Simultaneous Elicitation of Committee and Voters' Preferences

B. Napolitano[1], O. Cailloux[1] and P. Viappiani[2]

[1] LAMSADE, Université Paris-Dauphine, Paris, France
[2] LIP6, Sorbonne Université, Paris, France

Advances in Economic Design : Games, voting, information and measurement
28 November 2019

Scenario

Setting: Incompletely specified profile and positional scoring rule

Scenario

Setting: Incompletely specified profile and positional scoring rule

Goal: Development of an incremental elicitation protocol based on minimax regret

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

Why?

- Voters: difficult or costly to order *all* alternatives
- Committee: difficult to *specify* a voting rule precisely and abstractly

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

Why?

- Voters: difficult or costly to order *all* alternatives
- Committee: difficult to *specify* a voting rule precisely and abstractly

How?

- *Minimax regret*: given the current knowledge, the alternatives with the lowest worst-case regret are selected as tied winners

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

Context

A	alternatives, $\lvert A \rvert = m$
N	voters
$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$	complete preferences profile
$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$	(convex) scoring vector that the committee has in mind

Context

A	alternatives, $\lvert A \rvert = m$
N	voters
$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$	complete preferences profile
$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$	(convex) scoring vector that the committee has in mind

W defines a Positional Scoring Rule $f_W(P) \subseteq A$ using scores $s^{W,P}(a),\ \forall\ a \in A$

Context

A alternatives, $|A| = m$

N voters

$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$ complete preferences profile

$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$ (convex) scoring vector that the committee has in mind

W defines a Positional Scoring Rule $f_W(P) \subseteq A$ using scores $s^{W,P}(a),\ \forall\ a \in A$

P and W exist in the minds of voters and committee but unknown to us

Questions

Questions

Questions to the voters

Comparison queries that ask a particular voter to compare two alternatives $a, b \in A$

$$a \succ_j b \quad ?$$

Questions

Questions to the voters

Comparison queries that ask a particular voter to compare two alternatives $a, b \in A$

$$a \succ_j b \quad ?$$

Questions to the committee

Queries relating the difference between the importance of consecutive ranks from r to $r + 2$

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2}) \quad ?$$

Our Knowledge

The answers to these questions define C_P and C_W that is our knowledge about P and W

Our Knowledge

The answers to these questions define C_P and C_W that is our knowledge about P and W

- $C_P \subseteq \mathcal{P}$ constraints on the profile given by the voters

Our Knowledge

The answers to these questions define C_P and C_W that is our knowledge about P and W

- $C_P \subseteq \mathcal{P}$ constraints on the profile given by the voters
- $C_W \subseteq \mathcal{W}$ constraints on the voting rule given by the committee

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b.

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P, C_W}(a, b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b.

We select the alternative which minimizes the maximal regret

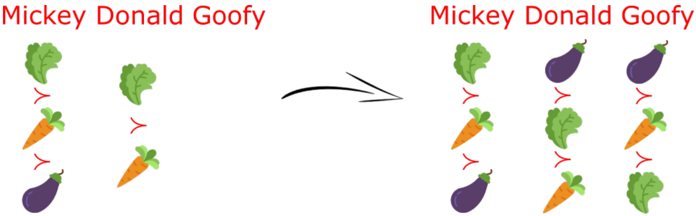
Pairwise Max Regret Computation

The computation of PMR^{C_P,C_W}(🥬, 🍆) can be seen as a game in which an adversary both:

Pairwise Max Regret Computation

The computation of $\mathrm{PMR}^{C_P,C_W}($, $)$ can be seen as a game in which an adversary both:

- **chooses a complete profile P** $\in \mathcal{P}$



- **chooses a feasible weight vector W** $\in \mathcal{W}$

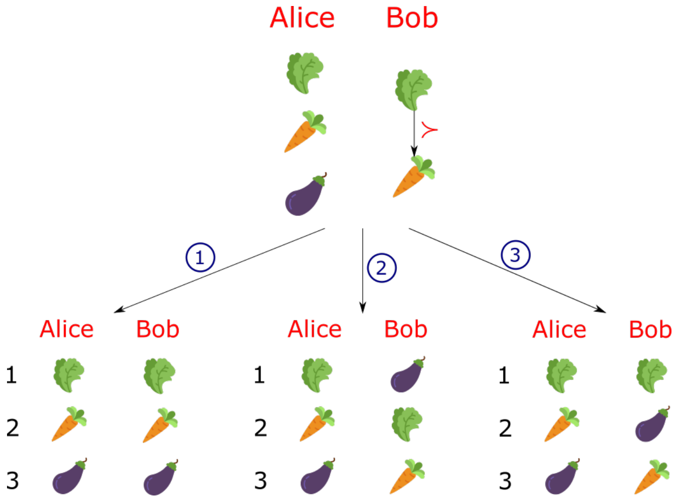
$$\mathbf{(1, ?, 0)} \longrightarrow \mathbf{(1, 0, 0)}$$

in order to maximize the difference of scores

Computing Minimax Regret: Example

Profile completion

Consider the following partial profile



Computing Minimax Regret: Example

Weight selection

Consider the following constraints on the scoring vector given by the committee

Computing Minimax Regret: Example

Minimax computing

Consider the completion number and the weight vector .

MR(🥕) = max {
- PMR(🥕🥬) = 19 → v = 3 w={10,1,0}
- PMR(🥕🍆) = 9 → v = 2 w={10,1,0}

MR(🥬) = $\boxed{-1}$

MR(🍆) = 20

$\boxed{\text{MMR}}$ = -1 → winner 🥬

Elicitation strategies

Elicitation strategies

- **Random**: equiprobably draws a question among the set of the possible ones;

Elicitation strategies

- **Random**: equiprobably draws a question among the set of the possible ones;
- **Extreme completions**: choses the question that reduces the most the uncertainty;

Elicitation strategies

- **Random**: equiprobably draws a question among the set of the possible ones;
- **Extreme completions**: choses the question that reduces the most the uncertainty;
- **Pessimistic**: selects the question that leads to minimal regret in the worst case;

Elicitation strategies

- **Random**: equiprobably draws a question among the set of the possible ones;
- **Extreme completions**: choses the question that reduces the most the uncertainty;
- **Pessimistic**: selects the question that leads to minimal regret in the worst case;
- **Two phase**: it asks a predefined sequence of questions to the committee and then it only asks questions about the voters.

Thank You!

C. Boutilier, R. Patrascu, P. Poupart, and D. Schuurmans.
Constraint-based Optimization and Utility Elicitation using the Minimax Decision Criterion.
Artifical Intelligence, 170(8–9):686–713, 2006.

Tyler Lu and Craig Boutilier.
Robust approximation and incremental elicitation in voting protocols.
In *Proceedings of IJCAI 2011*, pages 287–293, 2011.

William E. Stein, Philip J. Mizzi, and Roger C. Pfaffenberger.
A stochastic dominance analysis of ranked voting systems with scoring.
European Journal of Operational Research, 74(1):78 – 85, 1994.

Paolo Viappiani.
Positional scoring rules with uncertain weights.
In *Scalable Uncertainty Management - 12th International Conference, SUM 2018, Milan, Italy, October 3-5, 2018, Proceedings*, pages 306–320, 2018.