Simultaneous Elicitation of Committee and Voters' Preferences

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Scenario

Setting: Incompletely specified profile and positional scoring rule

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Goal: Development of an incremental elicitation protocol based on minimax regret

Motivation and approach

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How?

- *Minimax regret*: given the current knowledge, the alternatives with the lowest worst-case regret are selected as tied winners

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

Context

A alternatives, $|A| = m$

N voters

$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$ complete preferences profile

$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$ (convex) scoring vector that the committee has in mind

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P and W exist in the minds of voters and committee but unknown to us

Questions

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Comparison queries that ask a particular voter to compare two alternatives $a, b \in A$

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Questions to the committee

Queries relating the difference between the importance of consecutive ranks from r to $r+2$

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2}) \quad ?$$

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- $C_P \subseteq \mathcal{P}$ constraints on the profile given by the voters
- $C_W \subseteq \mathcal{W}$ constraints on the voting rule given by the committee

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b.

We select the alternative which minimizes the maximal regret

Pairwise Max Regret Computation

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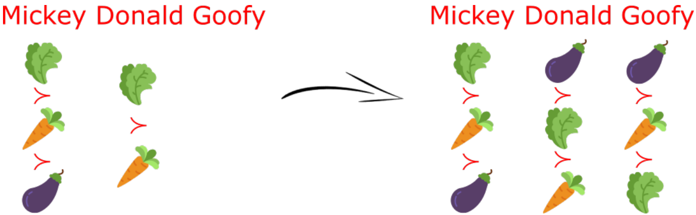
- **chooses a complete profile P** $\in \mathcal{P}$



- **chooses a feasible weight vector W** $\in \mathcal{W}$

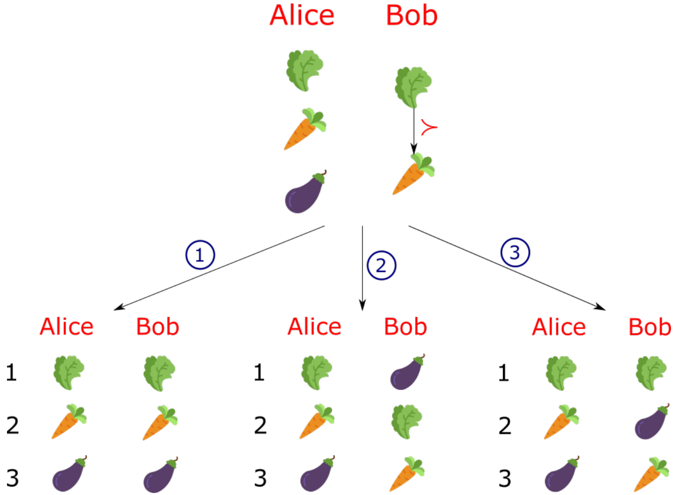
$$(\mathbf{1}, ?, \mathbf{0}) \longrightarrow (\mathbf{1}, \mathbf{0}, \mathbf{0})$$

in order to maximize the difference of scores

Computing Minimax Regret: Example

Profile completion

Consider the following partial profile



Computing Minimax Regret: Example

Weight selection

Consider the following constraints on the scoring vector given by the committee

$$w_1 \geq 2 \cdot w_2$$
$$w_2 > w_3$$
$$w_1 - w_2 \geq w_2 - w_3$$

Computing Minimax Regret: Example

Minimax computing

MR() = max {

PMR( ) = 19 → v = 3   w={10,1,0}

PMR( ) = 9 → v = 2   w={10,1,0}

MR() = $\boxed{-1}$

MR() = 20

$\boxed{\text{MMR}}$ = -1 → winner 

Elicitation strategies

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- **Extreme completions**: choses the question that reduces the most the uncertainty;
- **Pessimistic**: selects the question that leads to minimal regret in the worst case;
- **Two phase**: it asks a predefined sequence of questions to the committee and then it only asks questions about the voters.

Thank You!

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