Simultaneous Elicitation of Committee and Voters' Preferences

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Classical Scenario

Setting: Voters specify preferences over alternatives and a committee defines the social choice rule to aggregate them

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Goal: Find a consensus choice

Our Scenario

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Goal: Develop an incremental elicitation strategy to acquire the most relevant information

Motivation and approach

Who?

- Imagine to be an *external observer* helping with the voting procedure

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- Committee: difficult to *specify* a voting rule precisely and abstractly

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Why?

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What?

- We want to reduce uncertainty, inferring (*eliciting*) the true preferences of voters and committee, in order to quickly converge to an optimal or a near-optimal alternative

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Approach

- Develop query strategies that interleave questions to the committee and questions to the voters
- Use *Minimax regret* to measure the quality of those strategies

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Assumptions

- We consider *positional scoring rules*, which attach weights to positions according to a scoring vector w
- We assume w to be *convex*

$$w_i - w_{i+1} \geq w_{i+1} - w_{i+2} \qquad \forall i$$

Related Works

Incomplete profile

- and known weights: Minimax regret to produce a robust winner approximation (*Lu and Boutilier 2011*, [2]; *Boutilier et al. 2006*, [1])

Uncertain weights

- and complete profile: dominance relations derived to eliminate alternatives always less preferred than others (*Stein et al. 1994*, [3])
- in positional scoring rules (*Viappiani 2018*, [4])

Context

A	alternatives, $\lvert A \rvert = m$
N	voters
$P = (\succ_j,\ j \in N),\ P \in \mathcal{P}$	complete preferences profile
$W = (\boldsymbol{w}_r,\ 1 \leq r \leq m),\ W \in \mathcal{W}$	(convex) scoring vector that the committee has in mind

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P and W exist in the minds of voters and committee but unknown to us

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Comparison queries that ask a particular voter to compare two alternatives $a, b \in A$

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Questions to the committee

Queries relating the difference between the importance of consecutive ranks from r to $r + 2$

$$w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2}) \quad ?$$

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- $C_W \subseteq \mathcal{W}$ constraints on the voting rule given by the committee

Minimax Regret

Given $C_P \subseteq \mathcal{P}$ and $C_W \subseteq \mathcal{W}$:

$$\mathrm{PMR}^{C_P,C_W}(a,b) = \max_{P \in C_P, W \in C_W} s^{P,W}(b) - s^{P,W}(a)$$

is the maximum difference of score between a and b under all possible realizations of the full profile *and* weights

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We care about the worst case loss: *maximal regret* between a chosen alternative a and best real alternative b

We select the alternative which minimizes the maximal regret

Pairwise Max Regret Computation

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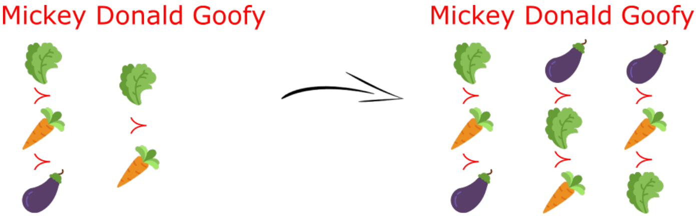

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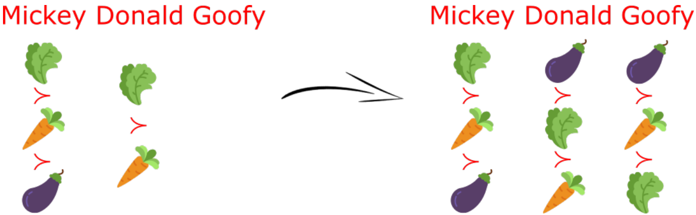


- **chooses a feasible weight vector W** $\in \mathcal{W}$

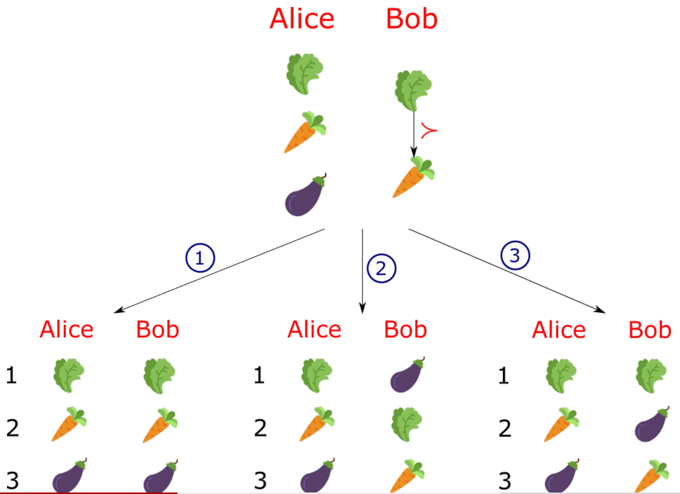
$$(\mathbf{1}, ?, \mathbf{0}) \longrightarrow (\mathbf{1}, \mathbf{0}, \mathbf{0})$$

in order to maximize the difference of scores

Computing Minimax Regret: Example

Profile completion

Consider the following partial profile



Computing Minimax Regret: Example

Weight selection

Consider the following constraint on the scoring vector given by the committee

$$w_1 \geq 2 \cdot w_2$$

and the convex assumption

$$w_1 - w_2 \geq w_2 - w_3$$

Computing Minimax Regret: Example

Minimax computing

MR(🥕) = max {
- PMR(🥕🥬) = 19 → v = 3 w={10,1,0}
- PMR(🥕🍆) = 9 → v = 2 w={10,1,0}

MR(🥬) = $\boxed{-1}$

MR(🍆) = 20

$\boxed{\text{MMR}}$ = -1 → winner 🥬

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The termination condition could be:

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- when the minimax regret is zero

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Decides with a probability of $\frac{1}{2}$ each whether to ask a question about

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- **weights**: it draws a rank $2 \leq r \leq m-2$ equiprobably, takes λ as the middle of the interval of values we are still uncertain about, and asks whether $w_r - w_{r+1} \geq \lambda(w_{r+1} - w_{r+2})$

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- **a preference ordering**: it draws equiprobably a voter whose preference is not known entirely and two alternatives that are incomparable in our current knowledge

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Pessimistic Strategy

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Assume that a question leads to the possible new knowledge states (C_P^1, C_W^1) and (C_P^2, C_W^2) depending on the answer, then the badness of the question in the worst case is:

$$\max_{i=1,2} \mathrm{MMR}(C_P^i, C_W^i)$$

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Note:

if the maximal MMR of two questions are equal, then prefers the one with the lowest MMR values associated to the opposite answer

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Two Phase Strategy

- **phase one:** asks a predefined sequence of $m - 2$ questions to the committee in order to gather informations about the weights (one per rank except the extremes)
- **phase two:** asks only questions to the voters, using the mechanism defined in the Pessimistic strategy

Elicitation strategies

Extreme completions Strategy

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Let x^* be the minimax regret optimal alternative, and $\bar{y}$, $\bar{P}$ and $\bar{W}$ the instantiations that, given the choice of x^*, maximize the regret. Define:

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$$\tau_{P_i} = \min_{\hat{P}_i \in C_P} s^{\hat{P}_i,\bar{W}}(\bar{y}) - s^{\hat{P}_i,\bar{W}}(x^*)$$

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The extreme completions strategy asks a question to whoever minimizes τ

Preliminary Results

k	Rnd± sd	T. ph.± sd	Pes.± sd
0	5.0±0	5.0±0	5.0±0
5	4.9±0.2	5.0±0.0	4.2±0.3
10	4.8±0.2	4.4±0.3	3.4±0.5
15	4.3±0.6	3.7±0.3	2.7±0.5
20	3.9±0.3	2.7±0.5	1.6±0.8
25	3.5±0.8	2.2±0.7	0.8±0.6
30	3.0±0.8	1.5±1.1	0.5±0.7

Table: Minimax regret in problems of size $(5, 5)$ after k questions.

Thank You!

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