Compromise and elicitation in social choice

A study of egalitarianism and incomplete information in voting

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Classical setting

Alternatives $=$ { , ,  } $\xleftarrow{\textit{prefers}}$ Agents $=$ { , ,  } Chair $=$  $\Rightarrow$ Voting Rule

Classical setting

Alternatives $= \{ \; , \; , \; \} \xleftarrow{\textit{prefers}}$ Agents $= \{ \; , \; , \; \}$ Chair $=$ $\Rightarrow$ Voting Rule

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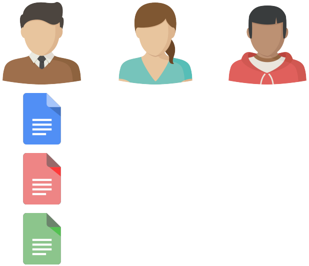

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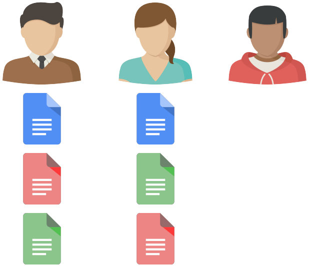

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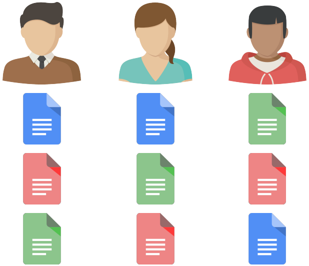

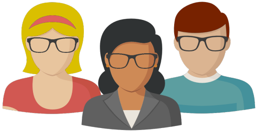

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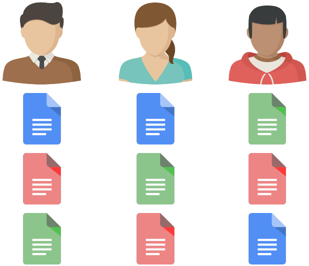
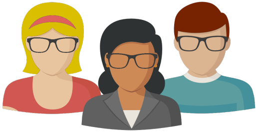
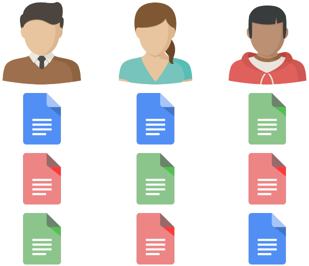


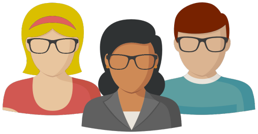


Borda

Classical setting

Alternatives = {  } $\xleftarrow{\textit{prefers}}$ Agents = {  } Chair =  $\Rightarrow$ Voting Rule



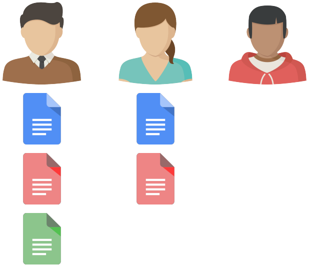


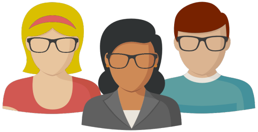
Borda

winner: 

Incomplete knowledge about profile

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Voting Rule





Borda

winner: ?

Incomplete knowledge about voting rule

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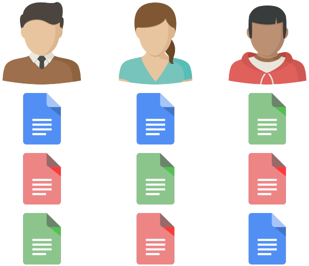
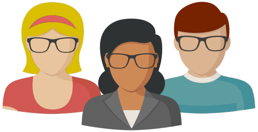


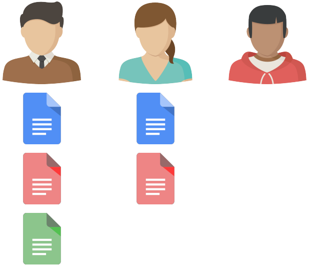


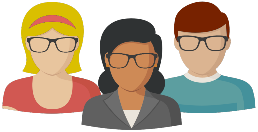
?

winner: ?

Research Question I:
Incomplete knowledge about profile and voting rule

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Voting Rule



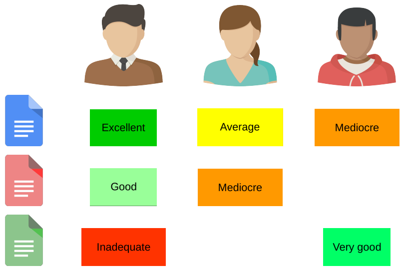


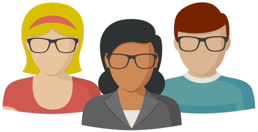
?

winner: ?

Research Question II:
Incomplete knowledge under Majority Judgment

Alternatives $= \{$, , $\}$ $\xleftarrow{\textit{prefers}}$ Agents $= \{$, , $\}$ Chair $=$ $\Rightarrow$ Voting Rule



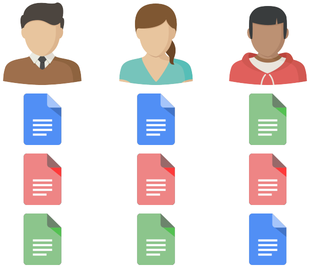


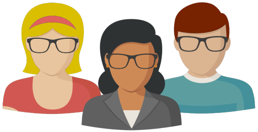
Majority Judgment

winner: ?

Research Question III:
Compromise from an equal-loss perspective

Alternatives $= \{\ ,\ ,\ \} \xleftarrow{\textit{prefers}}$ Agents $= \{\ ,\ ,\ \}$ Chair $=$ $\Rightarrow$ Voting Rule





What is a compromise?

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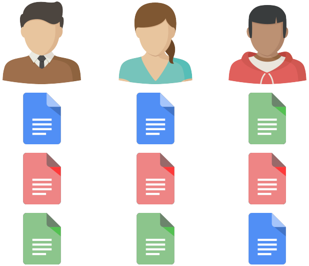
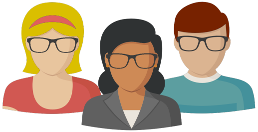
Compromise from an equal-loss perspective

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = ⇒ Voting Rule





What is a compromise?

maybe  ?

Outline

Outline

Notation

$\mathcal{A}$	set of alternatives, $\lvert\mathcal{A}\rvert = m$
N	set of voters, $\lvert N\rvert = n$
$\mathcal{L}(\mathcal{A})$	set of all linear orderings given $\mathcal{A}$
$\succ_i \in \mathcal{L}(\mathcal{A})$	preference ranking of voter $i \in N$
$P = (\succ_1, \ldots, \succ_n) \in \mathcal{L}(\mathcal{A})^N$	a profile
$\mathscr{P}^*(\mathcal{A})$	possible winners (non-empty subsets of $\mathcal{A}$)
$f : \mathcal{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$	a Social Choice Rule

Outline

Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

Setting: Incompletely specified preferences and social choice rule

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Goal: Reduce uncertainty

- eliciting pref. of agents and chair incrementally and simultaneously
- quickly converge to an optimal or a near-optimal alternative

Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

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Approach:

Napolitano, B., Cailloux, O., and Viappiani, P. (2021). Simultaneous elicitation of scoring rule and agent preferences for robust winner determination.
In *Proceedings of Algorithmic Decision Theory - 7th International Conference, ADT 2021*

- Develop strategies interleaving questions to the chair and agents
- Use *Minimax regret* to measure the quality of those strategies

Related Works

Incomplete profile

- and known rule: Minimax regret to produce a robust winner approximation [Lu and Boutilier, 2011, Boutilier et al., 2006]

Uncertain rule

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others [Stein et al., 1994]
- considering positional scoring rules [Viappiani, 2018]

Context

$P = (\succ_i,\ i \in N) \in \mathcal{P}$ complete preferences profile

$W = (W_r,\ 1 \le r \le m) \in \mathcal{W}$ **convex** scoring vector

P and W exist in the minds of agents and chair but unknown to us

W defines a **Positional Scoring Rule** $f_W(P) \subseteq A$

Questions

Questions to the agents

Comparison queries that ask a particular agent to compare two alternatives in $\mathcal{A}$

 :  $\succ$  ?

Questions to the chair

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

 : $W_{r_1} \rightarrow W_{r_2} > W_{r_2} \rightarrow W_{r_3}$?

The answers to these questions define $\mathbf{C_P}$ and $\mathbf{C_W}$ that is our knowledge about P and W

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

The *Maximum Regret* MR of an alternative a is the highest possible loss when selecting a as a winner under all possible completions of C_P and C_W

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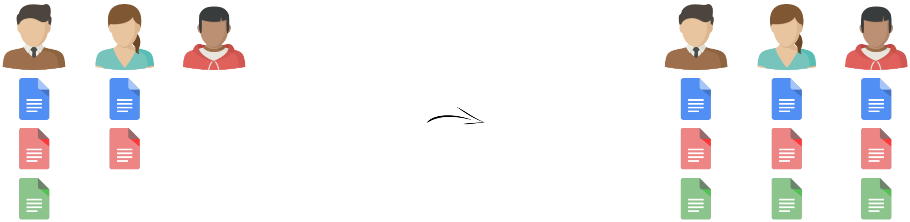


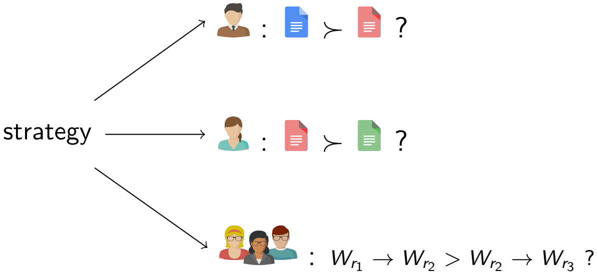
Minimax Regret

The alternative that currently minimizes the maximum regret is used:

- as winner recommendation when the elicitation process stops
- to guide elicitation strategies

Elicitation strategies

At each step, the strategy selects a question to ask either to one of the agents about her preferences or to the chair about the voting rule



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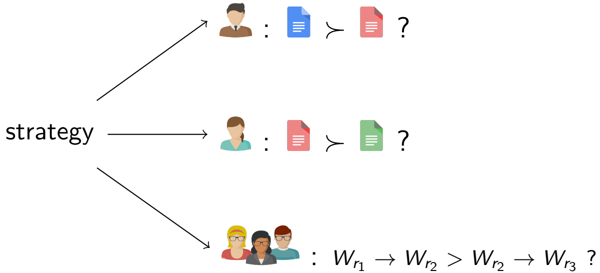


Termination:

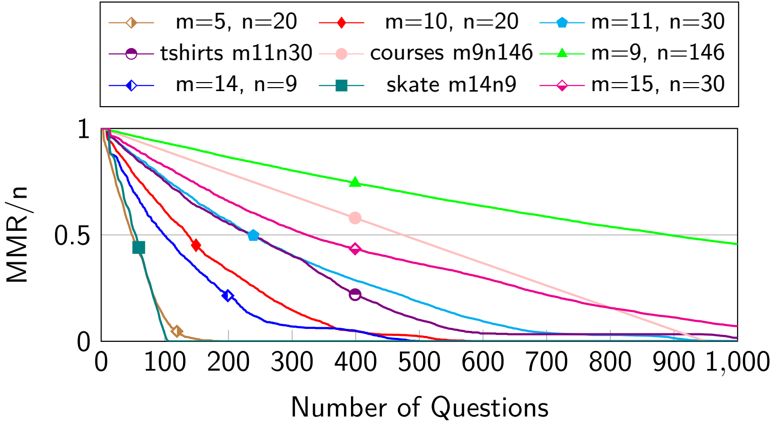
- when the minimax regret is lower than a threshold
- when the minimax regret is zero

Elicitation strategies: Pessimistic Strategy

- It selects first $n + (m - 2)$ candidate questions: one per each agent and one per each rank (excluding the first and the last one which are known)
- It selects the question that leads to minimal regret in the worst case from the set candidate questions

Empirical Evaluation: Pessimistic for different datasets

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets (some from PrefLib)

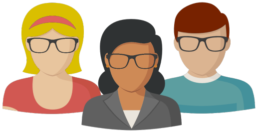


Outline

Incomplete knowledge: profile

Alternatives $= \{\ ,\ ,\ \} \xleftarrow{\textit{prefers}}$ Agents $= \{\ ,\ ,\ \}$ Chair $=$ $\Rightarrow$ Voting Rule

Excellent	Average	Mediocre
Good	Mediocre	
Inadequate		Very good



Majority Judgment

Majority Judgment

Voters judges candidates assigning grades from an ordinal scale. The winner is the candidate with the highest median of the grades received. [Balinski and Laraki, 2011]

Excellent	Very good	Good	Average	Mediocre	Inadequate	To be Rejected

Majority Judgment

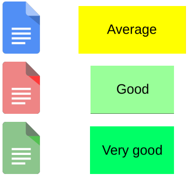
Alternatives $= \{$  $\}$ $\xleftarrow{\textit{prefers}}$ Agents $= \{$  $\}$ Chair $=$  $\Rightarrow$ Majority Judgment

Excellent	Average	Mediocre
Good	Mediocre	Good
Inadequate	Excellent	Very good

Majority Judgment

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Majority Judgment

Median



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	Excellent	Average	Mediocre
	Good	Mediocre	Good
	Inadequate	Excellent	Very good

Median

	Average
	Good
	Very good

winner:

Majority Judgment: Incomplete Knowledge

Alternatives $= \{\ ,\ ,\ \} \xleftarrow{\textit{prefers}}$ Agents $= \{\ ,\ ,\ \}$ Chair $=$ $\Rightarrow$ Majority Judgment

	Excellent	Average	Mediocre
	Good	Mediocre	
	Inadequate		Very good

Majority Judgment: Incomplete Knowledge

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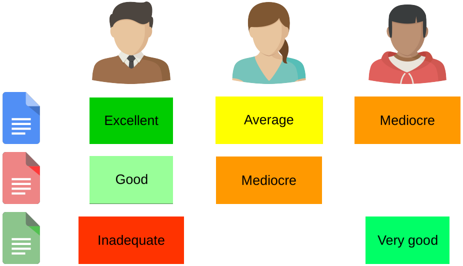

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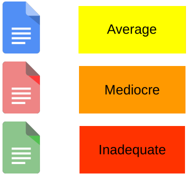

Majority Judgment: Incomplete Knowledge

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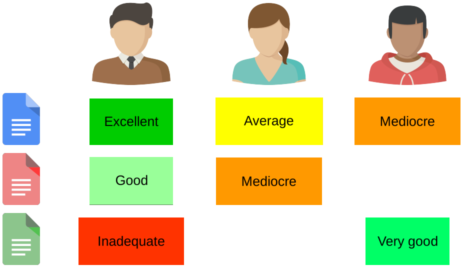

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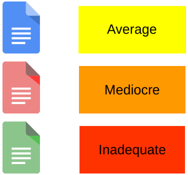


winner: 

Majority Judgment: Uses

In the last few years MJ has being adopted by a progressively larger number of french political parties including: Le Parti Pirate, Génération(s), LaPrimaire.org, France Insoumise and La République en Marche.

LaPrimaire.org is a french political initiative whose goal is to select an independent candidate for the french presidential election using MJ as voting rule.

Majority Judgment: Generalizing LaPrimaire.org

The procedure consists of two rounds:

1: each voter expresses her judgment on 5 random candidates. The 5 ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the 5 finalists. The one with the best median is the winner

Majority Judgment: Generalizing LaPrimaire.org

The procedure consists of two rounds:

1: each voter expresses her judgment on k random candidates. The k ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the k finalists. The one with the best median is the winner

Incomplete Knowledge

Remark

If a winner of the complete profile is among the k finalists then it will also be a winner of the incomplete profile

Probability of missing the winner

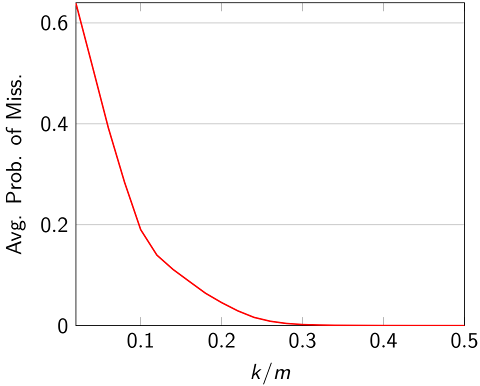
The probability of asking a voter i to evaluate the alternative j in k questions is:

$$\mathcal{P}(i_j) = \frac{\binom{m-1}{k-1}}{\binom{m}{k}} = \frac{k}{m}$$

After asking k questions to each voter the average size of grades known for each alternative is

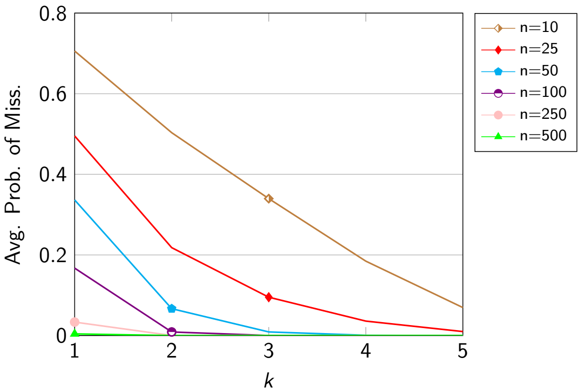
$$n \cdot \frac{k}{m}$$

Figure: Avg probability of missing the winner under uniform distribution of preferences, for $n = 100$, $m = 50$ and $k \in [\![1, 25]\!]$



Experimental results

Figure: Avg probability of missing the winner using a real case distribution of preferences, given $m = 12$ and $k \in [\![1, 5]\!]$



Outline

Compromising as an equal loss principle

Setting: Several voters express their preferences over a set of alternatives

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Goal: Find a procedure determining a collective choice that promotes a notion of compromise

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Goal: Find a procedure determining a collective choice that promotes a notion of compromise

Approach:

Cailloux, O., Napolitano, B., and Sanver, M. R. (2022). Compromising as an equal loss principle. *Review of Economic Design*

- Define a compromise from an equal loss perspective
- Propose classes of rules reflecting this concept

Related Works

- **Majoritarian Compromise**: picks the alternatives that receive the support of the majority of voters at the highest possible quality, breaking ties according to the quantity of support [Sertel, 1986]

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Related Works

- **Majoritarian Compromise**: picks the alternatives that receive the support of the majority of voters at the highest possible quality, breaking ties according to the quantity of support [Sertel, 1986]
- **q-approval FB**: picks the alternatives that receive the support of q voters at the highest possible quality, no tie-breaking
- **Fallback Bargaining**: q-approval with $q = n$ [Brams and Kilgour, 2001]

Motivation

$\mathcal{A} = \{$ , ,  $\}$ $n = 100$

51		$\succ$		$\succ$	
49		$\succ$		$\succ$	

Motivation

$\mathcal{A} = \{ \; , \; , \; \}$ $n = 100$

$$\begin{array}{lccccc} \mathbf{51} & & \succ & & \succ & \\ \mathbf{49} & & \succ & & \succ & \end{array}$$

- Plurality: $\{ \; \}$

Motivation

$\mathcal{A} = \{$ , ,  $\}$ $n = 100$

51		$\succ$		$\succ$	
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- Plurality: $\{$$\}$
- MC: $\{$$\}$

Motivation

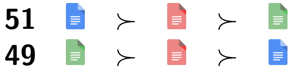
$\mathcal{A} = \{$ , ,  $\}$ $n = 100$

51  $\succ$  $\succ$ 
49  $\succ$  $\succ$ 

- Plurality: $\{$$\}$
- MC: $\{$$\}$
- FB_q
  - $q \in \{1, \ldots, 49\}$: $\{$, $\}$
  - $q \in \{50, 51\}$: $\{$$\}$
  - $q \in \{52, \ldots, 100\}$: $\{$$\}$

Motivation

$\mathcal{A} = \{$ , ,  $\}$ $n = 100$



Observations

-  receives unanimous support when each voter falls back one step from her ideal point
- almost all the SCRs studied impose a willingness to compromise, but do not ensure a compromise

Motivation

$\mathcal{A} = \{ \; , \; , \; \}$ $n = 100$

$$\begin{array}{llllll} \mathbf{51} & & \succ & & \succ & \\ \mathbf{49} & & \succ & & \succ & \end{array}$$

Observations

- receives unanimous support when each voter falls back one step from her ideal point
- almost all the SCRs studied impose a willingness to compromise, but do not ensure a compromise

Thesis

is a better compromise when egalitarianism is a major concern

Equal loss

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

Equal loss

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

51 $\succ$ $\succ$

49 $\succ$ $\succ$

$\lambda_P(\ ) = (0, 2)$

$\lambda_P(\ ) = (1, 1)$

$\lambda_P(\ ) = (2, 0)$

Equal loss

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+$ a spread measure

Equal loss

$$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N \text{ a loss vector}$$
$$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+ \text{ a spread measure}$$

Σ is the set of spread measures σ such that

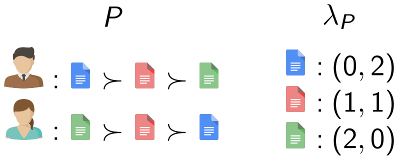
$$\sigma(l) = 0 \iff l_i = l_j,\ \forall i, j \in N, \quad \forall l \in [\![0, m-1]\!]^N$$

.

Equal loss

$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{a \in \mathcal{A} \mid \forall b \in \mathcal{A} : \sigma(\lambda_P(a)) \leq \sigma(\lambda_P(b))\}$$

$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P)$ denotes the "most egalitarian" alternatives, i.e. those in $\mathcal{A}$ whose loss vectors are the most equally distributed according to σ



$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{\ \} \quad \forall \sigma \in \Sigma$$

Egalitarian compromises

An SCR is Egalitarian Compromise Compatible iff at each profile, it selects some "most egalitarian" alternatives

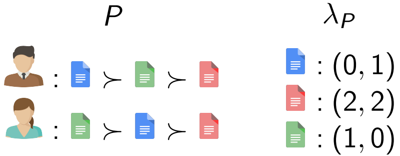
Egalitarian compromise compatibility

An SCR f is ECC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) \neq \emptyset$$

Egalitarian compromises and Pareto dominance

ECC rules are *very* egalitarian



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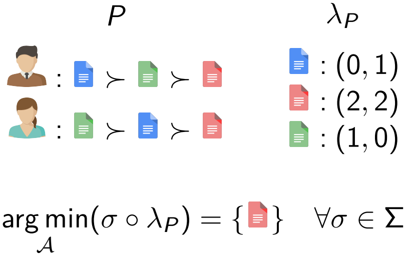

Theorem

$ECC \cap Paretian = \emptyset$ *(for $n, m \geq 2$)*

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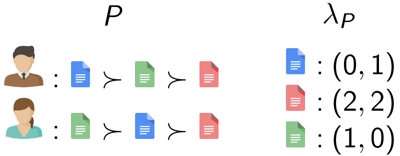


$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{$$  $$\} \quad \forall \sigma \in \Sigma$$

Theorem

ECC $\cap$ *Paretian* $= \emptyset$ *(for $n, m \geq 2$)*

$f \in \text{ECC} \Rightarrow$  $\in f(P)$, $f \in \text{Paretian} \Rightarrow$  $\notin f(P)$

Paretian compromises

An SCR is Paretian Compromise Compatible iff at each profile, it selects some "most egalitarian" alternatives *among the Pareto optimal ones*

Paretian compromise compatibility

An SCR f is PCC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{PO(P)}(\sigma \circ \lambda_P) \neq \emptyset$$

Results

For at least three voters and no restrictions on Σ:

	ECC	PCC
Condorcet procedures	No	No
Scoring rules	No	No
Antiplurality	No	Yes
BK compromises	No	No
Fallback bargaining	No	Yes

Restricting Σ

We consider a restriction $\bar{\Sigma} \subset \Sigma$ imposing a minimal condition for which

$$(m-3, m-1, m-2, \ldots, m-2)$$

is more equal than

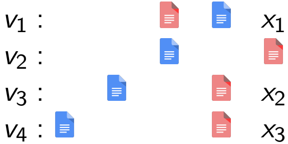
$$(m-2, m-3, \ldots, 1, 0, \ldots, 0)$$

Restricting Σ

Theorem

Under $\bar{\Sigma}$, AP and FB are not PCC.

Proof for $m = 5, n = 4$.



$\lambda_P($$) = (2, 4, 3, 3)$

$\lambda_P($$) = (3, 2, 1, 0)$

-  is the only alternative never last, thus for both rules: $f(P) = \{$$\}$
- $(\bar{\sigma} \circ \lambda_P)($$) < (\bar{\sigma} \circ \lambda_P)($$)$, $\forall \bar{\sigma} \in \bar{\Sigma}$
- and  $\in PO(P)$, thus  $\notin \arg\min_{PO(P)}(\bar{\sigma} \circ \lambda_P)$ $\square$

Results

For at least three voters and **no** restrictions on Σ:

	ECC	PCC
Condorcet procedures	No	No
Scoring rules	No	No
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Fallback bargaining	No	Yes

Results

For at least three voters **with** restrictions $\bar{\Sigma}$ on Σ:

	ECC	PCC
Condorcet procedures	No	No
Scoring rules	No	No
Antiplurality	No	No
BK compromises	No	No
Fallback bargaining	No	No

Results

For at least three voters **with** restrictions $\bar{\Sigma}$ on Σ:

	ECC	PCC
Condorcet procedures	No	No
Scoring rules	No	No
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Fallback bargaining	No	No

Similar results for two voters

Outline

Conclusions

Considering incomplete knowledge, we:

- introduced a simultaneous and incremental elicitation approach for agents and chair preferences
- developed several strategies and released our framework for further experiments and improvements
- analyzed the elicitation strategy used in a real voting scenario using MJ

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Considering incomplete knowledge, we:

- introduced a simultaneous and incremental elicitation approach for agents and chair preferences
- developed several strategies and released our framework for further experiments and improvements
- analyzed the elicitation strategy used in a real voting scenario using MJ

Considering a classical setting, we:

- revised the concept of compromise on an equal loss perspective
- proved that almost all SCRs fail to ensure a compromise
- defined new classes of voting rules reflecting this notion

Future work

Considering incomplete knowledge of agents and chair preferences:

- more strategies with different heuristics can be implemented
- the elicitation of the rule can be expanded to more than scoring rules and the convexity constraint can be relaxed
- the conversion of questions into profiles can be used in other settings

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Considering incomplete knowledge using MJ:

- the manipulability of the elicitation process using MJ can be explored
- steps toward explicability and axiomatization can be taken

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Considering incomplete knowledge using MJ:

- the manipulability of the elicitation process using MJ can be explored
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Considering a notion of compromise:

- the cardinal setting can be analyzed including intensity of preferences
- new definitions of compromise can be conceived
- the trade-off between equity and efficiency can be explored

Thank You!

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Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_2 - W_3 \geq 2\,(W_3 - W_4) \quad ?$$

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$$s(a) \geq s(b)$$

#1	**#2**
–	–
a	–
b	**b**
–	**a**

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#**1**	#**2**	#**3**	#**3**
c	*d*	*a*	*b*
a	*c*	*b*	*a*
b	**b**	*c*	*d*
d	**a**	*d*	*c*

Empirical Evaluation

Pessimistic reaching "low enough" regret

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \le n/10}$	$q_a^{MMR \le n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 | 5.0 | 5.8]	5.3	[5.4 | 6.2 | 7.2]
m10n20	10	20	0.0	[13.9 | 16.1 | 18.4]	32.0	[19.7 | 21.8 | 24.7]
m11n30	11	30	0.0	[16.6 | 19.0 | 22.3]	45.2	[23.1 | 25.7 | 28.9]
tshirts	11	30	0.0	[13.1 | 16.6 | 19.6]	43.2	[28.2 | 32.0 | 35.6]
courses	9	146	0.0	[6.0 | 7.0 | 7.0]	0.0	[6.8 | 7.0 | 7.0]
m14n9	14	9	5.4	[30.3 | 33.5 | 36.7]	64.1	[37.6 | 40.5 | 44.3]
skate	14	9	0.0	[11.4 | 11.6 | 12.3]	0.0	[11.5 | 11.8 | 12.8]
m15n30	15	30	0.0	[25.0 | 29.5 | 33.7]		

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	ca $\pm$ sd	ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0

FB and AP are PCC

Theorem

FB and Antiplurality are PCC. *(for $n, m \geq 3$)*

Proof sketch.

Define $\sigma^{\text{discrete}}(l) = 1 \iff l$ is not constant.
If some $a \in PO(P)$ has a constant loss vector, e. g.

$a_1 \succ a_2 \succ a_3 \succ a_4$

$a_3 \succ a_2 \succ a_1 \succ a_4$

there is exactly one such alternative, $FB(P) = \{a\}$ and it is never last so $a \in AP(P)$.
Otherwise, σ does not discriminate among $PO(P)$, thus Paretianism suffices. $\square$