Compromise and elicitation in social choice

A study of egalitarianism and incomplete information in voting

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Ph.D. Thesis Defense, 09 December 2022

LAMSADE
UMR CNRS 7243
laboratoire d 'analyse et modélisation de systèmes pour l'aide à la décision

Classical setting

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule

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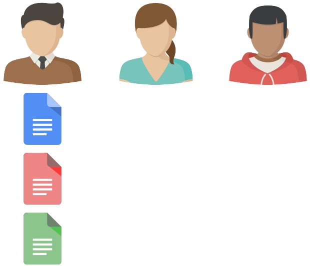

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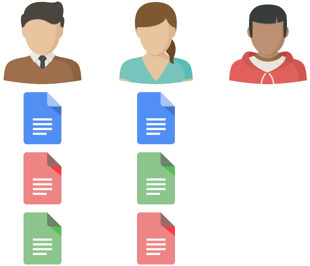

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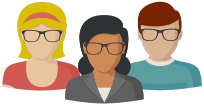

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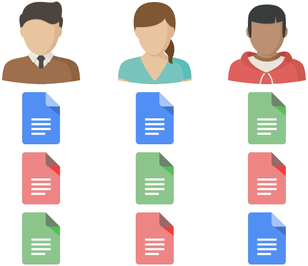

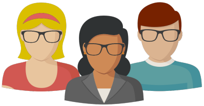

Borda

Classical setting

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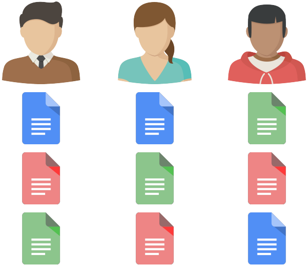

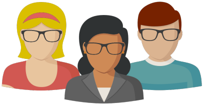

Borda

winner: 

Incomplete knowledge about profile

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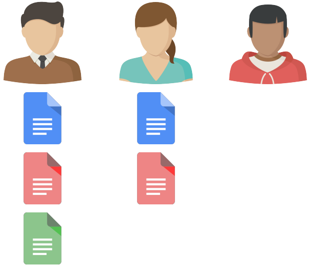
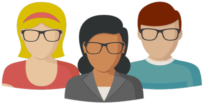


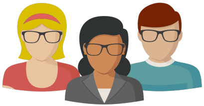


Borda

winner: ?

Incomplete knowledge about voting rule

Agents $= \{$, , $\}$, Altern. $= \{$, , $\}$, Chair $=$ $\Rightarrow$ Voting Rule

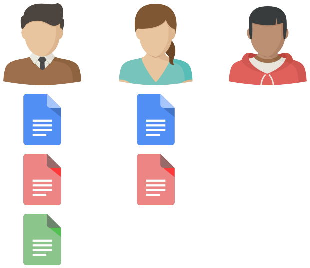


?

winner: ?

Research Question I:
Incomplete knowledge about profile and voting rule

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule

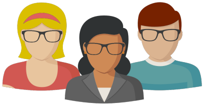


?

winner: ?

Research Question II:
Incomplete knowledge under Majority Judgment

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule



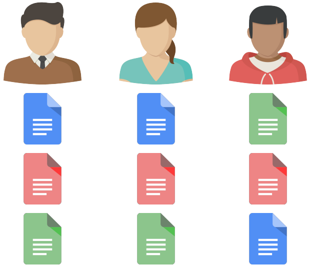
Majority Judgment

winner: ?

Research Question III:

Compromise from an equal-loss perspective

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule



What is a compromise?

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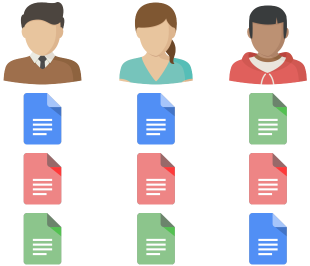

What is a compromise?

maybe  ?

Outline

Outline

Notation

$\mathcal{A}$	set of alternatives, $\lvert\mathcal{A}\rvert = m$
N	set of voters, $\lvert N\rvert = n$
$\mathcal{L}(\mathcal{A})$	set of all linear orderings given $\mathcal{A}$
$\succ_i \in \mathcal{L}(\mathcal{A})$	preference ranking of voter $i \in N$
$P = (\succ_1, \ldots, \succ_n) \in \mathcal{L}(\mathcal{A})^N$	a profile
$\mathscr{P}^*(\mathcal{A})$	possible winners (non-empty subsets of $\mathcal{A}$)
$f : \mathcal{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$	a Social Choice Rule

Outline

Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

Setting: Incompletely specified preferences and social choice rule

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Goal: Reduce uncertainty, inferring (*eliciting*) incrementally and simultaneously the true preferences of agents and chair to quickly converge to an optimal or a near-optimal alternative

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Approach:

Beatrice Napolitano, Olivier Cailloux, and Paolo Viappiani. Simultaneous elicitation of scoring rule and agent preferences for robust winner determination.
In *Proceedings of Algorithmic Decision Theory - 7th International Conference, ADT 2021*, 2021

- Develop query strategies that interleave questions to the chair and to the agents
- Use *Minimax regret* to measure the quality of those strategies

Related Works

Incomplete profile

- and known rule: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [6]; *Boutilier et al. 2006*, [3])

Uncertain rule

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [10])
- considering positional scoring rules (*Viappiani 2018*, [11])

Context

$P = (\succ_i,\ i \in N),\ P \in \mathcal{P}$	complete preferences profile unknown to us
$W = (W_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$	**convex** scoring vector that the chair has in mind

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W defines a **Positional Scoring Rule** $f_W(P) \subseteq A$

P and W exist in the minds of agents and chair but unknown to us

Questions

Two types of questions:

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Questions to the agents

Comparison queries that ask a particular agent i to compare two alternatives $a, b \in \mathcal{A}$

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Two types of questions:

Questions to the agents

Comparison queries that ask a particular agent i to compare two alternatives $a, b \in \mathcal{A}$

Questions to the chair

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

The answers to these questions define $\mathbf{C_P}$ and $\mathbf{C_W}$ that is our knowledge about P and W

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

The *Maximum Regret* MR of an alternative a is the highest possible loss when selecting a as a winner under all possible completions of C_P and C_W

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$$\mathrm{MMR}^{C_P, C_W} = \min_{a \in \mathcal{A}} \mathrm{MR}^{C_P, C_W}(a)$$

Minimax Regret

The alternative that *minimizes* the maximum regret is used:

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- as winner recommendation when the elicitation process stops

Minimax Regret

The alternative that *minimizes* the maximum regret is used:

- as winner recommendation when the elicitation process stops
- to guide elicitation strategies

Elicitation strategies

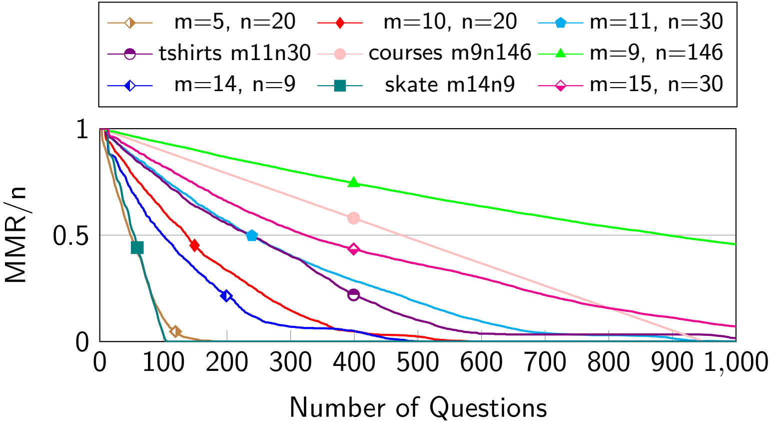
At each step, the strategy selects a question to ask either to one of the agents about her preferences or to the chair about the voting rule

The termination condition could be:

- when the minimax regret is lower than a threshold
- when the minimax regret is zero

- It selects first $n + (m - 2)$ candidate questions: one per each agent and one per each rank (excluding the first and the last one which are known)
- It selects the question that leads to minimal regret in the worst case from the set candidate questions

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets.



Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_2 - W_3 \geq 2\,(W_3 - W_4) \quad ?$$

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$$W_2 \;-\; W_3 \geq 2\;W_3 - 2\;W_4$$

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$$\begin{aligned} W_2 \ - \ W_3 &\geq 2\ W_3 - 2\ W_4 \\ W_2 + 2\ W_4 &\geq 3\ W_3 \end{aligned}$$

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#1	#2
–	–
a	–
b	**b**
–	**a**

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#1	**#2**	**#3**	**#3**
c	*d*	*a*	*b*
a	*c*	*b*	*a*
b	**b**	*c*	*d*
d	**a**	*d*	*c*

Outline

Incomplete knowledge: profile

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule

Majority Judgment

Majority Judgment

Voters judges candidates assigning grades from an ordinal scale. The winner is the candidate with the highest median of the grades received. (Balinski and Laraki 2011, [1])

Excellent	Very good	Good	Average	Mediocre	Inadequate	To be Rejected

Majority Judgment

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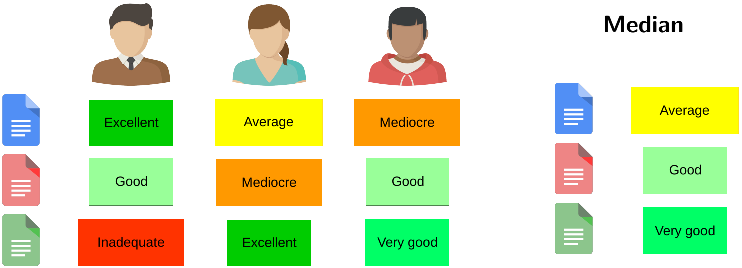

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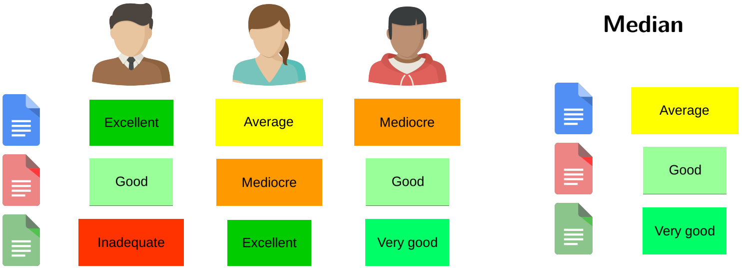

winner: 

Majority Judgment: Incomplete Knowledge

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Majority Judgment

	Excellent	Average	Mediocre
	Good	Mediocre	
	Inadequate		Very good

Majority Judgment: Incomplete Knowledge

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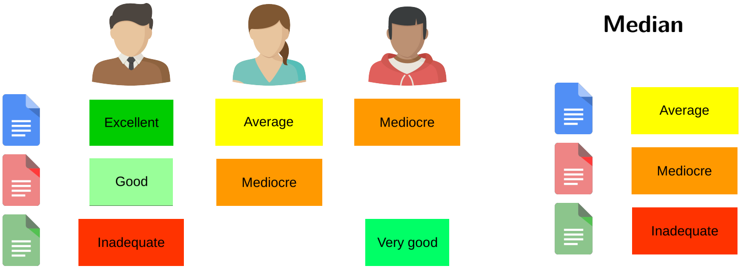

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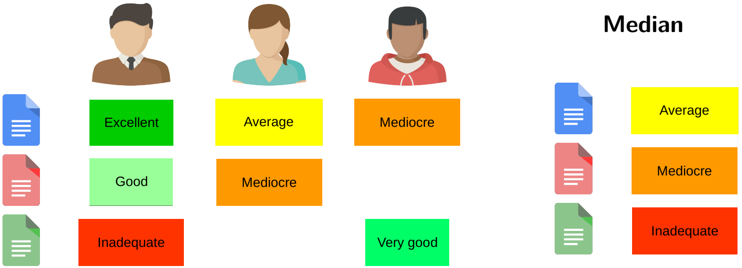

winner: 

In the last few years MJ has being adopted by a progressively larger number of french political parties including: Le Parti Pirate, Génération(s), LaPrimaire.org, France Insoumise and La République en Marche. [7]

LaPrimaire.org is a french political initiative whose goal is to select an independent candidate for the french presidential election using MJ as voting rule.

The procedure consists of two rounds:

1: each voter expresses her judgment on five random candidates. The five ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the five finalists. The one with the best median is the winner

The procedure consists of two rounds:

1: each voter expresses her judgment on k random candidates. The k ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the k finalists. The one with the best median is the winner

Incomplete Knowledge

Remark

If a winner of the complete profile is among the k finalists then it will also be a winner of the incomplete profile

Theorem

There exist an incomplete profile and one of its completion that do not share the same sets of winners

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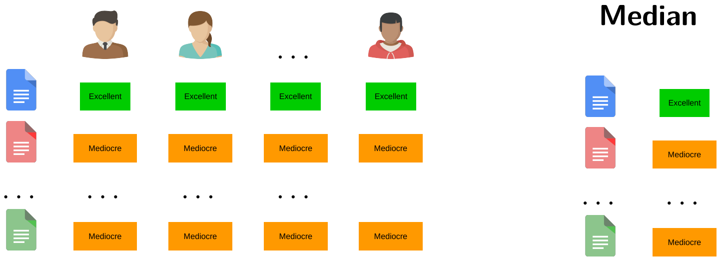

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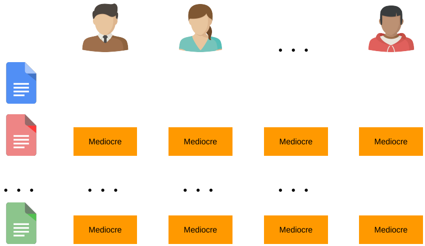
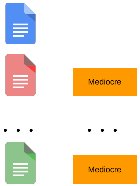


Median



Probability of missing the winner

Theorem

By asking each voter to evaluate k equiprobably picked alternatives, the probability that an alternative j is never asked about is $(1 - \frac{k}{m})^n$

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For $k = 3$, $n = m = 10$ this is $0.7^{10} \approx 0.0282\%$

Probability of missing the winner

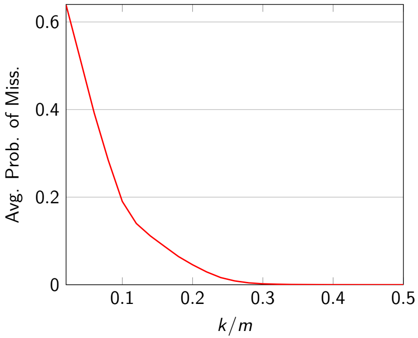
Theorem

By asking each voter to evaluate k equiprobably picked alternatives, the probability that an alternative j is never asked about is $(1 - \frac{k}{m})^n$

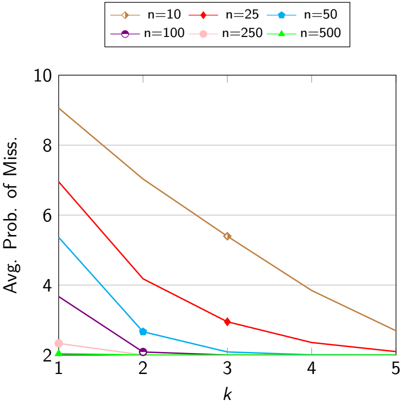
What about real scenarios?

Experimental results

(a) Avg prob. of missing the winner under uniform distribution of preferences, for $n = 100, m = 50$ and $k \in [\![1, 25]\!]$



(b) Avg prob. of missing the winner using a real case distribution of preferences, given $m = 12$ several n and $k \in [\![1, 5]\!]$



Outline

Compromising as an equal loss principle

Setting: Several voters express their preferences over a set of alternatives

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Goal:Find a procedure determining a collective choice that promotes a notion of compromise

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Approach:

Olivier Cailloux, Beatrice Napolitano, and M. Remzi Sanver. Compromising as an equal loss principle.
Review of Economic Design, May 2022

- Define a compromise from an equal loss perspective
- Propose classes of rules reflecting this concept

Conclusions

For at least three voters and no restrictions on Σ:

	ECC	PCC
Condorcet procedures	No	No
Scoring rules	No	No
Antiplurality	No	Yes
BK compromises	No	No
Fallback bargaining	No	Yes (PC)

Conclusions

For at least three voters and with restrictions on Σ:

	ECC	PCC
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Related Works

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- **Median Voting Rule**: picks all alternatives receiving a majority of support at the highest possible quality (Bassett and Persky, 1999 [2])

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- **Fallback Bargaining**: bargainers fall back to less and less preferred alternatives until they reach a unanimous agreement (Brams and Kilgour, 2001 [4])
- **q-approval FB**: picks the alternatives which receive the support of q voters at the highest possible quality, breaking ties according to the quantity of support

Motivation: A simple example

$n = 100, \mathcal{A} = \{a, b, c\}$

$$\begin{array}{lccccc} \mathbf{51} & a & \succ & b & \succ & c \\ \mathbf{49} & c & \succ & b & \succ & a \end{array}$$

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- FB_q, $q \in \{1, ..., \frac{n}{2} + 1\}$: $\{a\}$

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Does b seem a better compromise?

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

Notation

Losses

$$
\begin{array}{ccc}
P & \qquad & \lambda_P \\
v_1 : a \succ b \succ c & & a : (0,2) \\
v_2 : c \succ b \succ a & & b : (1,1) \\
 & & c : (2,0)
\end{array}
$$

Given $P = (\succ_i)_{i \in N}$:

- $\lambda_{\succ_i}(x) = |\{y \in \mathcal{A} \mid y \succ_i x\}| \in [\![0, m-1]\!]$ the loss of i when choosing $x \in \mathcal{A}$ instead of her favorite alternative
- $\lambda_P(x)$ associates to each voter her loss when choosing x

Notation

Losses

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+$ a spread measure

Notation

Losses

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector
$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+$ a spread measure

Σ is the set of spread measures σ such that

$$\sigma(l) = 0 \iff l_i = l_j,\ \forall i, j \in N, \quad \forall l \in [\![0, m-1]\!]^N$$

.

Given $X \subseteq \mathcal{A}$

$$\arg\min_X(\sigma \circ \lambda_P) = \{x \in X \mid \forall y \in X : \sigma(\lambda_P(x)) \leq \sigma(\lambda_P(y))\}$$

$\arg\min_X(\sigma \circ \lambda_P)$ denotes the alternatives in X whose loss vectors are the most equally distributed according to σ

$$
\begin{array}{c}
P \\
v_1 : a \succ b \succ c \\
v_2 : c \succ b \succ a
\end{array}
\qquad\qquad
\begin{array}{c}
\lambda_P \\
a : (0,2) \\
b : (1,1) \\
c : (2,0)
\end{array}
$$

$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{b\} \quad \forall \sigma \in \Sigma$$

Egalitarian compromises

An SCR is Egalitarian Compromise Compatible iff at each profile, it selects some "less unequal" alternatives

Egalitarian compromise compatibility

An SCR f is ECC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) \neq \emptyset$$

Egalitarian compromises and Pareto dominance

ECC rules are *very* egalitarian

P	λ_P
$v_1 : a \succ c \succ b$	$a : (0, 1)$
$v_2 : c \succ a \succ b$	$b : (2, 2)$
	$c : (1, 0)$

$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{b\} \quad \forall \sigma \in \Sigma$$

Egalitarian compromises and Pareto dominance

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Theorem

ECC $\cap$ *Paretian* $= \emptyset$ *(for $n, m \geq 2$)*

Egalitarian compromises and Pareto dominance

ECC rules are *very* egalitarian

$$\begin{array}{cc} P & \lambda_P \\ \begin{array}{l} v_1 : a \succ c \succ b \\ v_2 : c \succ a \succ b \\ \\ \end{array} & \begin{array}{r} a : (0,1) \\ b : (2,2) \\ c : (1,0) \end{array} \end{array}$$

$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{b\} \quad \forall \sigma \in \Sigma$$

Theorem

ECC $\cap$ *Paretian* $= \emptyset$ *(for $n, m \geq 2$)*

$f \in \text{ECC} \Rightarrow b \in f(P)$, $f \in \text{Paretian} \Rightarrow b \notin f(P)$

Paretian compromises

An SCR is Paretian Compromise Compatible iff at each profile, it selects some “less unequal” alternatives *among the Pareto optimal ones*

Paretian compromise compatibility

An SCR f is PCC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \underset{PO(P)}{\arg\min}(\sigma \circ \lambda_P) \neq \emptyset$$

Paretian compromises

An SCR is Paretian Compromise Compatible iff at each profile, it selects some "less unequal" alternatives *among the Pareto optimal ones*

Paretian compromise compatibility

An SCR f is PCC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{PO(P)}(\sigma \circ \lambda_P) \neq \emptyset$$

Recall:

Egalitarian compromise compatibility

An SCR f is ECC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) \neq \emptyset$$

FB and AP are PCC

Theorem

FB and Antiplurality are PCC. *(for $n, m \geq 3$)*

Proof sketch.

Define $\sigma^{\text{discrete}}(l) = 1 \iff l$ is not constant.
If some $a \in PO(P)$ has a constant loss vector, e. g.

$a_1 \succ a_2 \succ a_3 \succ a_4$
$a_3 \succ a_2 \succ a_1 \succ a_4$

there is exactly one such alternative, $FB(P) = \{a\}$ and it is never last so $a \in AP(P)$.
Otherwise, σ does not discriminate among $PO(P)$, thus Paretianism suffices. □

Restricting Σ

Definition (Condition $C_{m,n}$)

Given $m \geq 4, n \geq \max\{4, m-1\}$, σ satisfies condition $C_{m,n}$ iff $\sigma(m-3, m-1, m-2, \ldots, m-2) < \sigma(m-2, m-3, \ldots, 1, 0, \ldots, 0)$.

$$
\begin{array}{lllllll}
v_1: & & & x & y & a_1 \\
v_2: & & & y & & x \\
v_3: & & y & & x & a_2 \\
v_4: & y & & & x & a_3
\end{array}
$$

Requires that:
$(\sigma \circ \lambda_P)(x) < (\sigma \circ \lambda_P)(y)$

Restricting Σ

Theorem

Under condition $C_{m,n}$, AP and FB are not PCC.

Proof for $m = 5, n = 4$.

$$\begin{array}{lllllll} v_1 : & & & x & y & a_1 \\ v_2 : & & & y & & x \\ v_3 : & & y & & x & a_2 \\ v_4 : & y & & & x & a_3 \end{array}$$

- y is the only alternative never last, thus for both rules: $f(P) = \{y\}$
- $(\sigma \circ \lambda_P)(x) < (\sigma \circ \lambda_P)(y)$
- and $x \in PO(P)$, thus $y \notin \arg\min_{PO(P)}(\sigma \circ \lambda_P)$ $\square$

Other results

Theorem

Condorcet consistent rules are neither ECC nor PCC *(for $m, n \geq 3$)*

Theorem

Scoring rules,except AP, are neither ECC nor PCC *(for $m \geq 3$ and large enough n)*

Theorem

FB_q rules with $q \in [\![1, n-1]\!]$ are neither ECC nor PCC *(for $m, n \geq 3$)*

Outline

C

onsidering a classical model in which the preferences of a set of voters over a set of alternatives are known, we defined two classes of voting rules able to reflect a notion of compromise in which egalitarianism, in the sense of conceding equally, is a major concern.

M

oreover, we stepped back from this standard perspective in which preferences are assumed to be known from the beginning and investigated the problem of preference elicitation in different settings.

Thank You!

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Empirical Evaluation

Pessimistic reaching "low enough" regret

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \leq n/10}$	$q_a^{MMR \leq n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 | 5.0 | 5.8]	5.3	[5.4 | 6.2 | 7.2]
m10n20	10	20	0.0	[13.9 | 16.1 | 18.4]	32.0	[19.7 | 21.8 | 24.7]
m11n30	11	30	0.0	[16.6 | 19.0 | 22.3]	45.2	[23.1 | 25.7 | 28.9]
tshirts	11	30	0.0	[13.1 | 16.6 | 19.6]	43.2	[28.2 | 32.0 | 35.6]
courses	9	146	0.0	[6.0 | 7.0 | 7.0]	0.0	[6.8 | 7.0 | 7.0]
m14n9	14	9	5.4	[30.3 | 33.5 | 36.7]	64.1	[37.6 | 40.5 | 44.3]
skate	14	9	0.0	[11.4 | 11.6 | 12.3]	0.0	[11.5 | 11.8 | 12.8]
m15n30	15	30	0.0	[25.0 | 29.5 | 33.7]		

Empirical Evaluation

Pessimistic chair first and then agents (and vice-versa)

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	ca $\pm$ sd	ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0