Compromise and elicitation in social choice

A study of egalitarianism and incomplete information in voting

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Classical setting

Alternatives = { , ,  } $\xleftarrow{\textit{prefers}}$ Agents = { , ,  } Chair =  $\Rightarrow$ Voting Rule

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Alternatives $= \{ \;,\;,\; \} \xleftarrow{\textit{prefers}}$ Agents $= \{ \;,\;,\; \}$ Chair $=$ $\Rightarrow$ Voting Rule

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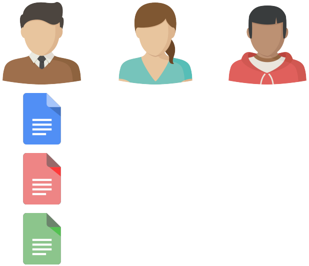

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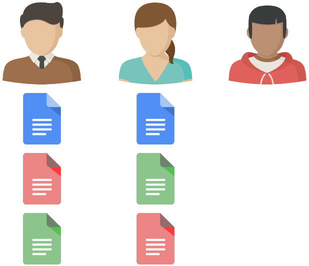

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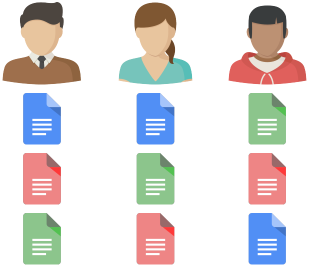

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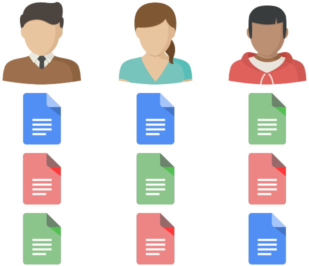

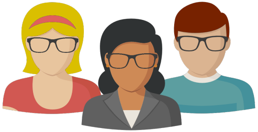

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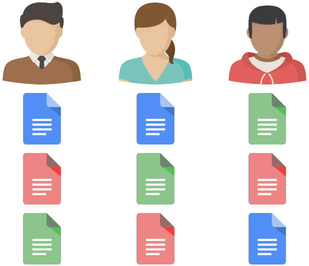
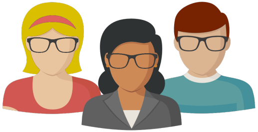
Alternatives $= \{\ ,\ ,\ \} \xleftarrow{\textit{prefers}}$ Agents $= \{\ ,\ ,\ \}$ Chair $=$ $\Rightarrow$ Voting Rule

Borda

Classical setting

Alternatives = {  } $\xleftarrow{\textit{prefers}}$ Agents = {  } Chair =  $\Rightarrow$ Voting Rule



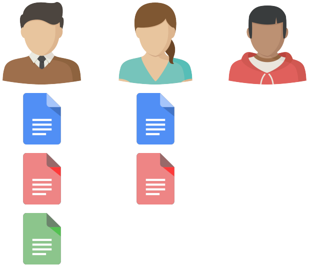


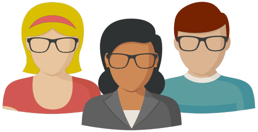
Borda

winner: 

Incomplete knowledge about profile

$\text{Alternatives} = \{\ ,\ ,\ \} \xleftarrow{\textit{prefers}} \text{Agents} = \{\ ,\ ,\ \} \quad \text{Chair} = \quad \Rightarrow \text{Voting Rule}$



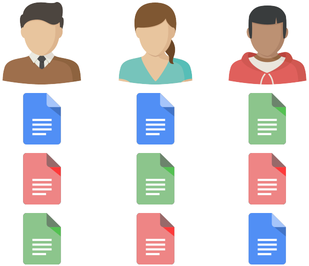


Borda

winner: ?

Incomplete knowledge about voting rule

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Voting Rule

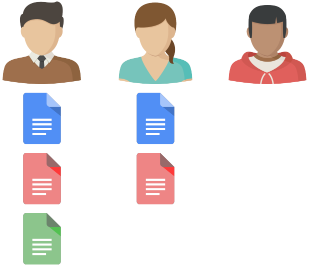


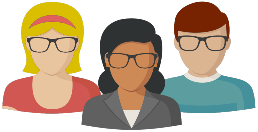
?

winner: ?

Research Question I:
Incomplete knowledge about profile and voting rule

Alternatives $= \{\ ,\ ,\ \} \xleftarrow{\textit{prefers}}$ Agents $= \{\ ,\ ,\ \}$ Chair $=$ $\Rightarrow$ Voting Rule



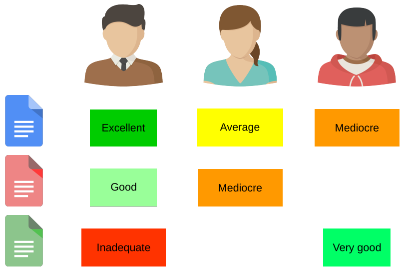


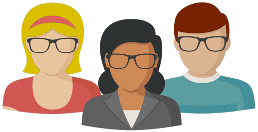
?

winner: ?

Research Question II:
Incomplete knowledge under Majority Judgment

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Voting Rule





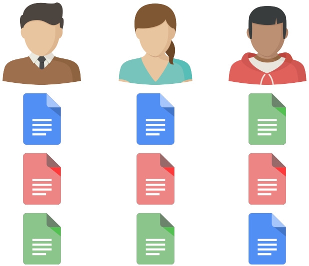
Majority Judgment

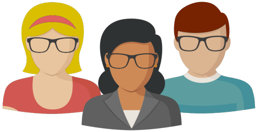
winner: ?

Research Question III:

Compromise from an equal-loss perspective

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Voting Rule





What is a compromise?

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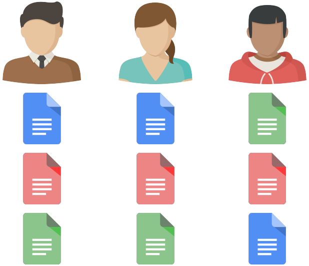

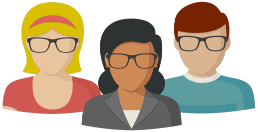

What is a compromise?

maybe  ?

Outline

Outline

Notation

$\mathcal{A}$	set of alternatives, $\lvert\mathcal{A}\rvert = m$
N	set of voters, $\lvert N\rvert = n$
$\mathcal{L}(\mathcal{A})$	set of all linear orderings given $\mathcal{A}$
$\succ_i \in \mathcal{L}(\mathcal{A})$	preference ranking of voter $i \in N$
$P = (\succ_1, \ldots, \succ_n) \in \mathcal{L}(\mathcal{A})^N$	a profile
$\mathscr{P}^*(\mathcal{A})$	possible winners (non-empty subsets of $\mathcal{A}$)
$f : \mathcal{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$	a Social Choice Rule

Outline

Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

Setting: Incompletely specified preferences and social choice rule

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Goal: Reduce uncertainty

- eliciting preferences of agents and chair incrementally and simultaneously
- quickly converge to an optimal or a near-optimal alternative

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Approach:

Napolitano, B., Cailloux, O., and Viappiani, P. (2021). Simultaneous elicitation of scoring rule and agent preferences for robust winner determination.
In *Proceedings of Algorithmic Decision Theory - 7th International Conference, ADT 2021*

- Develop strategies interleaving questions to the chair and agents
- Use *Minimax regret* to measure the quality of those strategies

Related Works

Incomplete profile

- and known rule: Minimax regret to produce a robust winner approximation [Lu and Boutilier, 2011, Boutilier et al., 2006]

Uncertain rule

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others [Stein et al., 1994]
- considering positional scoring rules [Viappiani, 2018]

Context

$P = (\succ_i,\ i \in N) \in \mathcal{P}$ complete preferences profile

$W = (W_r,\ 1 \leq r \leq m) \in \mathcal{W}$ **convex** scoring vector

P and W exist in the minds of agents and chair but unknown to us

W defines a **Positional Scoring Rule** $f_W(P) \subseteq A$

Questions

Questions to the agents

Comparison queries that ask a particular agent to compare two alternatives in $\mathcal{A}$

 :  $\succ$  ?

Questions to the chair

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

 : $W_{r_1} \rightarrow W_{r_2} > W_{r_2} \rightarrow W_{r_3}$?

The answers to these questions define $\mathbf{C_P}$ and $\mathbf{C_W}$ that is our knowledge about P and W

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

The *Maximum Regret* MR of an alternative a is the highest possible loss when selecting a as a winner under all possible completions of C_P and C_W

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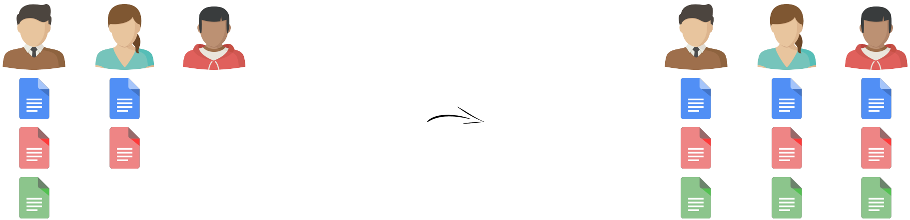


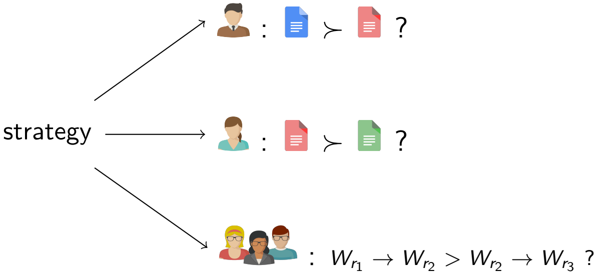
Minimax Regret

The alternative that currently minimizes the maximum regret is used:

- as winner recommendation when the elicitation process stops
- to guide elicitation strategies

Elicitation strategies

At each step, the strategy selects a question to ask either to one of the agents about her preferences or to the chair about the voting rule



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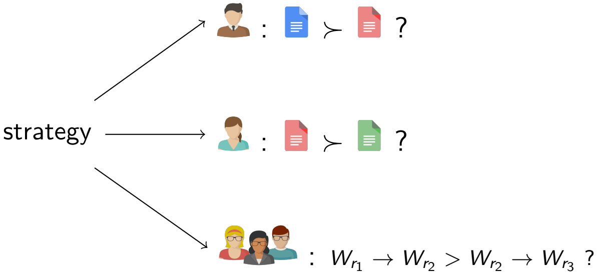


Termination:

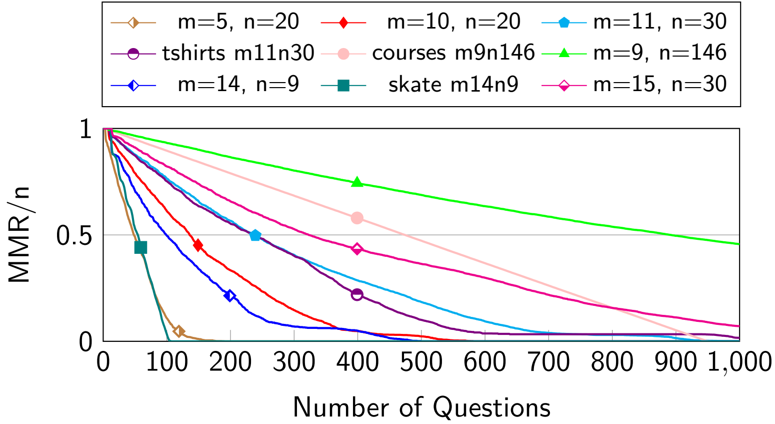
- when the minimax regret is lower than a threshold
- when the minimax regret is zero

Elicitation strategies: Pessimistic Strategy

- It selects first $n + (m - 2)$ candidate questions: one per each agent and one per each rank (excluding the first and the last one which are known)
- It selects the question that leads to minimal regret in the worst case from the set candidate questions

Empirical Evaluation: Pessimistic for different datasets

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets (some from PrefLib)



Outline

Incomplete knowledge: profile

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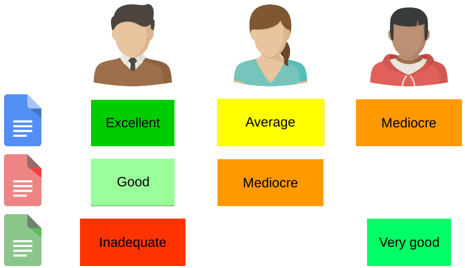

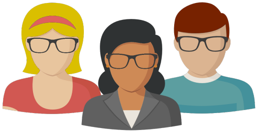

Majority Judgment

Majority Judgment

Voters judges candidates assigning grades from an ordinal scale. The winner is the candidate with the highest median of the grades received. [Balinski and Laraki, 2011]

Excellent	Very good	Good	Average	Mediocre	Inadequate	To be Rejected

Majority Judgment

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Majority Judgment

	Excellent	Average	Mediocre
	Good	Mediocre	Good
	Inadequate	Excellent	Very good

Majority Judgment

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = $\Rightarrow$ Majority Judgment

	Excellent	Average	Mediocre
	Good	Mediocre	Good
	Inadequate	Excellent	Very good

Median

	Average
	Good
	Very good

Majority Judgment

Alternatives = { , , } $\xleftarrow{\textit{prefers}}$ Agents = { , , } Chair = ⇒ Majority Judgment

Excellent	Average	Mediocre
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Inadequate	Excellent	Very good

Median

Average
Good
Very good

winner:

Majority Judgment: Incomplete Knowledge

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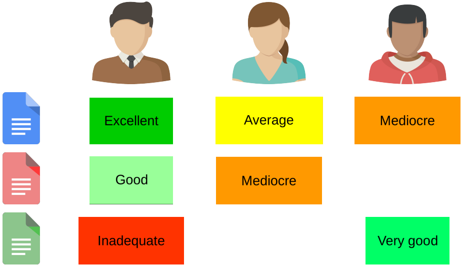

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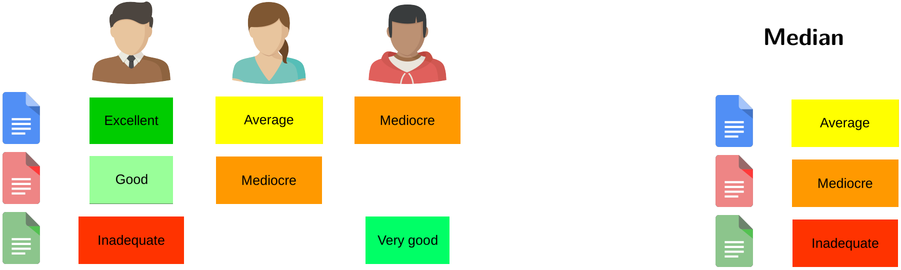


winner: 

Majority Judgment: Uses

In the last few years MJ has being adopted by a progressively larger number of french political parties including: Le Parti Pirate, Génération(s), LaPrimaire.org, France Insoumise and La République en Marche.

LaPrimaire.org is a french political initiative whose goal is to select an independent candidate for the french presidential election using MJ as voting rule.

Majority Judgment: Generalizing LaPrimaire.org

The procedure consists of two rounds:

1: each voter expresses her judgment on 5 random candidates. The 5 ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the 5 finalists. The one with the best median is the winner

Majority Judgment: Generalizing LaPrimaire.org

The procedure consists of two rounds:

1: each voter expresses her judgment on k random candidates. The k ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the k finalists. The one with the best median is the winner

Incomplete Knowledge

Remark

If a winner of the complete profile is among the k finalists then it will also be a winner of the incomplete profile

Probability of missing the winner

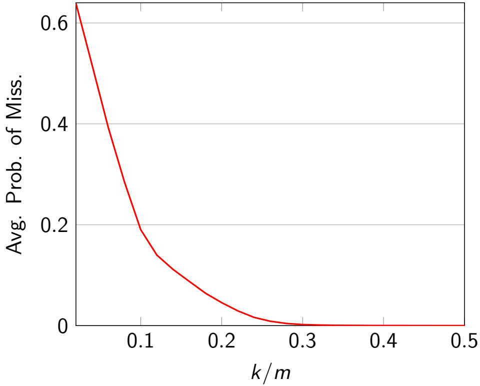
The probability of asking a voter i to evaluate the alternative j in k questions is:

$$\mathcal{P}(i_j) = \frac{\binom{m-1}{k-1}}{\binom{m}{k}} = \frac{k}{m}$$

After asking k questions to each voter the average size of grades known for each alternative is

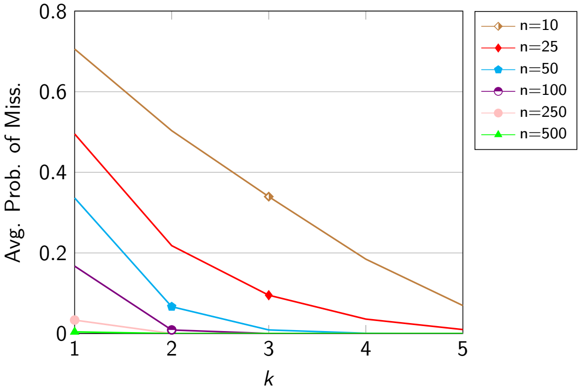
$$n \cdot \frac{k}{m}$$

Figure: Avg probability of missing the winner under uniform distribution of preferences, for $n = 100$, $m = 50$ and $k \in [\![1, 25]\!]$



Experimental results

Figure: Avg probability of missing the winner using a real case distribution of preferences, given $m = 12$ and $k \in [\![1, 5]\!]$



Outline

Compromising as an equal loss principle

Setting: Several voters express their preferences over a set of alternatives

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Goal: Find a procedure determining a collective choice that promotes a notion of compromise

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Approach:

Cailloux, O., Napolitano, B., and Sanver, M. R. (2022). Compromising as an equal loss principle. *Review of Economic Design*

- Define a compromise from an equal loss perspective
- Propose classes of rules reflecting this concept

Related Works

- **Majoritarian Compromise**: picks the alternatives that receive the support of the majority of voters at the highest possible quality, breaking ties according to the quantity of support [Sertel, 1986]

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Related Works

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- **q-approval FB**: picks the alternatives that receive the support of q voters at the highest possible quality, no tie-breaking
- **Fallback Bargaining**: q-approval with $q = n$ [Brams and Kilgour, 2001]

Motivation

$\mathcal{A} = \{ \;,\; ,\; \}$ $n = 100$

51 $\succ$ $\succ$

49 $\succ$ $\succ$

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$\mathcal{A} = \{$ , ,  $\}$ $n = 100$

51  $\succ$  $\succ$ 

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- Plurality: $\{$$\}$

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- Plurality: $\{$$\}$
- MC: $\{$$\}$

Motivation

$\mathcal{A} = \{ \;,\;,\; \}$ $n = 100$

$$\begin{array}{llllll} \textbf{51} & & \succ & & \succ & \\ \textbf{49} & & \succ & & \succ & \end{array}$$

- Plurality: $\{\;\}$
- MC: $\{\;\}$
- FB_q
  - $q \in \{1, \ldots, 49\}$: $\{\;,\;\}$
  - $q \in \{50, 51\}$: $\{\;\}$
  - $q \in \{52, \ldots, 100\}$: $\{\;\}$

Motivation

$\mathcal{A} = \{ \;,\;,\; \}$ $n = 100$

51 $\succ \quad \succ$

49 $\succ \quad \succ$

Observations

- receives unanimous support when each voter falls back one step from her ideal point
- almost all the SCRs studied impose a willingness to compromise, but do not ensure a compromise

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Observations

- receives unanimous support when each voter falls back one step from her ideal point
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Thesis

is a better compromise when egalitarianism is a major concern

Equal loss

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

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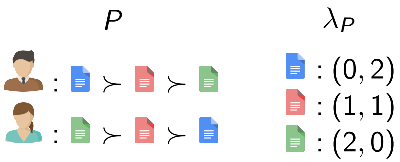

Equal loss

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+$ a spread measure

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$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+$ a spread measure

Σ is the set of spread measures σ such that

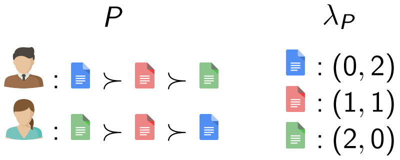
$$\sigma(l) = 0 \iff l_i = l_j,\ \forall i, j \in N, \quad \forall l \in [\![0, m-1]\!]^N$$

Equal loss

$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{a \in \mathcal{A} \mid \forall b \in \mathcal{A} : \sigma(\lambda_P(a)) \leq \sigma(\lambda_P(b))\}$$

$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P)$ denotes the "most egalitarian" alternatives, i.e. those in $\mathcal{A}$ whose loss vectors are the most equally distributed according to σ

Equal loss



$$\underset{\mathcal{A}}{\arg\min}(\sigma \circ \lambda_P) = \{$$  $$\} \quad \forall \sigma \in \Sigma$$

Egalitarian compromises

An SCR is Egalitarian Compromise Compatible iff at each profile, it selects some "most egalitarian" alternatives

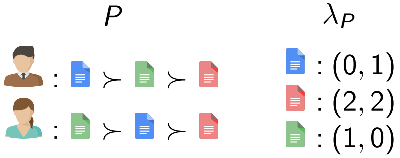
Egalitarian compromise compatibility

An SCR f is ECC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) \neq \emptyset$$

Egalitarian compromises and Pareto dominance

ECC rules are *very* egalitarian



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Egalitarian compromises and Pareto dominance

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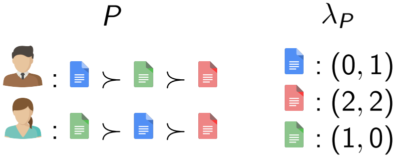


$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{$$  $$\} \quad \forall \sigma \in \Sigma$$

Theorem

ECC $\cap$ *Paretian* $= \emptyset$ *(for $n, m \geq 2$)*

$f \in \text{ECC} \Rightarrow$  $\in f(P)$, $f \in \text{Paretian} \Rightarrow$  $\notin f(P)$

Paretian compromises

An SCR is Paretian Compromise Compatible iff at each profile, it selects some "most egalitarian" alternatives *among the Pareto optimal ones*

Paretian compromise compatibility

An SCR f is PCC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{PO(P)}(\sigma \circ \lambda_P) \neq \emptyset$$

Results

For at least three voters and no restrictions on Σ:

	ECC	PCC
Condorcet procedures	No	No
Scoring rules	No	No
Antiplurality	No	Yes
BK compromises	No	No
Fallback bargaining	No	Yes

Restricting Σ

We consider a restriction $\bar{\Sigma} \subset \Sigma$ imposing a minimal condition for which

$$(m-3, m-1, m-2, \dots, m-2)$$

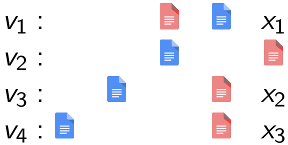
is more equal than

$$(m-2, m-3, \dots, 1, 0, \dots, 0)$$

Theorem

Under $\bar{\Sigma}$, AP and FB are not PCC.

Proof for $m = 5, n = 4$.



$$\lambda_P(\ ) = (2, 4, 3, 3)$$
$$\lambda_P(\ ) = (3, 2, 1, 0)$$

- is the only alternative never last, thus for both rules: $f(P) = \{\ \}$
- $(\bar{\sigma} \circ \lambda_P)(\ ) < (\bar{\sigma} \circ \lambda_P)(\ ), \forall \bar{\sigma} \in \bar{\Sigma}$
- and $\ \in PO(P)$, thus $\ \notin \arg\min_{PO(P)}(\bar{\sigma} \circ \lambda_P)$

$\square$

Results

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Results

For at least three voters **with** restrictions $\bar{\Sigma}$ on Σ:

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	ECC	PCC
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Similar results for two voters

Outline

Conclusions

Considering incomplete knowledge, we:

- introduced a simultaneous and incremental elicitation approach for agents and chair preferences
- developed several strategies and released our framework for further experiments and improvements
- analyzed the elicitation strategy used in a real voting scenario using MJ

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Considering a classical setting, we:

- revised the concept of compromise from an equal loss perspective
- proved that almost all SCRs fail to ensure a compromise
- defined new classes of voting rules reflecting this notion

Future work

Considering incomplete knowledge of agents and chair preferences:

- more strategies with different heuristics can be implemented
- the elicitation of the rule can be expanded to more than scoring rules and the convexity constraint can be relaxed
- the conversion of questions into profiles can be used in other settings

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Considering incomplete knowledge using MJ:

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- steps toward explicability and axiomatization can be taken

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Considering incomplete knowledge using MJ:

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Considering a notion of compromise:

- the cardinal setting can be analyzed including intensity of preferences
- new definitions of compromise can be conceived
- the trade-off between equity and efficiency can be explored

Thank You!

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Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_2 - W_3 \geq 2\,(W_3 - W_4) \quad ?$$

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#1	**#2**
–	–
a	–
b	**b**
–	**a**

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#**1**	#**2**	#**3**	#**3**
c	*d*	*a*	*b*
a	*c*	*b*	*a*
b	**b**	*c*	*d*
d	**a**	*d*	*c*

Empirical Evaluation

Pessimistic reaching "low enough" regret

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR\leq n/10}$	$q_a^{MMR\leq n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 | 5.0 | 5.8]	5.3	[5.4 | 6.2 | 7.2]
m10n20	10	20	0.0	[13.9 | 16.1 | 18.4]	32.0	[19.7 | 21.8 | 24.7]
m11n30	11	30	0.0	[16.6 | 19.0 | 22.3]	45.2	[23.1 | 25.7 | 28.9]
tshirts	11	30	0.0	[13.1 | 16.6 | 19.6]	43.2	[28.2 | 32.0 | 35.6]
courses	9	146	0.0	[6.0 | 7.0 | 7.0]	0.0	[6.8 | 7.0 | 7.0]
m14n9	14	9	5.4	[30.3 | 33.5 | 36.7]	64.1	[37.6 | 40.5 | 44.3]
skate	14	9	0.0	[11.4 | 11.6 | 12.3]	0.0	[11.5 | 11.8 | 12.8]
m15n30	15	30	0.0	[25.0 | 29.5 | 33.7]		

Empirical Evaluation

Pessimistic chair first and then agents (and vice-versa)

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	ca $\pm$ sd	ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0

FB and AP are PCC

Theorem

FB and Antiplurality are PCC. *(for $n, m \geq 3$)*

Proof sketch.

Define $\sigma^{\text{discrete}}(I) = 1 \iff I$ is not constant.
If some $a \in PO(P)$ has a constant loss vector, e. g.

$a_1 \succ a_2 \succ a_3 \succ a_4$
$a_3 \succ a_2 \succ a_1 \succ a_4$

there is exactly one such alternative, $FB(P) = \{a\}$ and it is never last so $a \in AP(P)$.
Otherwise, σ does not discriminate among $PO(P)$, thus Paretianism suffices. $\square$