Elicitation and explanation for voting rules

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Outline

1. Notation
2. Compromising as an equal loss principle
3. Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination
4. Setting
5. Preference Elicitation under Majority Judgment

Outline

Context

$\mathcal{A}$ alternatives, $|\mathcal{A}| = m$

N voters, $|N| = n$

$\mathcal{L}(\mathcal{A})$, $\succ_i \in \mathcal{L}(\mathcal{A})$ a linear order over $\mathcal{A}$

$P \in \mathcal{L}(\mathcal{A})^N$ a profile

$\mathscr{P}^*(\mathcal{A})$ the possible winners (the non-empty subsets of $\mathcal{A}$)

$f : \mathcal{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$ a SCR

Outline

Context

Introducing the problem

Setting: Several voters express their preferences over a set of alternatives

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Goal: Find a procedure determining a collective choice that promotes a notion of compromise

Context
Related Works

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- **Fallback Bargaining**: bargainers fall back to less and less preferred alternatives until they reach a unanimous agreement
- **q-approval FB**: picks the alternatives which receive the support of q voters at the highest possible quality, breaking ties according to the quantity of support

Context

Motivation: A simple example

$n = 100, \mathcal{A} = \{a, b, c\}$

51	a	b	c
49	c	b	a

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Does b seem a better compromise?

Notation

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

Notation

Losses

$$P \qquad\qquad \lambda_P$$

$$\begin{array}{ll} v_1 : a \succ b \succ c & a : (0,2) \\ v_2 : c \succ b \succ a & b : (1,1) \\ & c : (2,0) \end{array}$$

Given $P = (\succ_i)_{i \in N}$:

- $\lambda_{\succ_i}(x) = |\{y \in \mathcal{A} \mid y \succ_i x\}| \in [\![0, m-1]\!]$ the loss of i when choosing $x \in \mathcal{A}$ instead of her favorite alternative
- $\lambda_P(x)$ associates to each voter her loss when choosing x

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Σ is the set of spread measures σ such that

$$\sigma(l) = 0 \iff l_i = l_j,\ \forall i, j \in N, \quad \forall l \in [\![0, m-1]\!]^N$$

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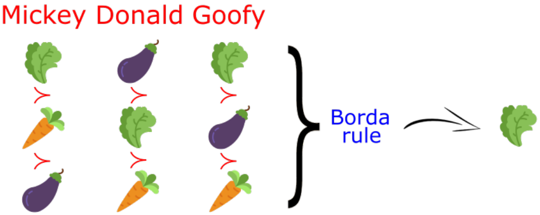
(Head of the)
Committee

$w = (w_1, w_2, w_3)$
$= (2, 1, 0)$

Voters



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Setting: Voters specify preferences over alternatives and a committee defines the social choice rule to aggregate them

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Our Scenario

Setting: Incompletely specified preferences and social choice rule

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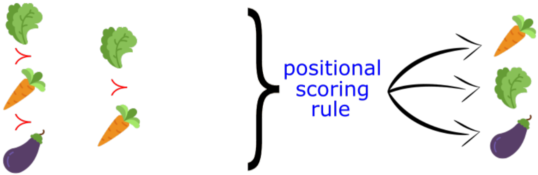
$w_1 \geq 2\, w_2$

Voters

Mickey Donald Goofy



Our Scenario

Setting: Incompletely specified preferences and social choice rule

Goal: Develop an incremental elicitation strategy to acquire the most relevant information

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

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What?

- We want to reduce uncertainty, inferring (*eliciting*) the true preferences of voters and committee, in order to quickly converge to an optimal or a near-optimal alternative

Motivation and approach

Approach

- Develop query strategies that interleave questions to the committee and questions to the voters
- Use *Minimax regret* to measure the quality of those strategies

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Assumptions

- We consider *positional scoring rules*, which attach weights to positions according to a scoring vector w
- We assume w to be *convex*

$$w_r - w_{r+1} \geq w_{r+1} - w_{r+2} \qquad \forall r$$

and that $w_1 = 1$ and $w_m = 0$

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

Context

A alternatives, $|A| = m$

N voters

$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$ complete preferences profile

$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$ (convex) scoring vector that the committee has in mind

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P and W exist in the minds of voters and committee but unknown to us

Questions

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Comparison queries that ask a particular voter to compare two alternatives $a, b \in A$

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Questions to the committee

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2}) \quad ?$$

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- $C_W \subseteq \mathcal{W}$ constraints on the voting rule given by the committee

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b

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$$\mathrm{MR}^{C_P,C_W}(a) = \max_{b \in A} \mathrm{PMR}^{C_P,C_W}(a,b)$$

We select the alternative which minimizes the maximal regret

$$\mathrm{MMR}^{C_P,C_W} = \min_{a \in A} \mathrm{MR}^{C_P,C_W}(a)$$

Pairwise Max Regret Computation

The computation of $\mathrm{PMR}^{C_P, C_W}($🥬, 🍆$)$ can be seen as a game in which an adversary both:

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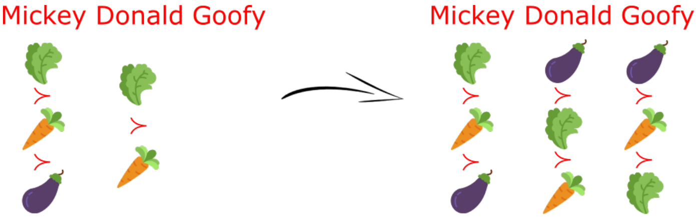

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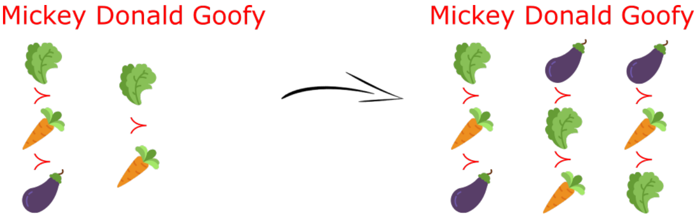


- **chooses a feasible weight vector** $\mathrm{W} \in \mathcal{W}$

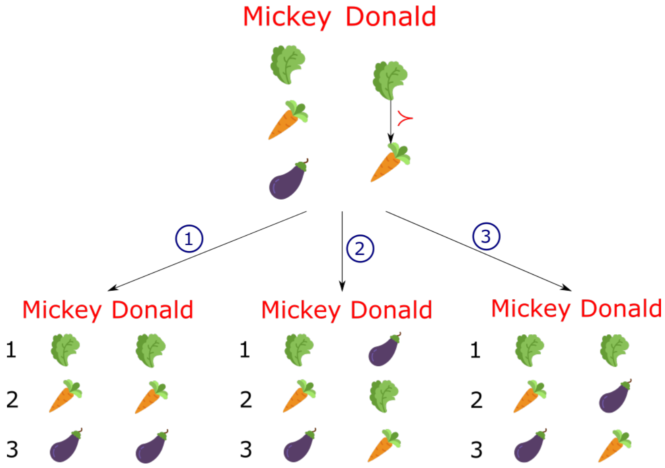
$$(1,?,0) \longrightarrow (1,0,0)$$

in order to maximize the difference of scores

Computing Minimax Regret: Example

Profile completion

Consider the following partial profile



Computing Minimax Regret: Example

Weight selection

Consider the following constraint on the scoring vector given by the committee

$$w_1 \geq 2 \cdot w_2$$

and the convex assumption

$$w_1 - w_2 \geq w_2 - w_3$$

Computing Minimax Regret: Example

Minimax computing

MR() = max { PMR( ) $\xrightarrow{v = ③ \quad w=\{1,0,0\}}$ = 2 ; PMR( ) $\xrightarrow{v = ② \quad w=\{1,0,0\}}$ = 1 }

MR() = $\boxed{0}$

MR() = 2

$\boxed{\text{MMR}}$ = 0 → winner 

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- when the minimax regret is zero

Elicitation strategies

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- **weights**: it draws a rank $2 \leq r \leq m-2$ equiprobably, takes λ as the middle of the interval of values we are still uncertain about, and asks whether $w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$

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- **a preference ordering**: it draws equiprobably a voter whose preference is not known entirely and two alternatives that are incomparable in our current knowledge

Elicitation strategies

Pessimistic Strategy

Elicitation strategies

Pessimistic Strategy

Assume that a question leads to the possible new knowledge states (C_P^1, C_W^1) and (C_P^2, C_W^2) depending on the answer, then the badness of the question in the worst case is:

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Note:

if the maximal MMR of two questions are equal, then prefers the one with the lowest MMR values associated to the opposite answer

Elicitation strategies

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- **phase one:** asks a predefined sequence of $m-2$ questions to the committee in order to gather informations about the weights (one per rank except the extremes)

Elicitation strategies

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- **phase one:** asks a predefined sequence of $m-2$ questions to the committee in order to gather informations about the weights (one per rank except the extremes)
- **phase two:** asks only questions to the voters, using the mechanism defined in the Pessimistic strategy

Elicitation strategies

Extreme Completions Strategy

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Let a^* be the minimax regret optimal alternative, and $\bar{b}$, $\bar{P}$ and $\bar{W}$ the instantiations that maximize the regret when a^* is chosen.

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$$\tau_W = \min_{W \in C_W} s^{\bar{P},W}(\bar{b}) - s^{\bar{P},W}(a^*)$$

$$\tau_{P_i} = \min_{\hat{P}_i \in C_P} s^{\hat{P}_i,\bar{W}}(\bar{b}) - s^{\hat{P}_i,\bar{W}}(a^*)$$

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The extreme completions strategy asks a question to whoever minimizes τ

Results

k	Rnd$\pm$ sd	T. ph.$\pm$ sd	Pes.$\pm$ sd
0	5.0$\pm$0	5.0$\pm$0	5.0$\pm$0
5	4.9$\pm$0.2	5.0$\pm$0.0	4.2$\pm$0.3
10	4.8$\pm$0.2	4.4$\pm$0.3	3.4$\pm$0.5
15	4.3$\pm$0.6	3.7$\pm$0.3	2.7$\pm$0.5
20	3.9$\pm$0.3	2.7$\pm$0.5	1.6$\pm$0.8
25	3.5$\pm$0.8	2.2$\pm$0.7	0.8$\pm$0.6
30	3.0$\pm$0.8	1.5$\pm$1.1	0.5$\pm$0.7

Table: Minimax regret in problems of size $(5, 5)$ after k questions.

Outline

Thank You!

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