Shaping spiking patterns through synaptic parameters revealed by wavelet bifurcation analysis

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Abstract— **We investigate different dynamical regimes of a small neuronal circuit. This circuit includes two cells which are interconnected by means of dynamical synapses (excitatory and inhibitory). On the individual level, each neuron is modelled by FitzHugh-Nagumo equations. To analyze complex patterns and transitions between them in this small circuit, we apply wavelet analysis. We show that this tool could be applicable to systems that are difficult to investigate using methods of classical bifurcation analysis.**

Keywords— FitzHugh-Nagumo model; synaptic coupling, wavelet analysis.

I. INTRODUCTION

Synchronous activity in neuronal networks generates different rhythmic patterns that are responsible for a large variety of physiological processes – from memory to controlling of rhythmic movement [1,2]. Such activity could be indicated by various oscillatory rhythms, which differ in frequency, amplitude, pattern complexity, simulation thresholds and localization in the brain [3]. Regimes that appear in neuronal networks could be spontaneous; the transitions between different rhythms are also possible in dependence on the initial conditions or system parameters [4-6].

Numerous experiments and computational simulations have shown that the emergence of different rhythms and switching between rhythms occur due to the control of coupling strengths [7] and could be also controlled by the synaptic kinetics [8-10]. It should be noted that classical methods of bifurcation analysis of these patterns and transitions between them sometimes are not applicable due to the extremal non-linearity of a system. Therefore, it is necessary to develop other, more convenient, tools for the study of network dynamics.

In the present work, we propose a model of a system formed by two neurons mutually coupled through chemical synapses. The oscillatory dynamics of this circuit is controlled by the synaptic kinetics and demonstrates different complex patterns observed in the experiments. Our goal is to analyse t such rhythmical patterns and transitions between these patterns with the tools of the continuous wavelet analysis with the Morlet wavelet specially adjusted to this particular goal.

II. APPROACH AND RESULTS

We describe two mutually coupled neurons by the modified FitzHugh-Nagumo equations:

$$\frac{dv_i}{dt} = v_i - \frac{v_i^3}{3} - u_i + I_{ext}\delta_{i,P} + \sum_j I_{syn}^{(ji)},$$
$$\frac{du_i}{dt} = \varepsilon_i\left(v_i + a - bu_i\right),$$

where the synaptic coupling is described as

$$I_{syn}^{(ji)} = G_{ji}s_{ji}\left(E_{ex,in} - v_i\right),$$
$$\frac{ds_{ji}}{dt} = \frac{A}{2}\left(1 + \tanh\frac{v_j}{v_{vsl}}\right)\left(1 - s_{ji}\right) - Bs_{ji}.$$

Here $I_{syn}^{(ji)}$ is the current from the cell j to the cell I; G_{ji} is the coupling strength between elements; s_{ji} is the synaptic variable, which depends on a threshold function defining two states of this variable: zero (non-active) or one (active); A, B and v_{vsl} are the parameters that determine synaptic kinetics

In order to study different rhythms, which arise in dependence on the synaptic variables, the authors have considered the response of the system to a variation of the synaptic parameters B_{ij} and B_{ji}, which determine kinetic constants of the closed state of excitatory and inhibitory synapses respectively. Therefore, the time decay constants B_{ij} and B_{ji} are changed on a grid from 0 to 1, where the starting point is the completely opened synapse state, whereas the ~~last~~ end point means the entirely closed state. The other parameters of synaptic current, namely A_{ji}, A_{ij} and v_{vsl}, for both cells do not affect rhythmic patterns emergence and were fixed.

As shown in experiments [7], there are at least three main rhythms that are differed in spike frequency and oscillations shape: the slow oscillated theta-rhythm (2-11 Hz), the gamma rhythm that introduces broadband (20-100 Hz) and the regime

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where the fast rhythm is modulated by the slow oscillations - theta/gamma rhythm. The authors have obtained a two-dimensional diagram (B_{ij} vs B_{ji}) of various rhythms which are differed by periods and shapes (Fig. 1a) shows an example of the revealed regimes).

To trace the time evolution of these regimes, we introduce the adiabatically slow varying parameter (straight line in Fig. 1a), $B = B_j^0 + k_j(t)t$, where

$$k(t) \approx \frac{dB_j}{dt} << \omega = \frac{2\pi}{T}$$

and apply the continuous wavelet transform

$$w(t,T) = \int_{-\infty}^{+\infty} f(t')\psi^*\left(\frac{t-t'}{2\pi/T}\right)dt'$$

with the Morlet wavelet

$$\psi(\xi) = \frac{T}{(2\pi)^{3/2}} e^{i\omega_o\xi} e^{-\frac{\xi^2}{2}}$$

the results in the picture in Fig. 1b.

Finally, a specific algorithm developed to reveal leading wavelet maxima lines during the direct and reverse time evolution of the parameter was applied to form a kind of the wavelet-bifurcation diagram shown in Fig. 1c.

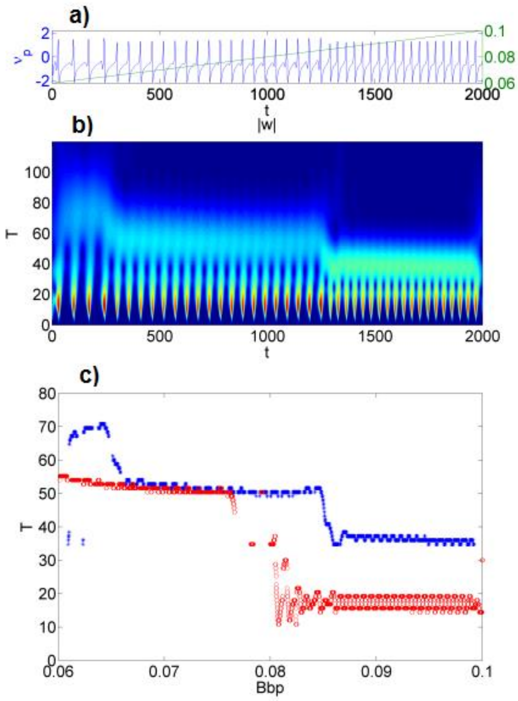

Fig.1 Dynamical transitions between regimes. The upper one (a): the slow rhythm transfers to the regular spiking at the B_{ji} variation that occurs continuously. The middle panel (b): the result of the wavelet transform. The lower panel (c) shows hysteretical transitions in dependence on the initial conditions and co-existing of several regimes. Other parameters were: $a=0.5,\ b=0.8,\ \varepsilon=0.3,\ G_{ji}=G_{ij}=0.5,\ I_{ext}=0.5,\ A=1, v_{vsl}=0.1,\ B_{ij}=0.19$

III. Conclusions

Thus, the authors have isolated main regimes corresponded to experimentally obtained regimes [1-4] and described an evolution/transformation between them using the wavelet transform tools. The authors have shown that a rich variety of synchronous regimes could be obtained in the system of two identical neurons where the coupling is non-symmetrical and determined by the kinetic parameters of their connection. It should be noted that the bifurcation analysis of periodical orbits, in this case, is very complicated and practically impossible due to the method's restriction in the conventional software (matcont, Logbif).

In this case, the wavelet analysis is a simple and universal method for studying complex and unclear transitions between different rhythms. Application of this method allows avoiding a lot of numerical mistakes that arise in the programming of "bifurcation analysis" code.

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