoord
4.3.1	10 - 10	$\checkmark m = 10 \tag{2}$
4.5.1	$m = 10x + 10$ $y - y_1 = m(x - x_1)$	\checkmark m = 10 \checkmark substitution into eqn of line /
	= 10(0) + 10 y - 3 = 10(x - 0)	vervanging in verg. van lyn
	=10 $y = 10x + 3$	✓ answer / antwoord
	, and the second	(3)
4.3.2	$y = (x+1)-2 \qquad \qquad y = x+k$	
	y = x - 1 OR / OF $-2 = -1 + k$	✓ substitution / vervanging
	k = -1	
	$\therefore y = x - 1$	✓ answer / antwoord
	·	(2)
4.4	$x \ge \frac{3}{2}$	✓✓ answer / antwoord (A)
	2	answer / antwoora (A) (2)
		[19]
		[17]

QUESTION 5/VRAAG 5

5.1	$h(x) = a^x$	
		✓ substitution / <i>vervanging</i>
	$\frac{1}{2} = a^{-1}$	
	$\therefore a=2$	✓ answer / antwoord (2)
5.2	$y=2^x$	(=)
	$x = 2^{y}$ $\therefore h^{-1}(x) : y = \log_2 x$	✓ interchanging x and y / omruil van x en y ✓ answer / antwoord
5.3	(3).9 10523	(2)
	$\frac{1}{h-1}$ 0 1 x	 ✓ y-intercept for h / y-afsnit vir h ✓ shape and asymptote of h vorm en asimptoot van h ✓ x-intercept for h -1 / x-afsnit vir h -1 ✓ shape and asymptote of h -1 vorm en asimptoot van h -1 (4)
5.4	$x>0$; $x\in\Box$	✓ answer / antwoord (1)
5.5	$x > 2$; $x \in \square$	✓ answer / antwoord
	$ \begin{array}{c} \mathbf{OR} / \mathbf{OF} \\ \log_2 x > 1 \\ \text{(algebraically/algebraïes)} \end{array} $	OR / OF
	$\therefore x > 2$	✓ answer / antwoord
5.6.1	$t(x) = \left(\frac{1}{2}\right)^x - 1$ $= 2^{-x} - 1$	(1)
	\Rightarrow reflection about the y-axis / refleksie om die y – as shift of 1 unit down / skuif van 1 eenheid af	✓ reflection / refleksie ✓ shift / skuif
<i>5 < 2</i>	. 1	(2)
5.6.2	y = -1	✓ answer / antwoord (1)
		[13]

QUESTION 6/VRAAG 6

6.1	$A = P(1-i)^n$	✓ formula / formule
	$A = 980000(1-9,2\%)^7$	✓ substitution / <i>vervanging</i>
	A = R498685,82	✓ answer / antwoord
6.2	$A = P(1+i)^n$	(3)
0.2	$20020, 28 = 13500(1+8,2\%)^{n}$ $1,4829837037 = 1,082^{n}$ $\therefore n = \log_{(1,082)} [1,4829837037]$ $n = 5 \text{ years} / jaar$	✓ substitution / vervanging ✓ simplification / vereenvoudiging ✓ use of logs / gebruik van logs ✓ answer / antwoord (4)
6.3	Amount in savings account / Bedrag in spaarrekening: $= 3500 \left(1 + \frac{0.07}{4}\right)^{8} \left(1 + \frac{0.08}{12}\right)^{36} + 5700 \left(1 + \frac{0.08}{12}\right)^{24}$ $= R11793,19$ OR / OF	✓ $n = 8$ and $/ en i = 0.07/4$ ✓ substitution $/ vervanging$ ✓ $n = 36$ and $/ en i = 0.08/12$ ✓ substitution $/ vervanging$ ✓ addition $/ optelling$ ✓ answer $/ antwoord$
	$A_1 = 3500 \left(1 + \frac{7\%}{4}\right)^8 = R4021,08624$	OR / OF
	$A_2 = 4021,08624 \left(1 + \frac{8\%}{12}\right)^{12} = R4354,834415$	✓ $n = 8$ and $/ en i = 0.07/4$ ✓ substitution $/ vervanging$
	$A_3 = 4354,83441 + 5700 = R10054,834415$	$\checkmark n = 12 \text{ and } / en \ i = 0.08/12$
	Final Amount / Finale Bedrag	✓ addition / optelling
	$=10054,83441\left(1+\frac{8\%}{12}\right)^{24}$	
	= R11793,19	✓ substitution / vervanging
		✓ answer / antwoord
		(6)
		[13]

QUESTION 7/VRAAG 7

QCLSII	ON //VRAAG /		Penalise 1 mark for incorrect notation in this question
7.1	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$		Penaliseer 1 punt vir verkeerde notasie in hierdie vraag
	$= \lim_{h \to 0} \frac{5 - 2(x+h)^2 - \left(5 - 2x^2\right)}{h}$		
	$= \lim_{h \to 0} \frac{5 - 2x^2 - 4xh - 2h^2 - 5 + 2x^2}{h}$	-	$\checkmark 5-2x^2-4xh-2h^2$
	$=\lim_{h\to 0}\frac{-4xh-2h^2}{h}$		✓ simplification / vereenvoudiging
	$=\lim_{h\to 0}\frac{h(-4x-2h)}{h}$		✓ factorisation / faktorisering
	$= \lim_{h \to 0} \left(-4x - 2h \right)$ $= -4x$	Answer ONLY: 0 m SLEGS antwoord: 0	
			✓ answer / antwoord
7.2.1	$f(x) = 2x^5 - 7\sqrt{x} + \frac{1}{x}$ $= 2x^5 - 7x^{\frac{1}{2}} + x^{-1}$		$\checkmark 2x^5 - 7x^{\frac{1}{2}} + x^{-1}$
	2		$\checkmark 10x^4$
	$f'(x) = 10x^4 - \frac{7}{2}x^{-\frac{1}{2}} - x^{-2}$ $= 10x^4 - \frac{7}{2\sqrt{x}} - \frac{1}{x^2}$		$\begin{array}{c c} \checkmark & -\frac{7}{2}x^{-\frac{1}{2}} \\ \checkmark & -x^{-2} \end{array}$
722	$2\sqrt{x}$ x^2		(4
7.2.2	$\frac{d}{dx} \left[\frac{2x^2 - x - 6}{2x + 3} \right]$		
	$\frac{d}{dx} \left[\frac{(2x+3)(x-2)}{(2x+3)} \right]$		✓ factors / faktore
	$\begin{vmatrix} \frac{d}{dx}[x-2] \\ = 1 \end{vmatrix}$		✓ simplification / vereenvoudiging
	_1		✓ answer / antwoord
			[11

QUESTION 8/VRAAG 8

8.1	$f'(x) = 3x^2 - 10x - 8 = 0$	$\checkmark f'(x) = 0$	
	(3x+2)(x-4) = 0	✓ factors / faktore	
	$x = -\frac{2}{3} \text{ or } / \text{ of } x = 4$	✓ x-values / x-waardes	
	$y = \frac{400}{27} (14,81) \text{ or } / \text{ of } y = -36$	✓ y-values / y-waardes	
	$L\left(-\frac{2}{3};\frac{400}{27}\right)$ and $/enM(4;-36)$		(4)
8.2	$f(x) = x^3 - 5x^2 - 8x + 12 = 0$		
	(x-6)(x-1)(x+2) = 0	✓ factors / faktore	
	$\therefore x = 6 \; ; \; x = 1 \; ; \; x = -2$	$\checkmark x = 1$	(2)
8.3		$\checkmark x = -2$	(3)
	(-0.67, 14.81) $(-2, 0)$ $(-2, 0)$ $(-2, 0)$ $(-2, 0)$ (-10) (-20) (-30) (-30) (-30) $(-4, -36)$	 ✓ x-intercepts / x-afsnitte ✓ y-intercept / y-afsnit ✓ turning points / draaipunte ✓ shape / vorm 	
	40		(4)
8.4	$m = \frac{0 - (-16)}{6 - 2} = 4$ $y - y_1 = m(x - x_1)$ $y - 0 = 4(x - 6)$ $y = 4x - 24$ $\therefore a = 4 \text{ and } / en \ q = -24$	$\checkmark a = 4$ $\checkmark q = -24$	(2)
8.5	f''(x) = 6x - 10	✓ f"(x)	
	f''(2) = 6(2) - 10	✓ substitution / vervanging	
8.6	$= 2 > 0$ $\Rightarrow \text{concave up} / \text{konkaaf op}$ $4x - 24 = -36$ $x = -3$	✓ conclusion / gevolgtrekking ✓ equating / gelyk stel ✓ $x = -3$	(3)
	$\therefore -3 \le x \le 2 \text{or } / of x \ge 6$	✓✓ answer /antwoord	(4) [20]

QUESTION 9/VRAAG 9

9.1	Hourly cost = fuel cost + other costs	
	$Uurlikse\ koste = brandstofkoste + ander\ koste$	$\checkmark 4x^2 + 1000$
	$=4x^2+1000$	
	Duration of trip/Tydsduur van reis = $\frac{\text{distance}/afstand}{\text{speed}/spoed} = \frac{500}{x}$	$\checkmark \frac{500}{x}$
	Total cost/Totale koste	
	= $(hourly cost/uurlikse koste) \times (number of hours/aantal ure)$	$\checkmark (4x^2 + 1000) \times \left(\frac{500}{x}\right)$
	$C(x) = (4x^2 + 1000) \times \left(\frac{500}{x}\right)$	
	$=2000x + \frac{500000}{x}$	(3)
9.2	GIV.) 2000 500 000 0	✓ C'(x)
	$C'(x) = 2000 - \frac{500000}{x^2} = 0$	$\checkmark C'(x) = 0$
	$2000x^2 - 500000 = 0$	✓ standard form / standaardvorm
	$2000x^2 = 500000$	✓ simplification / vereenvoudiging
	$x^2 = 250$	✓ answer / antwoord
	$x = \sqrt{250} = 15,81 km / h$	(5)
		[8]

QUESTION 10/VRAAG 10

10.1		
	P(A or / of B) = P(A) + P(B)	✓ rule / reël
	0,64 = 3P(B) + P(B)	✓ substitution / vervanging ✓ answer / antwoord
	0,64 = 4P(B)	(3)
	$\therefore P(B) = 0.16$	
10.2.1	12% Fall DF	
	Dry	
	37%	✓ 37% and / en 63% ✓ 12% and / en 88%
	88% Not Fall DN	✓ 36% and / en 64%
		✓ outcomes / uitkomste
	36% Fall WF	
	63%	
	0370	(4)
	Wet	(4)
	64% Not Fall WN	
10.2.2	(37% ×88%)+(63% ×64%)	
		✓ (37%×88%)+(63%×64%)
	$=0,7288 \approx \frac{73}{100}$	✓ answer / antwoord (2)
10.3.1	182 + x + 4 + 30 = 240 OR/OF $x + 182 = 206$ OR/OF $x + 4 = 20$	
	x = 240 - 216 $x = 24$ $x = 24$	√ answer / antwoord
	x = 24	$\begin{array}{c} \bullet \text{ answer } \wedge \text{ antwoord} \\ \end{array} \tag{2}$
10.3.2	For independent events	
	Vir onafhanklike gebeurtenisse	
	$P(R) \times P(C) = P(R \cap C)$	
	$P(R \text{ and } / \text{en } C) = \frac{24}{240} = 0.10$	
	$P(R) \times P(C)$ 240	$\checkmark P(R \cap C) = 0.10$
	$= \left(\frac{206}{240}\right) \times \left(\frac{28}{240}\right)$	$\int D(D) \times D(C) = 0.10$
	= 0.10	$\checkmark P(R) \times P(C) = 0.10$
	∴ Yes, the events are independent. Ja, die gebeurtenisse is onafhanklik.	✓ conclusion / gevolgtrekking
	· ·	(3)
		[14]
		TOTAL/TOTAAL: 150