

# Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

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**LAMSADE**

UMR CNRS 7243

laboratoire d'analyse et modélisation de systèmes pour l'aide à la décision

# Introducing the problem

Agents = { , ,  }, Alternatives = { , ,  }, Chair =   $\Rightarrow$  Positional Scoring Rule

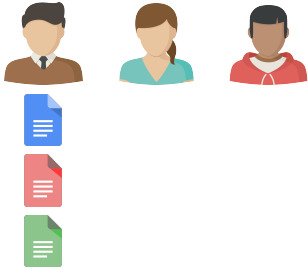
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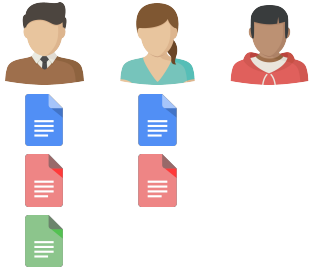
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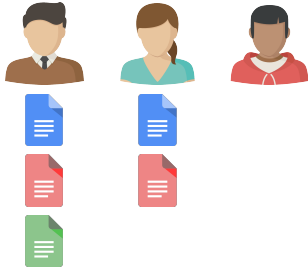
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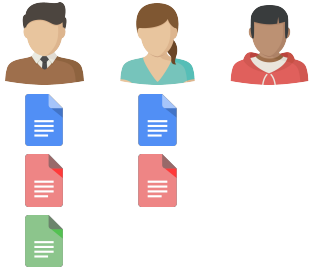
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$$\text{weight}(1^{\text{st}}) \geq 2 \cdot \text{weight}(2^{\text{nd}})$$

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**Setting:** Incompletely specified preferences and social choice rule



# Introducing the problem

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- Agents: difficult or costly to order *all* alternatives
- Chair: difficult to *specify* a voting rule precisely

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**Approach:**

- Develop query strategies that interleave questions to the chair and to the agents
- Use *Minimax regret* to measure the quality of those strategies

### Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

### Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

# Assumptions

- We consider *Positional Scoring Rules*, which attach weights to positions according to a scoring vector  $W$
- We assume  $W$  to be *convex*

$$W_r - W_{r+1} \geq W_{r+1} - W_{r+2}$$

for all positions  $r$ , and that  $W_1 = 1$  and  $W_m = 0$

# Notation

$A$  alternatives,  $|A| = m$

$N$  agents (*voters*)

$P = (\succ_j, j \in N)$ ,  $P \in \mathcal{P}$  complete preferences profile

$W = (W_r, 1 \leq r \leq m)$ ,  $W \in \mathcal{W}$  (convex) scoring vector that the chair has in mind

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$P$  and  $W$  exist in the minds of agents and chair but unknown to us



# Questions

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## Questions to the agents

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## Questions to the chair

Queries relating the difference between the importance of consecutive ranks from  $r$  to  $r + 2$

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2}) \quad ?$$

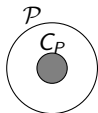
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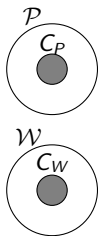
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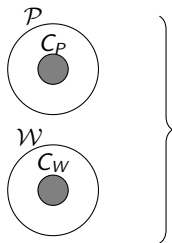
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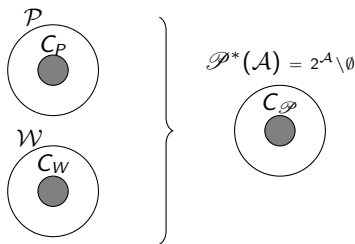
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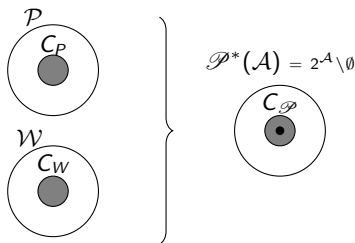




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# Minimax Regret

Given  $C_P \subseteq \mathcal{P}$  and  $C_W \subseteq \mathcal{W}$ :

$$\text{PMR}^{C_P, C_W}(a, b) = \max_{P \in C_P, W \in C_W} s^{P, W}(b) - s^{P, W}(a)$$

is the maximum difference of score between  $a$  and  $b$  under all possible realizations of the full profile *and* weights

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**We select the alternative which *minimizes* the maximum regret**

$$\text{MMR}^{C_P, C_W} = \min_{a \in \mathcal{A}} \text{MR}^{C_P, C_W}(a)$$

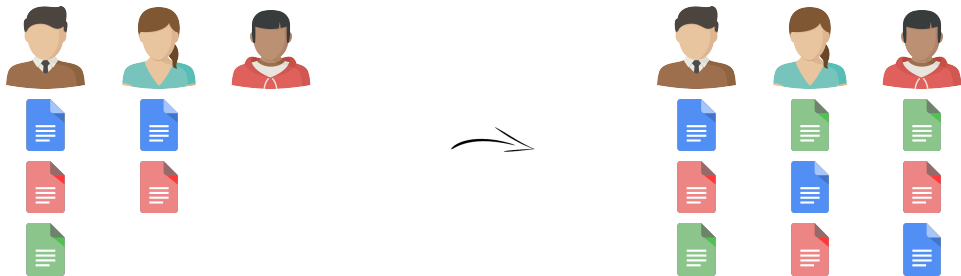
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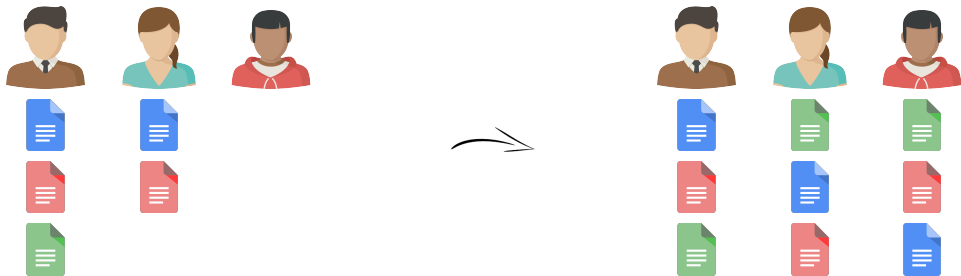
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- chooses a complete profile  $\mathbf{P} \in \mathcal{P}$



- chooses a feasible weight vector  $\mathbf{W} \in \mathcal{W}$

$$(1, ?, 0) \longrightarrow (1, 0, 0)$$

in order to maximize the difference of scores

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The termination condition could be:

- when the minimax regret is lower than a threshold
- when the minimax regret is zero

# Elicitation strategies

## Pessimistic Strategy

Assume that a question leads to the possible new knowledge states  $(C_P^1, C_W^1)$  and  $(C_P^2, C_W^2)$  depending on the answer, then the badness of the question in the worst case is:

$$\max_{i=1,2} \text{MMR}(C_P^i, C_W^i)$$

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### Note:

if the maximal MMR of two questions are equal, then prefers the one with the lowest MMR values associated to the opposite answer

# Elicitation strategies

## Pessimistic Strategy: Candidate questions

Let  $(a^*, \bar{b}, \bar{P}, \bar{W})$  be the current solution of the minimax regret

We select  $n + (m - 2)$  candidate questions:

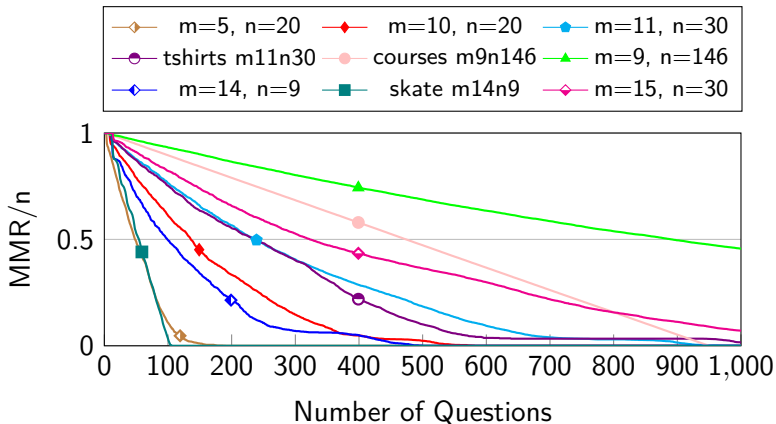
- **One question per agent:** For each agent  $i$ , either:
  - $a^* \succ_j^{\bar{P}} \bar{b}$  : we ask about an incomparable alternative that can be placed above  $a^*$  by the adversary to increase  $\text{PMR}(a^*, \bar{b})$
  - $\bar{b} \succ_j^{\bar{P}} a^*$  : we ask about an incomparable alternative that can be placed between  $a^*$  and  $\bar{b}$  by the adversary to increase  $\text{PMR}(a^*, \bar{b})$
  - $a^*$  and  $\bar{b}$  are incomparable: we ask to compare them
- **One question per rank** (excluding the first and the last one which are known): For each rank  $r$  take the middle of the interval of values for  $\lambda$  that are still possible and asks whether

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2})$$

# Empirical Evaluation

Pessimistic for different datasets

**Figure:** Average MMR (normalized by  $n$ ) after  $k$  questions with Pessimistic strategy for different datasets.





# Empirical Evaluation

Pessimistic reaching "low enough" regret

**Table:** Questions asked by Pessimistic strategy on several datasets to reach  $\frac{n}{10}$  regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \leq n/10}$	$q_a^{MMR \leq n/10}$			$q_c^{MMR=0}$	$q_a^{MMR=0}$		
m5n20	5	20	0.0	[ 4.3	5.0	5.8 ]	5.3	[ 5.4	6.2	7.2 ]
m10n20	10	20	0.0	[ 13.9	16.1	18.4 ]	32.0	[ 19.7	21.8	24.7 ]
m11n30	11	30	0.0	[ 16.6	19.0	22.3 ]	45.2	[ 23.1	25.7	28.9 ]
tshirts	11	30	0.0	[ 13.1	16.6	19.6 ]	43.2	[ 28.2	32.0	35.6 ]
courses	9	146	0.0	[ 6.0	7.0	7.0 ]	0.0	[ 6.8	7.0	7.0 ]
m14n9	14	9	5.4	[ 30.3	33.5	36.7 ]	64.1	[ 37.6	40.5	44.3 ]
skate	14	9	0.0	[ 11.4	11.6	12.3 ]	0.0	[ 11.5	11.8	12.8 ]
m15n30	15	30	0.0	[ 25.0	29.5	33.7 ]				

# Empirical Evaluation

Pessimistic chair first and then agents (and vice-versa)

**Table:** Average MMR in problems of size (10, 20) after 500 questions, among which  $q_c$  to the chair.

$q_c$	ca $\pm$ sd	ac $\pm$ sd
0	0.6 $\pm$ 0.5	0.6 $\pm$ 0.5
15	0.5 $\pm$ 0.5	0.5 $\pm$ 0.5
30	0.3 $\pm$ 0.5	0.3 $\pm$ 0.4
50	0.0 $\pm$ 0.1	0.0 $\pm$ 0.1
100	0.1 $\pm$ 0.2	0.1 $\pm$ 0.1
200	2.3 $\pm$ 1.4	2.1 $\pm$ 1.8
300	5.2 $\pm$ 2.4	6.8 $\pm$ 0.6
400	10.9 $\pm$ 0.9	12.2 $\pm$ 1.0
500	20.0 $\pm$ 0.0	20.0 $\pm$ 0.0

Thank You!



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Constraint-based optimization and utility elicitation using the minimax decision criterion.  
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William E. Stein, Philip J. Mizzi, and Roger C. Pfaffenberger.

A stochastic dominance analysis of ranked voting systems with scoring.  
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Positional scoring rules with uncertain weights.  
In *Scalable Uncertainty Management*, 2018.

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Queries relating the difference between the importance of consecutive ranks

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#1	#2
—	—
<b>a</b>	—
<b>b</b>	<b>b</b>
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<b>a</b>	<i>c</i>	<i>b</i>	<i>a</i>
<b>b</b>	<b>b</b>	<i>c</i>	<i>d</i>
<i>d</i>	<b>a</b>	<i>d</i>	<i>c</i>