Simultaneous Elicitation of Committee and Voters' Preferences

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Classical Scenario

Setting: Voters specify preferences over alternatives and a committee defines the social choice rule to aggregate them

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Goal: Find a consensus choice

Our Scenario

Setting: Incompletely specified preferences and social choice rule

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Goal: Develop an incremental elicitation strategy to acquire the most relevant information

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

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- Voters: difficult or costly to order *all* alternatives
- Committee: difficult to *specify* a voting rule precisely and abstractly

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Why?

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What?

- We want to reduce uncertainty, inferring (*eliciting*) the true preferences of voters and committee, in order to quickly converge to an optimal or a near-optimal alternative

Motivation and approach

Approach

- Develop query strategies that interleave questions to the committee and questions to the voters
- Use *Minimax regret* to measure the quality of those strategies

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Assumptions

- We consider *positional scoring rules*, which attach weights to positions according to a scoring vector w
- We assume w to be *convex*

$$w_r - w_{r+1} \geq w_{r+1} - w_{r+2} \qquad \forall r$$

and that $w_1 = 1$ and $w_m = 0$

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

Context

A alternatives, $|A| = m$

N voters

$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$ complete preferences profile

$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$ (convex) scoring vector that the committee has in mind

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P and W exist in the minds of voters and committee but unknown to us

Questions

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Questions to the voters

Comparison queries that ask a particular voter to compare two alternatives $a, b \in A$

$$a \succ_j b \quad ?$$

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Questions to the committee

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2}) \quad ?$$

Questions

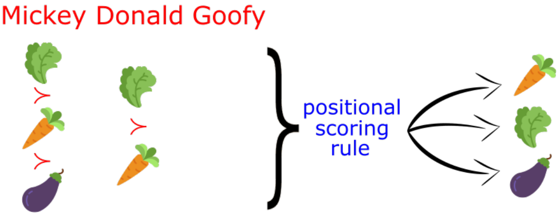

(Head of the)
Committee
w1 ≥ 2 w2
Voters
Mickey Donald Goofy
≻
≻
≻
positional scoring rule

Questions

(Head of the)
Committee

$w_1 \geq 2\, w_2$

Voters



Question to a voter:  $\succ_{donald}$  ?

Questions

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Question to the committee: $w_1 \geq 2.3 \cdot w_2$?

Our Knowledge

The answers to these questions define C_P and C_W that is our knowledge about P and W

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- $C_P \subseteq \mathcal{P}$ constraints on the profile given by the voters
- $C_W \subseteq \mathcal{W}$ constraints on the voting rule given by the committee

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P, C_W}(a, b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b

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$$\mathrm{MR}^{C_P, C_W}(a) = \max_{b \in A} \mathrm{PMR}^{C_P, C_W}(a, b)$$

We select the alternative which minimizes the maximal regret

$$\mathrm{MMR}^{C_P, C_W} = \min_{a \in A} \mathrm{MR}^{C_P, C_W}(a)$$

Pairwise Max Regret Computation

The computation of PMR^{C_P,C_W}(🥬, 🍆) can be seen as a game in which an adversary both:

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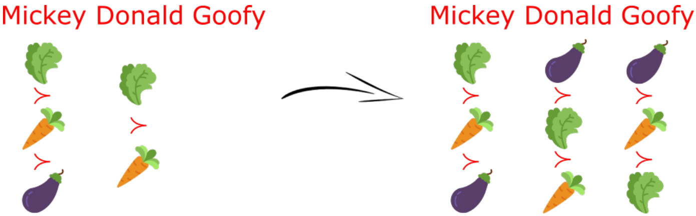
- **chooses a complete profile** $\mathrm{P} \in \mathcal{P}$



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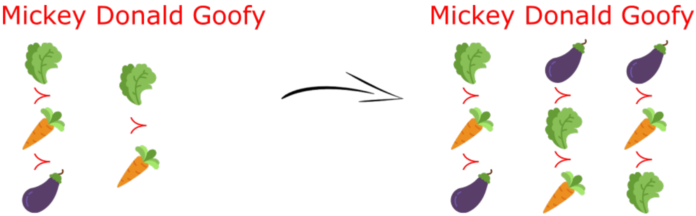


- **chooses a feasible weight vector** $\mathrm{W} \in \mathcal{W}$

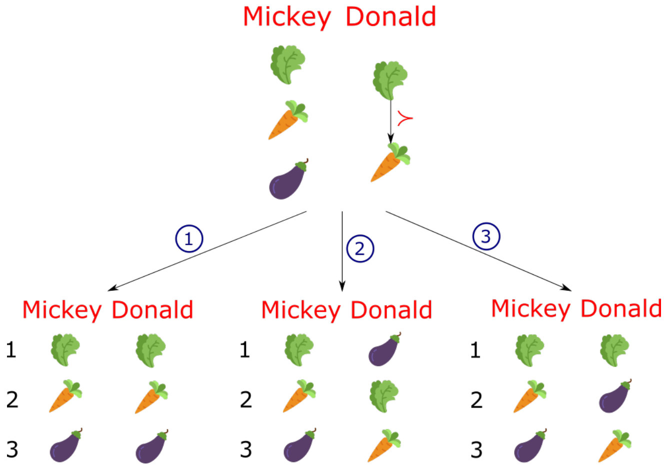
$$(1, ?, 0) \longrightarrow (1, 0, 0)$$

in order to maximize the difference of scores

Computing Minimax Regret: Example

Profile completion

Consider the following partial profile



Computing Minimax Regret: Example

Weight selection

Consider the following constraint on the scoring vector given by the committee

$$w_1 \geq 2 \cdot w_2$$

and the convex assumption

$$w_1 - w_2 \geq w_2 - w_3$$

Computing Minimax Regret: Example

Minimax computing

MR() = max {
- PMR( ) $\xrightarrow{v = 3 \quad w=\{1,0,0\}}$ = 2
- PMR( ) $\xrightarrow{v = 2 \quad w=\{1,0,0\}}$ = 1

MR() = $\boxed{0}$

MR() = 2

$\boxed{\text{MMR}}$ = 0 → winner 

Elicitation strategies

At each step, the strategy selects a question to ask either to one of the voters about her preferences or to the committee about the voting rule

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The termination condition could be:

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- when the minimax regret is zero

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- **weights**: it draws a rank $2 \leq r \leq m-2$ equiprobably, takes λ as the middle of the interval of values we are still uncertain about, and asks whether $w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$

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- **a preference ordering**: it draws equiprobably a voter whose preference is not known entirely and two alternatives that are incomparable in our current knowledge

Elicitation strategies

Pessimistic Strategy

Elicitation strategies

Pessimistic Strategy

Assume that a question leads to the possible new knowledge states (C_P^1, C_W^1) and (C_P^2, C_W^2) depending on the answer, then the badness of the question in the worst case is:

$$\max_{i=1,2} \mathrm{MMR}(C_P^i, C_W^i)$$

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Note:

Other aggregators than max can be used

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  - a^* and $\bar{b}$ are incomparable: we ask to compare them

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It uses the same criterion as the pessimistic strategy, but limiting it to a small set of $n+1$ candidate questions:

- **One question to the committee:** Consider W_τ the weight vector that minimize the PMR in the worst case.
  We ask about the position $r = \underset{i=[\![1,m-1]\!]}{\operatorname{argmax}} |\bar{W}(i) - W_\tau(i)|$

Elicitation strategies

Two Phase Strategy

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Using the mechanism defined in the Limited Pessimistic strategy:

Elicitation strategies

Two Phase Strategy

Using the mechanism defined in the Limited Pessimistic strategy:

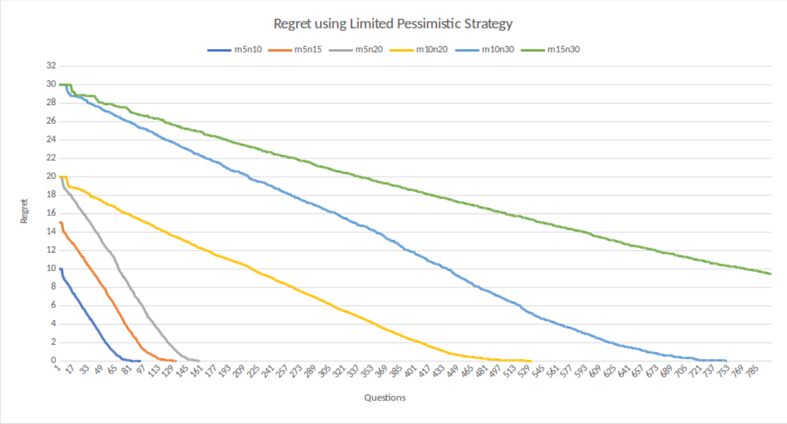
- **phase one:** asks p questions to the committee in order to gather informations about the weights
- **phase two:** asks $k - p$ questions to the voters

Results

k	Rnd$\pm$ sd	Pes.$\pm$ sd	L. pes.$\pm$ sd
0	5.0$\pm$0	5.0$\pm$0	5.0$\pm$0
5	5.0$\pm$0.1	3.7$\pm$0.0	4.4$\pm$0.6
10	4.7$\pm$0.4	3.3$\pm$0.4	3.3$\pm$0.4
15	4.4$\pm$0.5	2.7$\pm$0.4	2.7$\pm$0.7
20	3.7$\pm$0.5	1.5$\pm$0.4	2.1$\pm$0.7
25	3.1$\pm$0.7	1.4$\pm$0.5	0.9$\pm$0.6
30	2.6$\pm$0.5	0.4$\pm$0.4	0.5$\pm$0.4

Table: Minimax regret in a setting with 5 alternatives and 5 voters, after k questions.

Results


Regret using Limited Pessimistic Strategy
m5n10
m5n15
m5n20
m10n20
m10n30
m15n30
Regret
Questions

Results

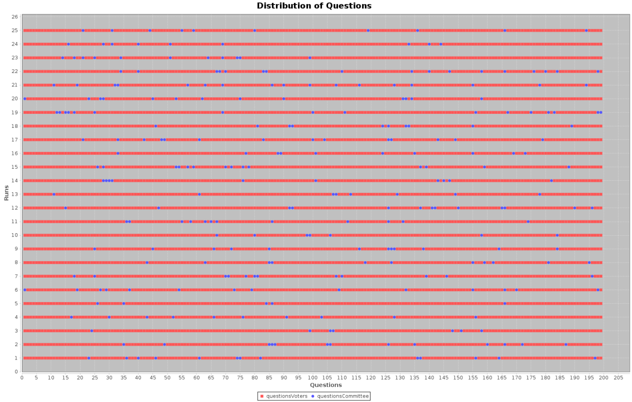


Figure: Distribution of the first 200 questions asked with Limited Pessimistic strategy in a setting with 10 alternatives and 20 voters, for 25 runs.

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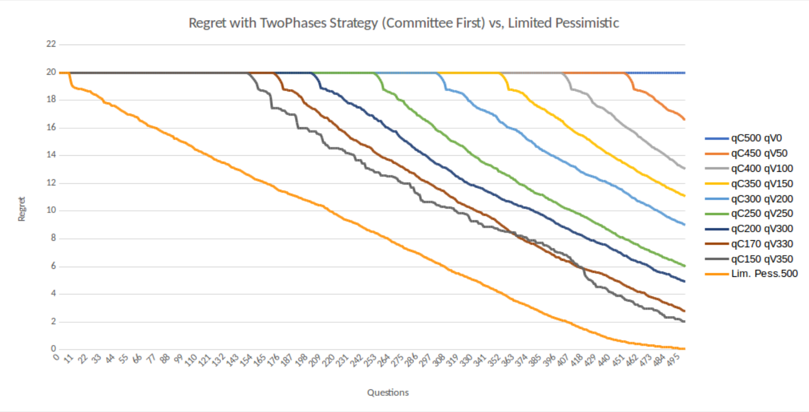

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- Introduced simultaneous elicitation of both committee and voters' preferences
- Proposed the use of minimax regret as a means of robust winner determination and as a guide to the process of elicitation
- Our experimental results suggest that this approach is effective and allows to quickly reduce worst regret significantly

Future Works

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- Further development of elicitation strategies, considering alternative heuristics

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- Test these strategies on partial specified profiles, ideally on real datasets

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- Extending elicitation to voting rules beyond scoring rules

Thank You!

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