SHAPING SPIKING PATTERNS THROUGH SYNAPTIC PARAMETERS REVEALED BY WAVELET BIFURCATION ANALYSIS

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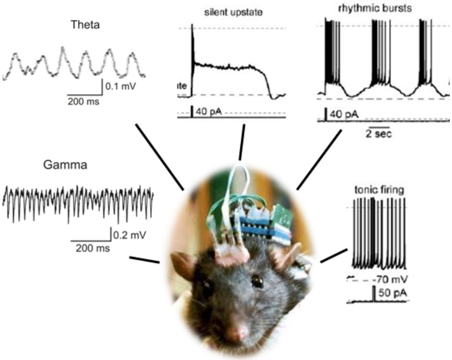
SINCHRONOUS ACTIVITY OF NEURONAL NETWORK

I. Rat brain: in *vitro* and in *vivo*



- Neuronal network (NN) in brain exhibits a rich variety of oscillatory states
- NN is able to switch between each other states
- Oscillatory elements are characterized by the different frequencies and amplitudes
- The shape of sinchronous regime is defined by the coupling type between elements

Kass JI., Mintz IM., *PNAS*, **103** (2006), 183

Hahn PJ., Durand DM., *J.Comp. Neurosc.*, **11** (2001),5

Gloveli T.,et al. *PNAS*, **102** (2005), 13295

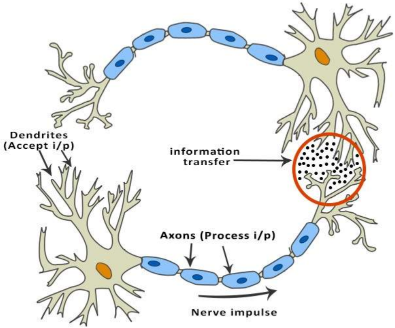
SIMPLE MODEL OF THE SINGLE NEURON

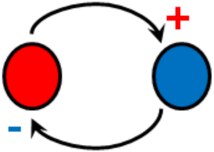
I. FitzHugh-Nagumo system [Biophysical Journal, 1961]

$$
\begin{aligned}
\frac{dv}{dt} &= v - \frac{v^3}{3} - u + I \\
\frac{du}{dt} &= \varepsilon(v + a - bu),
\end{aligned}
$$

- v- membrane potential or activator; allows regenerative self-excitation via a positive feedback (in fact, it introduces the voltage-like variable measurable in experiments)
- u- recovery variable or inhibitor,provides negative slower feedback
- I- input signal (experimental injection of external current into the membrane)
- a, b- mimic ion kinetic
- ε- time scale of oscillations

TWO COUPLED FHN EQUATIONS





$$\frac{dv_i}{dt} = v_i - \frac{v_i^3}{3} - u_i + I_{\text{ext}}\delta_{i,P} + \sum_j I_{\text{syn}}^{(ji)},$$

$$\frac{du_i}{dt} = \varepsilon_i\,(v_i + a - b\,u_i).$$

COUPLING

$$I_{\text{syn}}^{(ji)} = G_{ji} s_{ji} \left(E_{\text{ex,in}} - v_i\right)$$

$$\frac{ds_{ji}}{dt} = \frac{A}{2}\left(1 + \tanh \frac{v_j}{v_{vsl}}\right)(1 - s_{ji}) - $$

$$-Bs_{ji}$$

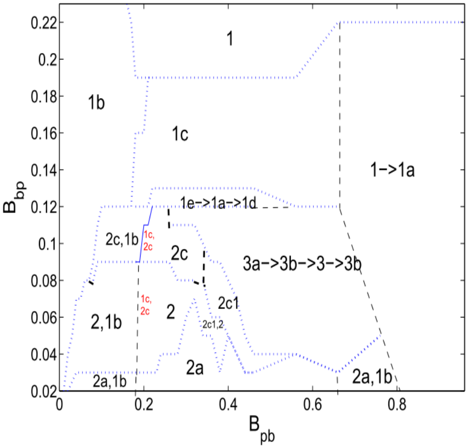
- $I_{\text{syn}}^{(ji)}$- the current from cell j to cell i
- G_{ji}- coupling strength between elements
- $s_{ji}(t)$- synaptic variable, that depends on a threshold function and defines two states of this variable: zero (non-active) or one (active).
- A, B and v_{vsl}- parameters that determine synaptic kinetics

N. Kopell, C. Borgers, D. Pervouchine, P. Malerba, A. Tort, In: *Hippocampal Microcircuits: A Computational Modeller's Resource Book.*, Chapter 15, Eds. V. Cutsuridis, B.P. Graham, S. Cobb, I. Vida. Springer 2010.

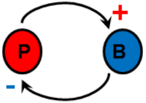
5. Lavrova A., Zaks M., Schimansky-Geier L., *Phys. Rev.E* **85** (2012) 041922

Goal: Can only synaptic parameters define a network dynamics?

Regimes diagramm



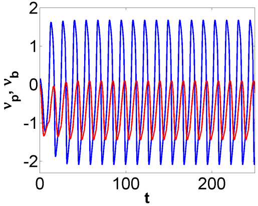
- parameters A and v_{vsl}- don't influence on the mini-network dynamics
- Parameters of the system: a=0.5, b=0.8, ε=0.3, G_{bp}=G_{bp}=0.5, I_{ext}=0.5, A=1, $v_{sl}=0.1$



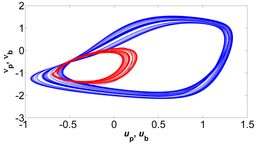
Regimes classification

Fast oscillations: $T \approx 15 - 25$ rel.units

1.Regular spiking

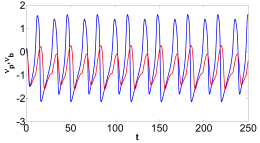


1a.Aperiodic spiking

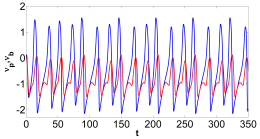


- 1b- cell B oscillates sub-threshold

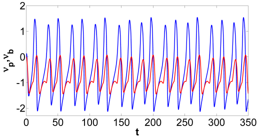
1c.Two-cycles spiking



1d.Three-cycles spiking

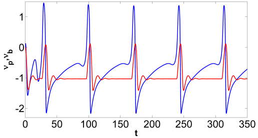


1e.Four-cycles spiking

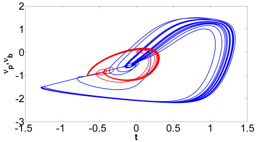


Slow oscillations: $T > 32$ rel.units

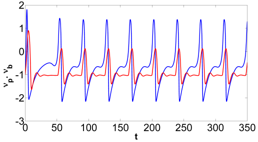
2.50 $< T <$ 300 rel.units



2a.Aperiodic spiking

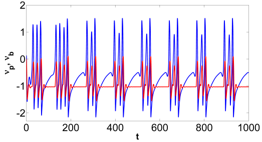


2c.30 $< T <$ 40 rel.units

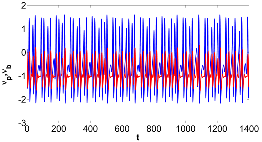


Bursting

3. Regular bursts

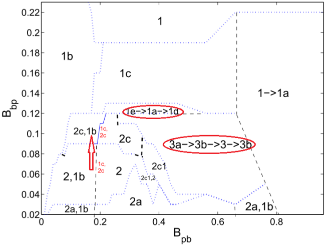


3a. Unregular bursts

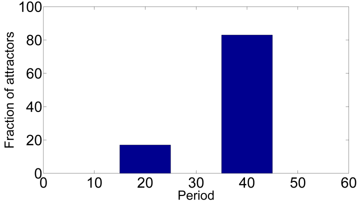


- 3b- medium regime:short breaks between spikes patches

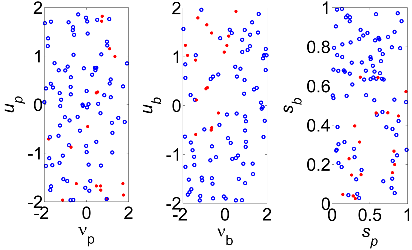
MULTISTABILITY



B_{bp}=0.1,B_{pb}=0.15



Distribution of initial conditions



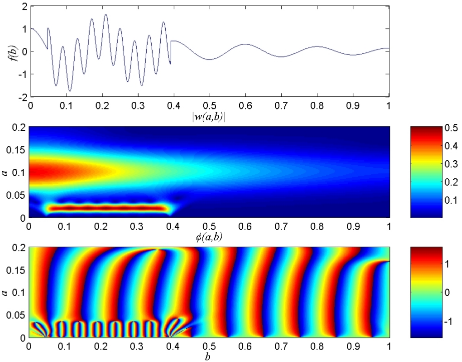
CWT WITH THE MORLET WAVELET

$$w(a,b) = \int_{-\infty}^{+\infty} f(t) e^{i\omega_0 \frac{t-b}{a}} e^{-\frac{(t-b)^2}{2a^2}} \frac{dt}{\sqrt{2\pi a}},$$

where
t, b – time variables: experimental time and shift time

a – scale variable

ω_0 – central frequency
Local period is defined as
$T = 2\pi a_{max}/\omega_0$



BACKGROUND FOR CWT ANALYSIS OF TRANSITIONS

The system with a slow varying parameter:

$$
\begin{aligned}
\frac{dv_i}{dt} &= v_i - \frac{v_i^3}{3} - u_i + I_{ext}\delta_{i,P} + \sum_j G_{ji}\left(E_{ex,in} - v_i\right) sji, \\
\frac{du_i}{dt} &= \varepsilon_i(v_i - a - bu_i) \\
\frac{ds_{ji}}{dt} &= \frac{A}{2}\left(1 + \tanh\frac{v_j}{v_{vsl}}\right) - \left[B_j^0 + k_j(t)t\right]s_{ji}
\end{aligned}
$$

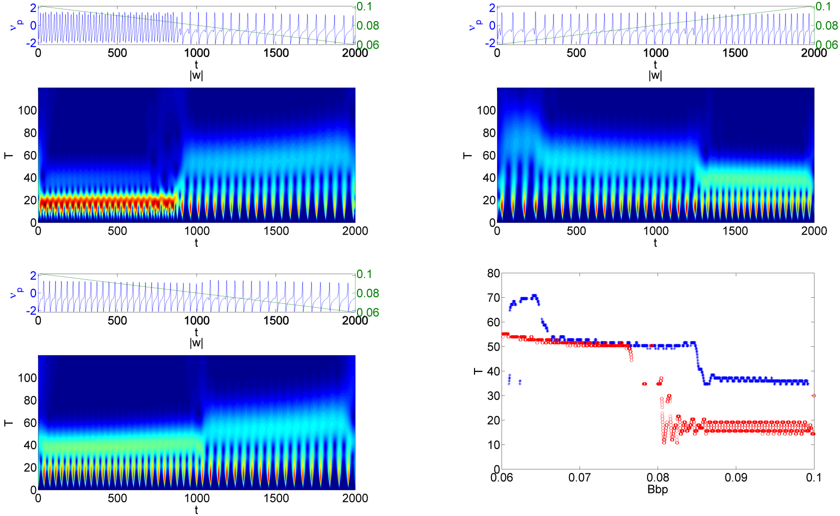
A slow (adiabatic) variation implies that

$$k(t) \approx \frac{dB_j}{dt} << \omega = \frac{2\pi}{T},$$

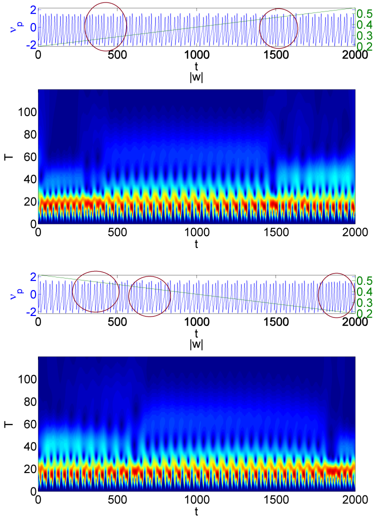
where T is the period of oscillations with a constant B_j.

Dynamical transitions between regimes

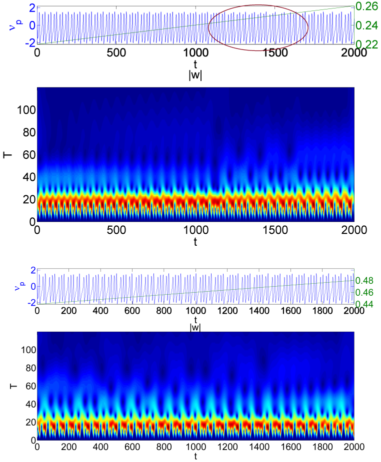
Transition 2c→2.$B_{pb} = 0.19$



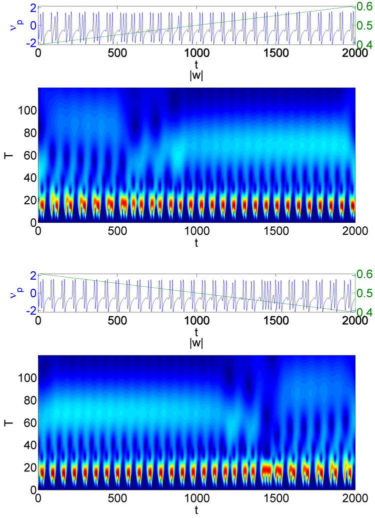
Transition from two-cycles spiking regime (1c) to four-cycles spiking rhythm (1d).$B_{bp} = 0.12$



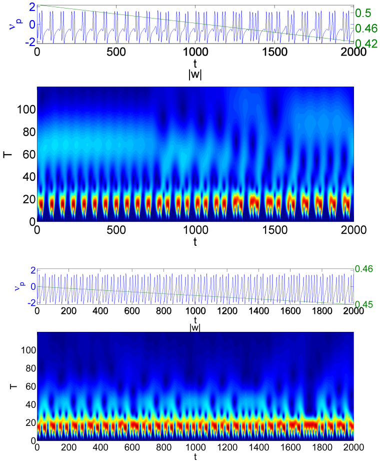
Chaotic scenario of 1e→1d transition



Transitions in bursting rhythm.$B_{bp} = 0.08$



Chaotic scenario of 2-burst to 3-burst transition



Conclusions and outlook

- A rich variety of synchronous regimes could be obtained in the system of two identical neurons where coupling is non-symmetrical and determined by the kinetic parameters of their connection.
- Bifurcation Analysis of periodical orbits in this case is very complicated and is practically impossible due to methods restriction in well-known programms (matcont, Logbif)
- Wavelet analysis is simple and universal method for studying complex and unclear transitions between different rhythms. Application of this method allows to avoid a lot of numerical mistakes that arise at the programming of ”bifurcation analysis” code