Simultaneous Elicitation of Scoring Rule and Agents Preferences for Robust Winner Determination

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Abstract

Social choice deals with the problem of determining a consensus choice from the preferences of different agents (voters). In the classical setting, the voting rule is fixed beforehand and full information concerning the preferences of the agents is provided. Recently, the assumption of full preference information has been questioned by a number of researchers and several methods for eliciting preferences have been proposed. In this paper we go one step further and we assume that both the voting rule and the agents preferences are partially specified. Focusing on positional scoring rules, we assume that the chair, while not able to give a precise definition of the rule, is capable of answering simple questions requiring to pick a winner from a specific example profile. Moreover, the preferences of the agents are incrementally acquired by asking comparison queries. In this setting, we propose a method for robust approximate winner determination with minimax regret. We also provide an interactive elicitation protocol based on minimax regret and develop several query strategies that interleave questions to the chair and questions to the agents in order to attempt to acquire the most relevant information in order to quickly converge to optimal or a near-optimal alternative.

1 Introduction

Aggregation of preference information is a central task in many computer systems (recommender systems, search engines, etc). In many situations, such as in group recommender systems, the goal is to find a consensus choice. It is therefore natural to look at methods from social choice and see how they can be adapted for group decision-making in a computerized setting.

The traditional approach to social choice assumes that both the social choice function and the full preference orderings of the agents are expressed beforehand. These represent two strong hypothesis. Requiring agents to express full preference orderings can be prohibitively costly (in terms of cognitive and communication cost). This observation has motivated a number of recent works considering social choice with partial preference orders [Xia and Conitzer, 2008, Pini et al., 2009, Konczak and Lang, 2005] and incremental elicitation [Kalech et al., 2011, Lu and Boutilier, 2011, Naamani-Dery et al., 2015] of agents' preferences.

Furthermore, it is often difficult for non-expert users to formalize a voting rule on the basis of some generic preferences over a desired aggregation method. Thus, the first hypothesis should also be relaxed. The work of Cailloux and Endriss [2014] provides elicitation methods for a quite general class of rules based on weak orders. When considering positional scoring rules, several authors [Stein et al., 1994, Llamazares and Peña, 2013, Viappiani, 2018] have worked on positional scoring rules with uncertain weights, assuming that the preferences of the agents are fully known.

In this paper we focus on positional scoring rules, that are a particularly common method used to aggregate rankings, and we assume that both the agents' preferences and the social choice rule are partially specified. We develop methods for computing the minimax-optimal alternative using positional scoring rules and we provide incremental elicitation methods to acquire relevant preference information. We then discuss several heuristics that determine queries, either to an agent or to the chair, that quickly reduce minimax regret. While previous works have considered either partial information about the agents' preferences or a partially specified aggregation method, we do not know of any work considering both sources of uncertainty at the same time.

The paper is organized as follows. In Section 2 we provide the necessary background and in Section 3 we introduce the minimax criterion, that selects a winner that minimizes the worst possible regret. Then, in Section 4 we provide an interactive elicitation protocol based on minimal regret; in Section 5 we present the empirical validation of our approach with simulations; and Section 6 provides some final thoughts.

2 Social choice with partial information

We now introduce some basic concepts. We consider a set A of m alternatives (products, restaurants, movies, public projects, job candidates, etc.) and a set $\{1, \dots, n\}$ of agents (voters). Each agent j comes from an infinite set $\mathbb{N}$ of potential agents and is associated to her "real" preference order $\succ_j \in \mathcal{L}(A)$ which is a linear order (a connected, transitive, asymmetric relation) over the alternatives. Following the social choice nomenclature, we call *profile* the association of a preference to each agent, considering a subset of agents from the set $\mathbb{N}$, and denote a profile by $(\succ_1, \ldots, \succ_n)$. A profile is equivalently represented by $\boldsymbol{v} = (v_1, \ldots, v_n)$ where $v_j(i) \in \{1, \ldots, m\}$ denotes the rank (position) of alternative i in the preference order $\succ_j$.

Let V be the set of possible preference profiles (the union, for any integer, n of the n-fold cartesian product of the linear orders over the alternatives). A social choice function $f : V \rightarrow \mathscr{P}^*(A)$ associates a profile with a set of winners, where $\mathscr{P}^*(A)$ represents the set of subsets of A except for the emptyset. (Sets are used for tied winners.) Among the many possible social choice functions, we consider *positional scoring rules (PSR)*, which attach weights to positions according to the vector $(w_1, \ldots, w_m)$ (also called the scoring vector). An alternative obtains a score that depends on the rank obtained in each of the preference orders:

$$s(x; \boldsymbol{v}, \boldsymbol{w}) = \sum_{j=1}^{n} w_{v_j(x)} = \sum_{r=1}^{m} \alpha_r^x w_r \tag{1}$$

where α_r^x is the number of times that alternative x was ranked in the r-th position. The winners are the alternatives gighest scores.

In this work we assume fixed, but unknown to us, a profile $\boldsymbol{v}^*$, representing the preferences $\succ_j^*$ of the agents, and a weight vector $\boldsymbol{w}^*$, representing the preferences of the chair, and we want to reason about partial preference information concerning those objects. At a given time, our knowledge of agent j's preference is encoded by a partial order over the alternatives, thus a transitive and asymmetric binary relation, denoted by $\succ_j^{\mathrm{p}}$. In this work we assume that preference information is truthful, i.e. $a \succ_j^{\mathrm{p}} b \Rightarrow a \succ_j^* b$. An incomplete profile $\boldsymbol{p} = (\succ_1^{\mathrm{p}}, \ldots, \succ_n^{\mathrm{p}})$ maps each agent to a partial preference.

A completion of $\succ_j^{\mathrm{p}}$ is any linear order $\succ$ that extends $\succ_j^{\mathrm{p}}$ and we indicate with $C(\succ_j^{\mathrm{p}}) = \{\succ \in \mathcal{L}(A) \mid \succ_j^{\mathrm{p}} \subseteq \succ\}$ the set of possible completions of $\succ_i^{\mathrm{p}}$. Then $C(\boldsymbol{p}) = C(\succ_1^{\mathrm{p}}) \times \ldots \times C(\succ_n^{\mathrm{p}})$ is the set of complete profiles extending p. Note that $\boldsymbol{v}^* \in C(\boldsymbol{p})$.

We also assume that the weights of the scoring rule are only partially specified. Therefore, the vector $(w_1, \ldots, w_m)$ is not known but we are given a set of constraints restraining the possible values that weights can take. We consider a decreasing sequence of weights:

$$1 = w_1 \geq w_2 \geq \ldots \geq w_m = 0. \tag{2}$$

This is a natural assumption, as it is better to be ranked first than second, second than third, etc. Without loss of generality, we consider that $w_1 = 1$ and $w_m = 0$.

The weights of a scoring rule can model different preferences of the chair. For instance, the weights can control the inclination to favor "extreme" alternatives (often at either the top or the bottom of the input rankings) at the expenses of "moderate" alternatives (that are more consistently in the middle part of the input rankings).

An important class of scoring rule is the one composed of weights that represent a convex sequence [Stein et al., 1994, Llamazares, 2016], meaning that the difference between the weight of the first position and the weight of the second position is at least as great as the difference between the weights of the second and third positions, etc.

$$\forall r \in \{1, \ldots, m-2\} : w_r - w_{r+1} \geq w_{r+1} - w_{r+2}. \tag{3}$$

The constraint above is often used when aggregating rankings in sport competitions. We use $\mathcal{W}$ to denote the set of convex weight vectors.

In general it can be difficult to set the weights in an appropriate way. We assume that in addition of basic requirements (monotonicity and convexity), the chair (the person or the organization that is supervising the voting process) is able to specify additional preferences about how the social choice function should behave. In this work we assume that the preferences of the chair are encoded with linear constraints about the vector $\boldsymbol{w}$, relating the value of the weights of different positions, and the set of these constraints is denoted by $\mathcal{C}_W$. Moreover, we use $W \subseteq \mathcal{W}$ to denote the set of weight vectors compatible with the preferences expressed by the chair about the scoring vector.

Of course it may be difficult for real decision makers to state preferences about the voting rule in such an abstract way. These additional preferences can be elicited by asking questions about concrete profiles, for instance, by showing a complete profile of a small synthetic election and asking who should be elected in this case, as shown in Section 4.

3 Robust winner determination

In this paper, we consider a setting where both the agents' preferences and the preferences of the chair about the voting rule are incomplete. Some authors have considered possible and necessary winners assuming a partial profile [Xia and Conitzer, 2008] or assuming an incompletely specified scoring rule [Viappiani, 2018]; however, we note that, in typical settings, there are no necessary winner and too many possible winners. In practice it is often useful to declare a winner given partial information.

As a decision criterion to determine a winner, we propose to use minimax regret. Minimax regret [Savage, 1954] is a decision criterion that has been used for robust optimization under data uncertainty [Kouvelis and Yu, 1997] as well as in decision-making with uncertain utility values [Salo and Hämäläinen, 2001, Boutilier et al., 2006]. Lu and Boutilier [2011] have adopted minimax regret for winner determination in social choice with the preferences of the agents that are only partially known, while the social choice function is predetermined and known.

In this work, we consider the simultaneous presence of uncertainty in agents' preferences and in weights. Using *maximum regret* to quantify the worst-case error, the alternatives that minimize this error are selected as tied winners, providing us with a form of robust optimization. Intuitively, the quality of a proposed alternative a is how far a is from the optimal one in the worst case, given the current knowledge.

The maximum regret is considered by assuming that an adversary can both 1) extend the partial profile $\boldsymbol{p}$ into a complete profile, and 2) instantiate the weights choosing among any weight vector in W, where $\boldsymbol{p}$ and W represent our knowledge so far. We formalize the notion of minimax regret in multiple steps. First of all, $\mathrm{Regret}(x, \boldsymbol{v}, \boldsymbol{w})$ is the "regret" of selecting x as a winner instead of choosing the optimal alternative under $\boldsymbol{v}$ and $\boldsymbol{w}$:

$$\mathrm{Regret}(x, \boldsymbol{v}, \boldsymbol{w}) = \max_{y \in A} s(y; \boldsymbol{v}, \boldsymbol{w}) - s(x; \boldsymbol{v}, \boldsymbol{w}).$$

The pairwise max regret $\mathrm{PMR}(x, y; \boldsymbol{p}, W)$ of x relative to y given partial profile $\boldsymbol{p}$ and the set of weights W is the worst-case loss of choosing x instead of y under all possible realizations of the full profile *and* all possible instantiations of the weights:

$$\mathrm{PMR}(x, y; \boldsymbol{p}, W) = \max_{\boldsymbol{w} \in W} \max_{\boldsymbol{v} \in C(\boldsymbol{p})} s(y; \boldsymbol{v}, \boldsymbol{w}) - s(x; \boldsymbol{v}, \boldsymbol{w}).$$

Max regret $\mathrm{MR}(x; \boldsymbol{p}, W)$ is the worst-case loss of x. That is the loss occurred as the result of an adversarial selection of the complete profile $\boldsymbol{v} \in C(\boldsymbol{p})$ and of the scoring vector $\boldsymbol{w} \in W$ that together maximize the loss between x and the true winner under $\boldsymbol{v}$ and $\boldsymbol{w}$.

$$\mathrm{MR}(x; \boldsymbol{p}, W) = \max_{y \in A} \mathrm{PMR}(x, y; \boldsymbol{p}, W) \tag{4}$$

$$= \max_{\boldsymbol{w} \in W} \max_{\boldsymbol{v} \in C(\boldsymbol{v})} \mathrm{Regret}(x, \boldsymbol{v}, \boldsymbol{w}). \tag{5}$$

Finally, $\mathrm{MMR}(\boldsymbol{p}, W)$ is the value of minimax regret under $\boldsymbol{p}$ and W, obtained when recommending a minimax optimal alternative $x^*_{\boldsymbol{p},W}$:

$$\mathrm{MMR}(\boldsymbol{p}, W) = \min_{x \in A} \mathrm{MR}(x; \boldsymbol{p}, W)$$

$$x^*_{\boldsymbol{p},W} \in A^*_{\boldsymbol{p},W} = \operatorname*{argmin}_{x \in A} \mathrm{MR}(x; \boldsymbol{p}, W)$$

By picking as consensus choice an alternative associated with minimax regret, we can provide a recommendation that gives worst-case guarantees, giving some robustness in face of uncertainty (due to both not knowing the agents' preferences and the weights used in the aggregation). In cases of ties in minimax regret, we can either decide to return all minimax alternatives $A^*_{\boldsymbol{p},W}$ as winners or to pick just one of them using some tie-breaking strategy.

Observe that if $\mathrm{MMR}(\boldsymbol{p}, W) = 0$, then $x^*_{\boldsymbol{p},W}$ is a necessary co-winner; this means that for any valid completion of the profile and any feasible $w \in W$, $x^*_{\boldsymbol{p},W}$ obtains a highest score.

We note that our notion of regret gives some cardinal meaning to the scores: instead of just being used to select winners under the corresponding PSR, their differences are considered as representing the regret of the chair.

Computation of minimax regret In order to compute pairwise maximum regret, and therefore minimax regret, we decompose the PMR into the contributions associated to each agent by adapting the reasoning from Lu and Boutilier [2011]. The setting is however more challenging due to the presence of uncertainty in the weights.

Recall that, in the computation of $s(x; \boldsymbol{v}, \boldsymbol{w})$, $w_{v_j(x)}$ represents the score that x obtains in the ranking v_j (see Equation 1). Since scoring rules are additively decomposable, we can consider separately the contribution of each agent to the total score. Thus, we can write the actual regret of choosing x instead of y as $s(y; \boldsymbol{v}, \boldsymbol{w}) - s(x; \boldsymbol{v}, \boldsymbol{w}) = \sum_{j=1}^{n} w_{v_j(y)} - w_{v_j(x)}$, and we can rewrite PMR as follows:

$$\mathrm{PMR}(x, y; \boldsymbol{p}, W) = \max_{\boldsymbol{w} \in W} \max_{\boldsymbol{v} \in C(\boldsymbol{p})} [s(y; \boldsymbol{v}, \boldsymbol{w}) - s(x; \boldsymbol{v}, \boldsymbol{w})]$$

$$= \max_{\boldsymbol{w} \in W} \sum_{j=1}^{n} \max_{v_j \in C(\succ^p_j)} [w_{v_j(y)} - w_{v_j(x)}].$$

Note that in general the inner max depends on the weights chosen by the outer max.

We are interested in computing $\mathrm{PMR}(x, y; \boldsymbol{p}, W)$. This represents the "worst" difference of score, thus the difference of score between y and x under the worst case preferences compatible with $\boldsymbol{p}$ and W, where the worst case is the one that maximizes this difference of score. We consider now a procedure for completing a partial profile that was first proposed by Lu and Boutilier [2011] when considering a fixed weight vector. As we will show, this procedure can also be used when the weight vector is not completely known.

Claim 1. *There exists a completion $\hat{\boldsymbol{v}} \in C(\boldsymbol{p})$ such that $\mathrm{PMR}(x, y; \boldsymbol{p}, W) = \max_{\boldsymbol{w} \in W} [s(y; \hat{\boldsymbol{v}}, \boldsymbol{w}) - s(x; \hat{\boldsymbol{v}}, \boldsymbol{w})]$ and such that the linear order $\hat{v}_j$ of each agent j satisfies:*

$$a \succ_j x \Leftrightarrow \neg(x \succeq^p_j a) \tag{6}$$

$$y \succ_j a \Leftrightarrow \neg(a \succeq^p_j y) \wedge \neg((x \succeq^p_j y) \wedge \neg(x \succeq^p_j a)). \tag{7}$$

Sketch of proof. Consider our knowledge $\succeq^{\mathrm{p}}_j$ about the preference of the agent j. The adversary's goal is to make the score of y as high as possible and the score of x as low as possible. To do this, he should complete $\succ^{\mathrm{p}}_j$ to $\succ_j$ by putting above x as many alternatives as he can, that is, all the alternatives except those that are known to be worse than x (those a such that $x \succeq^{\mathrm{p}}_j a$); and similarly, he should put below y all the alternatives he can. Two conditions must be excluded for a to go below y. The alternatives such that $a \succeq^{\mathrm{p}}_j y$ can't be put below y. Furthermore, the first objective must take priority over the second one: when an alternative should go above x according to the first objective (because $\neg(x \succeq^{\mathrm{p}}_j a)$), and x is known to be better than y (thus $x \succeq^{\mathrm{p}}_j y$), then a should be put above x (irrespective of whether $a \succeq^{\mathrm{p}}_j y$), which will move both x and y one rank lower than if a had been put below y. This maximizes the adversary's interests: because the weight vector is convex, the difference of scores will be lower when both alternatives are ranked lower (Equation 3), and that difference of scores is in favor of x when $x \succ^{\mathrm{p}}_j y$, thus to be minimized from the point of view of the adversary. □

Let $\succeq^{\mathrm{p}}_j(x)$ designate the set of alternatives that are known to be considered by j as less good than or equal to x, and $\prec^{\mathrm{p}}_j(y)$ be the set of alternatives known to be considered by j as strictly better than y.

Claim 2. *The rank of x in the PMR-maximizing linear orders of agent j is $\hat{v}_j(x) = 1 + |A| - |\succeq^p_j(x)|$, and the rank of y is $\hat{v}_j(y) = 1 + |\prec^p_j(y)| + |\beta|$, where $|\beta| = |A \setminus (\succeq^p_j(x) \cup \prec^p_j(y))|$ if $(x \succeq^p_j y)$ and $|\beta| = 0$ otherwise.*

Proof. The rank of x is directly obtained from eq. (6). The rank of y is obtained by complementing eq. (7), obtaining $a \succeq_j y \Leftrightarrow (a \succeq_j^{\mathrm{p}} y) \vee ((x \succeq_j^{\mathrm{p}} y) \wedge \neg(x \succeq_j^{\mathrm{p}} a))$, and, observing that $a \succ_j y \Leftrightarrow a \neq y \wedge a \succeq_j y$, obtaining that $a \succ_j y$ if and only if

$$(a \neq y) \wedge [(a \succeq_j^{\mathrm{p}} y) \vee ((x \succeq_j^{\mathrm{p}} y) \wedge \neg(x \succeq_j^{\mathrm{p}} a))], \quad (8)$$

or equivalently, if and only if

$$(a \succ_j^{\mathrm{p}} y) \vee ((x \succeq_j^{\mathrm{p}} y) \wedge \neg(x \succeq_j^{\mathrm{p}} a)). \quad (9)$$

Indeed, (8) $\Rightarrow$ (9), and (9) $\Rightarrow$ (8) because $(x \succeq_j^{\mathrm{p}} y) \wedge \neg(x \succeq_j^{\mathrm{p}} a) \Rightarrow a \neq y$ (as when $a = y$, $(x \succeq_j^{\mathrm{p}} y)$ and $\neg(x \succeq_j^{\mathrm{p}} a)$ are opposite claims). Suffices now to rewrite eq. (9) to let the two disjuncts designate disjoint sets:

$$a \succ_j y \Leftrightarrow (a \succ_j^{\mathrm{p}} y) \vee ((x \succeq_j^{\mathrm{p}} y) \wedge \neg(x \succeq_j^{\mathrm{p}} a) \wedge \neg(a \succ_j^{\mathrm{p}} y)).$$

□

The claim can also be understood by observing that in the case $(x \succeq_j^{\mathrm{p}} y)$, β is the number of alternatives incomparable with both x and y.

Claim 3. *The* PMR *can be written as:*

$$\mathrm{PMR}(x, y; \boldsymbol{p}, W) = \max_{w \in W} \sum_{j=1}^{n} w_{\hat{v}_j(y)} - w_{\hat{v}_j(x)} = \quad (10)$$

$$= \max_{w \in W} \sum_{r=1}^{m} (\hat{\alpha}_r^y - \hat{\alpha}_r^x) w_i. \quad (11)$$

where $\hat{\alpha}_r^y$ (resp. $\hat{\alpha}_r^x$) is the number of times y (resp. x) is at rank r in the complete profile $\hat{\boldsymbol{v}}$.

The last claim shows that PMR is linear in the weights. Recall that the preferences of the chair are encoded with linear constraints $\mathcal{C}_W$. The pairwise max regret $\mathrm{PMR}(x, y; \boldsymbol{p}, W)$ can be obtained as the solution of the following linear program defined on the variables $w_1, \ldots, w_m$, which represent the weights attached to different positions.

$$\max_{\boldsymbol{w}} \sum_{r=1}^{m} (\hat{\alpha}_r^y - \hat{\alpha}_r^x) w_r$$

s.t. eq. (2) and eq. (3) and $\mathcal{C}_W$. Note that given our choice $w_1 = 1$ and $w_m = 0$, there are only $m - 2$ variables (we leave w_1 and w_m in the LP just for clarity of presentation).

The max regret $\mathrm{MR}(x; \boldsymbol{p}, W)$ is determined by considering the pairwise regret of x with all other alternatives in A. Optimal alternatives w.r.t. minimax regret are the ones with least max regret. Observe that, whenever the PMR of an alternative x (against some other alternative y) exceeds the best MR value found so far, we don't need to further evaluate x. This idea can be exploited further by adopting a minimax-search tree [Braziunas, 2011].

4 Interactive Elicitation

We propose an incremental elicitation method based on minimax regret. At each step, the system may ask a question either to one of the agents about her preferences or to the chair about the voting rule. The goal is to acquire relevant information to reduce minimax regret as quickly as possible. As termination condition of elicitation, we can check whether minimax regret is lower than a threshold; if we wish optimality, we can perform elicitation until minimax regret drops to zero.

The remainder of this Section is structured as follows. First of all, we discuss the different types of questions that can be asked to the agents and to the chair, and the way responses are handled. Then, we describe different strategies to determine informative queries to ask next, with the goal of reducing $\mathrm{MMR}(\boldsymbol{p}, W)$ quickly.

Question types We distinguish between questions asked to the agents and questions asked to the chair. As *questions asked to the agents* it is natural to consider comparison queries asking to compare two alternatives. The effect of a response to a question asked to an agent is the increase in our knowledge about the agents rankings, thus augmenting the partial profile $\boldsymbol{p}$. If agent j answers a comparison query stating that alternative a is preferred to b, then the partial order $\succ_j^{\mathrm{p}}$ is augmented with $a \succ_j^{\mathrm{p}} b$ and by transitive closure.

A bit more discussion is needed about *questions asked to the chair*. Such questions aim at refining our knowledge about the scoring rule; a response gives us a constraint on the weight vector $\boldsymbol{w}$. In particular, we want to acquire constraints of the type:

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$$

for $r \in \{1, \ldots, m-2\}$, relating the difference between the importance of ranks r and $r+1$ with the difference between ranks $r+1$ and $r+2$.

Building concrete questions for the chair As we assume the weights constitute the utility components of the chair, it might be reasonable to assume that the chair is able to answer such abstract questions in our setting. However, it is important to make sure that a question can also, in principle, be asked in a more concrete way, in terms of winners of example profiles. This permits to test how the chair understands the question and to relate the preference of the chair to her choice behavior in the economic sense. Furthermore this will be necessary for an ordinal extension of our work where the scores would not be considered as cardinal utilities. Thus, our task is to build a profile, given λ and $r \leq m - 2$, in such a way that the set of (tied) winners picked by the chair reveals whether $w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$.

Claim 4. *Given a rational $\lambda = p/q$ and a rank r between* 1 *and $m-2$, the profile named P' in the ensuing description is such that, for any weight vector $\boldsymbol{w} \in \mathcal{W}$, $a \in f(P')$ iff $w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$ and $b \in f(P')$ iff $w_r - w_{r+1} \leq \lambda(w_{r+1} - w_{r+2})$, where f is the PSR parameterized with $\boldsymbol{w}$.*

Proof. Observe that the question may be defined equivalently as $q \cdot w_r + p \cdot w_{r+2} \geq (p+q) \cdot w_{r+1}$, where p, q are natural numbers. As a first attempt to make this question concrete, we could define a profile P containing $p + q$ agents in such a way that an alternative a receives q times the rank r and

p times the rank $r+2$, and an alternative b receives $p+q$ times the rank $r+1$. Observing that the score of a in that profile is exactly the left hand side of the question, and that the score of b is the right hand side, intuition suggests that observing whether the chair picks a or b as winner will let us determine which side is greater; equality occurring when the chair declares a and b as tied for the victory. However, we still need to complete the profile, thus, come up with other $m-2$ alternatives defined so that each agent in the resulting profile has placed exactly one alternative in each rank. And we must ensure, doing this, that these other alternatives are not better than a or b: if the chair picks a different alternative c, it tells us that the chair prefers c to both a and b, but it generally does not answer the question we are interested in. Taking $p=q=1, m=4, r=2$, we see that such a completion may be impossible. Luckily, this problem can be worked around, at the price of increasing the number of agents.

First build a temporary profile P of $p+q$ agents with a and b ranked as just described. Complete it with $m-2$ alternatives ranked in arbitrary orders so that the resulting profile has complete strict rankings for each of the $p+q$ agent. Now we will make the other alternatives as bad as desired by adding agents to P that appreciate a and b more than the other alternatives, thus building a profile P' in which, whatever the weights, c may not have a better score than a. Observe that if we add δ agents (with δ a natural number) that put a at first rank and b at second rank, and δ agents that put b at first rank and a at second rank, we do not change the difference of the scores of a and b, and thus, observing the chair choosing a or b still answers the question.

We can prove that, for $\delta=p+q$, and whatever the weight vector $\boldsymbol{w} \in \mathcal{W}$, no alternative, except possibly b, has a better score than a. (In fact, as the construction will exhibit, they are all worst than both a and b, but the weaker fact is enough for our claim.) Pick any alternative c that is not a or b. Define c'_t as the number of times c gets rank t or better in P', and define a'_t similarly. Now $a'_1 \geq \delta, a'_t \geq 2\delta$ for $2 \leq t < m, a'_{r+2} = q+p+2\delta = a'_m$. To obtain upper bounds for c'_t, assume c is ranked first by the $p+q$ agents in P. By our construction, the 2δ new agents of P' give to c the ranks $(3,m)$, or $(4,m-1)$, … Observe that the score of c is maximal for attribution $(3,m)$, by convexity of the weights. Therefore, in the best case for c, $c'_1 \leq p+q, c'_2 \leq p+q, c'_t \leq p+q+\delta$ for $3 \leq t < m, c'_m \leq p+q+2\delta$. Observe that $a'_t \geq c'_t, 1 \leq t \leq m$ (with strict inequality for $t=2$). Because weights are non increasing, this guarantees, with $\delta=p+q$, that the score of a is not lower than the one of c. □

At this stage, we are satisfied that a procedure exists to transform our abstract questions to questions about winners of a PSR. Further studies would investigate, for example, what is lost in terms of elicitation efficiency when we are forced to restrain to realistic concrete questions, meaning, questions involving small profiles; or investigate the relationship between our understanding of scores as (cardinal) utility and (ordinal) scores as definition of a PSR.

Elicitation strategies We develop several elicitation strategies for simultaneous elicitation of agents' preferences and of the scoring rule. While it is of course possible to first fully elicit the agents' preferences and afterwards elicit weights, we also want to propose interleaved approaches and compare them experimentally. Indeed, it can be beneficial to split efforts asking questions to the chair and to agents, depending on which is estimated to be more informative. We define here the various strategies we tested experimentally. A strategy tells us, given the current partial knowledge $(\boldsymbol{p}, W)$, which question should be asked next.

The *Random* strategy gives a baseline for comparison and informs about the difficulty of an elicitation problem. This strategy first decides with a probability of $1/2$ each whether it will ask a question about weights or a question about a preference ordering (unless the profile is already known entirely). If it opted for a question about weights, it draws one rank in $2 \leq r \leq m-2$ equiprobably, takes the middle of the interval of values for λ that are still possible considering our knowledge so far, and asks whether $w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$. The intervals are initialized to $[1,n]$. If it opted for a question to agents, it draws equiprobably among the agents whose preference is not known entirely; picks an alternative a randomly with equal probability among those involved in some incomparabilities in $\succ^{\mathrm{p}}_j$; and picks an alternative b with equal probability among those incomparable with a in $\succ^{\mathrm{p}}_j$.

The *Pessimistic* strategy selects the question that leads to minimal regret in the worst case, considering both possible answers to the question. Assume that a question leads to the possible new knowledge states $(\boldsymbol{p}_1, W_1)$ and $(\boldsymbol{p}_2, W_2)$, depending on the answer. Then the badness of the question in the worst case is: $\max_{i=1,2} \mathrm{MMR}(\boldsymbol{p}_i, W_i)$. This badness measure gives a way of picking questions among a set of possible questions, by picking one that minimizes ths measure of minimax regret *a posteriori*. However, to avoid the absorption property of max, we adopt the $\mathrm{leximax}$ criterion as an aggregator: if the maximal MMR of two questions are equal, then it considers the other MMR values, that are the one associated to the opposite answer, preferring the question with the lowest value. We expect this strategy to perform very well, but only for very small problem instances: its complexity is in $O(n^2m^5)$, because we consider $O(m^2)$ questions for each agent and need for each question to compute MMR twice, whose complexity is $O(nm^3)$. For small dimensions, it estimates an upper bound to the possible quality of a strategy.

The *Limited pessimistic* strategy uses the same criterion as the pessimistic strategy, but limiting it to a small set of $n+1$ candidate questions: one per agent, and one to the chair. It uses the heuristic proposed by Lu and Boutilier [2011] to define the candidate questions to the agents. Considering agent j, either $x^* \succ^{\mathrm{p}}_j \bar{y}$, or $\bar{y} \succ^{\mathrm{p}}_j x^*$, or x^* and $\bar{y}$ are incomparable in $\succ^{\mathrm{p}}_j$. As shown in the proof of Claim 1, in order to increase $\mathrm{PMR}(x^*, \bar{y})$, an adversary should place as many alternatives as he can above x^* in the first case, and between $\bar{y}$ and x^* in the second. An intuitive way to reduce the pairwise max regret is, therefore, to directly ask questions involving such alternatives, hoping for an answer that would prevent the adversary to play his best game. If x^* and $\bar{y}$ are incomparable, then we just ask the agent j to compare them. To define the candidate question to the chair,

k	Rnd ± sd	Pes. ± sd	L. pes. ± sd
0	5.0 ± 0	5.0 ± 0	5.0 ± 0
5	5.0 ± 0.1	3.7 ± 0.0	4.4 ± 0.6
10	4.7 ± 0.4	3.3 ± 0.4	3.3 ± 0.4
15	4.4 ± 0.5	2.7 ± 0.4	2.7 ± 0.7
20	3.7 ± 0.5	1.5 ± 0.4	2.1 ± 0.7
25	3.1 ± 0.7	1.4 ± 0.5	0.9 ± 0.6
30	2.6 ± 0.5	0.4 ± 0.4	0.5 ± 0.4

Table 1: Minimax regret in problems of size $(5, 5)$ after k questions.

k	Rnd ± sd	L. pes. ± sd
0	20.0 ± 0	20.0 ± 0
20	20.0 ± 0.0	18.8 ± 0.1
40	19.9 ± 0.1	17.7 ± 0.3
60	19.8 ± 0.2	17.0 ± 0.5
80	19.4 ± 0.2	15.7 ± 0.4
100	18.9 ± 0.4	14.6 ± 0.7

Table 2: Regret in problems of size $(10, 20)$ after k questions.

consider $\boldsymbol{w}^\tau = \mathrm{argmin}_{\boldsymbol{w} \in W}\, s(\bar{y}; \bar{\boldsymbol{v}}, \boldsymbol{w}) - s(x^*; \bar{\boldsymbol{v}}, \boldsymbol{w})$, the weight vector that minimize the PMR in the worst case. We compare $\bar{\boldsymbol{w}}$ and $\boldsymbol{w}^\tau$ component-wise to find the position r that is most valuable to ask about (the one that maximizes $|\bar{\boldsymbol{w}}_r - \boldsymbol{w}^\tau_r|$), and choose λ as the middle of the interval of possible values for that rank. We then evaluate, for each candidate question, the minimax regret for the two possible answers; finally we choose the best according to $\mathrm{leximax}$.

Lastly, the *Two phases* strategy is developed in order to investigate the effect of varying the proportion of questions asked to agents and to the chair. It first asks p questions to the chair, then $k - p$ questions to the agents, using in both cases the same method as *Limited pessimistic* to select the specific question. Note that when asking first only questions to the chair, if the obtained knowledge approximates well the scoring vector, then in the second part of the elicitation we fall into the more classical setting of incompleteness of preferences assuming a known the voting rule.

5 Empirical Evaluation

We test our strategies using randomly generated datasets. Our first goal is to see, with a small problem size ($m = 5, n = 5$) and the (time consuming) Pessimistic strategy, if an important lowering of the maximal regret can be achieved with a reasonable number of questions. We also want to estimate how "hard" such a problem is, by using the Random strategy as a baseline. Third, we want to estimate the loss (in terms of worst regret) when switching to the faster Limited pessimistic strategy. Fourth, we want to see, on a bigger problem size ($m = 10, n = 20$), how many questions must be asked for that strategy to achieve a significant reduction of the worst regret. Finally, we want to evaluate, thanks to the Two phases strategy, the impact of varying the proportion of questions asked to the agents (with respect to questions asked to the chair) on the reduction of the worst regret.

p	Two ph. ± sd
0	20.0 ± 0
25	18.9 ± 0.1
50	17.3 ± 0.5
75	15.8 ± 0.6
100	14.7 ± 0.6
125	13.0 ± 0.5
150	12.0 ± 0.5
175	10.0 ± 0.6
200	11.5 ± 0.6

Table 3: Regret in problems of size $(10, 20)$ after 200 questions.

We use the following protocol. Having picked a problem size (m, n), a number of questions k and a strategy, we randomly generate an "oracle", containing the true preferences of the agents (i.e. the linear orders) and the weights associated with the chair's scoring rule. We start with empty knowledge ($\boldsymbol{p} = (\emptyset_j), W = \mathcal{W}$) about the preference orderings of the agents or the weight differences favored by the chair. We obtain the first question to be asked using the selected strategy, as described above. We use the oracle to answer the question and update our knowledge, which is thus used to obtain the next question. This is repeated until k answers have been obtained. Then we compute the resulting MMR. We repeat this whole experiment 10 times, for a given (m, n, k), and report the average resulting MMR and the standard deviation.

The results are displayed in tables 1 to 3, where strategies are designated by Rnd for Random, Pes. for Pessimistic, L. pes. for Limited pessimistic and 2 ph. for Two phases.

We observe that the Pessimistic strategy is able to reduce the regret almost completely after 30 questions. We also see that Limited pessimistic performs almost as well as Pessimistic; this is good news since (for $m = 5$ and $n = 5$), the former strategy is much faster and takes only $2.6s$ for a complete elicitation session, while the latter takes $16s$.

6 Conclusions

In this paper we have considered a social choice setting with partial information about the agent's preferences and a partially specified voting rule. In this setting, we have proposed the use of minimax regret both as a means of robust winner determination as well as a guide to the process of simultaneous elicitation of preferences and voting rule. Our experimental results suggest that regret-based elicitation is effective and allow to quickly reduce worst regret, but they also show that starting with zero knowledge became a pretentious approach really quickly when increasing the size of the problem. Future works will therefore include the test of these strategies on partial specified profiles, ideally on real datasets. An other important direction is the extension to voting rules beyond scoring rules. As part of the contribution of this work, we publish an open-source library that allows to reproduce our experiments, and more (url not displayed for anonymity reasons).

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