Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

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Introducing the problem

Agents = { , ,  }, Alternatives = { , ,  }, Chair =  $\Rightarrow$ Positional Scoring Rule

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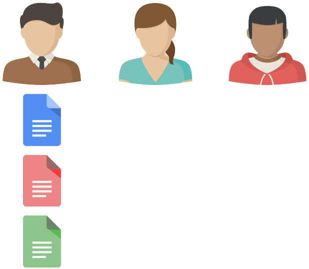

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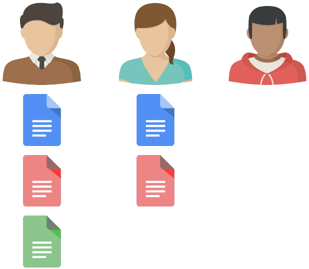

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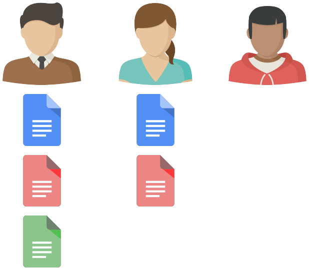

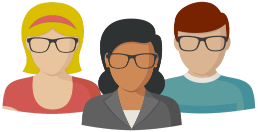

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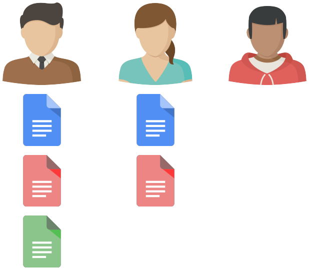

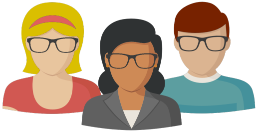

$\text{weight}(1^{\text{st}}) \geq 2 \cdot \text{weight}(2^{\text{nd}})$

Introducing the problem

Setting: Incompletely specified preferences and social choice rule

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- Agents: difficult or costly to order *all* alternatives
- Chair: difficult to *specify* a voting rule precisely

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Goal: Reduce uncertainty, inferring (*eliciting*) incrementally and simultaneously the true preferences of agents and chair to quickly converge to an optimal or a near-optimal alternative

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Approach:

- Develop query strategies that interleave questions to the chair and to the agents
- Use *Minimax regret* to measure the quality of those strategies

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

Assumptions

- We consider *Positional Scoring Rules*, which attach weights to positions according to a scoring vector W
- We assume W to be *convex*

$$W_r - W_{r+1} \geq W_{r+1} - W_{r+2}$$

for all positions r, and that $W_1 = 1$ and $W_m = 0$

Notation

A alternatives, $|A| = m$

N agents (*voters*)

$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$ complete preferences profile

$W = (W_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$ (convex) scoring vector that the chair has in mind

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P and W exist in the minds of agents and chair but unknown to us

Questions

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Questions to the agents

Comparison queries that ask a particular agent j to compare two alternatives $a, b \in \mathcal{A}$

$$a \succ_j b \quad ?$$

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Questions to the chair

Queries relating the difference between the importance of consecutive ranks from r to $r + 2$

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2}) \quad ?$$

Current Knowledge

The answers to these questions define C_P and C_W that is our knowledge about P and W

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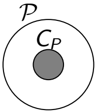

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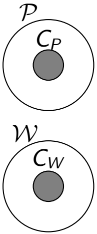
- $C_W \subseteq \mathcal{W}$ constraints on the voting rule given by the chair



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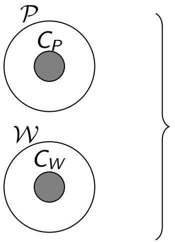

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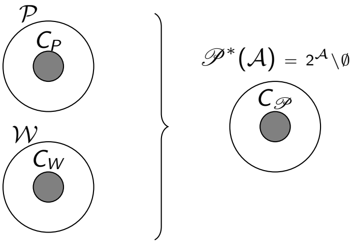

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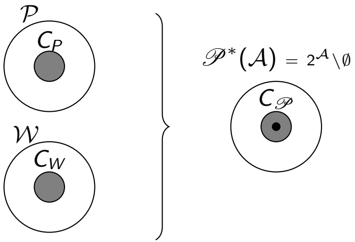

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P, C_W}(a, b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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We select the alternative which *minimizes* the maximal regret

$$\mathrm{MMR}^{C_P, C_W} = \min_{a \in \mathcal{A}} \mathrm{MR}^{C_P, C_W}(a)$$

Pairwise Max Regret Computation

The computation of $\mathrm{PMR}^{C_P, C_W}($$,$ $)$ can be seen as a game in which an adversary both:

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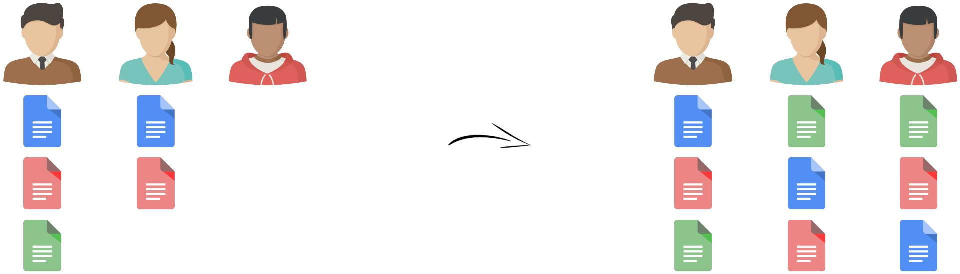

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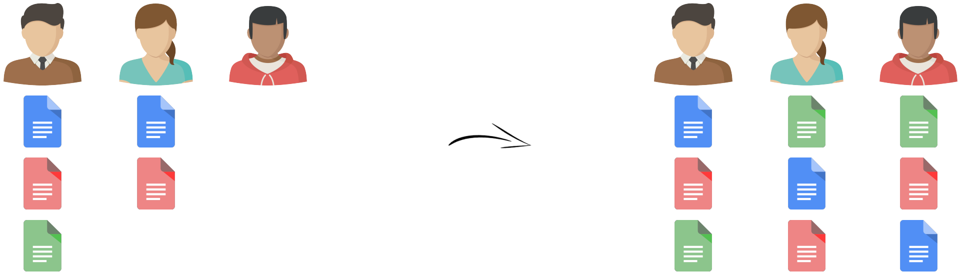

- **chooses a feasible weight vector W** $\in \mathcal{W}$

$$\mathbf{(1, ?, 0)} \longrightarrow \mathbf{(1, 0, 0)}$$

in order to maximize the difference of scores

Elicitation strategies

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The termination condition could be:

- when the minimax regret is lower than a threshold
- when the minimax regret is zero

Assume that a question leads to the possible new knowledge states (C_P^1, C_W^1) and (C_P^2, C_W^2) depending on the answer, then the badness of the question in the worst case is:

$$\max_{i=1,2} \mathrm{MMR}(C_P^i, C_W^i)$$

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Pessimistic Strategy

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Note:

if the maximal MMR of two questions are equal, then prefers the one with the lowest MMR values associated to the opposite answer

Let $(a^*, \bar{b}, \bar{P}, \bar{W})$ be the current solution of the minimax regret

We select $n+1$ candidate questions:

- **One question per agent:** For each agent i, either:
  - $a^* \succ_j^{\bar{P}} \bar{b}$: we ask about an incomparable alternative that can be placed above a^* by the adversary to increase PMR(a^*,$\bar{b}$)
  - $\bar{b} \succ_j^{\bar{P}} a^*$: we ask about an incomparable alternative that can be placed between a^* and $\bar{b}$ by the adversary to increase PMR(a^*,$\bar{b}$)
  - a^* and $\bar{b}$ are incomparable: we ask to compare them
- **One question to the chair:** Consider W_τ the weight vector that minimize the PMR in the worst case.
  We ask about the position $r = \underset{i=[\![1,m-1]\!]}{\arg\max} |\bar{W}(i) - W_\tau(i)|$

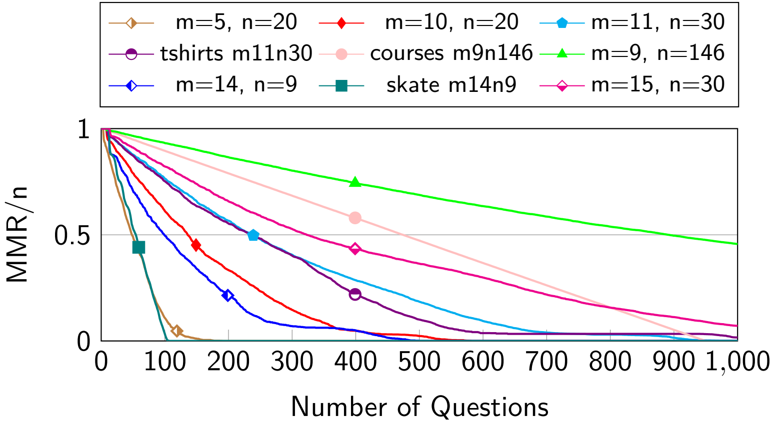

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets.

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \leq n/10}$	$q_a^{MMR \leq n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 | 5.0 | 5.8]	5.3	[5.4 | 6.2 | 7.2]
m10n20	10	20	0.0	[13.9 | 16.1 | 18.4]	32.0	[19.7 | 21.8 | 24.7]
m11n30	11	30	0.0	[16.6 | 19.0 | 22.3]	45.2	[23.1 | 25.7 | 28.9]
tshirts	11	30	0.0	[13.1 | 16.6 | 19.6]	43.2	[28.2 | 32.0 | 35.6]
courses	9	146	0.0	[6.0 | 7.0 | 7.0]	0.0	[6.8 | 7.0 | 7.0]
m14n9	14	9	5.4	[30.3 | 33.5 | 36.7]	64.1	[37.6 | 40.5 | 44.3]
skate	14	9	0.0	[11.4 | 11.6 | 12.3]	0.0	[11.5 | 11.8 | 12.8]
m15n30	15	30	0.0	[25.0 | 29.5 | 33.7]		

Empirical Evaluation

Pessimistic chair first and then agents (and vice-versa)

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	2 ph. ca $\pm$ sd	2 ph. ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0

Thank You!

Icon made by Eucalyp from www.flaticon.com

Craig Boutilier, Relu Patrascu, Pascal Poupart, and Dale Schuurmans.
Constraint-based optimization and utility elicitation using the minimax decision criterion.
Artificial Intelligence, 2006.

Tyler Lu and Craig Boutilier.
Robust approximation and incremental elicitation in voting protocols.
In *Proc. of IJCAI'11*, 2011.

William E. Stein, Philip J. Mizzi, and Roger C. Pfaffenberger.
A stochastic dominance analysis of ranked voting systems with scoring.
EJOR, 1994.

Paolo Viappiani.
Positional scoring rules with uncertain weights.
In *Scalable Uncertainty Management*, 2018.

Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2}) \quad ?$$

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$$\begin{aligned} W_2 \;-\; W_3 &\geq 2\;W_3 - 2\;W_4 \\ W_2 + 2\;W_4 &\geq 3\;W_3 \end{aligned}$$

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$$s(a) \geq s(b)$$

#1	**#2**
–	–
a	–
b	**b**
–	**a**

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#**1**	#**2**	#**3**	#**3**
c	*d*	*a*	*b*
a	*c*	*b*	*a*
b	**b**	*c*	*d*
d	**a**	*d*	*c*