Elicitation and explanation for voting rules

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Pré-soutenance de thèse
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Plan of the thesis and questions

- Final dissertation by October 2021, defense by December 2021
- Status of the works:
  - Compromise: Submitted to Social Choice and Welfare but rejected; under submission to Review of Economic Design;
  - Elicitation PSR: Submitted to IJCAI20, AAMAS21 and IJCAI21 but rejected; under revision at ADT21;
  - Elicitation MJ: ongoing work, plan to have a final draft before the defense.
- Given the current status of my works, is the plan feasible?
- Any suggestions on the dissertation structure ?

Outline

1. Notation
2. Compromising as an equal loss principle
3. Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination
4. Preference Elicitation under Majority Judgment

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Context

$\mathcal{A}$	alternatives, $\lvert\mathcal{A}\rvert = m$
N	voters, $\lvert N\rvert = n$
$\mathcal{L}(\mathcal{A})$, $\succ_i \in \mathcal{L}(\mathcal{A})$	a linear order over $\mathcal{A}$
$P \in \mathcal{L}(\mathcal{A})^N$	a profile
$\mathscr{P}^*(\mathcal{A})$	the possible winners (the non-empty subsets of $\mathcal{A}$)
$f : \mathcal{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$	a SCR

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Introducing the problem

Setting: Several voters express their preferences over a set of alternatives

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Goal: Find a procedure determining a collective choice that promotes a notion of compromise

Context

Related Works

- **Plurality**: selects the alternatives considered as best by the highest number of voters

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- **Fallback Bargaining**: bargainers fall back to less and less preferred alternatives until they reach a unanimous agreement
- **q-approval FB**: picks the alternatives which receive the support of q voters at the highest possible quality, breaking ties according to the quantity of support

Context

Motivation: A simple example

$n = 100, \mathcal{A} = \{a, b, c\}$

51	a	b	c
49	c	b	a

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Does b seem a better compromise?

Notation

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

Notation

Losses

P	λ_P
$v_1 : a \succ b \succ c$	$a : (0, 2)$
$v_2 : c \succ b \succ a$	$b : (1, 1)$
	$c : (2, 0)$

Given $P = (\succ_i)_{i \in N}$:

- $\lambda_{\succ_i}(x) = |\{y \in \mathcal{A} \mid y \succ_i x\}| \in [\![0, m-1]\!]$ the loss of i when choosing $x \in \mathcal{A}$ instead of her favorite alternative
- $\lambda_P(x)$ associates to each voter her loss when choosing x

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Σ is the set of spread measures σ such that

$$\sigma(l) = 0 \iff l_i = l_j,\ \forall i, j \in N, \quad \forall l \in [\![0, m-1]\!]^N$$

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Classical Scenario

Setting: Voters specify preferences over alternatives and a committee defines the social choice rule to aggregate them

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Our Scenario

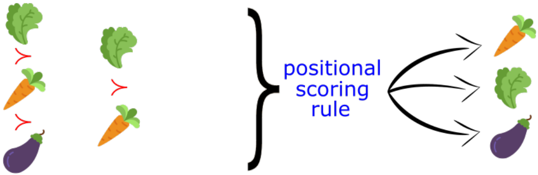
Setting: Incompletely specified preferences and social choice rule

(Head of the)
Committee

$w_1 \geq 2\, w_2$

Voters

Mickey Donald Goofy



Our Scenario

Setting: Incompletely specified preferences and social choice rule

Goal: Develop an incremental elicitation strategy to acquire the most relevant information

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

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What?

- We want to reduce uncertainty, inferring (*eliciting*) the true preferences of voters and committee, in order to quickly converge to an optimal or a near-optimal alternative

Motivation and approach

Approach

- Develop query strategies that interleave questions to the committee and questions to the voters
- Use *Minimax regret* to measure the quality of those strategies

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Assumptions

- We consider *positional scoring rules*, which attach weights to positions according to a scoring vector w
- We assume w to be *convex*

$$w_r - w_{r+1} \geq w_{r+1} - w_{r+2} \qquad \forall r$$

and that $w_1 = 1$ and $w_m = 0$

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

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A	alternatives, $\lvert A \rvert = m$
N	voters
$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$	complete preferences profile
$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$	(convex) scoring vector that the committee has in mind

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P and W exist in the minds of voters and committee but unknown to us

Questions

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Questions to the voters

Comparison queries that ask a particular voter to compare two alternatives $a, b \in A$

$$a \succ_j b \quad ?$$

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Questions to the committee

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2}) \quad ?$$

Our Knowledge

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- $C_W \subseteq \mathcal{W}$ constraints on the voting rule given by the committee

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b

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$$\mathrm{MR}^{C_P, C_W}(a) = \max_{b \in A} \mathrm{PMR}^{C_P, C_W}(a, b)$$

We select the alternative which minimizes the maximal regret

$$\mathrm{MMR}^{C_P, C_W} = \min_{a \in A} \mathrm{MR}^{C_P, C_W}(a)$$

Pairwise Max Regret Computation

The computation of PMR^{C_P,C_W}(🥬, 🍆) can be seen as a game in which an adversary both:

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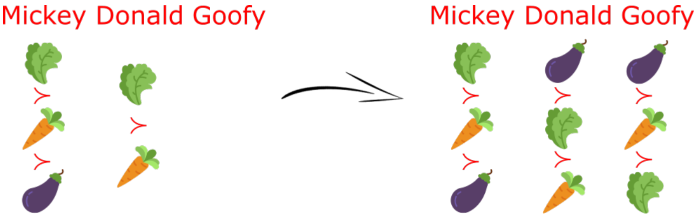

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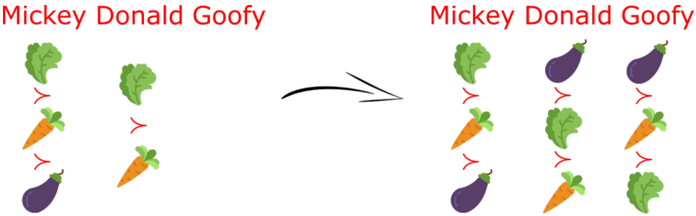


- **chooses a feasible weight vector** $\mathrm{W} \in \mathcal{W}$

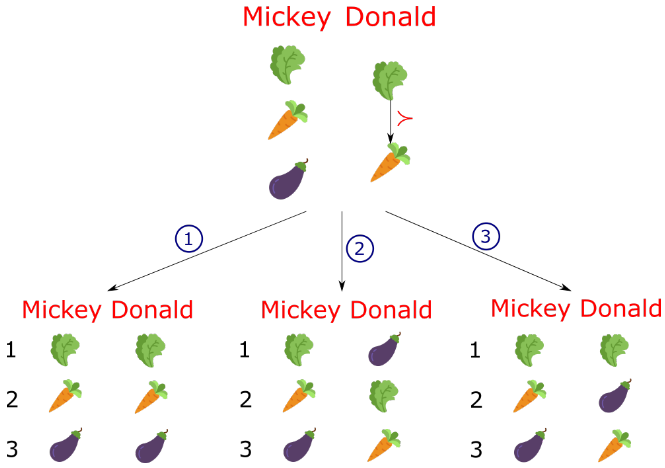
$$(1, ?, 0) \longrightarrow (1, 0, 0)$$

in order to maximize the difference of scores

Computing Minimax Regret: Example

Profile completion

Consider the following partial profile



Computing Minimax Regret: Example

Weight selection

Consider the following constraint on the scoring vector given by the committee

$$w_1 \geq 2 \cdot w_2$$

and the convex assumption

$$w_1 - w_2 \geq w_2 - w_3$$

Computing Minimax Regret: Example

Minimax computing

MR() = max {

PMR( ) $\xrightarrow{v = 3 \quad w=\{1,0,0\}}$ = 2

PMR( ) $\xrightarrow{v = 2 \quad w=\{1,0,0\}}$ = 1

MR() = $\boxed{0}$

MR() = 2

$\boxed{\text{MMR}}$ = 0 → winner 

Elicitation strategies

At each step, the strategy selects a question to ask either to one of the voters about her preferences or to the committee about the voting rule

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Elicitation strategies

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- **weights**: it draws a rank $2 \leq r \leq m-2$ equiprobably, takes λ as the middle of the interval of values we are still uncertain about, and asks whether $w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$

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- **a preference ordering**: it draws equiprobably a voter whose preference is not known entirely and two alternatives that are incomparable in our current knowledge

Elicitation strategies

Pessimistic Strategy

Elicitation strategies

Pessimistic Strategy

Assume that a question leads to the possible new knowledge states (C_P^1, C_W^1) and (C_P^2, C_W^2) depending on the answer, then the badness of the question in the worst case is:

$$\max_{i=1,2} \mathrm{MMR}(C_P^i, C_W^i)$$

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Note:

if the maximal MMR of two questions are equal, then prefers the one with the lowest MMR values associated to the opposite answer

Elicitation strategies

Two Phase Strategy

Elicitation strategies

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- **phase one:** asks a predefined sequence of $m - 2$ questions to the committee in order to gather informations about the weights (one per rank except the extremes)

Elicitation strategies

Two Phase Strategy

- **phase one:** asks a predefined sequence of $m-2$ questions to the committee in order to gather informations about the weights (one per rank except the extremes)
- **phase two:** asks only questions to the voters, using the mechanism defined in the Pessimistic strategy

Elicitation strategies

Extreme Completions Strategy

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Let a^* be the minimax regret optimal alternative, and $\bar{b}$, $\bar{P}$ and $\bar{W}$ the instantiations that maximize the regret when a^* is chosen.

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Let a^* be the minimax regret optimal alternative, and $\bar{b}$, $\bar{P}$ and $\bar{W}$ the instantiations that maximize the regret when a^* is chosen. Define:

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$$\tau_{P_i} = \min_{\hat{P}_i \in C_P} s^{\hat{P}_i,\bar{W}}(\bar{b}) - s^{\hat{P}_i,\bar{W}}(a^*)$$

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MMR $-\tau_W$ estimates the contribution to the regret of our uncertainty about the weights; MMR $-\tau_{P_i}$ estimates the uncertainty about the profile

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The extreme completions strategy asks a question to whoever minimizes τ

Results

k	Rnd$\pm$ sd	T. ph.$\pm$ sd	Pes.$\pm$ sd
0	5.0$\pm$0	5.0$\pm$0	5.0$\pm$0
5	4.9$\pm$0.2	5.0$\pm$0.0	4.2$\pm$0.3
10	4.8$\pm$0.2	4.4$\pm$0.3	3.4$\pm$0.5
15	4.3$\pm$0.6	3.7$\pm$0.3	2.7$\pm$0.5
20	3.9$\pm$0.3	2.7$\pm$0.5	1.6$\pm$0.8
25	3.5$\pm$0.8	2.2$\pm$0.7	0.8$\pm$0.6
30	3.0$\pm$0.8	1.5$\pm$1.1	0.5$\pm$0.7

Table: Minimax regret in problems of size $(5, 5)$ after k questions.

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Thank You!

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