Elicitation and explanation for voting rules

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Outline

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Context

$\mathcal{A}$ set of alternatives, $|\mathcal{A}| = m$

N set of voters, $|N| = n$

$\mathcal{L}(\mathcal{A})$ set of all linear orderings given $\mathcal{A}$

$\succ_i \in \mathcal{L}(\mathcal{A})$ preference ranking of voter $i \in N$

$P = (\succ_1, \dots, \succ_n) \in \mathcal{L}(\mathcal{A})^N$ a profile

$\mathscr{P}^*(\mathcal{A})$ possible winners (non-empty subsets of $\mathcal{A}$)

$f : \mathcal{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$ a Social Choice Rule

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Introducing the problem

Setting: Several voters express their preferences over a set of alternatives

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Introducing the problem

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Goal: Find a procedure determining a collective choice that promotes a notion of compromise

Context

Related Works

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- **Fallback Bargaining**: bargainers fall back to less and less preferred alternatives until they reach a unanimous agreement (Brams and Kilgour, 2001 [5])
- **q-approval FB**: picks the alternatives which receive the support of q voters at the highest possible quality, breaking ties according to the quantity of support

Context

Motivation: A simple example

$n = 100, \mathcal{A} = \{a, b, c\}$

$$\begin{array}{lccccc} 51 & a & \succ & b & \succ & c \\ 49 & c & \succ & b & \succ & a \end{array}$$

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Does b seem a better compromise?

Notation

Losses

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

Notation

Losses

P	λ_P
$v_1 : a \succ b \succ c$	$a : (0, 2)$
$v_2 : c \succ b \succ a$	$b : (1, 1)$
	$c : (2, 0)$

Given $P = (\succ_i)_{i \in N}$:

- $\lambda_{\succ_i}(x) = |\{y \in \mathcal{A} \mid y \succ_i x\}| \in [\![0, m-1]\!]$ the loss of i when choosing $x \in \mathcal{A}$ instead of her favorite alternative
- $\lambda_P(x)$ associates to each voter her loss when choosing x

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Σ is the set of spread measures σ such that

$$\sigma(l) = 0 \iff l_i = l_j,\ \forall i, j \in N, \quad \forall l \in [\![0, m-1]\!]^N$$

.

Notation

Minimizing losses

Given $X \subseteq \mathcal{A}$

$$\arg\min_X(\sigma \circ \lambda_P) = \{x \in X \mid \forall y \in X : \sigma(\lambda_P(x)) \leq \sigma(\lambda_P(y))\}$$

$\arg\min_X(\sigma \circ \lambda_P)$ denotes the alternatives in X whose loss vectors are the most equally distributed according to σ

Notation

Minimizing losses

$$\begin{array}{cc} P & \lambda_P \\ v_1 : a \succ b \succ c & a : (0,2) \\ v_2 : c \succ b \succ a & b : (1,1) \\ & c : (2,0) \end{array}$$

$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{b\} \quad \forall \sigma \in \Sigma$$

Egalitarian compromises

An SCR is Egalitarian Compromise Compatible iff at each profile, it selects some “less unequal” alternatives

Egalitarian compromise compatibility

An SCR f is ECC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) \neq \emptyset$$

Egalitarian compromises and Pareto dominance

ECC rules are *very* egalitarian

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Theorem

$ECC \cap Paretian = \emptyset$ *(for $n, m \geq 2$)*

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Theorem

ECC $\cap$ *Paretian* $= \emptyset$ *(for $n, m \geq 2$)*

$f \in \text{ECC} \Rightarrow b \in f(P)$, $f \in \text{Paretian} \Rightarrow b \notin f(P)$

Paretian compromises

An SCR is Paretian Compromise Compatible iff at each profile, it selects some "less unequal" alternatives *among the Pareto optimal ones*

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An SCR f is PCC iff

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Recall:

Egalitarian compromise compatibility

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$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \underset{\mathcal{A}}{\arg\min}(\sigma \circ \lambda_P) \neq \emptyset$$

FB and AP are PCC

Theorem

FB and Antiplurality are PCC. *(for $n, m \geq 3$)*

Proof sketch.

Define $\sigma^{\text{discrete}}(l) = 1 \iff l$ is not constant.
If some $a \in PO(P)$ has a constant loss vector, e. g.

$a_1 \succ a_2 \succ a_3 \succ a_4$

$a_3 \succ a_2 \succ a_1 \succ a_4$

there is exactly one such alternative, $FB(P) = \{a\}$ and it is never last so $a \in AP(P)$.
Otherwise, σ does not discriminate among $PO(P)$, thus Paretianism suffices. □

Restricting Σ

Definition (Condition $C_{m,n}$)

Given $m \geq 4, n \geq \max\{4, m-1\}$, σ satisfies condition $C_{m,n}$ iff $\sigma(m-3, m-1, m-2, \ldots, m-2) < \sigma(m-2, m-3, \ldots, 1, 0, \ldots, 0)$.

$$\begin{array}{lllllll}
v_1: & & & x & y & a_1 \\
v_2: & & & y & & x \\
v_3: & & y & & x & a_2 \\
v_4: & y & & & x & a_3
\end{array}$$

Requires that:

$$(\sigma \circ \lambda_P)(x) < (\sigma \circ \lambda_P)(y)$$

Restricting Σ

Theorem

Under condition $C_{m,n}$, AP and FB are not PCC.

Proof for $m = 5, n = 4$.

v_1 :			x	y	a_1
v_2 :			y		x
v_3 :		y		x	a_2
v_4 :	y			x	a_3

- y is the only alternative never last, thus for both rules: $f(P) = \{y\}$
- $(\sigma \circ \lambda_P)(x) < (\sigma \circ \lambda_P)(y)$
- and $x \in PO(P)$, thus $y \notin \arg\min_{PO(P)}(\sigma \circ \lambda_P)$ □

Other results

Theorem

Condorcet consistent rules are neither ECC nor PCC (for $m, n \geq 3$)

Theorem

Scoring rules,except AP, are neither ECC nor PCC (for $m \geq 3$ and large enough n)

Theorem

FB_q *rules with* $q \in [\![1, n-1]\!]$ *are neither ECC nor PCC* (for $m, n \geq 3$)

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Introducing the problem

Setting: Incompletely specified preferences and social choice rule

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Goal: Develop an incremental elicitation strategy to quickly acquire the most relevant information

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

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Why?

- Voters: difficult or costly to order *all* alternatives
- Committee: difficult to *specify* a voting rule precisely

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What?

- We want to reduce uncertainty, inferring (*eliciting*) the true preferences of voters and committee, in order to quickly converge to an optimal or a near-optimal alternative

Motivation and approach

Approach

- Develop query strategies that interleave questions to the committee and questions to the voters
- Use *Minimax regret* to measure the quality of those strategies

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Assumptions

- We consider *positional scoring rules*, which attach weights to positions according to a scoring vector w
- We assume w to be *convex*

$$w_r - w_{r+1} \geq w_{r+1} - w_{r+2} \qquad \forall r$$

and that $w_1 = 1$ and $w_m = 0$

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [6]; *Boutilier et al. 2006*, [4])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [8])
- in positional scoring rules (*Viappiani 2018*, [9])

Notation

$P \in \mathcal{P}$ complete preferences profile

$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$ (convex) scoring vector that the committee has in mind

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W defines a Positional Scoring Rule $f_W(P) \subseteq \mathcal{A}$ using scores $s^{W,P}(a),\ \forall\ a \in \mathcal{A}$

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P and W exist in the minds of voters and committee but unknown to us

Questions

Two types of questions:

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Questions to the voters

Comparison queries that ask a particular voter to compare two alternatives $a, b \in \mathcal{A}$

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Questions to the committee

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2}) \quad ?$$

Current Knowledge

The answers to these questions define C_P and C_W that is our knowledge about P and W

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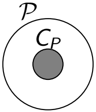

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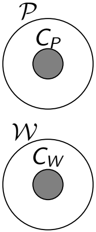

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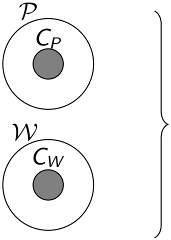

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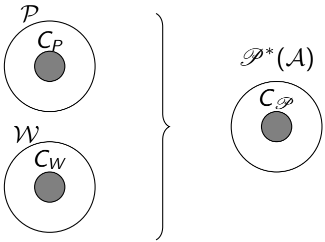

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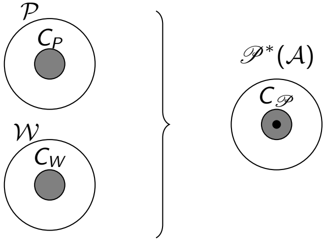

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b

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$$\mathrm{MR}^{C_P, C_W}(a) = \max_{b \in \mathcal{A}} \mathrm{PMR}^{C_P, C_W}(a, b)$$

We select the alternative which minimizes the maximal regret

$$\mathrm{MMR}^{C_P, C_W} = \min_{a \in \mathcal{A}} \mathrm{MR}^{C_P, C_W}(a)$$

Pairwise Max Regret Computation

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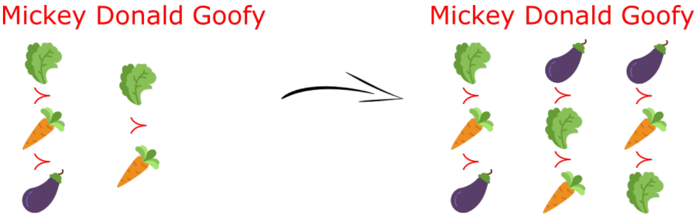

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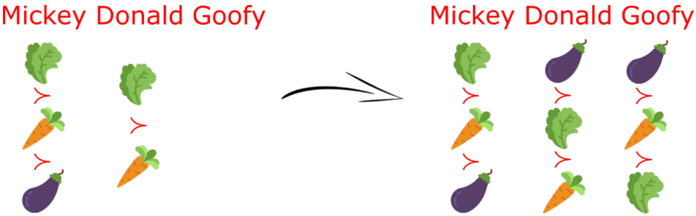

- **chooses a feasible weight vector** $\mathrm{W} \in \mathcal{W}$

$$(1, ?, 0) \longrightarrow (1, 0, 0)$$

in order to maximize the difference of scores

Elicitation strategies

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The termination condition could be:

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- when the minimax regret is zero

Elicitation strategies

Pessimistic Strategy

Assume that a question leads to the possible new knowledge states (C_P^1, C_W^1) and (C_P^2, C_W^2) depending on the answer, then the badness of the question in the worst case is:

$$\max_{i=1,2} \mathrm{MMR}(C_P^i, C_W^i)$$

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Note:

if the maximal MMR of two questions are equal, then prefers the one with the lowest MMR values associated to the opposite answer

Elicitation strategies

Pessimistic Strategy: Candidate questions

Let $(a^*, \bar{b}, \bar{P}, \bar{W})$ be the current solution of the minimax regret

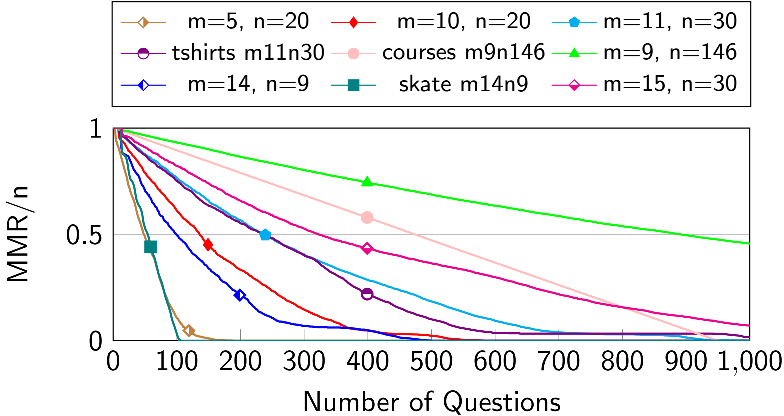
We select $n + 1$ candidate questions:

- **One question per voter:** For each voter i, either:
  - $a^* \succ_j^{\bar{P}} \bar{b}$: we ask about an incomparable alternative that can be placed above a^* by the adversary to increase PMR(a^*,$\bar{b}$)
  - $\bar{b} \succ_j^{\bar{P}} a^*$: we ask about an incomparable alternative that can be placed between a^* and $\bar{b}$ by the adversary to increase PMR(a^*,$\bar{b}$)
  - a^* and $\bar{b}$ are incomparable: we ask to compare them
- **One question to the committee:** Consider W_τ the weight vector that minimize the PMR in the worst case.
  We ask about the position $r = \underset{i=[\![1,m-1]\!]}{\arg\max} |\bar{W}(i) - W_\tau(i)|$

Empirical Evaluation

Pessimistic for different datasets

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets.



Empirical Evaluation

Pessimistic reaching "low enough" regret

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \leq n/10}$	$q_a^{MMR \leq n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 — 5.0 — 5.8]	5.3	[5.4 — 6.2 — 7.2]
m10n20	10	20	0.0	[13.9 — 16.1 — 18.4]	32.0	[19.7 — 21.8 — 24.7]
m11n30	11	30	0.0	[16.6 — 19.0 — 22.3]	45.2	[23.1 — 25.7 — 28.9]
tshirts	11	30	0.0	[13.1 — 16.6 — 19.6]	43.2	[28.2 — 32.0 — 35.6]
courses	9	146	0.0	[6.0 — 7.0 — 7.0]	0.0	[6.8 — 7.0 — 7.0]
m14n9	14	9	5.4	[30.3 — 33.5 — 36.7]	64.1	[37.6 — 40.5 — 44.3]
skate	14	9	0.0	[11.4 — 11.6 — 12.3]	0.0	[11.5 — 11.8 — 12.8]
m15n30	15	30	0.0	[25.0 — 29.5 — 33.7]		

Empirical Evaluation

Pessimistic committee first and then voters (and vice-versa)

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	2 ph. ca $\pm$ sd	2 ph. ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0

Outline

1. Notation
2. Compromising as an equal loss principle
3. Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination
4. Preference Elicitation under Majority Judgment

Context

Introducing the problem

Setting: Voters judges a random subset of alternatives and the winner is elected with the Majority Judgment rule

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Goal: Analyse the impact of the randomness in the outcome and find a more efficient elicitation procedure

Majority Judgment

Voters judges candidates assigning grades from an ordinal scale. The winner is the candidate with the highest median of the grades received. (Balinski and Laraki 2011, [2])

Excellent	Very good	Good	Average	Mediocre	Inadequate	To be Rejected

Majority Judgment

Uses

In the last few years MJ has being adopted by a progressively larger number of french political parties including: Le Parti Pirate, Génération(s), LaPrimaire.org, France Insoumise and La République en Marche. [1]

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LaPrimaire.org is a french political initiative whose goal is to select an independent candidate for the french presidential election using MJ as voting rule.

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LaPrimaire.org

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2: each voter expresses her judgment on all the five finalists. The one with the best median is the winner.

Notation

$\Delta = \{\alpha, \beta, \dots\}$ a *common language* (a set of strictly ordered grades)

$\alpha \geq \beta$ indicates that α is a better or equivalent grade than β

$P = \Delta^{m \times n}$ a profile is a m by n matrix of grades

ρ ordering function that given a vector of grades returns the vector ordered by decreasing grades

$f : \Delta^{m \times n} \rightarrow \Delta^m$ grading function

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$f : \Delta^{m \times n} \to \Delta^m$	grading function

A social grading function (SGF) is a grading function f that is neutral, anonymous, unanimous, monotonic, independent of irrelevant alternatives (IIA) and continuous.

Notation

The *majority-grade* is a SGF that associates to a profile P a vector of median grade values

$$f_{maj}(P)_i = \rho(P_i)_{\lfloor \frac{n}{2} \rfloor + 1},\ \forall i \in \mathcal{A}$$

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The winner function $F_{maj}(P) = \arg\max_{i \in \mathcal{A}} f_{maj}(P)_i$ selects the alternatives with the highest median grade as winners

Example

Complete Profile

P

	j_1	j_2	j_3
a	*Excellent*	*Excellent*	*Inadequate*
b	*Mediocre*	*Mediocre*	*Mediocre*
c	*Mediocre*	*Mediocre*	*Inadequate*
d	*Average*	*Average*	*Average*
e	*Average*	*Mediocre*	*Inadequate*
f	*Average*	*Mediocre*	*Mediocre*

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$f_{maj}(P)$

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$\mathrm{f_{maj}(P)}$

a	*Excellent*
b	*Mediocre*
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f	*Mediocre*

$$F_{maj}(P) = \{a\}$$

Example

Incomplete Profile

$\bar{\mathrm{P}}$

	j_1	j_2	j_3
a	*Excellent*		*Inadequate*
b	*Mediocre*	*Mediocre*	*Mediocre*
c	*Mediocre*	*Mediocre*	*Inadequate*
d	*Average*	*Average*	
e	*Average*	*Mediocre*	*Inadequate*
f		*Mediocre*	*Mediocre*

Example

Incomplete Profile

$\bar{\mathrm{P}}$

	j_1	j_2	j_3
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f		*Mediocre*	*Mediocre*

$\mathrm{f}_{\bar{\mathrm{maj}}}(\bar{\mathrm{P}})$

a	*Inadequate*
b	*Mediocre*
c	*Mediocre*
d	*Average*
e	*Mediocre*
f	*Mediocre*

Example

Incomplete Profile

	$\bar{P}$				$f_{\bar{maj}}(\bar{P})$
	j_1	j_2	j_3		
a	*Excellent*		*Inadequate*	a	*Inadequate*
b	*Mediocre*	*Mediocre*	*Mediocre*	b	*Mediocre*
c	*Mediocre*	*Mediocre*	*Inadequate*	c	*Mediocre*
d	*Average*	*Average*		d	*Average*
e	*Average*	*Mediocre*	*Inadequate*	e	*Mediocre*
f		*Mediocre*	*Mediocre*	f	*Mediocre*

$$a \notin F_{\bar{maj}}(\bar{P})$$

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- The random selection of questions is fair in terms of probability of being asked about a certain candidate i, but is it fair in terms of i being elected?
- Can we select the next question using a minimax regret notion instead of randomly selecting a candidate?

Thank You!

Plan of the thesis and questions

- Final dissertation by October 2021, defense by December 2021
- Status of the works:
  - Compromise: Rejected from Social Choice and Welfare; under submission to Review of Economic Design;
  - Elicitation PSR: Rejected from IJCAI20, AAMAS21 and IJCAI21; under revision at ADT21;
  - Elicitation MJ: ongoing work, plan to have a final draft before the defense.
- Given the current status of my works, is the plan feasible?
- Any suggestions on the dissertation structure ?

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