

Python编程与人工智能实践

算法篇: 无监督聚类

k-means 与 GMM (高斯混合模型)

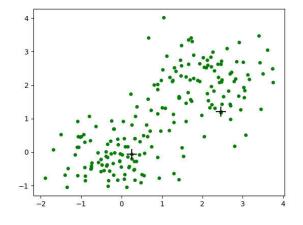
于泓 鲁东大学 信息与电气工程学院 2021.3.20



K-means (k均值聚类)

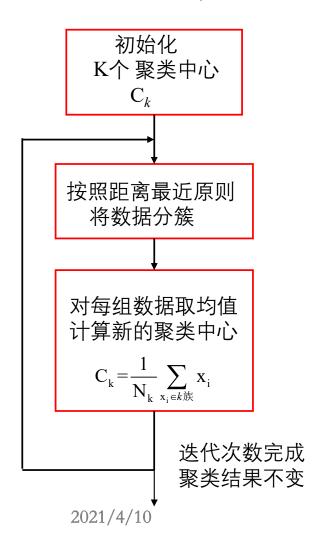
- 与有监督学习相比,无监督学习的样本没有任何标记。 无监督学习的算法需要自动找到这些没有标记的数据里 面的数据结构和特征。
- 聚类:把数据集分成一个个的簇 (cluster)

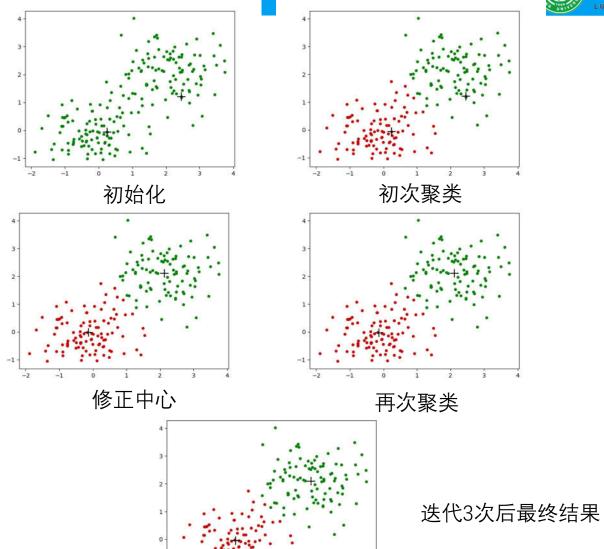
自动把数据分堆?





K-mean 算法





3



```
# 构造聚类中心
# dataset [N,D]
# K 聚类中心的数目 [K,D]
                                                            def creat centers(dataset,K):
def creat centers (dataset, K):
                                                                N,D = np.shape(dataset)
    val max = np.max(dataset,axis=0)
                                                                index = np.random.permutation(N)
    val min = np.min(dataset,axis=0)
                                                                centers = dataset[index[:2],:]
    centers = np.linspace(val min, val max, num=K+2)
                                                                 return centers
    return centers[1:-1,:]
                                                                      随机取值
      在X的值域内均匀取值
# keams 绘图
                                                           随机取K个点
# dataset (N,D)
# lab (N,)
# dic colors K 种颜色
# centers (K,D)
def draw kmeans (dataset, lab, centers, dic colors=None, name="0.jpg"):
   plt.cla()
                                                                                  绘图函数
   vals lab = set(lab.tolist())
                                                                                 显示中间结果
   for i,val in enumerate(vals lab):
       index = np.where(lab==val)[0]
       sub dataset = dataset[index,:]
       plt.scatter(sub dataset[:,0],sub dataset[:,1],s=16., color=dic colors[i])
   for i in range(np.shape(centers)[0]):
       plt.scatter(centers[i,0],centers[i,1],color="k",marker="+",s = 200.)
   plt.savefig(name)
    2021/4/10
```

```
def run kmeans(dataset,K, m = 20,dic colors=None, b draw=False):
   N,D = np.shape (dataset)
   # print(N,D)
   # 确定初始化聚类中心
   centers = creat centers (dataset, K)
   lab = np.zeros(N)
   if b draw:
                                                                          对聚类中心进行更新
       draw kmeans(dataset, lab, centers, dic colors, name="int.jpg")
                                                        # 绘图
   # 进行m轮迭代
                                                        if b draw:
   labs = np.zeros(N) # 初始聚类结果
                                                            draw kmeans (dataset, labs new, centers,
   for it in range(m):
                                                                       dic colors,name=str(it)+" oldcenter.jpg")
       # 计算每个点距离中心的距离
       distance = np.zeros([N,K])
                                                        # 计算新的聚类中心
       for k in range(K):
                                                        for k in range(K):
           center = centers[k,:]
                                                            index = np.where(labs new==k)[0]
                                                            centers[k,:] = np.mean(dataset[index,:],axis=0)
           # 计算欧式距离
           diff = np.tile(center, (N, 1)) - dataset
                                                        # 绘图
           sgrDiff = diff ** 2
                                                        if b draw:
           sqrDiffSum = sqrDiff.sum(axis=1)
                                                            draw kmeans (dataset, labs new, centers,
           distance[:,k] = sqrDiffSum
                                                                       dic colors,name=str(it)+" newcenter.jpg")
       # 距离排序,进行聚类
                                                        # 如果聚类结果和上次相同,退出
       labs new = np.argmin(distance,axis=1)
                                                        if np.sum(labs new-labs) == 0:
       error = np.sum(np.min(distance,axis=1))/N
                                                            return labs new
       print("第 %d 次聚类 距离误差 %.2f"%(it,error))
                                                            labs = labs new
         按照聚类中心进行聚类
                                                    return labs
```

2021/4/10

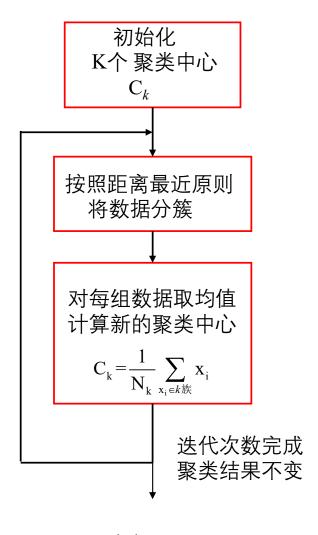
5

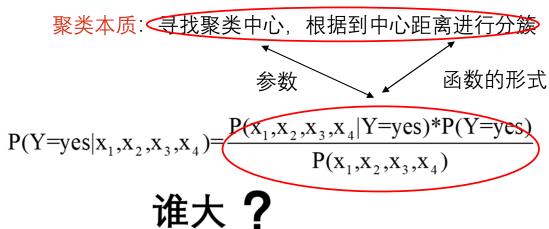


```
if __name__ =="__main__":
    a = np.random.multivariate_normal([2,2], [[.5,0],[0,.5]], 100)
    b = np.random.multivariate_normal([0,0], [[0.5,0],[0,0.5]], 100)
    dataset = np.r_[a,b]
    lab_ture = np.r_[np.zeros(100),np.ones(100)].astype(int)
    dic_colors={0:(0.,0.5,0.),1:(0.8,0,0)}
    labs = run_kmeans(dataset,K=2, m = 20,dic_colors=dic_colors,b_draw=True)

    yuhong@admin2:/home/sdo/machinelearning/k-mean$ python test_kmeans.py
    200 2
    第 0 次聚类 距离误差 5.47
    第 1 次聚类 距离误差 1.42
    第 2 次聚类 距离误差 0.97
    第 3 次聚类 距离误差 0.97
```







$P(Y=no|x_1,x_2,x_3,x_4) = \frac{P(x_1,x_2,x_3,x_4|Y=no)*P(Y=no)}{P(x_1,x_2,x_3,x_4)}$

利用函数
$$f(\mathbf{x}; C_k) = e^{-(\mathbf{x}-C_k)(x-C_k)^T}$$

拟合第C簇数据的分布



高斯分布

高斯分布(正态分布)是一个常见的连续概率分布,在统计领域中有非常重要的作用

$$N(x; \mu, \delta) = \frac{1}{\delta \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\delta^2}}$$

(多维)

$$N(\mathbf{x}; \mathbf{m}, \Sigma) = \frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x} - \mathbf{m})\Sigma^{-1}(\mathbf{x} - \mathbf{m})^{T}}$$

 \mathbf{x} : [1, D]

m:[1,D]

 Σ : [D, D]

2021/4/10

如何利用高斯函数来拟合一组数据的分布?

如何求解高斯分布的参数?

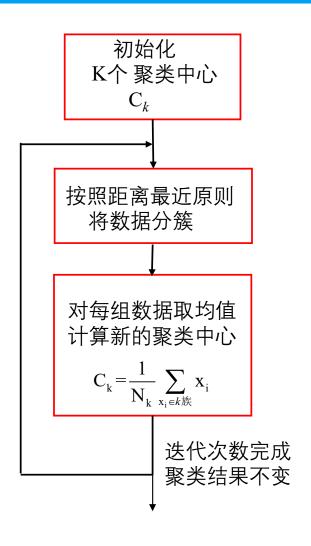
假设有一组数据 xi, 令下式最大:

上式的解为:

$$\mathbf{m} = \frac{1}{N} \sum_{i=1}^{N} \mathbf{x}_{i}$$

$$\Sigma = \frac{1}{N} \sum_{i=1}^{N} (\mathbf{x}_{i} - \mathbf{m}) (\mathbf{x}_{i} - \mathbf{m})^{T}$$





利用加权的高斯函数拟合第k簇数据的分布

$$W_{k}N(x;m_{k},\Sigma_{k}) \sum_{k=1}^{K} W_{k}=1$$

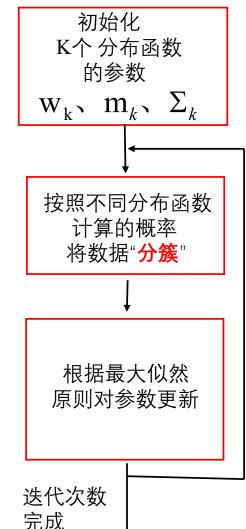
$$P(Y|X) = \frac{P(X|Y)*P(Y)}{P(X)}$$

$$W_{k}N(x_{i};m_{k},\Sigma_{k})$$

$$r_{ik} = \frac{\mathbf{w}_k N(\mathbf{x}_i; \mathbf{m}_k, \boldsymbol{\Sigma}_k)}{\sum_{k=1}^K \mathbf{w}_k N(\mathbf{x}_i; \mathbf{m}_k, \boldsymbol{\Sigma}_k)}$$

$$\mathbf{m}_{k} = \frac{\sum_{i=1}^{N} \mathbf{r}_{ik} \mathbf{x}_{i}}{\sum_{i=1}^{N} \mathbf{r}_{ik}}, \quad \sum_{k} = \frac{\sum_{i=1}^{N} \mathbf{r}_{ik} (\mathbf{x}_{i} - \mathbf{m}_{k}) (\mathbf{x}_{i} - \mathbf{m}_{k})^{T}}{\sum_{i=1}^{N} \mathbf{r}_{ik}}$$

$$\mathbf{w}_{k} = \frac{\sum_{i=1}^{N} \mathbf{r}_{ik}}{\mathbf{N}},$$





```
import matplotlib.pyplot as plt
                                                           # GMM 参数初始化
                                                           # dataset: [N,D] 训练数据
# 计算一个高斯的pdf
                                                           # K: 高斯成分的个数
# x: 数据 [N,D]
                                                          def inti GMM (dataset, K):
# sigma 方差 [D,D]
                                                               N,D = np.shape(dataset)
# mu 均值 [1,D]
                                                               val max = np.max(dataset,axis=0)
def getPdf(x,mu,sigma,eps = 1e-12):
                                                               val min = np.min(dataset,axis=0)
    N,D = np.shape(x)
                                                               centers = np.linspace(val min, val max, num=K+2)
    if D==1:
                                                               mus = centers[1:-1,:]
         sigma = sigma+eps
                                                               sigmas = np.array([0.5*np.eye(D) for i in range(K)])
         A = 1.0 / (sigma)
                                                               ws = 1.0/K * np.ones(K)
         det = np.fabs(sigma[0])
    else:
                                                               return mus, sigmas, ws
         sigma = sigma + eps*np.eye(D)
         A = np.linalq.inv(sigma)
         det = np.fabs(np.linalq.det(sigma))
    # 计算系数
    factor = (2.0 * np.pi)**(D / 2.0) * (det)**(0.5)
    # 计算 pdf
    dx = x - mu
    pdf = [(np.exp(-0.5*np.dot(np.dot(dx[i],A),dx[i]))+eps)/factor for i in range(N)]
    return pdf
                                                     N(\mathbf{x}; \mathbf{m}, \Sigma) = \frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x} - \mathbf{m})\Sigma^{-1}(\mathbf{x} - \mathbf{m})^{T}}
```

import numpy as np

```
def train GMM step(dataset, mus, sigmas, ws):
    N,D = np.shape (dataset)
    K,D = np.shape(mus)
    # 计算样本在每个成分上的pdf
    pdfs = np.zeros([N,K])
    for k in range(K):
        pdfs[:,k] = getPdf(dataset,mus[k],sigmas[k])
    # 获取r
    r = pdfs*np.tile(ws, (N, 1))
    r sum = np.tile(np.sum(r,axis=1,keepdims=True),(1,K))
   r = r/r_sum
   ·# 进行参数的更新
   for k in range(K):
       r k = r[:,k]
       N k = np.sum(r k)
       r k = r k[:,np.newaxis] #[N,1]
       # 更新mu
       mu = np.sum(dataset*r k,axis=0)/N k #[D,1]
       # 更新sigma
       dx = dataset - mu
       sigma = np.zeros([D,D])
       for i in range(N):
           sigma = sigma + r k[i,0]*np.outer(dx[i],dx[i])
       sigma = sigma/N k
       # 更新w
       w = N k/N
       mus[k]= mu
       sigmas[k]=sigma
       ws[k]=w
   return mus, sigmas, ws
```



$$r_{ik} = \frac{\mathbf{w}_k N(\mathbf{x}_i; \mathbf{m}_k, \boldsymbol{\Sigma}_k)}{\sum_{k=1}^K \mathbf{w}_k N(\mathbf{x}_i; \mathbf{m}_k, \boldsymbol{\Sigma}_k)}$$

$$m_{k} = \frac{\sum_{i=1}^{N} r_{ik} x_{i}}{\sum_{i=1}^{N} r_{ik}},$$

$$\sum_{k=1}^{N} r_{ik} (x_{i} - m_{k}) (x_{i} - m_{k})^{T}$$

$$\sum_{k=1}^{N} r_{ik} (x_{i} - m_{k}) (x_{i} - m_{k})^{T}$$

$$\mathbf{w}_{k} = \frac{\sum_{i=1}^{N} \mathbf{r}_{ik}}{\mathbf{N}},$$



GMM 训练

```
def train_GMM(dataset,K=2,m=10):
    mus,sigmas,ws = inti_GMM(dataset,K)

for i in range(m):
    print("Step ",i)
    mus,sigms,ws,r =train_GMM_step(dataset,mus,sigmas,ws)
    return mus,sigms,ws
```

计算输入数据在每个高斯成分上

的似然值

```
def getlogPdfFromeGMM(datas,mus,sigmas,ws):
    N,D = np.shape(datas)
    K,D = np.shape(mus)

weightedlogPdf = np.zeros([N,K])

for k in range(K):
    temp = getPdf(datas,mus[k],sigmas[k],eps = 1e-12)
    weightedlogPdf[:,k] = np.log(temp) + np.log(ws[k])

return weightedlogPdf, np.sum(weightedlogPdf,axis=1)
```

利用GMM进行聚类(比较似然值)

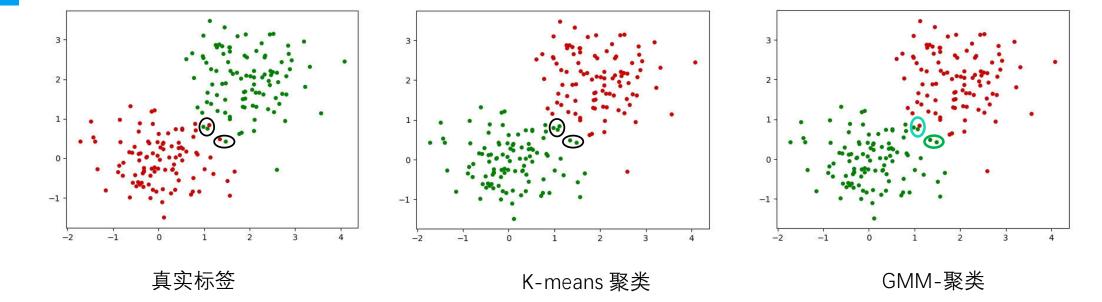
```
def clusterByGMM(datas,mus,sigmas,ws):
    weightedlogPdf,_ = getlogPdfFromeGMM(datas,mus,sigmas,ws)
    labs = np.argmax(weightedlogPdf,axis=1)
    return labs
```



```
def draw cluster(dataset, lab, dic colors, name="0.jpg"):
    plt.cla()
    vals lab = set(lab.tolist())
    for i,val in enumerate(vals lab):
        index = np.where(lab==val)[0]
        sub dataset = dataset[index,:]
        plt.scatter(sub dataset[:,0],sub dataset[:,1],s=16., color=dic colors[i])
                                            if __name__=="__main__":
    plt.savefig(name)
                                                    聚类测试 1
                                                1 1 1
                                                dic colors=\{0: (0.,0.5,0.), 1: (0.8,0,0)\}
                          测试程序
                                                a = np.random.multivariate normal([2,2], [[.5,0],[0,.5]], 100)
                          比较 GMM
                                                b = np.random.multivariate normal([0,0], [[0.5,0],[0,0.5]], 100)
                                                dataset = np.r [a,b]
                          与k-means
                                                lab ture = np.r [np.zeros(100), np.ones(100)].astype(int)
  yuhong@admin2:/home/sdo/machinelearning/GMM$ python (
    0.01103085 -0.0445471
                                                # 训练GMM
   mus, sigmas, ws = train GMM (dataset, K=2, m=10)
   [[0.52244123 0.11045438]
                                                print(mus)
   [0.11045438 0.46578895]]
                                                print(sigmas)
   [[0.43072345 0.00740219]
                                                print(ws)
   [0.00740219 0.43643166]]]
                                                # 进行聚类
  [0.50825447 0.49174553]
                                                labs GMM = clusterByGMM (dataset, mus, sigmas, ws)
  第 0 次聚类 距离误差 0.97
    1 次聚类 距离误差 0.89
                                                # k-menas 比较
    2 次聚类 距离误差 0.89
                                                labs kmeans = run kmeans (dataset, K=2, m = 20)
                                                # 画结果
                                                draw cluster (dataset, lab ture, dic colors, name="c ture1.jpg")
                                                draw cluster(dataset, labs GMM, dic colors, name="c GMM1.jpg")
```

draw cluster (dataset, labs kmeans, dic colors, name="c kmeans1.jpg")





$$f(\mathbf{x}; C_k) = e^{-(\mathbf{x}-C_k)(\mathbf{x}-C_k)^T} \qquad N(\mathbf{x}; \mathbf{m}, \Sigma) = \frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x}-\mathbf{m})\Sigma^{-1}(\mathbf{x}-\mathbf{m})^T}$$

都默认特征维度 之间弱相关

```
a = np.random.multivariate_normal([2,2], [[.5,0],[0,.5]], 100)
b = np.random.multivariate_normal([0,0], [[0.5,0],[0,0.5]], 100)
dataset = np.r [a,b]
```

TTT



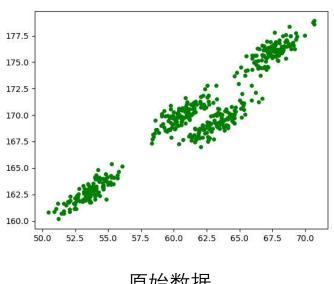
```
聚类测试 2
1.1.1
with open('Clustering qmm.csv','r') as f:
    lines = f.read().splitlines()[1:]
lines = [ line.split(",") for line in lines]
dataset = np.array(lines).astype(np.float)
lab ture = np.ones(np.shape(dataset)[0])
dic colors=\{0:(0.,0.5,0.),1:(0.8,0,0),2:(0.5,0.5,0),3:(0,0.5,0.5)\}
# 训练GMM
mus, sigmas, ws = train GMM (dataset, K=1, m=100)
# 进行聚类
labs GMM = clusterByGMM(dataset, mus, sigmas, ws)
# k-menas 比较
labs kmeans = run kmeans (dataset, dic colors, K=4, m = 20)
# 画结果
draw cluster(dataset,lab ture,dic colors,name="c ture2.jpg")
draw cluster (dataset, labs GMM, dic colors, name="c GMM2.jpg")
draw cluster(dataset, labs kmeans, dic colors, name="c kmeans2.jpg")
```

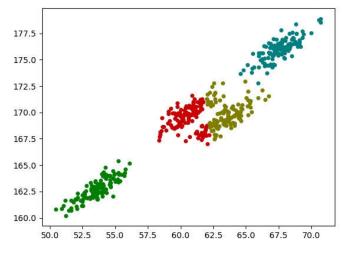
```
📔 Clustering_gmm.csv 🛛 🔡 test_knn.py 🛣 🔡 test_kmeans.py 🔀 🔡 sompak_gu
    Weight, Height
    67.06292382432797,176.08635470037433
    68.80409404055906,178.38866853397775
    60.93086316752808,170.28449576512674
  5 59.73384301263917,168.69199180312273
  6 65.43123003070372,173.7636790317747
    61.577160332549624,168.0917512363048
  8 63.34186626427547,170.64251602924492
  9 61.04164336121978,170.09668165680276
 10 62.63362334719166,171.8629715737515
 11 53.4078596166848,162.756843114429
 12 62.93820030525223,168.71007855359954
 13 68.55485709616576,176.4737467967919
 14 65.16304259531378,171.17658203536783
 15 53.44601710597588,162.91516701320765
 16 60.65937348586088,170.6476590778724
    59.1765543543734,169.19080975295822
 18 67.16384091157683,176.33706853049466
 19 60.62703390449537,169.8481191146251
 20 53.96476394988538,162.49055420493335
 21 60.35999202109157,169.89078752485602
```

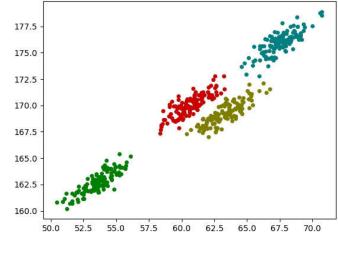
22 59.856871023856,168.85468537336587

```
第 0 次聚类 距离误差 4.70
第 1 次聚类 距离误差 2.48
第 2 次聚类 距离误差 2.34
第 3 次聚类 距离误差 2.28
第 4 次聚类 距离误差 2.27
第 5 次聚类 距离误差 2.27
```









原始数据

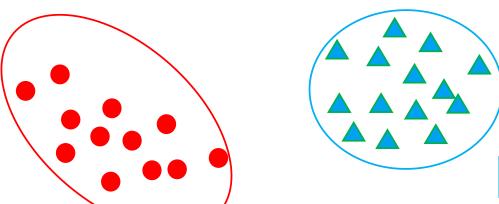
K-means聚类

GMM聚类

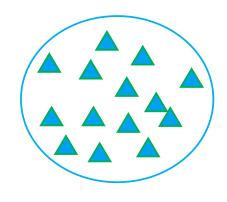


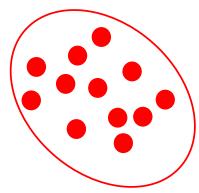
$f_{cir}(X;\lambda) = w_1 N_{cir1}(X;\lambda_{cir1}) + w_2 N_{cir2}(X;\lambda_{cir1})$

GMM 用于分类任务



 $f_{tri}(X;\lambda) = w_1 N_{tri1}(X;\lambda_{tri1}) + w_2 N_{tri2}(X;\lambda_{tri1})$







鸾尾花数据集 (Iris)

```
1 5.1,3.5,1.4,0.2,Iris-setosa
 2 4.9,3.0,1.4,0.2, Iris-setosa
 3 4.7,3.2,1.3,0.2, Iris-setosa
 4 4.6,3.1,1.5,0.2, Iris-setosa
 5 5.0,3.6,1.4,0.2, Iris-setosa
 6 5.4,3.9,1.7,0.4, Iris-setosa
 7 4.6,3.4,1.4,0.3, Iris-setosa
 8 5.0, 3.4, 1.5, 0.2, Iris-setosa
 9 4.4,2.9,1.4,0.2, Iris-setosa
10 4.9,3.1,1.5,0.1, Iris-setosa
11 5.4,3.7,1.5,0.2, Iris-setosa
12 4.8, 3.4, 1.6, 0.2, Iris-setosa
13 4.8,3.0,1.4,0.1, Iris-setosa
14 4.3,3.0,1.1,0.1,Iris-setosa
15 5.8, 4.0, 1.2, 0.2, Iris-setosa
16 5.7,4.4,1.5,0.4, Iris-setosa
17 5.4,3.9,1.3,0.4, Iris-setosa
18 5.1,3.5,1.4,0.3, Iris-setosa
19 5.7,3.8,1.7,0.3, Iris-setosa
20 5.1,3.8,1.5,0.3, Iris-setosa
21 5.4,3.4,1.7,0.2, Iris-setosa
22 5.1, 3.7, 1.5, 0.4, Iris-setosa
23 4.6, 3.6, 1.0, 0.2, Iris-setosa
24 5.1,3.3,1.7,0.5,Iris-setosa
```

数据集分割

```
1 1 1
    分类测试
1 1 1
file data = 'iris.data'
# 数据读取
data = np.loadtxt(file data, dtype = np.float, delimiter = ',',usecols=(0,1,2,3))
lab = np.loadtxt(file data,dtype = str, delimiter = ',',usecols=(4))
# 分为训练集和测试集
N = 150
N \text{ train} = 100
N test = 50
perm = np.random.permutation(N)
index train = perm[:N train]
index test = perm[N train:]
data train = data[index train,:]
lab train = lab[index train]
data test = data[index test,:]
lab test = lab[index test]
```



为每一类花训练一个GMM

```
# 获取 训练标签类型
unique labs = np.unique(lab train).tolist()
models = \{\}
# 进行GMM 训练,为每类数据训练一个GMM
for lab in unique labs:
   # 进行数据筛选
    index = np.where(lab train==lab)[0]
   dataset = data train[index,:]
    # 利用训练的数据训练 GMM
   mus, sigmas, ws = train GMM (dataset, K=2, m=20)
   models[lab]={}
   models[lab]["ws"]=ws
   models[lab]["mus"]=mus
   models[lab]["sigmas"]=sigmas
                                       N right =0
```

结果分析

计算测试数据在每个模型上的似然值

```
# 进测试
            pdfs = np.zeros([N test,len(unique labs)])
            index2lab = {}
            # 计算每条测试数据在不同GMM上的logpdf
            for i, lab in enumerate(unique labs):
                index2lab[i]= lab
                ws = models[lab]["ws"]
                mus = models[lab]["mus"]
                sigmas=models[lab]["sigmas"]
                # 计算每条测试数据在这个GMM上的pdf
                ,pdf = getlogPdfFromeGMM(data test,mus,sigmas,ws)
                pdfs[:,i] = pdf
            # 选取最大似然值 实现分类
            det labs index = np.argmax(pdfs,axis = 1).tolist()
# 将分类结果转为字符串
det labs str = [index2lab[i] for i in det labs index]
# 进行测试结果输出并统计准确率
for i, lab str in enumerate (det labs str):
   print("测试数据 %d 真实标签 %s 检测标签 %s"*(i,lab test[i],lab str))
   if lab str == lab test[i]:
       N \text{ right} = N \text{ right+1}
print("准确率为 %.2f%%"%(N right*100/N test))
```



```
则试数据 18 具头标金 1ris-virginica 检测标金 1ris-virginica
测试数据 19 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 20 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 21 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 22 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 23 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 24 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 25 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 26 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 27 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 28 真实标签 Iris-virginica 检测标签 Iris-virginica
测试数据 29 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 30 真实标签 Iris-virginica 检测标签 Iris-virginica
测试数据 31 真实标签 Iris-virginica 检测标签 Iris-virginica
测试数据 32 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 33 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 34 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 35 真实标签 Iris-virginica 检测标签 Iris-versicolor
测试数据 36 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 37 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 38 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 39 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 40 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 41 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 42 真实标签 Iris-virginica 检测标签 Iris-virginica
测试数据 43 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 44 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 45 真实标签 Iris-versicolor 检测标签 Iris-versicolor
测试数据 46 真实标签 Iris-setosa 检测标签 Iris-setosa
测试数据 47 真实标签 Iris-virginica 检测标签 Iris-virginica
测试数据 48 真实标签 Iris-virginica 检测标签 Iris-virginica
测试数据 49 真实标签 Iris-virginica 检测标签 Iris-virginica
准确率为 98.00%
```

```
检测结果
```

```
{'Iris-setosa': {'ws': array([0.39402107, 0.60597893]), 'mus': array(
[[4.81564238, 3.29694636, 1.38701516, 0.22828414],
       [5.03736233, 3.49279251, 1.51673888, 0.25380625]]), 'sigmas':
array([[[ 0.13375879, 0.10682633, 0.03618834, 0.00809864],
         0.10682633, 0.18285167, 0.03188169, -0.00863784],
                      0.03188169, 0.01343078, 0.00346548],
         0.03618834,
        [0.00809864, -0.00863784, 0.00346548, 0.0069349]],
       [[ 0.04630264, 0.05756617, -0.00830517,
                                                0.009006631,
        [0.05756617, 0.09713673, 0.00572862, 0.01644271],
                      0.00572862, 0.03167733, 0.00408086],
        [-0.00830517,
        [ 0.00900663, 0.01644271, 0.00408086, 0.01373466]]])}, 'Ir
is-versicolor': {'ws': array([0.46999148, 0.53000852]), 'mus': array(
[[5.64017004, 2.71789978, 4.01803327, 1.22334826],
       [6.24832999, 2.82563431, 4.51466062, 1.42693318]]), 'sigmas':
array([[[0.06239092, 0.02193021, 0.07514929, 0.00607901],
        [0.02193021, 0.0311503, 0.03886546, 0.01669104],
        [0.07514929, 0.03886546, 0.18662625, 0.02757355],
        [0.00607901, 0.01669104, 0.02757355, 0.01447079]],
       [[0.16558975, 0.04527669, 0.08135174, 0.00879371],
        [0.04527669, 0.09309925, 0.04504553, 0.03768444],
        [0.08135174, 0.04504553, 0.13301317, 0.05059577],
        [0.00879371, 0.03768444, 0.05059577, 0.03542002]]])}, 'Iris-v
irginica': {'ws': array([0.66787162, 0.33212838]), 'mus': array([[6.3
1165038, 2.98856619, 5.34603353, 2.07506742],
       [7.23001901, 2.95608353, 6.12742422, 1.88250241]]), 'sigmas':
array([[[0.22493145, 0.06224243, 0.12642455, 0.0500324 ],
        [0.06224243, 0.05823747, 0.05723793, 0.04267045],
        [0.12642455, 0.05723793, 0.14490017, 0.05790027],
        [0.0500324, 0.04267045, 0.05790027, 0.06889828]],
       [[0.32173969, 0.16267393, 0.30498326, 0.1131243],
        [0.16267393, 0.20520604, 0.11800457, 0.04541396],
        [0.30498326, 0.11800457, 0.32946502, 0.12940313],
        [0.1131243, 0.04541396, 0.12940313, 0.05878717]]])}
```

训练得到的GMM模型