Compromise and elicitation in social choice

A study of egalitarianism and incomplete information in voting

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LAMSADE
UMR CNRS 7243
laboratoire d 'analyse et modélisation de systèmes pour l'aide à la décision

Classical setting

Agents = { , ,  }, Altern. = { , ,  }, Chair =  $\Rightarrow$ Voting Rule

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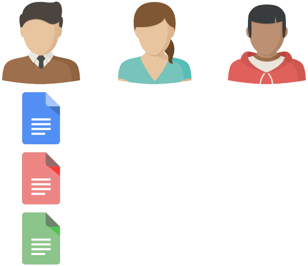

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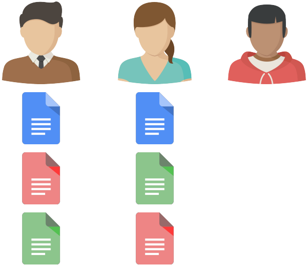

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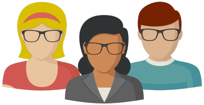

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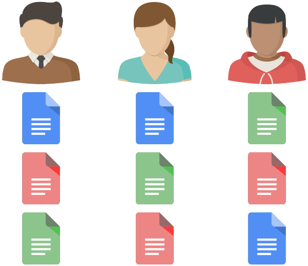

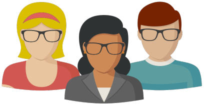

Borda

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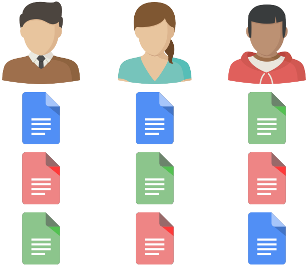

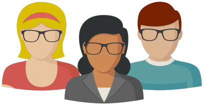

Borda

winner: 

Incomplete knowledge about profile

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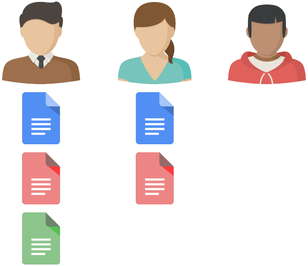

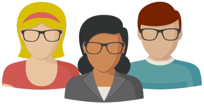

Borda

winner: ?

Incomplete knowledge about voting rule

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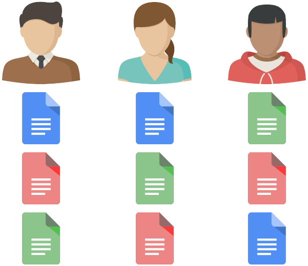

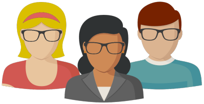

?

winner: ?

Research Question I:
Incomplete knowledge about profile and voting rule

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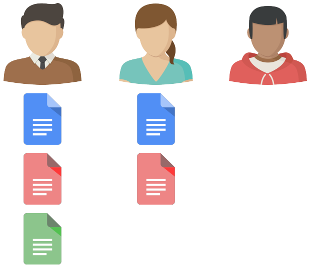


?

winner: ?

Research Question II:

Incomplete knowledge under Majority Judgment

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule

, Good   , Excell.

, Aver.   , To rej.

, Med.

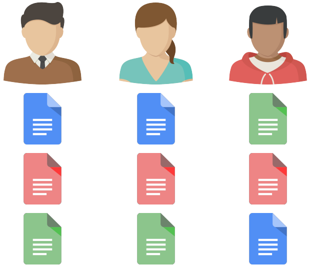
Majority Judgment

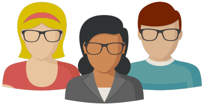
winner: ?

Research Question III:

Compromise from an equal-loss perspective

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule





What is a compromise?

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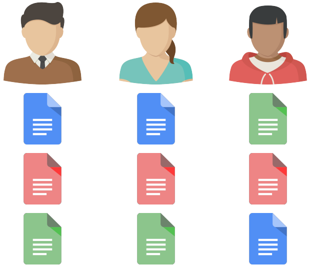

What is a compromise?

maybe  ?

Outline

1. Notation
2. Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination
3. Majority Judgment winner determination under incomplete information
4. Compromising as an equal loss principle
5. Conclusions

Outline

Notation

$\mathcal{A}$	set of alternatives, $\lvert\mathcal{A}\rvert = m$
N	set of voters, $\lvert N\rvert = n$
$\mathcal{L}(\mathcal{A})$	set of all linear orderings given $\mathcal{A}$
$\succ_i \in \mathcal{L}(\mathcal{A})$	preference ranking of voter $i \in N$
$P = (\succ_1, \ldots, \succ_n) \in \mathcal{L}(\mathcal{A})^N$	a profile
$\mathscr{P}^*(\mathcal{A})$	possible winners (non-empty subsets of $\mathcal{A}$)
$f : \mathcal{L}(\mathcal{A})^N \to \mathscr{P}^*(\mathcal{A})$	a Social Choice Rule

Outline

Related Works

Incomplete profile

- and known rule: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [6]; *Boutilier et al. 2006*, [4])

Uncertain rule

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [9])
- considering positional scoring rules (*Viappiani 2018*, [10])

Simultaneous Elicitation of Scoring Rule and Agent Preferences for Robust Winner Determination

Setting: Incompletely specified preferences and social choice rule

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Approach:

Beatrice Napolitano, Olivier Cailloux, and Paolo Viappiani. Simultaneous elicitation of scoring rule and agent preferences for robust winner determination.
In *Proceedings of Algorithmic Decision Theory - 7th International Conference, ADT 2021*

- Develop query strategies that interleave questions to the chair and to the agents
- Use *Minimax regret* to measure the quality of those strategies

Notation

A	alternatives, $\lvert A\rvert = m$
N	agents (*voters*)
$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$	complete preferences profile
$W = (W_r,\ 1 \le r \le m),\ W \in \mathcal{W}$	**convex** scoring vector that the chair has in mind

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W defines a **Positional Scoring Rule** $f_W(P) \subseteq A$ using scores $s^{W,P}(a),\ \forall\ a \in A$

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W defines a **Positional Scoring Rule** $f_W(P) \subseteq A$ using scores $s^{W,P}(a),\ \forall\ a \in A$

P and W exist in the minds of agents and chair but unknown to us

Questions

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Questions to the agents

Comparison queries that ask a particular agent j to compare two alternatives $a, b \in \mathcal{A}$

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Questions to the agents

Comparison queries that ask a particular agent j to compare two alternatives $a, b \in \mathcal{A}$

Questions to the chair

Queries relating the difference between the importance of consecutive ranks from r to $r + 2$

The answers to these questions define $\mathbf{C_P}$ and $\mathbf{C_W}$ that is our knowledge about P and W

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P, C_W}(a, b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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$$\mathrm{MMR}^{C_P, C_W} = \min_{a \in \mathcal{A}} \mathrm{MR}^{C_P, C_W}(a)$$

We select the alternative that *minimizes* the maximum regret

Elicitation strategies

At each step, the strategy selects a question to ask either to one of the agents about her preferences or to the chair about the voting rule

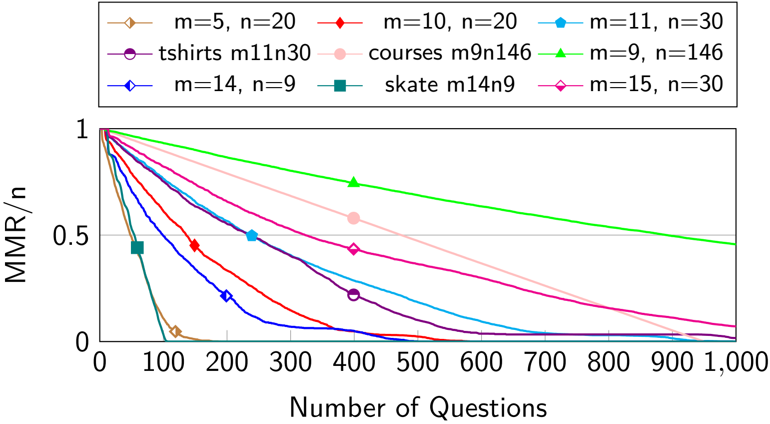
The termination condition could be:

- when the minimax regret is lower than a threshold
- when the minimax regret is zero

- It selects first $n + (m - 2)$ candidate questions: one per each agent and one per each rank (excluding the first and the last one which are known)

- It selects the question that leads to minimal regret in the worst case from the set candidate questions

Figure: Average MMR (normalized by n) after k questions with Pessimistic strategy for different datasets.



Building concrete questions for the chair

Queries relating the difference between the importance of consecutive ranks

$$W_r - W_{r+1} \geq \lambda(W_{r+1} - W_{r+2}) \quad ?$$

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Queries relating the difference between the importance of consecutive ranks

$$W_2 - W_3 \geq 2\,(W_3 - W_4) \quad ?$$

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$$\begin{aligned} W_2 \ - \ W_3 &\geq 2\ W_3 - 2\ W_4 \\ W_2 + 2\ W_4 &\geq 3\ W_3 \\ s(a) &\geq s(b) \end{aligned}$$

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$$s(a) \geq s(b)$$

#1	#2
–	–
a	–
b	**b**
–	**a**

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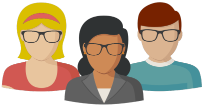
#**1**	#**2**	#**3**	#**3**
c	*d*	*a*	*b*
a	*c*	*b*	*a*
b	**b**	*c*	*d*
d	**a**	*d*	*c*

Outline

Incomplete knowledge: profile

Agents = { , , }, Altern. = { , , }, Chair = $\Rightarrow$ Voting Rule



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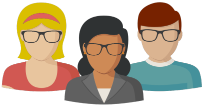

Majority Judgment

Majority Judgment

Voters judges candidates assigning grades from an ordinal scale. The winner is the candidate with the highest median of the grades received. (Balinski and Laraki 2011, [2])

Excellent	Very good	Good	Average	Mediocre	Inadequate	To be Rejected

Majority Judgment

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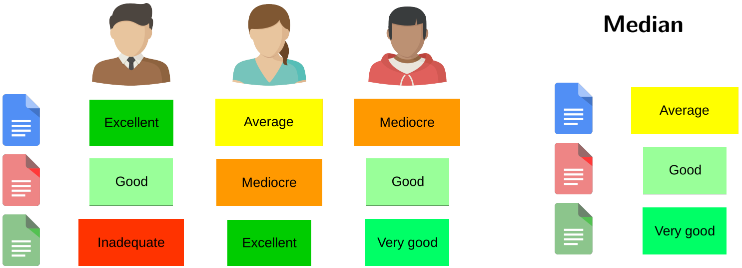

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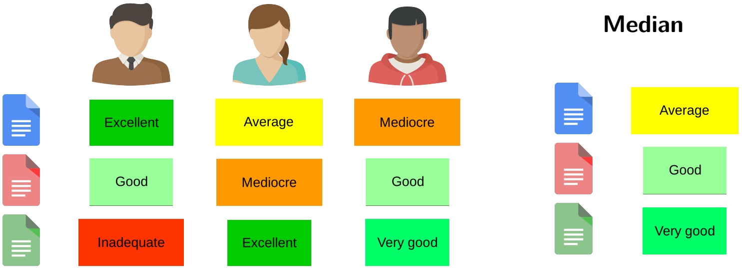

winner: 

Majority Judgment: Incomplete Knowledge

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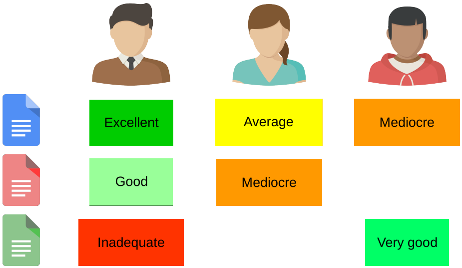

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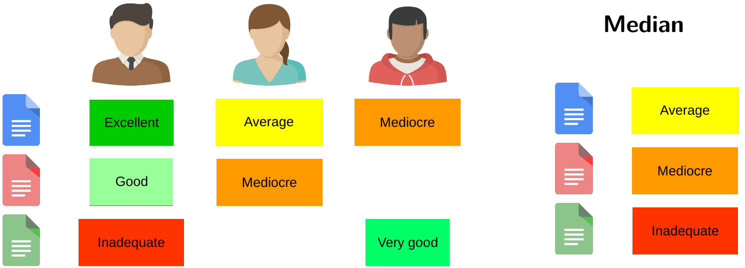

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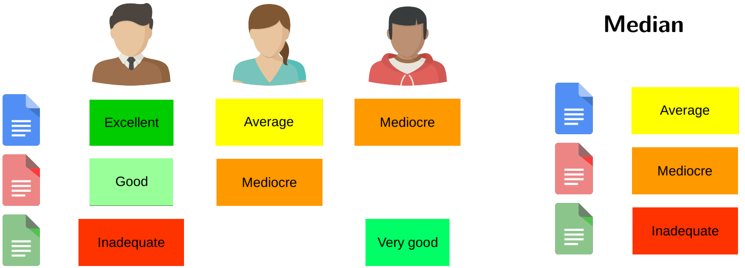

winner:

In the last few years MJ has being adopted by a progressively larger number of french political parties including: Le Parti Pirate, Génération(s), LaPrimaire.org, France Insoumise and La République en Marche. [1]

LaPrimaire.org is a french political initiative whose goal is to select an independent candidate for the french presidential election using MJ as voting rule.

The procedure consists of two rounds:

1: each voter expresses her judgment on five random candidates. The five ones with the highest medians qualify for the second round

2: each voter expresses her judgment on all the five finalists. The one with the best median is the winner

Majority Judgment
LaPrimaire.org

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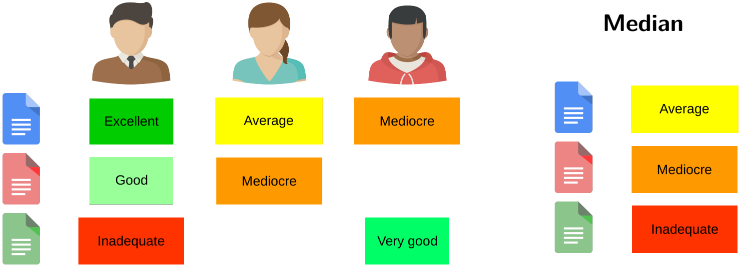

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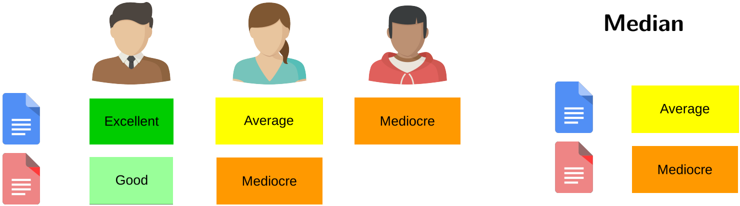

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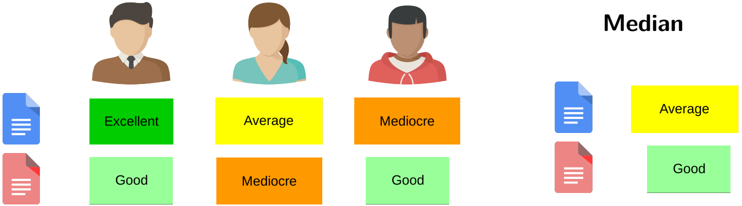

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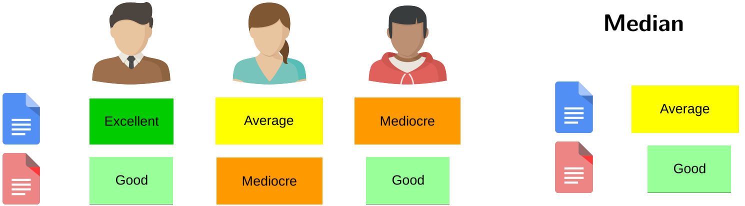

winner: 

Research Questions

- What is the probability of selecting a winner different from the one selected in case of complete knowledge?
- Can we elicit voters preferences using a minimax regret notion?
- What is the best trade-off between communication cost and optimal result?
- What is the voting rule applied on the resulting incomplete profile? What are its properties?

Outline

Setting: Several voters express their preferences over a set of alternatives

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Goal: Find a procedure determining a collective choice that promotes a notion of compromise

- **Plurality**: selects the alternatives considered as best by the highest number of voters

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- **Fallback Bargaining**: bargainers fall back to less and less preferred alternatives until they reach a unanimous agreement (Brams and Kilgour, 2001 [5])
- **q-approval FB**: picks the alternatives which receive the support of q voters at the highest possible quality, breaking ties according to the quantity of support

Motivation: A simple example

$n = 100, \mathcal{A} = \{a, b, c\}$

$$\begin{array}{lccccc} \mathbf{51} & a & \succ & b & \succ & c \\ \mathbf{49} & c & \succ & b & \succ & a \end{array}$$

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- Plurality: $\{a\}$
- MVR: $\{a\}$
- MC: $\{a\}$
- FB: $\{b\}$
- FB_q, $q \in \{1, ..., \frac{n}{2} + 1\}$: $\{a\}$

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Does b seem a better compromise?

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

Notation

Losses

$$
\begin{array}{ccc}
P & \qquad & \lambda_P \\
v_1 : a \succ b \succ c & & a : (0,2) \\
v_2 : c \succ b \succ a & & b : (1,1) \\
 & & c : (2,0)
\end{array}
$$

Given $P = (\succ_i)_{i \in N}$:

- $\lambda_{\succ_i}(x) = |\{y \in \mathcal{A} \mid y \succ_i x\}| \in [\![0, m-1]\!]$ the loss of i when choosing $x \in \mathcal{A}$ instead of her favorite alternative
- $\lambda_P(x)$ associates to each voter her loss when choosing x

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector

Notation
Losses

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector
$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+$ a spread measure

$\lambda_P : \mathcal{A} \to [\![0, m-1]\!]^N$ a loss vector
$\sigma : [\![0, m-1]\!]^N \to \mathbb{R}^+$ a spread measure

Σ is the set of spread measures σ such that

$$\sigma(l) = 0 \iff l_i = l_j,\ \forall i, j \in N, \quad \forall l \in [\![0, m-1]\!]^N$$

.

Given $X \subseteq \mathcal{A}$

$$\arg\min_{X}(\sigma \circ \lambda_P) = \{x \in X \mid \forall y \in X : \sigma(\lambda_P(x)) \leq \sigma(\lambda_P(y))\}$$

$\arg\min_X(\sigma \circ \lambda_P)$ denotes the alternatives in X whose loss vectors are the most equally distributed according to σ

$$
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\end{array}
$$

$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{b\} \quad \forall \sigma \in \Sigma$$

Egalitarian compromises

An SCR is Egalitarian Compromise Compatible iff at each profile, it selects some "less unequal" alternatives

Egalitarian compromise compatibility

An SCR f is ECC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) \neq \emptyset$$

Egalitarian compromises and Pareto dominance

ECC rules are *very* egalitarian

P	λ_P
$v_1 : a \succ c \succ b$	$a : (0, 1)$
$v_2 : c \succ a \succ b$	$b : (2, 2)$
	$c : (1, 0)$

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Theorem

ECC $\cap$ *Paretian* $= \emptyset$ *(for* $n, m \geq 2$*)*

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$$\arg\min_{\mathcal{A}}(\sigma \circ \lambda_P) = \{b\} \quad \forall \sigma \in \Sigma$$

Theorem

ECC $\cap$ *Paretian* $= \emptyset$ *(for $n, m \geq 2$)*

$f \in \text{ECC} \Rightarrow b \in f(P)$, $f \in \text{Paretian} \Rightarrow b \notin f(P)$

Paretian compromises

An SCR is Paretian Compromise Compatible iff at each profile, it selects some "less unequal" alternatives *among the Pareto optimal ones*

Paretian compromise compatibility

An SCR f is PCC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \underset{PO(P)}{\arg\min}(\sigma \circ \lambda_P) \neq \emptyset$$

Paretian compromises

An SCR is Paretian Compromise Compatible iff at each profile, it selects some "less unequal" alternatives *among the Pareto optimal ones*

Paretian compromise compatibility

An SCR f is PCC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{PO(P)} (\sigma \circ \lambda_P) \neq \emptyset$$

Recall:

Egalitarian compromise compatibility

An SCR f is ECC iff

$$\exists \sigma \in \Sigma \mid \forall P \in \mathcal{L}(\mathcal{A})^N : f(P) \cap \arg\min_{\mathcal{A}} (\sigma \circ \lambda_P) \neq \emptyset$$

FB and AP are PCC

Theorem

FB and Antiplurality are PCC. *(for $n, m \geq 3$)*

Proof sketch.

Define $\sigma^{\text{discrete}}(l) = 1 \iff l$ is not constant.
If some $a \in PO(P)$ has a constant loss vector, e. g.

$a_1 \succ a_2 \succ a_3 \succ a_4$
$a_3 \succ a_2 \succ a_1 \succ a_4$

there is exactly one such alternative, $FB(P) = \{a\}$ and it is never last so $a \in AP(P)$.
Otherwise, σ does not discriminate among $PO(P)$, thus Paretianism suffices. □

Restricting Σ

Definition (Condition $C_{m,n}$)

Given $m \geq 4, n \geq \max\{4, m-1\}$, σ satisfies condition $C_{m,n}$ iff $\sigma(m-3, m-1, m-2, \ldots, m-2) < \sigma(m-2, m-3, \ldots, 1, 0, \ldots, 0)$.

$$\begin{array}{lllllll} v_1: & & & x & y & a_1 \\ v_2: & & & y & & x \\ v_3: & & y & & x & a_2 \\ v_4: & y & & & x & a_3 \end{array}$$

Requires that:
$(\sigma \circ \lambda_P)(x) < (\sigma \circ \lambda_P)(y)$

Theorem

Under condition $C_{m,n}$, AP and FB are not PCC.

Proof for $m = 5, n = 4$.

$$
\begin{array}{lccccc}
v_1: & & & x & y & a_1 \\
v_2: & & & y & & x \\
v_3: & & y & & x & a_2 \\
v_4: & y & & & x & a_3
\end{array}
$$

- y is the only alternative never last, thus for both rules: $f(P) = \{y\}$
- $(\sigma \circ \lambda_P)(x) < (\sigma \circ \lambda_P)(y)$
- and $x \in PO(P)$, thus $y \notin \arg\min_{PO(P)}(\sigma \circ \lambda_P)$ □

Other results

Theorem

Condorcet consistent rules are neither ECC nor PCC (for $m, n \geq 3$)

Theorem

Scoring rules,except AP, are neither ECC nor PCC (for $m \geq 3$ and large enough n)

Theorem

FB_q *rules with* $q \in [\![1, n-1]\!]$ *are neither ECC nor PCC* (for $m, n \geq 3$)

Outline

Thank You!

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Empirical Evaluation

Pessimistic reaching "low enough" regret

Table: Questions asked by Pessimistic strategy on several datasets to reach $\frac{n}{10}$ regret, columns 4 and 5, and zero regret, last two columns.

dataset	m	n	$q_c^{MMR \leq n/10}$	$q_a^{MMR \leq n/10}$	$q_c^{MMR=0}$	$q_a^{MMR=0}$
m5n20	5	20	0.0	[4.3 | 5.0 | 5.8]	5.3	[5.4 | 6.2 | 7.2]
m10n20	10	20	0.0	[13.9 | 16.1 | 18.4]	32.0	[19.7 | 21.8 | 24.7]
m11n30	11	30	0.0	[16.6 | 19.0 | 22.3]	45.2	[23.1 | 25.7 | 28.9]
tshirts	11	30	0.0	[13.1 | 16.6 | 19.6]	43.2	[28.2 | 32.0 | 35.6]
courses	9	146	0.0	[6.0 | 7.0 | 7.0]	0.0	[6.8 | 7.0 | 7.0]
m14n9	14	9	5.4	[30.3 | 33.5 | 36.7]	64.1	[37.6 | 40.5 | 44.3]
skate	14	9	0.0	[11.4 | 11.6 | 12.3]	0.0	[11.5 | 11.8 | 12.8]
m15n30	15	30	0.0	[25.0 | 29.5 | 33.7]		

Empirical Evaluation

Pessimistic chair first and then agents (and vice-versa)

Table: Average MMR in problems of size $(10, 20)$ after 500 questions, among which q_c to the chair.

q_c	ca $\pm$ sd	ac $\pm$ sd
0	0.6 ± 0.5	0.6 ± 0.5
15	0.5 ± 0.5	0.5 ± 0.5
30	0.3 ± 0.5	0.3 ± 0.4
50	0.0 ± 0.1	0.0 ± 0.1
100	0.1 ± 0.2	0.1 ± 0.1
200	2.3 ± 1.4	2.1 ± 1.8
300	5.2 ± 2.4	6.8 ± 0.6
400	10.9 ± 0.9	12.2 ± 1.0
500	20.0 ± 0.0	20.0 ± 0.0