NATIONAL SENIOR CERTIFICATE
NASIONALE SENIORSERTIFIKAAT

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2023

MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 12 pages.
Hierdie nasienriglyn bestaan uit 12 bladsye.

QUESTION 1/*VRAAG 1*

1.1	16	✓✓ answer / *antwoord*	(2)
1.2	$sd = 5{,}83$	✓✓answer / *antwoord*	(2)
1.3	$16 + 5{,}83 = 21{,}83$ ∴ 2 above/*bokant*.	✓✓answer / *antwoord*	(2)
1.4	$16 - 5{,}83 = 10{,}17$ $\dfrac{3}{11} \times 100 = 27{,}27\%$	✓✓	(2)
			[8]

QUESTION 2/*VRAAG 2*

	Hours / *Ure*	**Number of dads** *Aantal pa's*	**Cumulative frequency** *Kumulatiewe frekwensie*
	$0 < x \leq 5$	1	1
	$5 < x \leq 10$	18	19
	$10 < x \leq 15$	24	43
	$15 < x \leq 20$	25	68
	$20 < x \leq 25$	18	86
	$25 < x \leq 30$	12	98
	$30 < x \leq 35$	1	99
	$35 < x \leq 40$	1	100

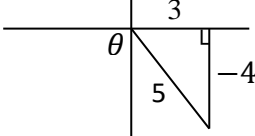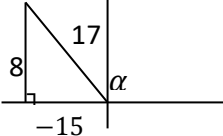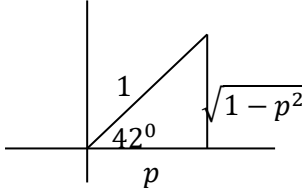| 2.1		✓ table / *tabel* ✓ anchor point / *ankerpunt* (0;0) ✓ (10; 19) (25; 86) ✓ (40; 100)	(4)
2.2	± 16 (Accept values from 14 to 18) / (*Aanvaar waardes vanaf 14 tot 18*)	✓✓ answer *antwoord*	(2)
2.3	$15 < x \leq 20$	✓ answer / *antwoord*	(1)
2.4	$\bar{x} = 16{,}8$	✓✓ answer / *antwoord*	(2)
2.5	Mean ≈ Median and lies in the Modal class. Data is symmetrical / Normal / Data not skewed. *Gemiddelde ≈ Mediaan en lê in die Modale klas* *Data is simmetries / Normaal / Data nie skeef nie.*	✓ ✓✓ reason / *rede*	(3)
			[12]

QUESTION 3/*VRAAG 3*

3.1	$y = -\frac{1}{3}x + 10 \qquad (m_1 \times m_2 = -1)$ $\frac{p}{2} = -\frac{1}{3}(4) + 10$ $p = \frac{52}{3}$	✓ equation of line / *verg. van lyn* ✓ substitution of point / *vervanging van punt* ✓ answer / *antwoord*	(3)
3.2	O(0;0) and/*en* $P(-2; p-1)$ and/*en* OP = 2 units/*eenhede.* $OP^2 = (-2-0)^2 + (p-1-0)^2$ $(2p)^2 = 4 + p^2 - 2p + 1$ $4p^2 = p^2 - 2p + 5$ $3p^2 + 2p - 5 = 0$ $(3p+5)(p-1) = 0$ $p = -\frac{5}{3}$ or $p = 1$ (*slegs*) $p = 1$ only	✓ substitution / *vervanging* ✓ simplification / *vereenvoudiging* ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ answer / *antwoord*	(5)
3.3.1	$m_{BD} = 2$	✓✓ answer / *antwoord*	(2)
3.3.2	Midpoint of BD : *Middelpunt van BD* $(0; -1)$ $y = -\frac{1}{2}x - 1$	✓ ✓ midpoint / *middelpunt* ✓ answer / *antwoord*	(3)
3.3.3	$x^2 + y^2 = 25$	✓✓ answer / *antwoord*	(2)
3.3.4	$x^2 + \left(-\frac{1}{2}x - 1\right)^2 = 25$ $x^2 + \frac{1}{4}x^2 + x + 1 = 25$ $4x^2 + x^2 + 4x + 4 = 100$ $5x^2 + 4x - 96 = 0$ $(5x + 24)(x - 4) = 0$ $x = -\frac{24}{5}$ or/*of* $x = 4$ $\therefore y = \frac{7}{5}$	✓ substitution / *vervanging* ✓ simplification / *vereenvoudiging* ✓ standard form / *standaardvorm* ✓ factorisation / *faktorisering* ✓ x-values / *x-waardes* ✓ answer / *antwoord*	(6)
			[21]

QUESTION 4 / *VRAAG 4*

4.1	$x^2 + y^2 = 16 \qquad\qquad y = 4 - 2x$ $x^2 + (4 - 2x)^2 = 16$ $x^2 + 16 - 16x + 4x^2 = 16$ $5x^2 - 16x = 0$ $x(5x - 16) = 0$ $x = 0 \quad$ or/*of* $\quad x = \dfrac{16}{5} = 3{,}2$ $y = 4 - 2(0) \qquad y = 4 - 2(3{,}2)$ $y = 4 \qquad\qquad\quad y = -2{,}4$	✓ $y = 4 - 2x$ ✓ substitution / *vervanging* ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ x-values / *x-waardes* ✓ substitution / *vervanging* ✓ y-values / *y-waardes*	(7)
4.2	$S(-3{,}2 \, ; 2{,}4)$	✓✓ answer / *antwoord*	(2)
4.3	$y = 4 - 2x$ $0 = 4 - 2x$ $2x = 4$ $x = 2$ $R(0 \, ; 2)$ ∴ radius = 2 units / *eenhede* $(x - 2)^2 + y^2 = 4$	✓ equating to 0 / *gelyk stel aan* 0 ✓ $x = 2$ ✓ radius = 2 units/*eenhede* ✓ answer / *antwoord*	(4)
4.4	$(x - y)^2 + y^2 - y = 12$ $(x - 6)^2 + y^2 - y + \dfrac{1}{4} = 12 + \dfrac{1}{4}$ $(x - 6)^2 + \left(y - \dfrac{1}{2}\right)^2 = \dfrac{49}{4}$ Centre/*Middelpunt* $\left(6 \, ; \dfrac{1}{2}\right)$ and/*en* $O(0 \, ; 0)$ $d = \sqrt{36 + \dfrac{1}{4}}$ $d \approx 6{,}02 \ / \ \dfrac{\sqrt{145}}{2}$	✓ completing the square *vierkantsvoltooiing* ✓ simplification / *vereenvoudiging* ✓ coordinates of midpoint *koördinate van middelpunt* ✓ substitution in distance formula *vervanging in afstand formule* ✓ answer / *antwoord*	(5)
			[18]

QUESTION 5/*VRAAG 5*

5.1	$5\cos\theta - 3 = 0$
	 $\cos\theta = \dfrac{3}{5}$
	
	 $\tan\alpha + \tan\theta$
	 $= -\dfrac{8}{15} + \left(-\dfrac{4}{3}\right)$
	 $= -\dfrac{28}{15}$
		$17\sin\alpha = 8$
		 $\sin\alpha = \dfrac{8}{17}$
		
			✓$\cos\theta = \dfrac{3}{5}$
			
			 ✓$\sin\alpha = \dfrac{8}{17}$
			
			 ✓ -4 in correct quadrant
			     in korrekte kwadrant
			 ✓-15 in correct quadrant
			     in korrekte kwadrant
			 ✓correct values / *korrekte waardes*
			
			 ✓ answer / *antwoord*
				(6)

5.2	$\cos 42^0 = p$ 			
5.2.1	$\sin 48^\circ = p$		✓✓answer / *antwoord*	(2)
5.2.2	$\sin(-2202^0)$ $= \sin(-42^0)$ $= -\sin 42^0$ $= -\sqrt{1 - p^2}$		✓$-\sin\left(42^\circ\right)$ ✓answer / *antwoord*	(2)
5.2.3	$\cos 84^0$ $= \cos 2(42^0)$ $= 2\cos^2 42^0 - 1$ $= 2p^2 - 1$		✓$\cos 2\left(42^\circ\right)$ ✓answer / *antwoord*	(2)
5.3	$\dfrac{\tan 300^0 + \cos(90^0 + x)}{\sin(180^0 - x) + 2\cos(-30^0)}$ $= \dfrac{-\tan 60^0 - \sin x}{\sin x + 2\left(\dfrac{\sqrt{3}}{2}\right)}$ $= \dfrac{-\sqrt{3} - \sin x}{\sin x + \sqrt{3}}$ $\dfrac{-\left(\sin x + \sqrt{3}\right)}{\left(\sin x + \sqrt{3}\right)}$ $= -1$		✓$-\tan 60^\circ$ ✓$-\sin x$ ✓$\sin x$ ✓$\dfrac{\sqrt{3}}{2}$ ✓ taking out of negative sign. *uithaal van negatiewe teken* ✓answer / *antwoord*	(6)

5.4	$\dfrac{1 - \sin 2x}{\cos 2x} = \dfrac{\cos x - \sin x}{\cos x + \sin x}$		
	$LHS = \dfrac{1 - 2\sin x \cos x}{\cos^2 x - \sin^2 x}$	✓ $2\sin x \cos x$ ✓ $\cos^2 x - \sin^2 x$	
	$LHS = \dfrac{\cos^2 x - 2\sin x \cos x + \sin^2 x}{\cos^2 x - \sin^2 x}$	✓ $1 = \cos^2 x + \sin^2 x$	
	$LHS = \dfrac{(\cos x - \sin x)(\cos x - \sin x)}{(\cos x - \sin x)(\cos x + \sin x)}$	✓factorising / _faktorisering_ ✓factorising / _faktorisering_	
	$LHS = \dfrac{\cos x - \sin x}{\cos x + \sin x}$		(5)
5.5	$\cos x - \sin x = \sqrt{2}$ $\dfrac{1}{\sqrt{2}}\cos x - \dfrac{1}{\sqrt{2}}\sin x = 1$ $\cos 45^0 \cos x - \sin 45^0 \sin x = 1$ $\cos(45^0 + x) = 1$ $45^0 + x = 0^0 + 360^0.k$ $x = -45^0 + 360^0.k$	✓ division by / _deling deur_ $\sqrt{2}$ ✓ $\dfrac{1}{\sqrt{2}} = \cos 45^0/\sin 45^0$ ✓ expansion rule / _reël_ ✓ $45^0 + x = 0^0 + 360^0.k$ ✓answer / _antwoord_	(5)
			[28]

QUESTION 6/*VRAAG 6*
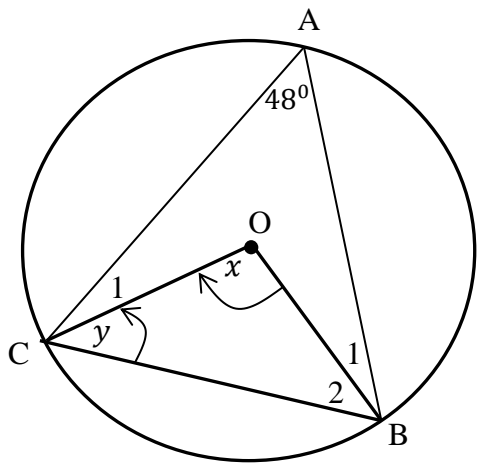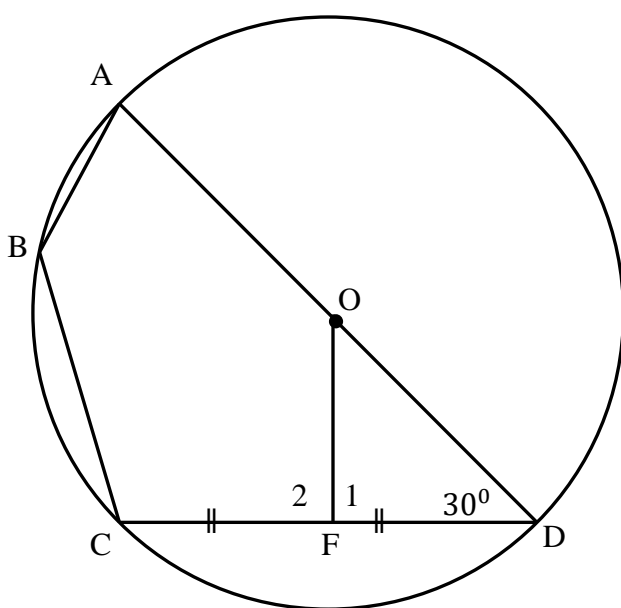
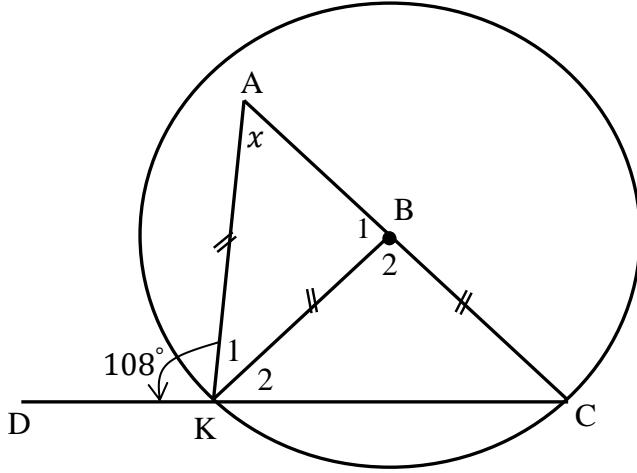| 6.1		✓shape / *vorm* ✓ intercepts / *afsnitte* ✓ starting and end points / *begin en eindpunte*	
			(3)
6.2	Period / *Periode* $= 180^0$	✓ answer / *antwoord*	
			(1)
6.3	$h(x) = \tan x + 2$	✓ answer / *antwoord*	(1)
6.4	$-135^0 \leq x < -90^0$	✓-135^0 ✓-90^0 ✓ answer / *antwoord*	(3)

6.5	$\cos B + 1 = \tan\dfrac{1}{2}B$
	Let/*Laat* $B = 2x$
	$\cos 2x + 1 = \tan\dfrac{1}{2}(2x)$
	$\cos 2x = \tan x - 1$
	$x = -135^0$ and/*en* $x = 45^0$
	$\therefore B = -270^0$ and/*en* $B = 90^0$
		✓ $\cos 2x + 1 = \tan\dfrac{1}{2}(2x)$
		✓ $\cos 2x = \tan x - 1$
		✓ both x-values
		  beide x-waardes
		✓ both B values /
		  beide B *waardes*
			(4)

			[12]

QUESTION 7/*VRAAG 7*

7.1	*In* ΔOAK In ΔKAT	✓ $\sin x = \frac{AK}{2}$	
	$\sin x = \frac{AK}{2}$ $\frac{AK}{\sin 2x} = \frac{KT}{\sin(90^0+x)}$	✓ $AK = 2\sin x$	
	$AK = 2\sin x$ $AK = \frac{KT2\sin x\cos x}{\cos x}$	✓ use of sine rule / *gebruik van sinusreël*	
	$AK = 2KT\sin x$	✓ $AK = 2KT\sin x$	
	$2\sin x = 2\,KT\sin x$	✓ $KT = 1$	
	$KT = 1$		
			(5)
7.2	In $\Delta\, KAT$		
	$T\hat{K}A = 90^0 - 3x$	✓$T\hat{K}A = 90^0 - 3x$	
	$\frac{AT}{\sin(90^0 - 3x)} = \frac{1}{\sin(90^0 + x)}$	✓ use of sine rule *gebruik van sinusreël*	
	$AT = \frac{\cos 3x}{\cos x}$		
			(2)
7.3	$AT = \frac{\cos 3x}{\cos x}$		
	$AT = \frac{\cos(2x + x)}{\cos x}$	✓ splitting of $\cos 3x$ and expansion *opbreek van* $\cos 3x$ *en uitbreiding*	
	$AT = \frac{\cos 2x\cos x - \sin 2x\sin x}{\cos x}$		
	$AT = \frac{\cos 2x\cos x - 2\sin x\cos x\sin x}{\cos x}$		
	$AT = \frac{\cos x(\cos 2x - 2\sin^2 x)}{\cos x}$	✓ common factor / *gemene faktor*	
	$AT = 1 - 2sin^2x - 2sin^2x$	✓ expansion of $\cos 2x$ *uitbreiding van* $\cos 2x$	
	$AT = 1 - 4\sin^2x$	✓ answer / *antwoord*	(4)
			[11]
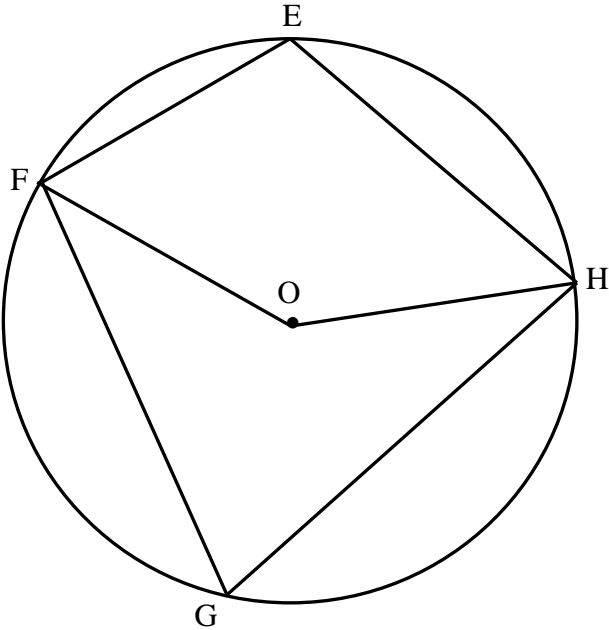
QUESTION 8/*VRAAG 8*

8.1		
8.1.1	$x = 96^0$ [∠ at centre = 2 × ∠ at circumference] [*middelpunte* ∠ = 2 × *omtrekshoek*]	✓ answer / *antwoord* ✓R (2)
8.1.2	$\hat{B}_2 = y$ [angles opp = sides] [*hoeke teenoor = sye*] $2y + 96^0 = 180^0$ [sum of angles of Δ] [*som van die hoeke van* Δ] $y = 42^0$	✓ S/R ✓answer / *antwoord* (2)
8.2		
8.2.1	$\hat{F}_1 = 90^0$ [line from centre bisects chord] [*lyn vanaf middelpunt halveer koord*]	✓ S ✓R (2)
8.2.2	$A\hat{B}C = 150^0$ [opp angles of cyclic quad] [*teenoorst. hoeke van koordevierhoek*]	✓ S ✓R (2)

8.3			
8.3.1	$\hat{B}_1 = x$ [angles opp equal sides] [*hoeke teenoor gelyke sye*]	✓S ✓R	(2)
8.3.2	$\hat{K}_2 + \hat{C} = x$ [ext angle of Δ] / [*buitehoek van* Δ] $\hat{C} = \hat{K}_2$ [angles opp equal sides] [*hoeke teenoor gelyke sye*] $\hat{C} = \dfrac{x}{2}$	✓S ✓R ✓S/R	(3)

8.3.3	$\hat{K}_1 = 180^0 - 2x$ [sum of angles of Δ]
	                    [*som van die hoeke van* Δ]
	 $108^0 + 180^0 - 2x + \dfrac{x}{2} = 180^0$ [adj angles on str line]
	            [*aangrensende hoeke op reguitlyn*]
	 $x = 72^0$
		✓S/R
		
		 ✓S ✓R
		
		 ✓answer / *antwoord*
			(4)

			[17]

QUESTION 9 / *VRAAG 9*

Construct radii OF and OH. $F\hat{O}H = 2 \times \hat{G}$ [angle at centre = 2 × angle at circum] Reflex $F\hat{O}H = 2 \times \hat{E}$ [angle at centre = 2 × angle at circum] $F\hat{O}H + F\hat{O}H = 2\hat{G} + 2\hat{E}$ but $F\hat{O}H + F\hat{O}H = 360^0$ [angles around a point] $2\hat{G} + 2\hat{E} = 360^0$ $\hat{G} + \hat{E} = 180^0$		✓S ✓S/R ✓S/R ✓S ✓S/R ✓S
Trek radiusse OF *en* OH. $F\hat{O}H = 2 \times \hat{G}$ [*middelpunte* ∠ = 2 × *omtrekshoek*] *Omwenteling* $F\hat{O}H = 2 \times \hat{E}$ [*middelpunte* ∠ = 2 × *omtrekshoek*] $F\hat{O}H + F\hat{O}H = 2\hat{G} + 2\hat{E}$ maar $F\hat{O}H + F\hat{O}H = 360^0$ [*hoeke rondom* 'n *punt*] $2\hat{G} + 2\hat{E} = 360^0$ $\hat{G} + \hat{E} = 180^0$		✓S ✓S/R ✓S/R ✓S ✓S/R ✓S
		[6]

QUESTION 10/VRAAG 10

10.1	$FE \perp DE$ [radius ⊥ tang] / [radius ⊥ raaklyn]
	 $F\hat{E}D = 90^0$
	 $F\hat{B}A = 90^0$ [line from centre bisects chord
	                [lyn vanaf middelpunt halveer koord]
	 ∴ EFBD = cyclic quad [opp angles = 180^0]
	    EFBD is 'n koordevierhoek [teenoorst. hoeke = 180°]
		✓ S/R
		 ✓ S
		 ✓ S/R
		 ✓ S/R
		
		 (4)

10.2	$\hat{C}_1 = 90^0$ [angle in semi-circle] / [hoek in semi-sirkel]
	 In ΔBCE and /en ΔFED
	          $\hat{C}_1 = F\hat{E}D$ [90^0]
	          $\hat{B}_2 = \hat{F}_1$ [angles in same segment]
	                      [hoeke in dieselfde segment]
	          $\hat{E}_2 = \hat{D}_1$ [3rd angle of Δ] / [3de hoek van Δ]
	      $\Delta BCE \;|||\; \Delta FED$ [∠; ∠; ∠]
		✓ S/R
		
		 ✓S/R
		 ✓S ✓R
		
		 ✓ S ✓R
		
		 (6)

10.3	$\dfrac{BC}{FE} = \dfrac{CE}{ED}$ [similar triangles] / [gelykvormige Δe] $BC = \dfrac{FA.CE}{ED}$	✓S ✓R ✓answer antwoord (3)

10.4	$F\hat{B}A = \hat{C}_1$ [proven] / [bewys]
	 ∴ $FB \parallel CE$ [corresponding angles equal]
	                  [ooreenkomstige hoeke gelyk]
	 $\dfrac{AC}{BC} = \dfrac{AE}{FE}$ [line parallel to one side of triangle]
	                  [lyn ewewydig aan een sy van driehoek]
	 $BC = \dfrac{AC.FE}{AE}$
		✓S/R
		 ✓S/R
		 ✓S ✓R
		
		 (4)

		[17]
	TOTAL/TOTAAL:	**150**