

## NATIONAL SENIOR CERTIFICATE/ NASIONALE SENIOR SERTIFIKAAT

GRADE/GRAAD 12

**SEPTEMBER 2022** 

# MATHEMATICS P2/WISKUNDE V2 MARKING GUIDELINE/NASIENRIGLYN

MARKS/PUNTE: 150

This marking guideline consists of 16 pages./ *Hierdie nasienriglyn bestaan uit 16 bladsye*.

## QUESTION 1/VRAAG 1

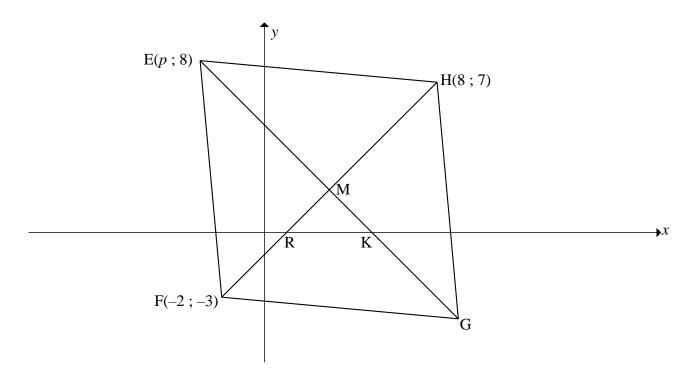
1.1	Distance of Jumps  Afstand van Spronge  (in cm)	Number of athletes  Aantal atlete	CF KF	✓ for cf [6 to 92] vir kf [6 tot 92]	
	$420 < d \le 460$	6	6	✓ for cf [94 to 100] vir kf [94 tot 100]	
	$460 < d \le 500$	14	20	, ,	
	$500 < d \le 540$	16	36		
	$540 < d \le 580$	42	78		
	$580 < d \le 620$	14	92		
	620 < d ≤ 660	2	94		
	660 < d ≤ 700	3	97		
	$700 < d \le 740$	2	99		
	$740 < d \le 780$	1	100		(2)
1.2				✓ anchor point	
	LONG JUMPERS VERSPRINGERS SE  120  120  400  400  DISTANCE OF JUMPS / AI	700 8	NGE	ankerpunt ✓ upper limits boonste limiete ✓ sf / ✓ smooth shape egalige vorm	(4)
1.3	The median jump is 553. Accept Dia median sprang is 553. Accept			✓✓ for answer vir antwoord	(2)
1.4	Die mediaan sprong is 553. Aanv Number jumped over 560 cm = 10			✓ for subtraction	(4)
	Therefore, it is 43% of the athlete	es.		vir aftrekking	
	Aantal wat oor 560 cm gespring h		43 atlete	✓ for the answer	(2)
	Dit is daarom 43% van die atlete.	•		vir die antwoord	(2) [10]

## QUESTION 2/VRAAG 2

Long jumper / Verspringer	1	2	3	4	5	6
x: Hours practised / Ure geoefen	4,5	2	3,5	4	8	3
y: Distance jumped / Afstand gespring (cm)	650	420	580	490	780	525

2.1	$a = 336,699$ $b = 56,992$ $\hat{y} = 336,699 + 56,992x$	✓ for/vir a	
		✓ for/vir b	
		$\checkmark$ for/vir a + bx	(3)
2.2	$\hat{y} = 336,699 + 56,992 (5.4) = 644,46 \text{ cm}$	✓ for substitution	
		vir vervanging	
		✓ for the answer	
		vir die antwoord	(2)
2.3	The more they practiced, the further they jumped.	✓✓ for the answer	
	Strong positive correlation.	vir die antwoord	
	Hoe meer hulle geoefen het, hoe verder het hulle gespring.		
	Sterk positiewe korrelasie.		(2)
2.4.1	The mean will decrease by 13 cm.	✓ for the answer	
	Die gemiddelde sal met 13 cm verminder.	vir die antwoord	(1)
2.4.2	The range will remain the same / No influence on range.	✓ for the answer	
	Die omvang sal dieselfde bly / Geen invloed op die	vir die antwoord	
	omvang.		(1)
2.4.3	The standard deviation remains the same.	✓ for the answer	, ,
	Die standaardafwyking bly dieselfde.	vir die antwoord	(1)
			[10]

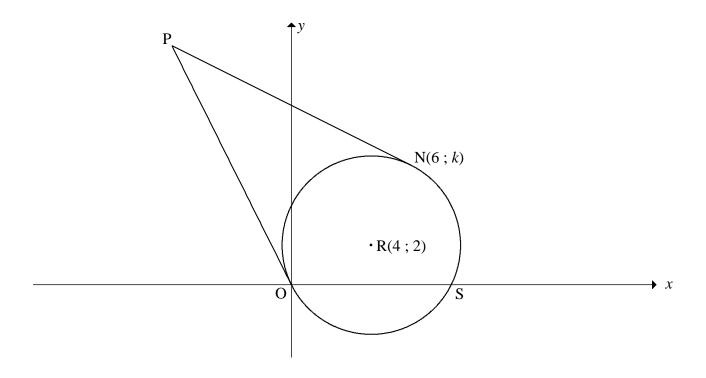
## QUESTION 3/VRAAG 3



3.1.1	M (3; 2)	✓ for/vir x ✓ for/vir y	(2)
3.1.2	$m_{FH} = \frac{7 - (-3)}{8 - (-2)} = 1$	✓ for subst. / vir vervanging ✓ for answer / vir antwoord	(2)
3.1.3	$m_{EG}$ = −1 (diagonals bisect at 90°) (hoeklyne halveer loodreg/by 90°) tan M $\hat{K}X$ = −1 M $\hat{K}X$ = 135° ∴M $\hat{K}R$ = 45°	✓ S ✓ S ✓ M $\hat{K}X = 135^{\circ}$ ✓ for answer / vir antwoord	(4)

3.2	FE = EH (sides of a rhombus = ) (sye van 'n rombus = ) FE <sup>2</sup> = EH <sup>2</sup> $(p+2)^2 + (8+3)^2 = (p-8)^2 + (8-7)^2$ $p^2 + 4p + 4 + 121 = p^2 - 16p + 64 + 1$ 20p = -120 p = -3	<ul> <li>✓ for equating / gelykstel         FE² = EH²</li> <li>✓ for squaring / kwadrering</li> <li>✓ for simplification         vir vereenvoudiging</li> <li>✓ for the answer         vir die antwoord</li> </ul>	
	OR/OF	OR/OF	
	E(p; 8) and Midpoint of HF / en Middelpunt van HF = $(3; 2)$ $m_{FH} = 1$	✓ for gradient of E to FH vir gradiënt van E na FH	
	gradient from E to midpoint of FH /	✓ statement / stelling	
	gradiënt vanaf E na middelpunt van FH		
	$= \frac{8-2}{p-3} = \frac{6}{p-3}$		
	FH is perpendicular to EG / FH is loodreg op EG	✓ for the product	
	$\therefore \frac{6}{p-3} \times 1 = -1$	vir die produk  ✓ for the answer	
	p-3	vir die antwoord	
	$\therefore p = -3$	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,	(4)
3.3	G(9; -4)	$\checkmark$ for/vir $x$ $\checkmark$ for/vir $y$	(2)
3.4	M(3;2)	✓ for coordinates of N	
	N(-9;2)	vir die koördinate van N	
	· · ( · )=/		
	MN = 12 units/eenhede	√√ for answer /	
		vir antwoord	(3)
			(3) [ <b>17</b> ]

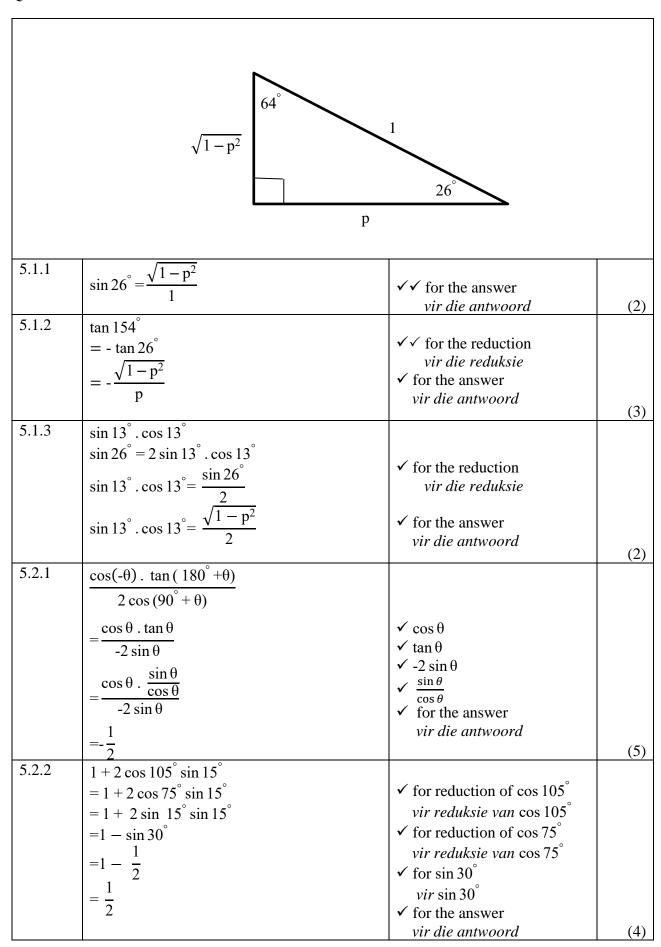
## QUESTION/VRAAG 4



4.1	$r^2 = (4-0)^2 + (2-0)^2$	✓ substitution / vervanging	
	$r^2 = 20$	$\checkmark$ for/vir $r^2$	
	$\therefore (x-4)^2 + (y-2)^2 = 20$	✓ for the equation/vir die vergelyking	(3)
4.2	$(6-4)^2 + (k-2)^2 = 20$	✓ substitution of / vervanging van N	
	$(k-2)^2 = 16$	✓ simplification / vereenvoudiging	
	$k - 2 = \pm 4$	✓ both answers for k /	
	k = 6  or/of  k = -2	beide antwoorde vir k	
	k = 6	$\checkmark$ selection of $k = 6$	
		keuse van $k = 6$	
	OR/OF	OR/OF	
	Sub: N(6; y) into the equation of the circle.		
	Verv. N(6; y) in die vergelyking van die sirkel. $(6-4)^2 + (y-2)^2 = 20$	✓ for substitution / vir vervanging	
		Z.C 1 1.C / ·	
	$4 + y^2 - 4y + 4 - 20 = 0$	✓ for standard form / vir	
	$y^2 - 4y - 12 = 0$	standaardvorm  ✓ for the factors / vir die faktore	
	(y-6)(y+2) = 0	101 the factors / vii the famore	
	y = 6  or/ of y = -2		
	$\therefore y = 6$	✓ for the answer / vir die antwoord	(4)

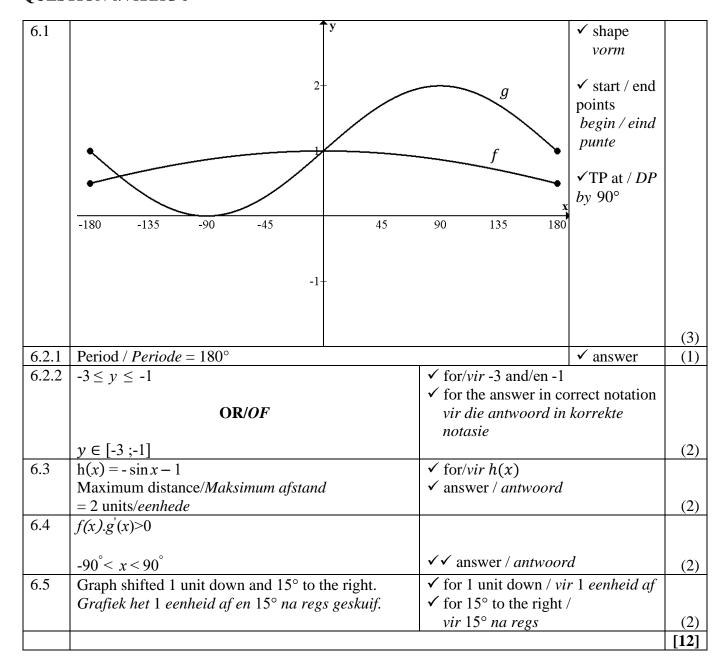
4.3	$m_{RN} = \frac{6-2}{6-4} = 2$ $m_{NP} = -\frac{1}{2}$ Equation of NP / Vergelyking van NP: $y-6 = -\frac{1}{2}(x-6)$	<ul> <li>✓ for gradient of RN         vir gradiënt van RN</li> <li>✓ for gradient of NP         vir gradiënt van NP</li> <li>✓ for substitution of N         vir vervanging van N</li> </ul>	
	$y = -\frac{1}{2}x + 9$	✓ for/vir $c = 9$ ✓ for answer / vir antwoord	(5)
4.4.1	$-2x = -\frac{1}{2}x + 9$	✓ for equating / vir gelykstelling	(-)
	$\begin{vmatrix} -\frac{3}{2}x = 9\\ -3x = 18 \end{vmatrix}$	✓ for the simplification vir die vereenvoudiging	
	$\therefore x = -6 \text{ and/} en \ y = 12$ $\therefore P(-6; 12)$	✓ for the answer / vir die antwoord	(3)
4.4.2	$RO = RN = \sqrt{4^2 + 2^2} = 2\sqrt{5} \text{ (radii/radiusse)}$	✓ use of distance formula gebruik van afstand formule	(3)
	PO = PN = $\sqrt{(-6)^2 + 12^2} = 6\sqrt{5}$ (tangents from same pt) (raaklyne vanaf dieselfde punt)	✓ for RO / RN answer vir RO / RN antwoord ✓ for PO / PN answer vir PO / PN antwoord	
	:. Perimeter of /Omtrek van PNRO = $2(2\sqrt{5}) + 2(6\sqrt{5})$ = $16\sqrt{5}$ or/of 35,78 units/eenhede	✓ for final answer vir finale antwoord	(4)
4.5	S(8;0)	✓✓ coordinates of S	
	T(12;-2)	koördinate van S ✓✓ coordinates of T koördinate van T	(4)
			[23]

#### **QUESTION 5/VRAAG 5**

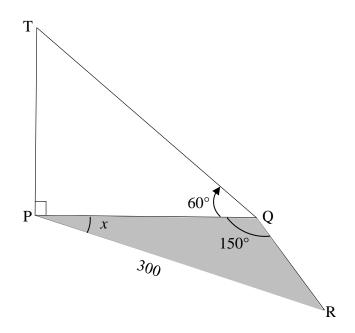


	Τ	1	
5.3.1	$\frac{1-\cos 2x-\sin x}{\cos x}=\tan x$		
	$\sin 2x - \cos x$		
	LHS:		
	$\frac{1-(1-2\sin^2 x)-\sin x}{2}$	$\checkmark$ expansion of $\cos 2x$	
	$2\sin x \cos x - \cos x$	$uitbreiding\ van\ \cos 2x$	
	$=\frac{2\sin^2 x - \sin x}{2}$	$\checkmark$ expansion of $\sin 2x$	
	$2\sin x \cos x - \cos x$	<i>uitbreiding van</i> $\sin 2x$	
	$=\frac{\sin x(2\sin x-1)}{1-x^2}$	✓ for the simplification	
	$\cos x(2\sin x - 1)$	vir die vereenvoudiging ✓ taking out HCF	
	$= \tan x = RHS$	uithaal van GGD	
			(4)
5.3.2	$\sin 2x = \cos x$	$\checkmark \text{ for/} vir \sin 2x = \cos x$	
		✓ for any 2 answers	
	$x = -90^{\circ} ; 30^{\circ} ; 90^{\circ} \text{ and/en } 150^{\circ}$	vir enige 2 antwoorde	
		✓ for any other 2 answers vir enige 2 antwoorde	(3)
5.4	$\sin^2 x + 2\sin x \cos x = 3\cos^2 x$	✓ for standard form	(3)
	$\sin^2 x + 2\sin x \cos x - 3\cos^2 x = 0$	vir standaardvorm	
	Divide every term by/Deel elke term deur $\cos^2 x$	$\checkmark$ for dividing by $\cos^2 x$	
	$\tan^2 x + 2\tan x - 3 = 0$	vir deling deur $\cos^2 x$	
	$(\tan + 3)(\tan x - 1) = 0$	✓ for the factors	
	$\tan x = -3 \text{ or/} of \tan x = 1$	$\checkmark$ for values of tan $x$	
	$x = 108,43^{\circ} + 180^{\circ}.k$ or/of $x = 45^{\circ} + 180^{\circ}.k$	$\frac{1}{\sqrt{1}}$ vir waardes van $\tan x$	
	where/waar $k \in \mathbb{Z}$	✓✓ for the answers	
		vir die antwoorde	
	OR/OF		
	$\sin^2 n + 2\sin n \cos n - 2\cos^2 n$	✓ for standard form	
	$\sin^2 x + 2\sin x \cos x = 3\cos^2 x$ $\sin^2 x + 2\sin x \cos x - 3\cos^2 x = 0$	vir standaardvorm	
	$\sin x + 2\sin x \cos x - 3\cos x - 0$ $(\sin x + 3\cos x)(\sin x - \cos x) = 0$	✓ for the factors	
	$\sin x = -3\cos x  \text{or/of } \sin x = \cos x$	vir die faktore	
	tan x = -3  or/ of tan x = 1	✓ for isolating $\sin x$	
	$x = 108,43^{\circ} + 180^{\circ}.k$	$\forall$ vir isolering van sin x $\forall$ for values of tan x	
	or/of $x = 45^{\circ} + 180^{\circ}$ . k	vir waardes van tan x	
	where/waar, $k \in \mathbb{Z}$	✓✓ for the answers	
		vir die antwoorde	
			(7)
			[30]

#### QUESTION 6/VRAAG 6

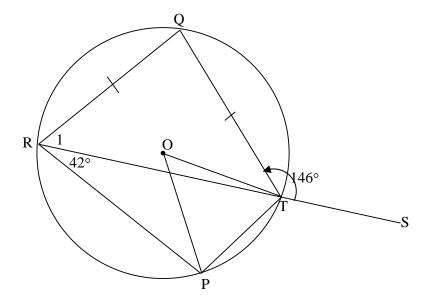


## QUESTION 7/VRAAG 7



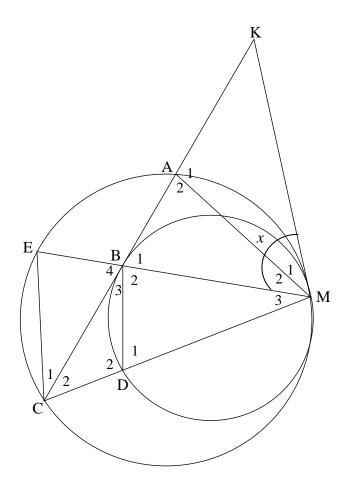
7.1	$\widehat{R}(30^{\circ} - x)$	✓ for answer / vir antwoord	(1)
7.2	$\frac{PQ}{\sin(30^\circ - x)} = \frac{300}{\sin 150^\circ}$	✓ for sine-rule  vir sinusreël	
	$\frac{PQ}{\sin(30^\circ - x)} = 600$	✓ for/vir 600	
	$PQ = 600 \sin(30^{\circ} - x)$	✓ for the answer vir die antwoord	(3)
7.3	$\tan 60^{\circ} = \frac{\text{TP}}{\text{PQ}}$	✓ for/vir tan 60°	
	TP = PQ tan 60° TP = $\sqrt{3}$ .600 sin(30° - x) TP = $\sqrt{3}$ .600. (sin 30° cos x-cos 30° sin x)	$ \sqrt[4]{\text{for/}vir} $ $ \sqrt{3}.600\sin(30^{\circ} - x) $	
	$TP = \sqrt{3} .600 \left( \frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x \right)$	✓ for expansion vir uitbreiding	
	$TP = \sqrt{3} .300(\cos x - \sin x)$	✓ for taking out common factor / vir uithaal van	(4)
		gemene faktor	(4) [ <b>8</b> ]

## QUESTION 8/VRAAG 8



8.1	$\hat{POT} = 84^{\circ} \ (\angle \text{ at centre}) / (Middelpunts \angle)$	✓ S ✓ R	(2)
8.2	$\hat{QTR} = 34^{\circ} \ (\angle s \text{ on a straight line})$	✓ S and/en R	
	$(\angle e \ op \ 'n \ reguitlyn)$ $\hat{R}_1 = 34^{\circ} (\angle s \ opp. = sides) / (\angle e \ teenoor = sye)$	✓ S and/en R	(2)
8.3	$R\widehat{Q}T = 112^{\circ}$	$\checkmark$ S and/en R $\checkmark$ S $\checkmark$ R	
	$\hat{RPT} = 68^{\circ} \text{ (opp. } \angle \text{s of cq)} / \text{ (teenoorst. } \angle e \text{ van } kv)$	V S V K	(3)
			[7]

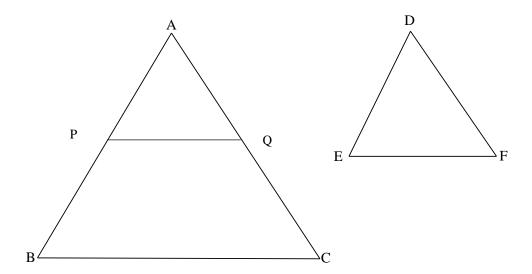
## QUESTION 9/VRAAG 9



9.1	$\widehat{B}_1 = x$	(tangents from common point) (raaklyne van gemene punt)	✓ S and/en R	
	$\widehat{B}_4 = x$	(vertically opposite angles) (regoorstaande hoeke)	✓ S and/en R	
	$\widehat{D}_1 = x$	(tan/chord theorem) (raaklyn/koord stelling)	✓ S ✓ R	
	$E\hat{C}M = x$	(tan/chord theorem) (raaklyn/koord stelling)	✓ S and/en R	(5)
9.2.1	$B\widehat{D}M = E\widehat{C}$	$\widehat{CD} = x \text{ (proven } / \text{ bewys)}$	✓S	` ′
	BD  EC	(corresponding angles =)	✓ R	
		(ooreenkomstige hoeke =)		
		(11111111111111111111111111111111111111		(2)
9.2.2	$\hat{A}_2 = \hat{E}$	(angles in the same segment)	✓ S ✓ R	
		(hoeke in dieselfde segment)		
	$\hat{B}_2 = \hat{E}$	(corresponding angles = , BD   EC)	✓ S and/en R	
	1 6	(ooreenkomstige hoeke =, $BD \parallel EC$ )		
	$\hat{A}_2 = \hat{B}_2$			(3)

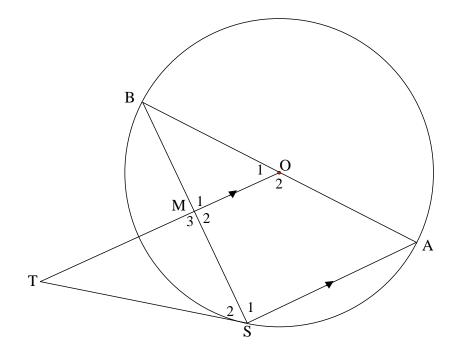
9.2.3	In ΔMEC: CE    DB (proven / bewys)		
	$\frac{ME}{MB} = \frac{MC}{MD} \text{ (prop. int. Thm, CE } \parallel DB\text{)}$	✓ S and/en R	
	(eweredigheid st, CE    DB)		
	$\therefore \frac{ME}{MC} = \frac{MB}{MD}$	✓ S	
	$\therefore ME \times MD = MC \times MB$		(2)
			[12]

## QUESTION 10/VRAAG 10



10.1	Construction: Mark off, on AB and AC, P and Q respectively such that AP = DE and AQ = DF.	✓ construction konstruksie	
	Konstruksie: Merk P en Q onderskeidelik op AB en AC af sodat AP = DE en AQ = DF.		
	In $\triangle$ PAQ and/en $\triangle$ EDF:		
	(1) PA = ED (construction / konstruksie)	✓ S and/en R	
	(2) $\hat{A} = \hat{D}$ (given / gegee)		
	(3) QA = FD (construction / konstruksie)		
	$\therefore \Delta PAQ \equiv \Delta EDF (SAS)$		
	$\therefore \hat{APQ} = \hat{E} (congruency / kongruensie)$	✓ S	
	But/Maar $\hat{\mathbf{B}} = \hat{\mathbf{E}}$ (given/gegee)	✓ S ✓ R	
	$\therefore A\hat{P}Q = \hat{E}$	✓ S	
	$\therefore$ PQ $\square$ BC (corresponding $\angle$ s = / ooreenkomstige. $\angle$ e =)		
	$\therefore \frac{AP}{AB} = \frac{AQ}{AC} \text{ (prop. int. thm / eweredigheid stelling)}$		
	But/Maar:		
	AP = DE  and/ en AQ = DF (construction/konstruksie)		
	$\therefore \frac{DE}{DE} = \frac{DF}{DE}$		
	··· AB AC		(6)

10.2



BÔM = Â (corresponding <s, <math="" as="">\square OM)   <math display="block">(ooreenkomstige \angle e, AS \square OM)</math> <math display="block">\therefore \hat{S}_3 = \hat{A}</math> <math display="block">\therefore TS \text{ is a tangent (conv. tan - chord thrm)}</math> <math display="block">TS \text{ is 'n raaklyn (omgekeerde raaklyn - koord stelling)}</math> <math display="block">10.2.2  \hat{S}_2 = 90^{\circ} (\angle s \text{ in a semi - circle})/(\angle \text{ in semi - sirkel})</math> <math display="block">\hat{M}_3 = 90^{\circ} (\text{corr. } \angle s / \text{ ooreenk. } \angle e, AS \square OM)</math> <math display="block">\therefore TS \text{ is diameter (conv. } \angle s \text{ in a semi - circle})</math> <math display="block">TS \text{ is 'n middellyn (omgek. } \angle e \text{ in semi - sirkel})</math> <math display="block">10.2.3  \text{In } \triangle ABS \text{ and/en } \triangle STM</math> <math display="block">(1)  \hat{A} = \hat{S}_3 \text{ (proven / bewys)}</math> <math display="block">(2)  \hat{B} = S\hat{T}M (\angle s \text{ in the same segment})/(\angle e \text{ in dies. segement})</math> <math display="block">(3)  \hat{S}_2 = \hat{M}_3 \text{ (proven / bewys)}</math> <math display="block">\triangle ABS \parallel \triangle STM (\angle \angle \angle \angle)</math> <math display="block">ABS \parallel \triangle STM (\angle \angle \angle \angle \angle)</math></s,>	(4)
∴ TS is a tangent (conv. tan - chord thrm) TS is 'n raaklyn (omgekeerde raaklyn - koord stelling)  10.2.2 $\hat{S}_2 = 90^\circ$ (∠s in a semi - circle)/(∠ in semi - sirkel) $\hat{M}_3 = 90^\circ \text{ (corr. } ∠s \text{ / ooreenk. } ∠e, \text{ AS} □ \text{ OM})$ ∴ TS is diameter (conv. ∠s in a semi - circle) TS is 'n middellyn (omgek. ∠e in semi - sirkel)  10.2.3 In ΔABS and/en ΔSTM $(1) \hat{A} = \hat{S}_3 \text{ (proven / bewys)}$ $(2) \hat{B} = \hat{S}TM \text{ (∠s in the same segment)}/(∠e in dies. segement)}$ $(3) \hat{S}_2 = \hat{M}_3 \text{ (proven / bewys)}$ $ABS = ABS = ASTM \text{ (∠s (AS))}$ $ABS = ABS = ASTM \text{ (∠s (AS))}$	
$\hat{M}_3 = 90^\circ$ (corr. ∠s / ooreenk. ∠e, AS□OM)  ∴ TS is diameter (conv. ∠s in a semi - circle)  TS is 'n middellyn (omgek. ∠e in semi – sirkel)  10.2.3 In $\triangle$ ABS and/en $\triangle$ STM  (1) $\hat{A} = \hat{S}_3$ (proven / bewys)  (2) $\hat{B} = \hat{S}$ TM (∠s in the same segment)/(∠e in dies. segement)  (3) $\hat{S}_2 = \hat{M}_3$ (proven / bewys) $\triangle$ ABS ASTM (∠((()))	(5)
∴ TS is diameter (conv. ∠s in a semi - circle)  TS is 'n middellyn (omgek. ∠e in semi - sirkel)  10.2.3 In $\triangle$ ABS and/en $\triangle$ STM  (1) $\hat{A} = \hat{S}_3$ (proven / bewys)  (2) $\hat{B} = \hat{S}$ TM (∠s in the same segment)/(∠e in dies. segement)  (3) $\hat{S}_2 = \hat{M}_3$ (proven / bewys) $A \triangle$ RSIII ASTM ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( (	(5)
10.2.3 In $\triangle$ ABS and/en $\triangle$ STM  (1) $\hat{A} = \hat{S}_3$ (proven / bewys)  (2) $\hat{B} = \hat{S}TM$ ( $\angle$ s in the same segment)/( $\angle$ e in dies. segement)  (3) $\hat{S}_2 = \hat{M}_3$ (proven / bewys)  (AABS ##ASTM ( $\angle$ ) ( $\angle$ )	(5)
(1) $\hat{A} = \hat{S}_3$ (proven / bewys) (2) $\hat{B} = \hat{S}TM$ ( $\angle S$ in the same segment)/( $\angle e$ in dies. segement) (3) $\hat{S}_2 = \hat{M}_3$ (proven / bewys)  AARSHII ASTIM ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( ( (	
(2) $\hat{B} = \hat{STM}$ ( $\angle s$ in the same segment)/( $\angle e$ in dies. segement) $\checkmark$ $\hat{S}$ (3) $\hat{S}_2 = \hat{M}_3$ (proven / bewys) $\checkmark$ $\hat{R}$ or/of $3^{rd}$	
(2) $\hat{B} = \hat{STM}$ (\(\angle s\) in the same segment)/(\(\angle e\) in dies. segement) \(\sim S\) (3) $\hat{S}_2 = \hat{M}_3$ (proven / bewys)  AABSHI ASTM (\(\alpha \) (\(\alpha \))	
V R or/of 3 <sup>th</sup>	
angle, 5 moen	(3)
$\begin{array}{ c c c c c }\hline 10.2.4 & \therefore \frac{AS}{SM} = \frac{SB}{MT} \text{ (similarity / gelykvormigheid)} & \checkmark S \\ \hline \end{array}$	
$AS \cdot MT = SM \cdot SB$	
But/Maar:  SR 2SM (Midneint them / green int CM□AS)	
SB = 2SM  (Midpoint thrm / prop.int. , OM  AS) $(Middelpunt stelling / Ewer., OM   AS)$	
$\therefore 2SM^2 = AS.MT$	(3)
	[21]
TOTAL/TOTAAL:	150