Preferences elicitation under incomplete knowledge

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LAMSADE
UMR CNRS 7243
laboratoire d'analyse et modélisation de systèmes pour l'aide à la décision

Classical setting

Agents = { , ,  }, Alternatives = { , ,  }, Chair =  $\Rightarrow$ Voting Rule

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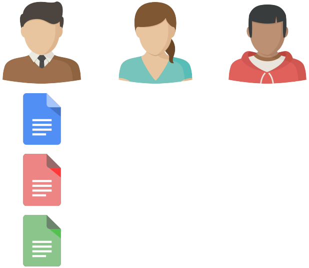

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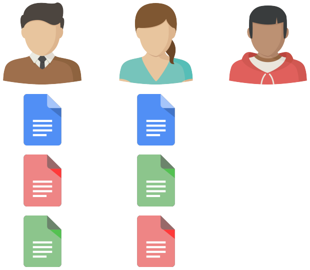

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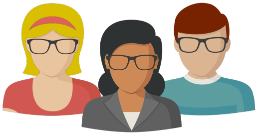

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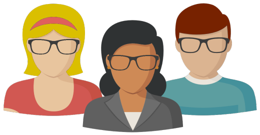

Borda

Classical setting

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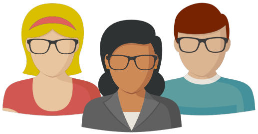

Borda

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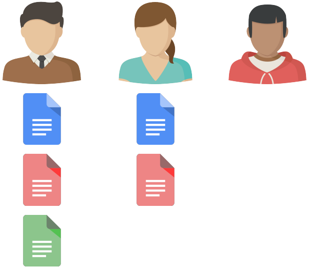

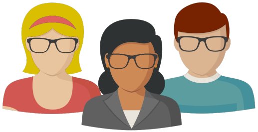

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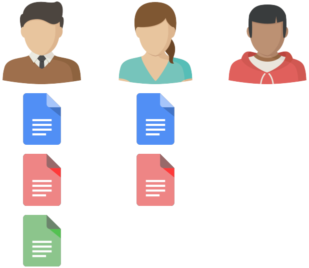

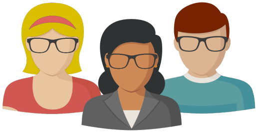

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Incomplete knowledge: voting rule

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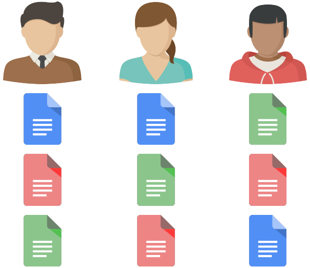

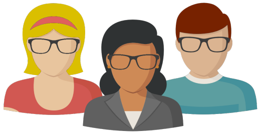

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Incomplete knowledge: voting rule

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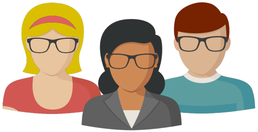

?

winner: ?

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [4]; *Boutilier et al. 2006*, [3])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [6])
- in positional scoring rules (*Viappiani 2018*, [7])

Incomplete knowledge: profile and voting rule

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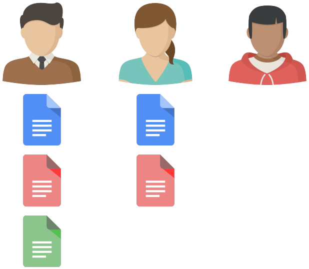

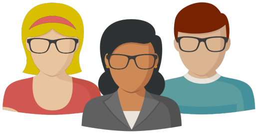

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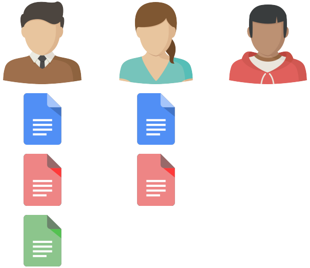

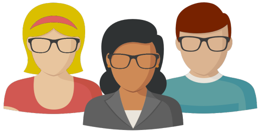

?

winner: ?

Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

Setting: Incompletely specified preferences and social choice rule

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Goal: Reduce uncertainty, inferring (*eliciting*) incrementally and simultaneously the true preferences of agents and chair to quickly converge to an optimal or a near-optimal alternative

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Approach:

Beatrice Napolitano, Olivier Cailloux, and Paolo Viappiani. Simultaneous elicitation of scoring rule and agent preferences for robust winner determination.
In *Proceedings of Algorithmic Decision Theory - 7th International Conference, ADT 2021*

- Develop query strategies that interleave questions to the chair and to the agents
- Use *Minimax regret* to measure the quality of those strategies

Assumptions

- We consider *Positional Scoring Rules*, which attach weights to positions according to a scoring vector W
- We assume W to be *convex*

$$W_r - W_{r+1} \geq W_{r+1} - W_{r+2}$$

for all positions r, and that $W_1 = 1$ and $W_m = 0$

Notation

A alternatives, $|A| = m$

N agents (*voters*)

$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$ complete preferences profile

$W = (W_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$ (convex) scoring vector that the chair has in mind

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P and W exist in the minds of agents and chair but unknown to us

Questions

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Questions to the agents

Comparison queries that ask a particular agent j to compare two alternatives $a, b \in \mathcal{A}$

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The answers to these questions define $\mathbf{C_P}$ and $\mathbf{C_W}$ that is our knowledge about P and W

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P, C_W}(a, b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

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We select the alternative that *minimizes* the maximum regret

Pairwise Max Regret Computation

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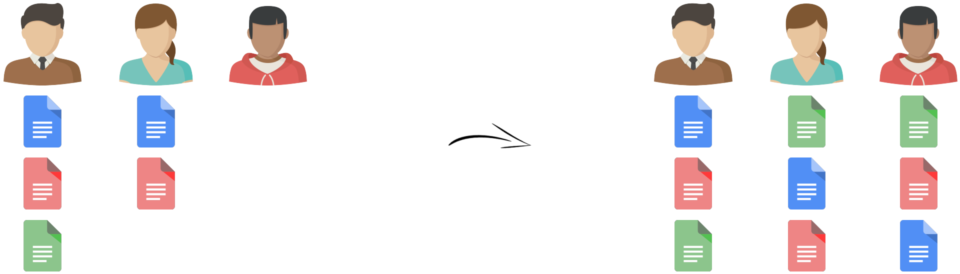

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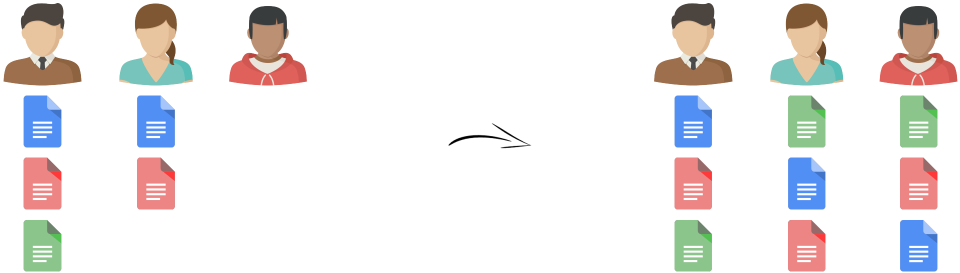


- **chooses a feasible weight vector W** $\in \mathcal{W}$

$$\mathbf{(1, ?, 0)} \longrightarrow \mathbf{(1, 0, 0)}$$

in order to maximize the difference of scores

Elicitation strategies

At each step, the strategy selects a question to ask either to one of the agents about her preferences or to the chair about the voting rule

The termination condition could be:

- when the minimax regret is lower than a threshold
- when the minimax regret is zero

Elicitation strategies

Pessimistic Strategy

It selects $n + (m-2)$ candidate questions:

- **One question per agent:** It selects incomparable alternatives that can be used by the "adversary" to increase the PMR
- **One question per rank** (excluding the first and the last one which are known)**:** For each rank r take the middle of the interval of values for λ that are still possible and asks whether

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2})$$

Elicitation strategies

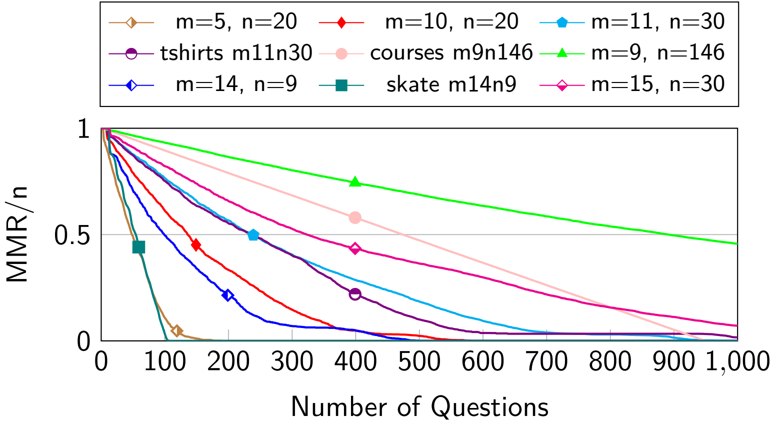
Pessimistic Strategy

Assume that a question leads to the possible new knowledge states (C_P^1, C_W^1) and (C_P^2, C_W^2) depending on the answer, then its score is:

$$\max_{i=1,2} \mathrm{MMR}(C_P^i, C_W^i)$$

The pessimistic strategy selects the question that leads to minimal regret in the worst case from a set of $n + (m-2)$ candidate questions

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets.



Incomplete knowledge: profile

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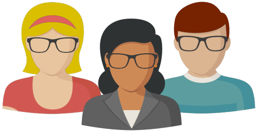

Majority Judgment

Majority Judgment

Voters judges candidates assigning grades from an ordinal scale. The winner is the candidate with the highest median of the grades received. (Balinski and Laraki 2011, [2])

Excellent	Very good	Good	Average	Mediocre	Inadequate	To be Rejected

Majority Judgment

Agents = { , , }, Alternatives = { , , }, Chair = $\Rightarrow$ Majority Judgment

Blue document	Excellent	Average	Mediocre
Red document	Good	Mediocre	Good
Green document	Inadequate	Excellent	Very good

Majority Judgment

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Median

Majority Judgment

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	Excellent	Average	Mediocre
	Good	Mediocre	Good
	Inadequate	Excellent	Very good

Median

	Average
	Good
	Very good

winner:

Majority Judgment

Incomplete Knowledge

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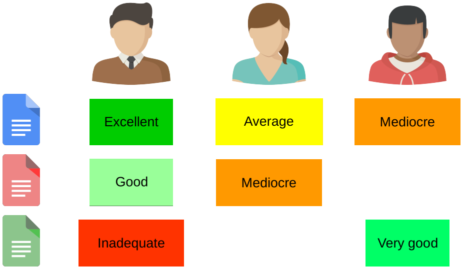


Agents = { , , }, Alternatives = { , , }, Chair = $\Rightarrow$ Majority Judgment

Excellent	Average	Mediocre
Good	Mediocre	
Inadequate		Very good

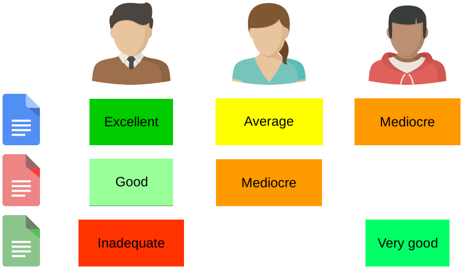
Median

Average
Mediocre
Inadequate

Majority Judgment

Incomplete Knowledge

Agents = { , , }, Alternatives = { , , }, Chair = ⇒ Majority Judgment



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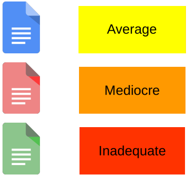

winner: 

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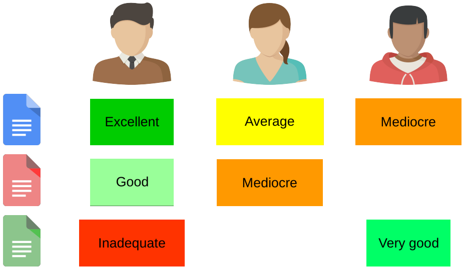
2: each voter expresses her judgment on all the five finalists. The one with the best median is the winner

Agents $= \{ \ , \ , \ \}$, Alternatives $= \{ \ , \ , \ \}$, Chair $=$ $\Rightarrow$ Majority Judgment



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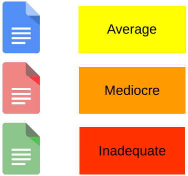

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Majority Judgment
LaPrimaire.org

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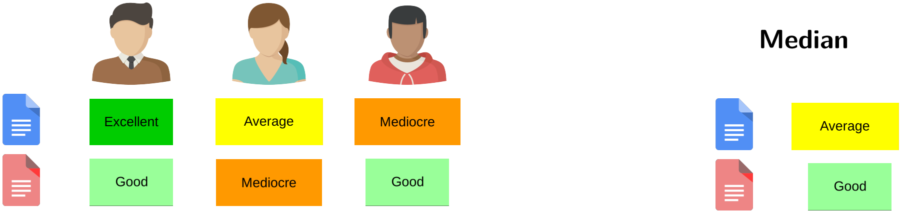

winner: 

Research Questions

- What is the probability of selecting a winner different from the one selected in case of complete knowledge?
- Can we elicit voters preferences using a minimax regret notion?
- What is the best trade-off between communication cost and optimal result?
- What is the voting rule applied on the resulting incomplete profile? What are its properties?

Thank You!

Icon made by Eucalyp from www.flaticon.com

Mieux voter.

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The MIT Press, 01 2011.

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Artificial Intelligence, 2006.

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Positional scoring rules with uncertain weights.
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Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2}) \quad ?$$

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$$W_2 + 2\ W_4 \geq 3\ W_3$$

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#1	**#2**
–	–
a	–
b	**b**
–	**a**

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$\#\mathbf{1}$	$\#\mathbf{2}$	$\#\mathbf{3}$	$\#\mathbf{3}$
c	d	a	b
$\mathbf{a}$	c	b	a
$\mathbf{b}$	$\mathbf{b}$	c	d
d	$\mathbf{a}$	d	c

Empirical Evaluation

Pessimistic reaching "low enough" regret

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \leq n/10}$	$q_a^{MMR \leq n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 | 5.0 | 5.8]	5.3	[5.4 | 6.2 | 7.2]
m10n20	10	20	0.0	[13.9 | 16.1 | 18.4]	32.0	[19.7 | 21.8 | 24.7]
m11n30	11	30	0.0	[16.6 | 19.0 | 22.3]	45.2	[23.1 | 25.7 | 28.9]
tshirts	11	30	0.0	[13.1 | 16.6 | 19.6]	43.2	[28.2 | 32.0 | 35.6]
courses	9	146	0.0	[6.0 | 7.0 | 7.0]	0.0	[6.8 | 7.0 | 7.0]
m14n9	14	9	5.4	[30.3 | 33.5 | 36.7]	64.1	[37.6 | 40.5 | 44.3]
skate	14	9	0.0	[11.4 | 11.6 | 12.3]	0.0	[11.5 | 11.8 | 12.8]
m15n30	15	30	0.0	[25.0 | 29.5 | 33.7]		

Empirical Evaluation

Pessimistic chair first and then agents (and vice-versa)

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	ca $\pm$ sd	ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0