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Hardware Architectures for Deep Learning

Calculating Data Motion

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Joel Emer and Vivienne Sze

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Massachusetts Institute of Technology
Electrical Engineering & Computer Science



1-D Convolution – Output Stationary



$$O_q = I_{q+s} \times F_s$$

Traversal order (fastest to slowest): S, Q

```
int i[W];      # Input activations
int f[S];      # Filter weights
int o[Q];      # Output activations

for q in [0, Q):
    for s in (0, S):
        o[q] += i[q+s]*f[s]
```

Output Stationary

Tensor: F[S]

Rank: S

0 1 2

8	5	2
---	---	---

Tensor: I[W]

Rank: W

0 1 2 3 4 5 6 7

1	1	2	3	3	2	7	6
---	---	---	---	---	---	---	---

Tensor: O[Q]

Rank: Q

0 1 2 3 4 5

0	0	0	0	0	0
---	---	---	---	---	---

An ISL Set

Input:

```
x = 5
Y = 3
```

```
set1 = isl.Set(f'{{ SetXY[x, y] :
    0 <= x <= {X-1} and
    0 <= y <= {Y-1} }}')
```

```
printSet(set1)
```

Space Name

Coordinates

Coordinate specification/constraints

Output:

```
set1 = { SetXY[x, y] : 0 <= x <= 4 and 0 <= y <= 2 }
```

set1

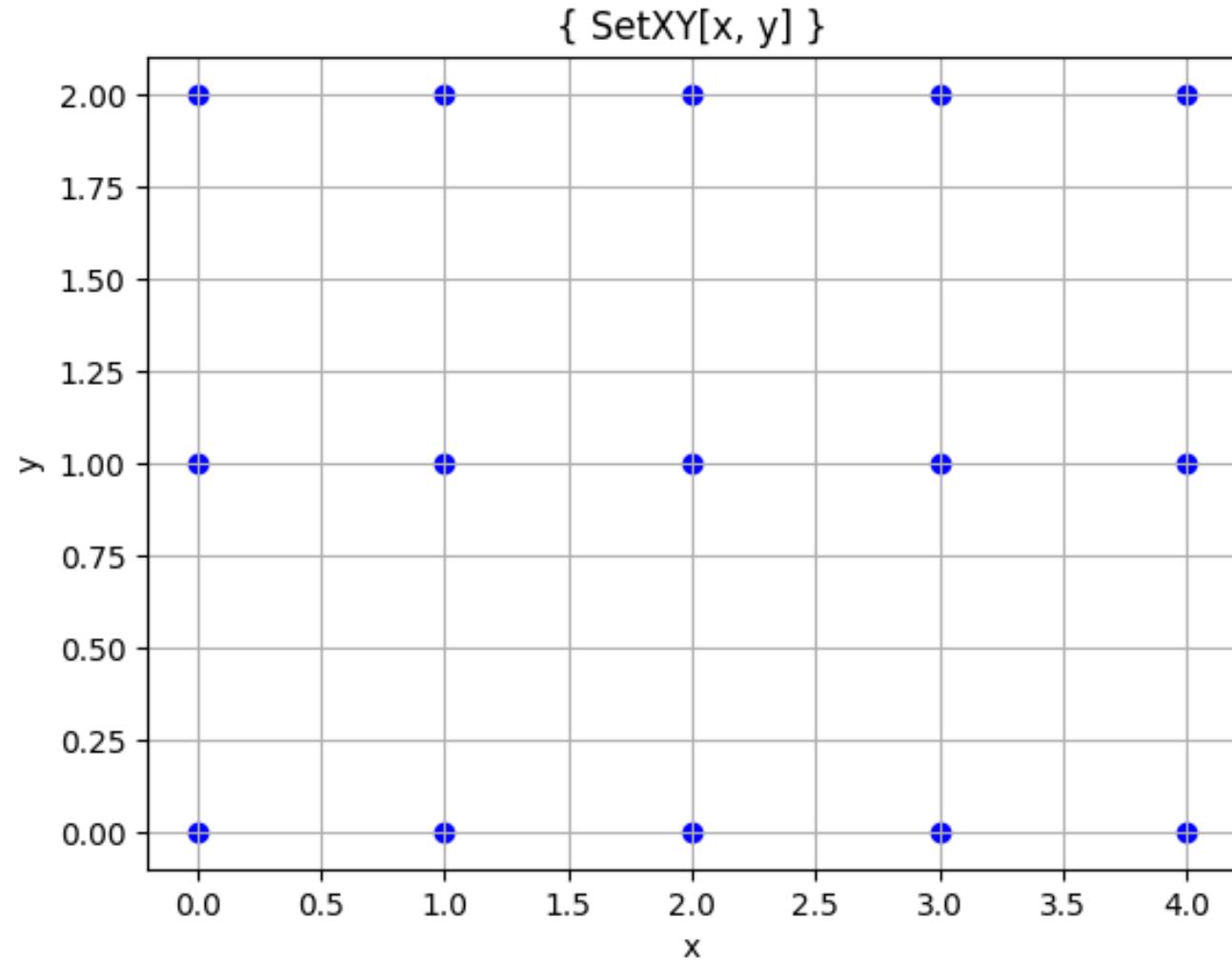
```
{ SetXY[0, 0]
{ SetXY[1, 0]
{ SetXY[2, 0]
{ SetXY[3, 0]
{ SetXY[4, 0]
{ SetXY[0, 1]
{ SetXY[1, 1]
{ SetXY[2, 1]
{ SetXY[3, 1]
{ SetXY[4, 1]
{ SetXY[0, 2]
{ SetXY[1, 2]
{ SetXY[2, 2]
{ SetXY[3, 2]
{ SetXY[4, 2]
```

An ISL Set

Input:

```
X = 5  
Y = 3  
  
set1 = isl.Set(f'{{ SetXY[x, y] :  
    0 <= x <= {X-1} and  
    0 <= y <= {Y-1} }}')  
  
plotSet(set1)
```

```
set1 = { SetXY[x, y] : 0 <= x <= 4 and 0 <= y <= 2 }
```



A different ISL Set

```
set1 = isl.Set(f'{{ SetXY[x, y] :  
    0 <= x <= {X-1} and  
    0 <= y <= {Y-1} }}')
```

Input:

```
set2 = isl.Set(f'{{ SetWX[x, y] :  
    0 <= x <= {X-1} and  
    0 <= y <= {Y-1} }}')
```

Output:

$\text{SetXY} = \{ \text{SetWX}[x, y] : 0 \leq x \leq 4 \text{ and } 0 \leq y \leq 2 \}$



Set has a different “spacename”

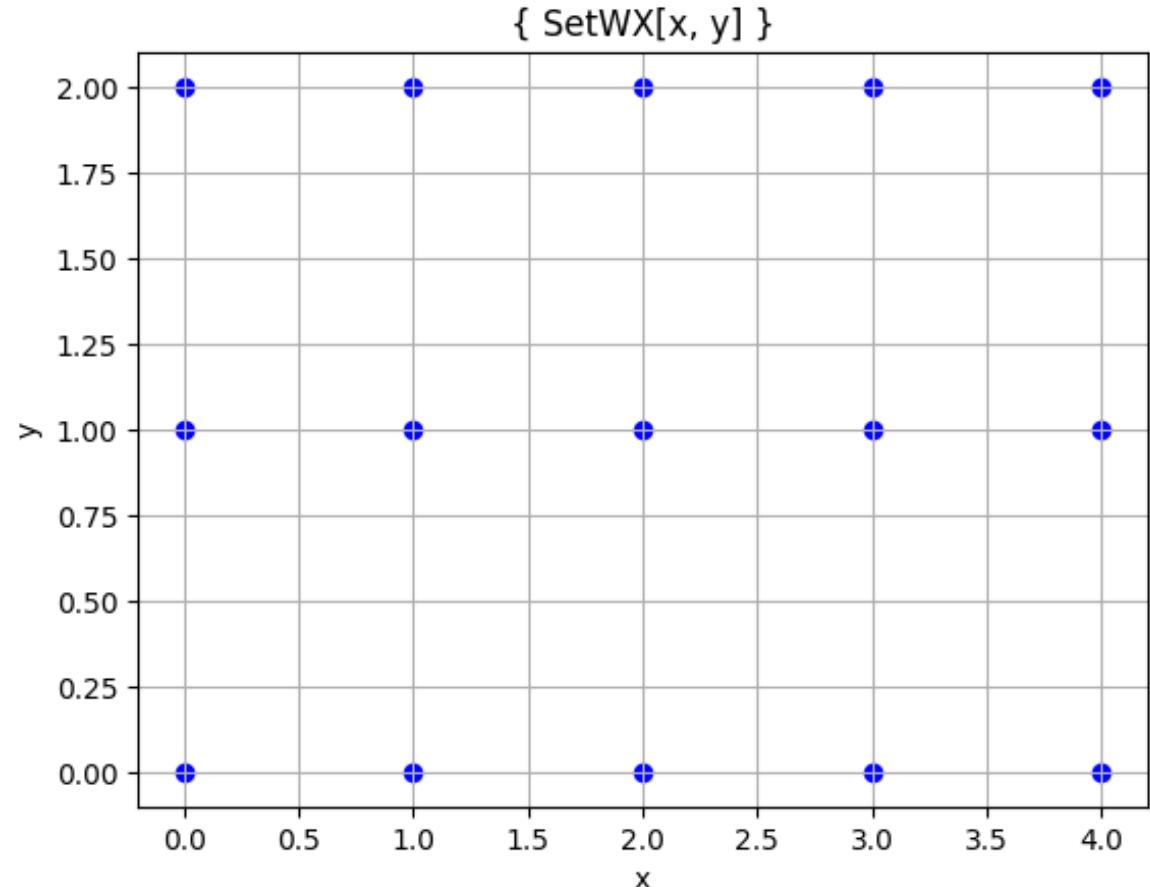
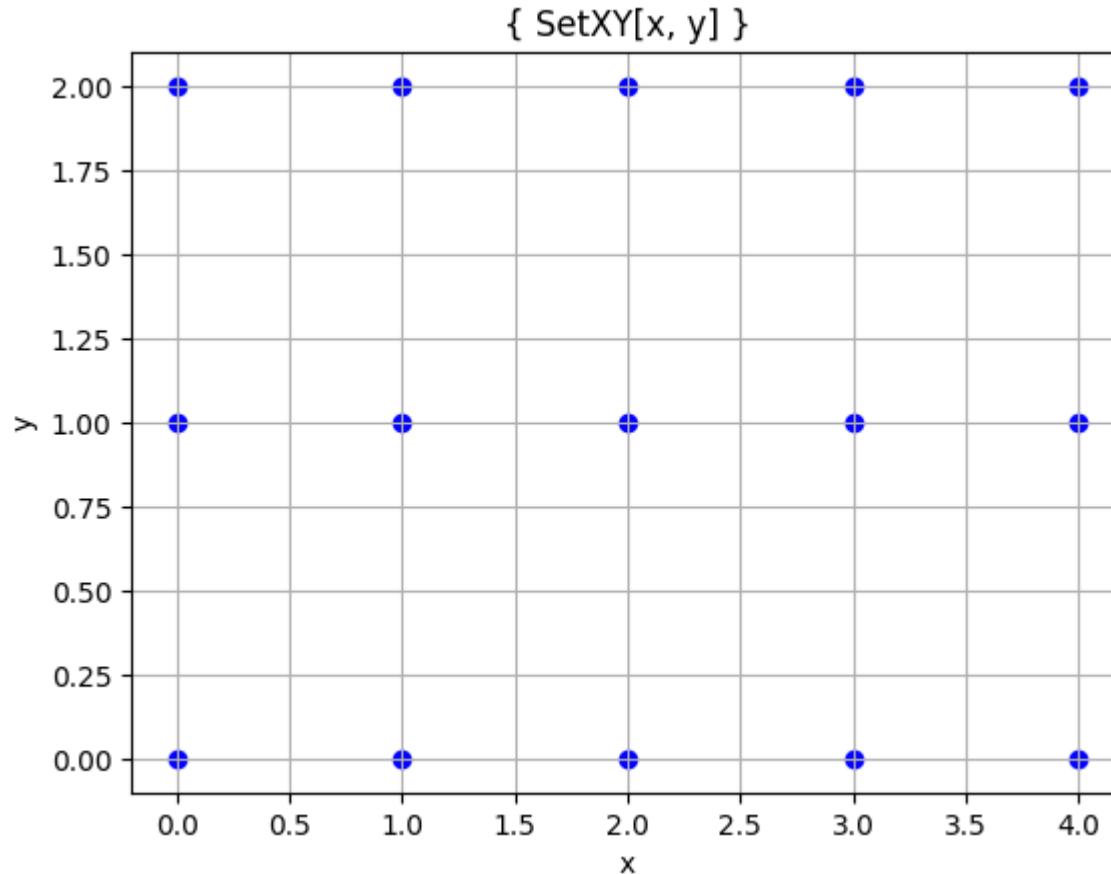
set2

```
{ SetWX[0, 0] }  
{ SetWX[1, 0] }  
{ SetWX[2, 0] }  
{ SetWX[3, 0] }  
{ SetWX[4, 0] }  
{ SetWX[0, 1] }  
{ SetWX[1, 1] }  
{ SetWX[2, 1] }  
{ SetWX[3, 1] }  
{ SetWX[4, 1] }  
{ SetWX[0, 2] }  
{ SetWX[1, 2] }  
{ SetWX[2, 2] }  
{ SetWX[3, 2] }  
{ SetWX[4, 2] }
```

A different ISL Set

set1 = { SetXY[x, y] : $0 \leq x \leq 4$ and $0 \leq y \leq 2$ }

set2 = { SetWX[x, y] : $0 \leq x \leq 4$ and $0 \leq y \leq 2$ }



Not the same!

Different Coordinate Order

```
set1 = isl.Set(f'{{ SetXY[x, y] :  
    0 <= x <= {X-1} and  
    0 <= y <= {Y-1} }}')
```

Input:

```
set3 = isl.Set(f'{{ SetXY[y, x] :  
    0 <= x <= {X-1} and  
    0 <= y <= {Y-1} }}')
```

Output:

```
Set3 = { SetXY[y, x] : 0 <= y <= 2 and 0 <= x <= 4 }
```

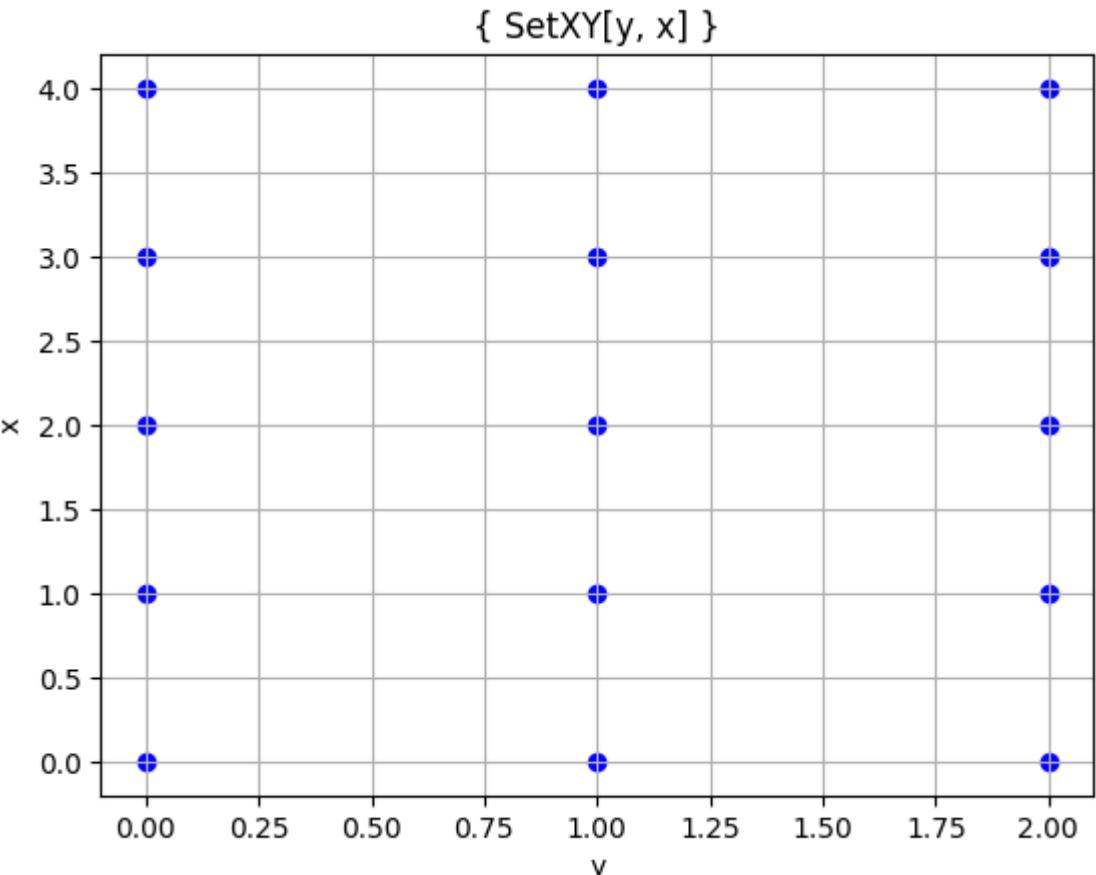
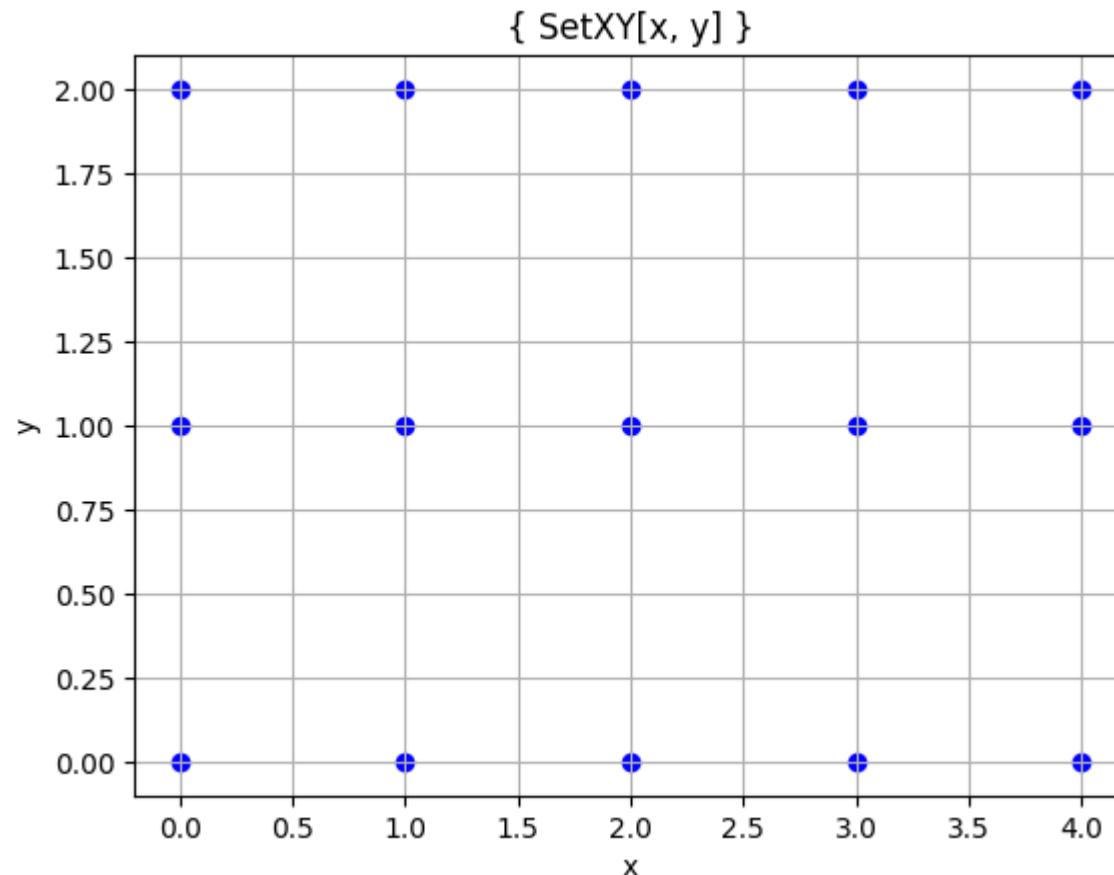
set3

```
{ SetXY[0, 0] }  
{ SetXY[0, 1] }  
{ SetXY[0, 2] }  
{ SetXY[0, 3] }  
{ SetXY[0, 4] }  
{ SetXY[1, 0] }  
{ SetXY[1, 1] }  
{ SetXY[1, 2] }  
{ SetXY[1, 3] }  
{ SetXY[1, 4] }  
{ SetXY[2, 0] }  
{ SetXY[2, 1] }  
{ SetXY[2, 2] }  
{ SetXY[2, 3] }  
{ SetXY[2, 4] }
```

ISL coordinate order matters

```
set1 = { SetXY[x, y] : 0 <= x <= 4 and 0 <= y <= 2 }
```

```
set3 = { SetXY[y, x] : 0 <= y <= 2 and 0 <= x <= 4 }
```



Not the same!

Different Constraints

Input:

```
set1 = isl.Set(f'{{ SetXY[x, y] :  
    0 <= x <= {X-1} and  
    0 <= y <= {Y-1} }}')
```

Output:

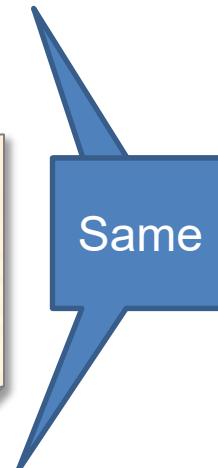
```
set1 = { SetXY[x, y] : 0 <= x <= 4 and 0 <= y <= 2 }
```

Input:

```
set4 = isl.Set(f'{{ SetXY[x, y] :  
    0 <= x < {X} and  
    0 <= y < {Y} }}')
```

Output:

```
set4 = { SetXY[x, y] : 0 <= x <= 4 and 0 <= y <= 2 }
```



set1

```
{ SetXY[0, 0] }  
{ SetXY[1, 0] }  
{ SetXY[2, 0] }  
{ SetXY[3, 0] }  
{ SetXY[4, 0] }  
{ SetXY[0, 1] }  
{ SetXY[1, 1] }  
{ SetXY[2, 1] }  
{ SetXY[3, 1] }  
{ SetXY[4, 1] }  
{ SetXY[0, 2] }  
{ SetXY[1, 2] }  
{ SetXY[2, 2] }  
{ SetXY[3, 2] }  
{ SetXY[4, 2] }
```

set4

```
{ SetXY[0, 0] }  
{ SetXY[1, 0] }  
{ SetXY[2, 0] }  
{ SetXY[3, 0] }  
{ SetXY[4, 0] }  
{ SetXY[0, 1] }  
{ SetXY[1, 1] }  
{ SetXY[2, 1] }  
{ SetXY[3, 1] }  
{ SetXY[4, 1] }  
{ SetXY[0, 2] }  
{ SetXY[1, 2] }  
{ SetXY[2, 2] }  
{ SetXY[3, 2] }  
{ SetXY[4, 2] }
```

set1 and set4 are the same

More Complex Constraints

```
set1 = isl.Set(f'{{ SetXY[x, y] :  
    0 <= x <= {X-1} and  
    0 <= y <= {Y-1} }}')
```

Input:

```
set5 = isl.Set(f'{{ SetXY[x, y] :  
    0 <= x <= {X-1} and  
    0 <= y <= x }}')
```

Output:

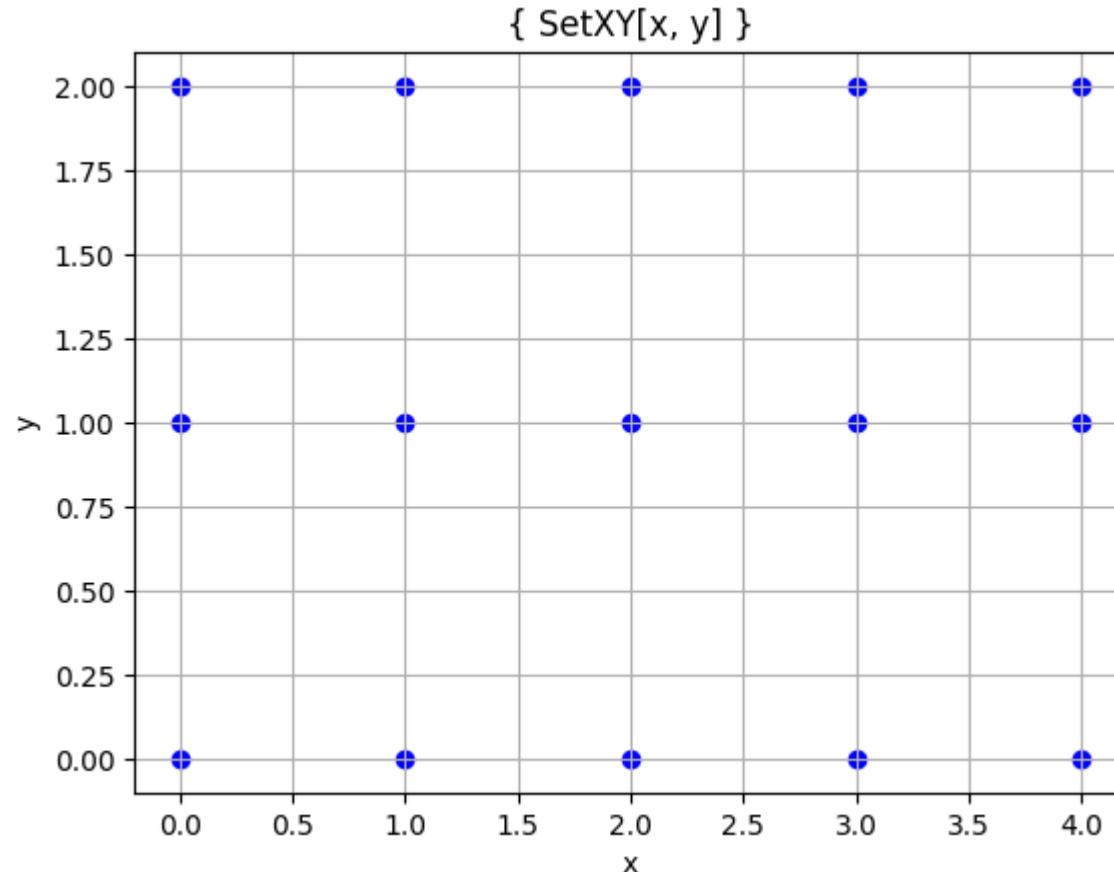
```
set5 = { SetXY[x, y] : 0 <= x <= 4 and 0 <= y <= x }
```

set5

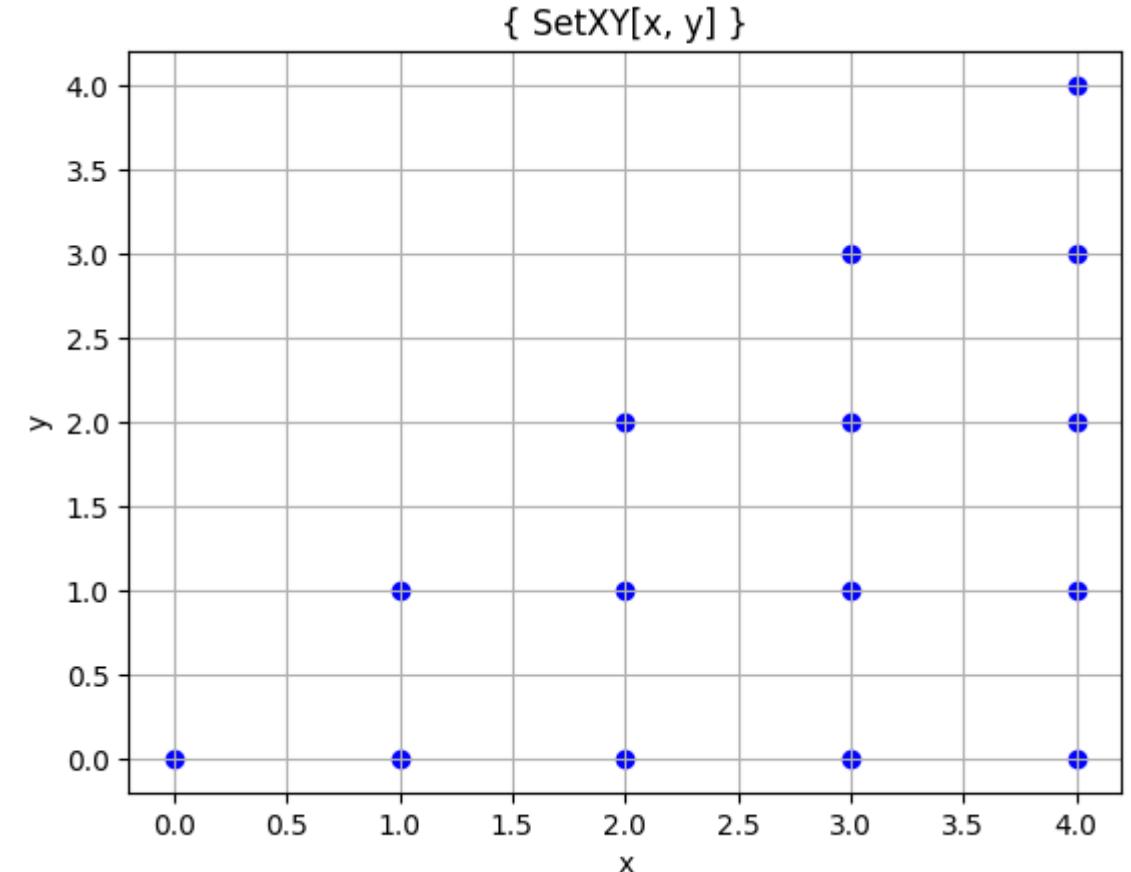
```
{ SetXY[0, 0] }  
{ SetXY[1, 0] }  
{ SetXY[1, 1] }  
{ SetXY[2, 0] }  
{ SetXY[2, 1] }  
{ SetXY[2, 2] }  
{ SetXY[3, 0] }  
{ SetXY[3, 1] }  
{ SetXY[3, 2] }  
{ SetXY[3, 3] }  
{ SetXY[4, 0] }  
{ SetXY[4, 1] }  
{ SetXY[4, 2] }  
{ SetXY[4, 3] }  
{ SetXY[4, 4] }
```

More Complex Constraints

set1 = { SetXY[x, y] : $0 \leq x \leq 4$ and $0 \leq y \leq 2$ }



set5 = { SetXY[x, y] : $0 \leq x \leq 4$ and $0 \leq y \leq x$ }



Defining an Iteration Space

Input:

```
X = 5  
Y = 3
```

```
ispaceXY = isl.Set(f'{{ IterationSpace[x,y] :  
    0 <= x < {X} and  
    0 <= y < {Y} }}')
```

Output:

```
ispaceXY = { IterationSpace[x, y] : 0 <= x <= 4 and 0 <= y <= 2 }
```

ispaceXY

```
{ IterationSpace[0, 0] }  
{ IterationSpace[1, 0] }  
{ IterationSpace[2, 0] }  
{ IterationSpace[3, 0] }  
{ IterationSpace[4, 0] }  
{ IterationSpace[0, 1] }  
{ IterationSpace[1, 1] }  
{ IterationSpace[2, 1] }  
{ IterationSpace[3, 1] }  
{ IterationSpace[4, 1] }  
{ IterationSpace[0, 2] }  
{ IterationSpace[1, 2] }  
{ IterationSpace[2, 2] }  
{ IterationSpace[3, 2] }  
{ IterationSpace[4, 2] }
```

Iteration Space

$$O_q = I_{q+s} \times F_s$$

Tensor: Ispace[S, Q]						
Rank: Q						
Rank: S						
0	1	1	1	1	1	1
1	1	1	1	1	1	1
2	1	1	1	1	1	1

What is the iteration space?

Cross product of Q x S

Defining an Iteration Space

$X = 5$
 $Y = 3$

```
ispaceXY = isl.Set(f'{{ IterationSpace[x,y] :  

    0 <= x < {X} and  

    0 <= y < {Y} }}')
```

Input:

$Q = 5$
 $S = 3$

```
ispace = isl.Set(f'{{ IterationSpace[q,s] :  

    0 <= q < {Q} and  

    0 <= s < {S} }}')
```

Output:

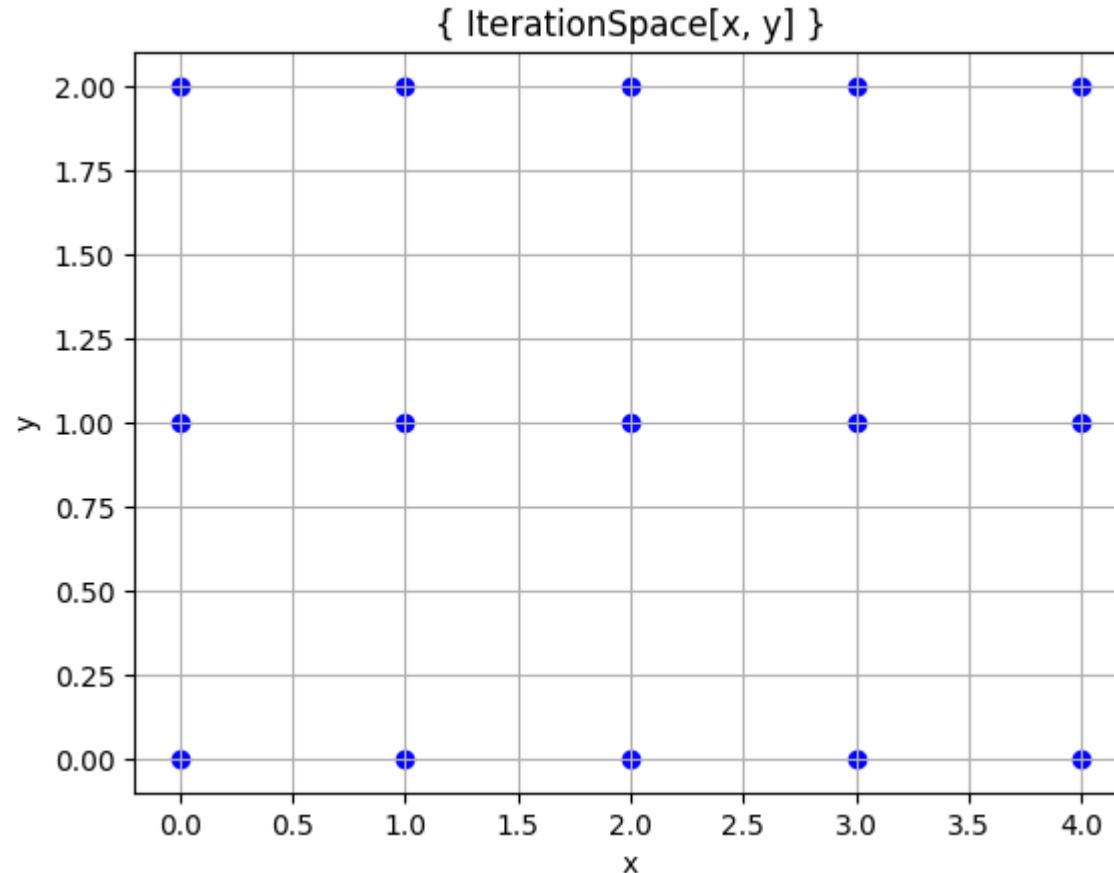
```
ispace = { IterationSpace[q, s] : 0 <= q <= 4 and 0 <= s <= 2 }
```

ispace

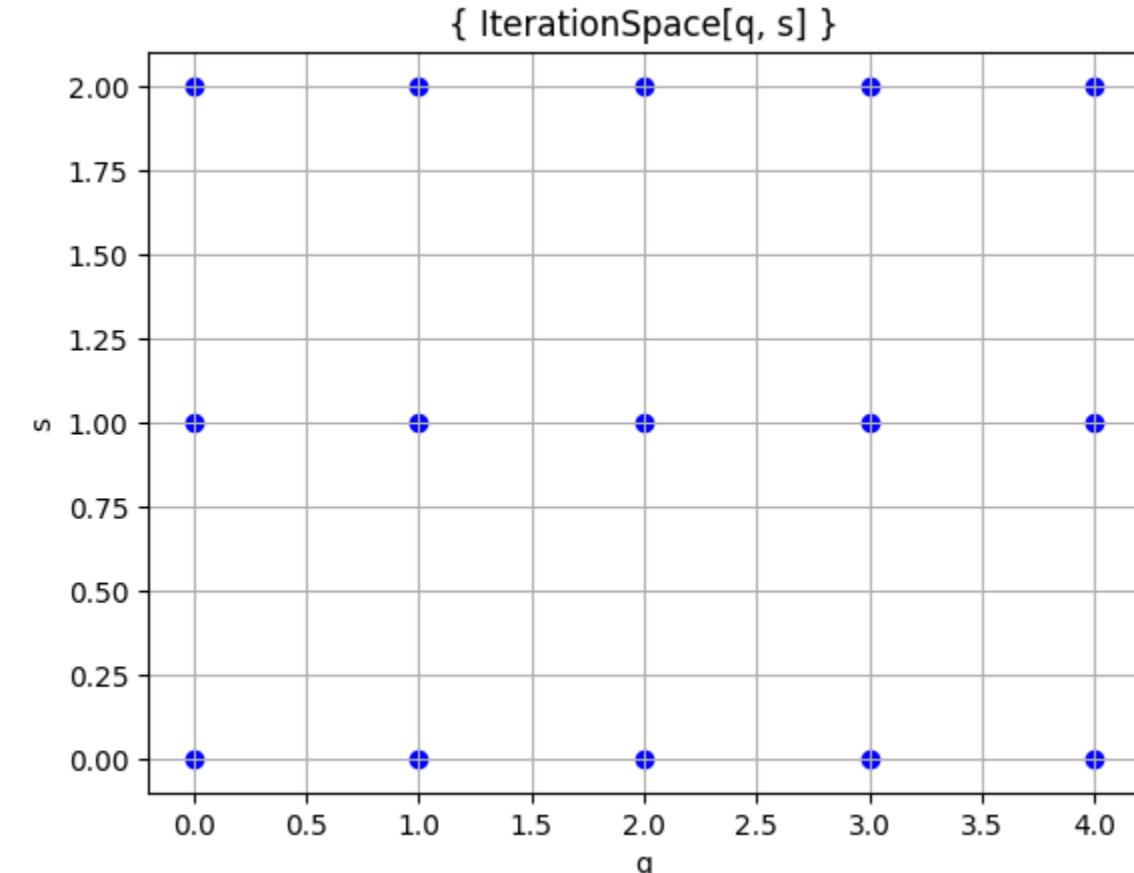
```
{ IterationSpace[0, 0] }
{ IterationSpace[1, 0] }
{ IterationSpace[2, 0] }
{ IterationSpace[3, 0] }
{ IterationSpace[4, 0] }
{ IterationSpace[0, 1] }
{ IterationSpace[1, 1] }
{ IterationSpace[2, 1] }
{ IterationSpace[3, 1] }
{ IterationSpace[4, 1] }
{ IterationSpace[0, 2] }
{ IterationSpace[1, 2] }
{ IterationSpace[2, 2] }
{ IterationSpace[3, 2] }
{ IterationSpace[4, 2] }
```

Coordinate Naming

{ IterationSpace[x, y] : $0 \leq x \leq 4$ and $0 \leq y \leq 2$ }



{ IterationSpace[q, s] : $0 \leq q \leq 4$ and $0 \leq s \leq 2$ }



Spaces are the same!

Maps – Relations Between Two Sets

```
unbounded_map = isl.Map(f'{{  
    SetXY[x,y] ->  
    SetXY_range[x1,y1] :  
        x1=x+1 and  
        y1=y+10}}')
```

Domain

Range

Relation

```
bounded_map = unbounded_map.  
intersect_domain(set1)
```

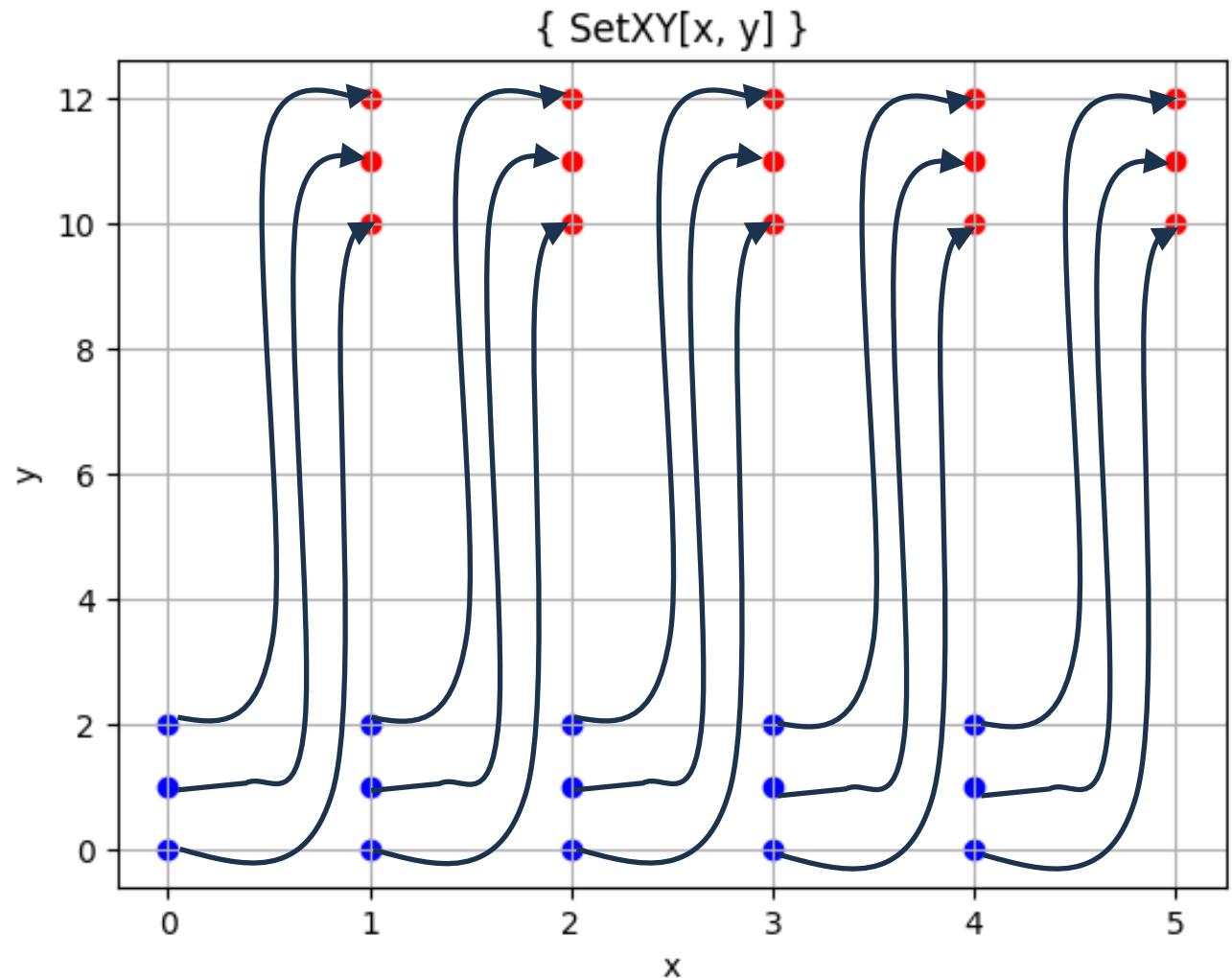
bounded_map

```
{ [SetXY[x = 0, y = 0] -> SetXY_range[x1 = 1, y1 = 10]] }  
{ [SetXY[x = 0, y = 1] -> SetXY_range[x1 = 1, y1 = 11]] }  
{ [SetXY[x = 0, y = 2] -> SetXY_range[x1 = 1, y1 = 12]] }  
{ [SetXY[x = 1, y = 0] -> SetXY_range[x1 = 2, y1 = 10]] }  
{ [SetXY[x = 1, y = 1] -> SetXY_range[x1 = 2, y1 = 11]] }  
{ [SetXY[x = 1, y = 2] -> SetXY_range[x1 = 2, y1 = 12]] }  
{ [SetXY[x = 2, y = 0] -> SetXY_range[x1 = 3, y1 = 10]] }  
{ [SetXY[x = 2, y = 1] -> SetXY_range[x1 = 3, y1 = 11]] }  
{ [SetXY[x = 2, y = 2] -> SetXY_range[x1 = 3, y1 = 12]] }  
{ [SetXY[x = 3, y = 0] -> SetXY_range[x1 = 4, y1 = 10]] }  
{ [SetXY[x = 3, y = 1] -> SetXY_range[x1 = 4, y1 = 11]] }  
{ [SetXY[x = 3, y = 2] -> SetXY_range[x1 = 4, y1 = 12]] }  
{ [SetXY[x = 4, y = 0] -> SetXY_range[x1 = 5, y1 = 10]] }  
{ [SetXY[x = 4, y = 1] -> SetXY_range[x1 = 5, y1 = 11]] }  
{ [SetXY[x = 4, y = 2] -> SetXY_range[x1 = 5, y1 = 12]] }
```

Maps

```
unbounded_map = isl.Map(f'{{  
    SetXY[x,y] ->  
    SetXY_range[x1,y1] :  
    x1=x+1 and  
    y1=y+10}})'
```

```
bounded_map = unbounded_map.  
intersect_domain(set1)
```



Iteration Space – Projections to Data

$$O_q = I_{q+s} \times F_s$$

Tensor: `Ispace[S, Q]`

Rank: `Q`

	0	1	2	3	4	5
0	1	1	1	1	1	1
1	1	1	1	1	1	1
2	1	1	1	1	1	1

Rank: `S`

Tensor: `F[S]`

Rank: `S`

0	1	2
8	5	2

Given a point (s, q) in the iteration space,
what weight does it map to?

F_s

Weight Projection

$$O_q = I_{q+s} \times F_s \quad \rightarrow$$

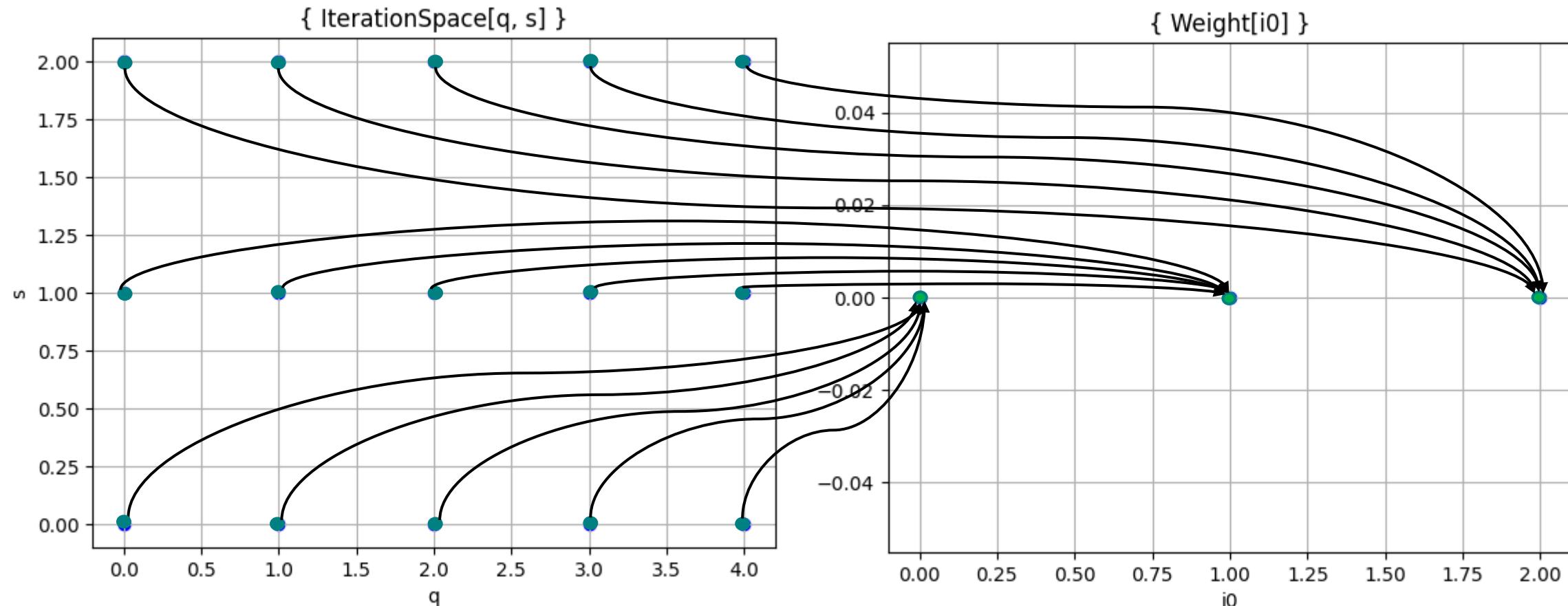
```
is2weight = isl.Map("{ IterationSpace[q,s] -> Weight[s] }")
```

```
{ [IterationSpace[q = 0, s = 0] -> Weight[0]] }
{ [IterationSpace[q = 0, s = 1] -> Weight[1]] }
{ [IterationSpace[q = 0, s = 2] -> Weight[2]] }
{ [IterationSpace[q = 1, s = 0] -> Weight[0]] }
{ [IterationSpace[q = 1, s = 1] -> Weight[1]] }
{ [IterationSpace[q = 1, s = 2] -> Weight[2]] }
{ [IterationSpace[q = 2, s = 0] -> Weight[0]] }
{ [IterationSpace[q = 2, s = 1] -> Weight[1]] }
{ [IterationSpace[q = 2, s = 2] -> Weight[2]] }
{ [IterationSpace[q = 3, s = 0] -> Weight[0]] }
{ [IterationSpace[q = 3, s = 1] -> Weight[1]] }
{ [IterationSpace[q = 3, s = 2] -> Weight[2]] }
{ [IterationSpace[q = 4, s = 0] -> Weight[0]] }
{ [IterationSpace[q = 4, s = 1] -> Weight[1]] }
{ [IterationSpace[q = 4, s = 2] -> Weight[2]] }
```

Weight Projection

$$O_q = I_{q+s} \times F_s \quad \rightarrow$$

```
is2weight = isl.Map("{ IterationSpace[q,s] -> Weight[s] }")
```



Note: Many-to-one relationship

Output Projection

$$O_q = I_{q+s} \times F_s \quad \rightarrow$$

```
is2output = isl.Map("{ IterationSpace[q,s] -> Output[q] }")
```

```
{ [IterationSpace[q = 0, s = 0] -> Output[0]] }
{ [IterationSpace[q = 0, s = 1] -> Output[0]] }
{ [IterationSpace[q = 0, s = 2] -> Output[0]] }
{ [IterationSpace[q = 1, s = 0] -> Output[1]] }
{ [IterationSpace[q = 1, s = 1] -> Output[1]] }
{ [IterationSpace[q = 1, s = 2] -> Output[1]] }
{ [IterationSpace[q = 2, s = 0] -> Output[2]] }
{ [IterationSpace[q = 2, s = 1] -> Output[2]] }
{ [IterationSpace[q = 2, s = 2] -> Output[2]] }
{ [IterationSpace[q = 3, s = 0] -> Output[3]] }
{ [IterationSpace[q = 3, s = 1] -> Output[3]] }
{ [IterationSpace[q = 3, s = 2] -> Output[3]] }
{ [IterationSpace[q = 4, s = 0] -> Output[4]] }
{ [IterationSpace[q = 4, s = 1] -> Output[4]] }
{ [IterationSpace[q = 4, s = 2] -> Output[4]] }
```

Input Projection

$$O_q = I_{q+s} \times F_S \quad \rightarrow$$

```
is2input = isl.Map("{ IterationSpace[q,s] -> Input[w] : w=q+s }")
```

```
{ [IterationSpace[q = 0, s = 0] -> Input[w = 0]] }
{ [IterationSpace[q = 0, s = 1] -> Input[w = 1]] }
{ [IterationSpace[q = 0, s = 2] -> Input[w = 2]] }
{ [IterationSpace[q = 1, s = 0] -> Input[w = 1]] }
{ [IterationSpace[q = 1, s = 1] -> Input[w = 2]] }
{ [IterationSpace[q = 1, s = 2] -> Input[w = 3]] }
{ [IterationSpace[q = 2, s = 0] -> Input[w = 2]] }
{ [IterationSpace[q = 2, s = 1] -> Input[w = 3]] }
{ [IterationSpace[q = 2, s = 2] -> Input[w = 4]] }
{ [IterationSpace[q = 3, s = 0] -> Input[w = 3]] }
{ [IterationSpace[q = 3, s = 1] -> Input[w = 4]] }
{ [IterationSpace[q = 3, s = 2] -> Input[w = 5]] }
{ [IterationSpace[q = 4, s = 0] -> Input[w = 4]] }
{ [IterationSpace[q = 4, s = 1] -> Input[w = 5]] }
{ [IterationSpace[q = 4, s = 2] -> Input[w = 6]] }
```

Compute Projection

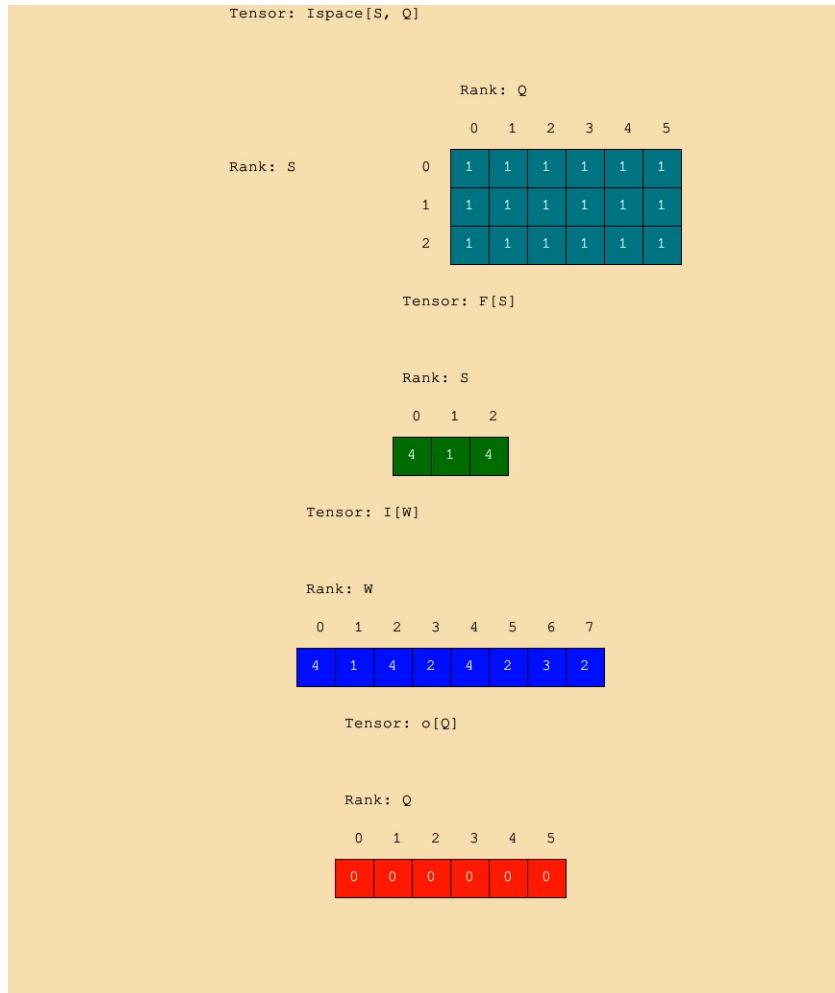
$$O_q = I_{q+s} \times F_s$$



```
is2compute = isl.Map("{ IterationSpace[q,s] -> MACC[q,s] }")
```

```
{ [IterationSpace[q = 0, s = 0] -> MACC[0, 0]] }
{ [IterationSpace[q = 0, s = 1] -> MACC[0, 1]] }
{ [IterationSpace[q = 0, s = 2] -> MACC[0, 2]] }
{ [IterationSpace[q = 1, s = 0] -> MACC[1, 0]] }
{ [IterationSpace[q = 1, s = 1] -> MACC[1, 1]] }
{ [IterationSpace[q = 1, s = 2] -> MACC[1, 2]] }
{ [IterationSpace[q = 2, s = 0] -> MACC[2, 0]] }
{ [IterationSpace[q = 2, s = 1] -> MACC[2, 1]] }
{ [IterationSpace[q = 2, s = 2] -> MACC[2, 2]] }
{ [IterationSpace[q = 3, s = 0] -> MACC[3, 0]] }
{ [IterationSpace[q = 3, s = 1] -> MACC[3, 1]] }
{ [IterationSpace[q = 3, s = 2] -> MACC[3, 2]] }
{ [IterationSpace[q = 4, s = 0] -> MACC[4, 0]] }
{ [IterationSpace[q = 4, s = 1] -> MACC[4, 1]] }
{ [IterationSpace[q = 4, s = 2] -> MACC[4, 2]] }
```

Iteration Space Traversal



Iteration Space to Timestamp

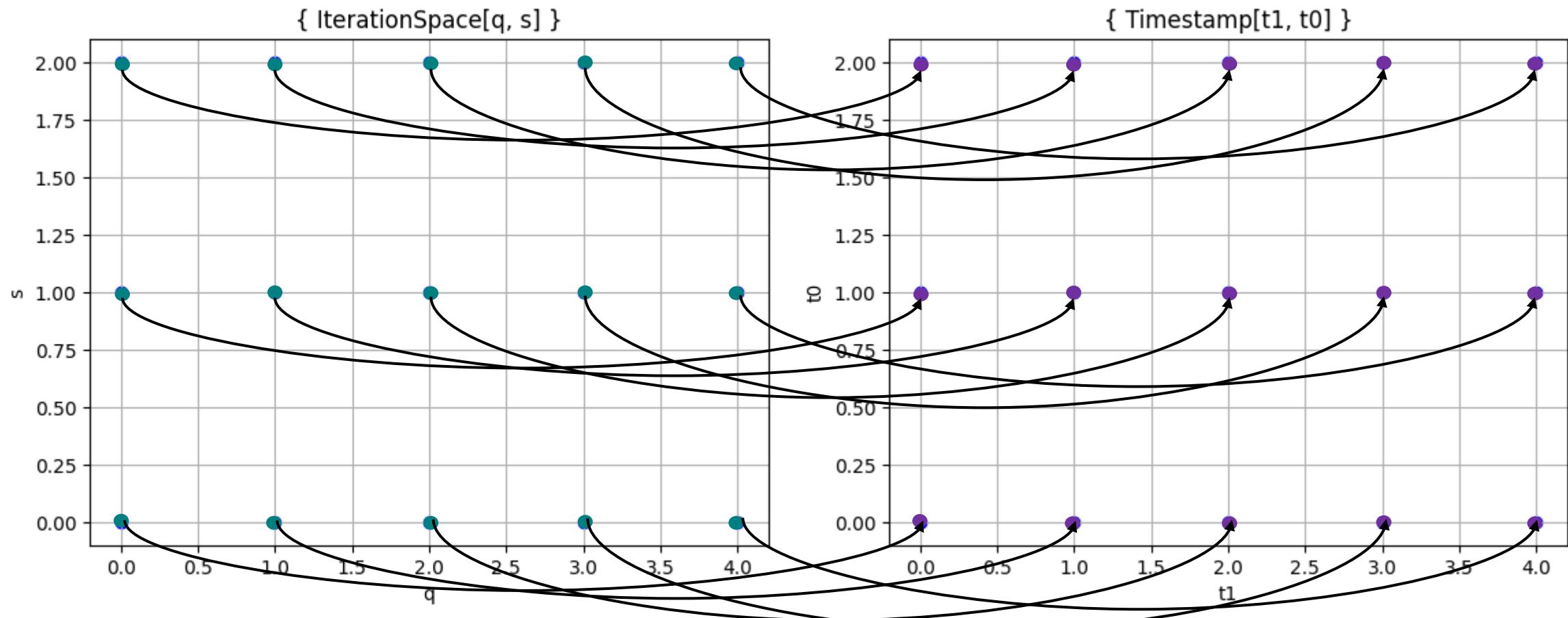
```
df_is2ts_unbounded = isl.Map(f'{{ IterationSpace[q,s] -> Timestamp[t1,t0] : t1=q and t0=s }}')
```

```
df_is2ts = df_is2ts_unbounded.intersect_domain(ispace)
```

```
{ [IterationSpace[q = 0, s = 0] -> Timestamp[t1 = 0, t0 = 0]] }
{ [IterationSpace[q = 0, s = 1] -> Timestamp[t1 = 0, t0 = 1]] }
{ [IterationSpace[q = 0, s = 2] -> Timestamp[t1 = 0, t0 = 2]] }
{ [IterationSpace[q = 1, s = 0] -> Timestamp[t1 = 1, t0 = 0]] }
{ [IterationSpace[q = 1, s = 1] -> Timestamp[t1 = 1, t0 = 1]] }
{ [IterationSpace[q = 1, s = 2] -> Timestamp[t1 = 1, t0 = 2]] }
{ [IterationSpace[q = 2, s = 0] -> Timestamp[t1 = 2, t0 = 0]] }
{ [IterationSpace[q = 2, s = 1] -> Timestamp[t1 = 2, t0 = 1]] }
{ [IterationSpace[q = 2, s = 2] -> Timestamp[t1 = 2, t0 = 2]] }
{ [IterationSpace[q = 3, s = 0] -> Timestamp[t1 = 3, t0 = 0]] }
{ [IterationSpace[q = 3, s = 1] -> Timestamp[t1 = 3, t0 = 1]] }
{ [IterationSpace[q = 3, s = 2] -> Timestamp[t1 = 3, t0 = 2]] }
{ [IterationSpace[q = 4, s = 0] -> Timestamp[t1 = 4, t0 = 0]] }
{ [IterationSpace[q = 4, s = 1] -> Timestamp[t1 = 4, t0 = 1]] }
{ [IterationSpace[q = 4, s = 2] -> Timestamp[t1 = 4, t0 = 2]] }
```

Iteration Space to Timestamp

```
df_is2ts_unbounded = isl.Map(f'{{ IterationSpace[q,s] -> Timestamp[t1,t0] : t1=q and t0=s }}')
df_is2ts = df_is2ts_unbounded.intersect_domain(ispace)
```



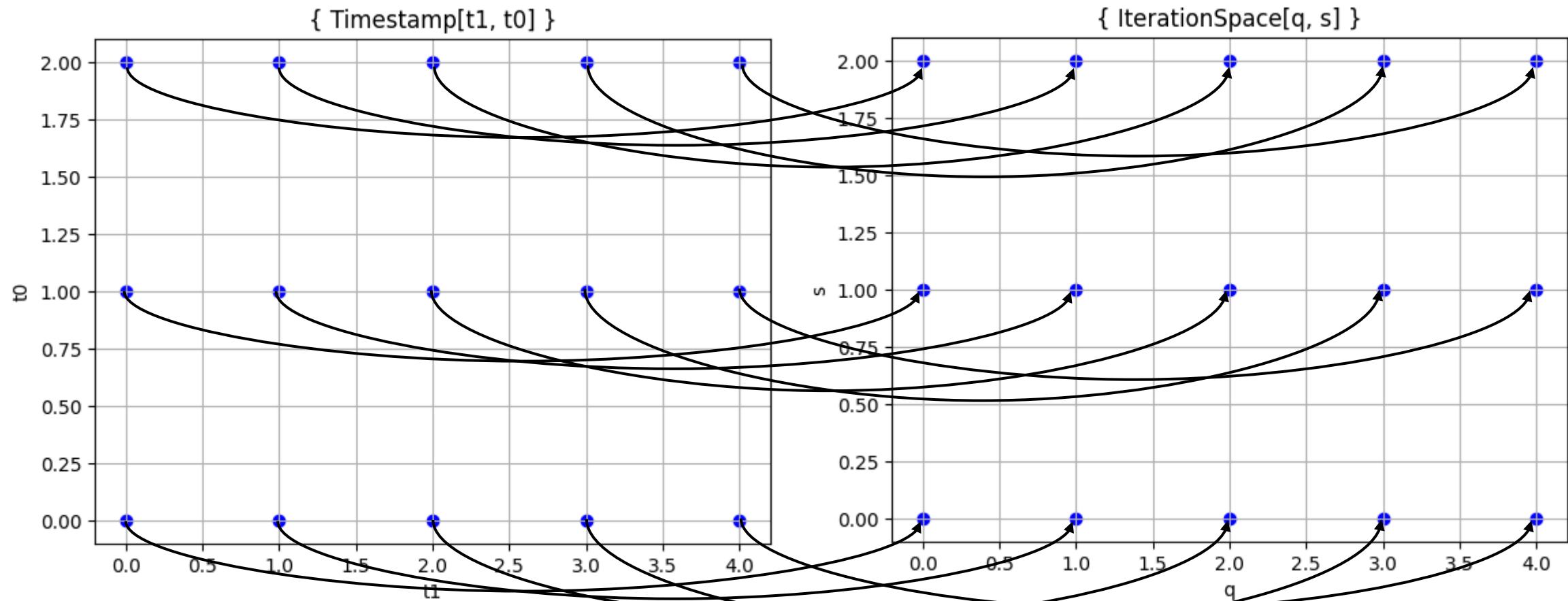
Timestamp to Iteration Space

```
df_ts2is = df_is2ts.reverse()
```

```
{ [Timestamp[t1 = 0, t0 = 0] -> IterationSpace[q = 0, s = 0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> IterationSpace[q = 0, s = 1]] }
{ [Timestamp[t1 = 0, t0 = 2] -> IterationSpace[q = 0, s = 2]] }
{ [Timestamp[t1 = 1, t0 = 0] -> IterationSpace[q = 1, s = 0]] }
{ [Timestamp[t1 = 1, t0 = 1] -> IterationSpace[q = 1, s = 1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> IterationSpace[q = 1, s = 2]] }
{ [Timestamp[t1 = 2, t0 = 0] -> IterationSpace[q = 2, s = 0]] }
{ [Timestamp[t1 = 2, t0 = 1] -> IterationSpace[q = 2, s = 1]] }
{ [Timestamp[t1 = 2, t0 = 2] -> IterationSpace[q = 2, s = 2]] }
{ [Timestamp[t1 = 3, t0 = 0] -> IterationSpace[q = 3, s = 0]] }
{ [Timestamp[t1 = 3, t0 = 1] -> IterationSpace[q = 3, s = 1]] }
{ [Timestamp[t1 = 3, t0 = 2] -> IterationSpace[q = 3, s = 2]] }
{ [Timestamp[t1 = 4, t0 = 0] -> IterationSpace[q = 4, s = 0]] }
{ [Timestamp[t1 = 4, t0 = 1] -> IterationSpace[q = 4, s = 1]] }
{ [Timestamp[t1 = 4, t0 = 2] -> IterationSpace[q = 4, s = 2]] }
```

Timestamp to Iteration Space

```
df_ts2is = df_is2ts.reverse()
```



Composing Maps

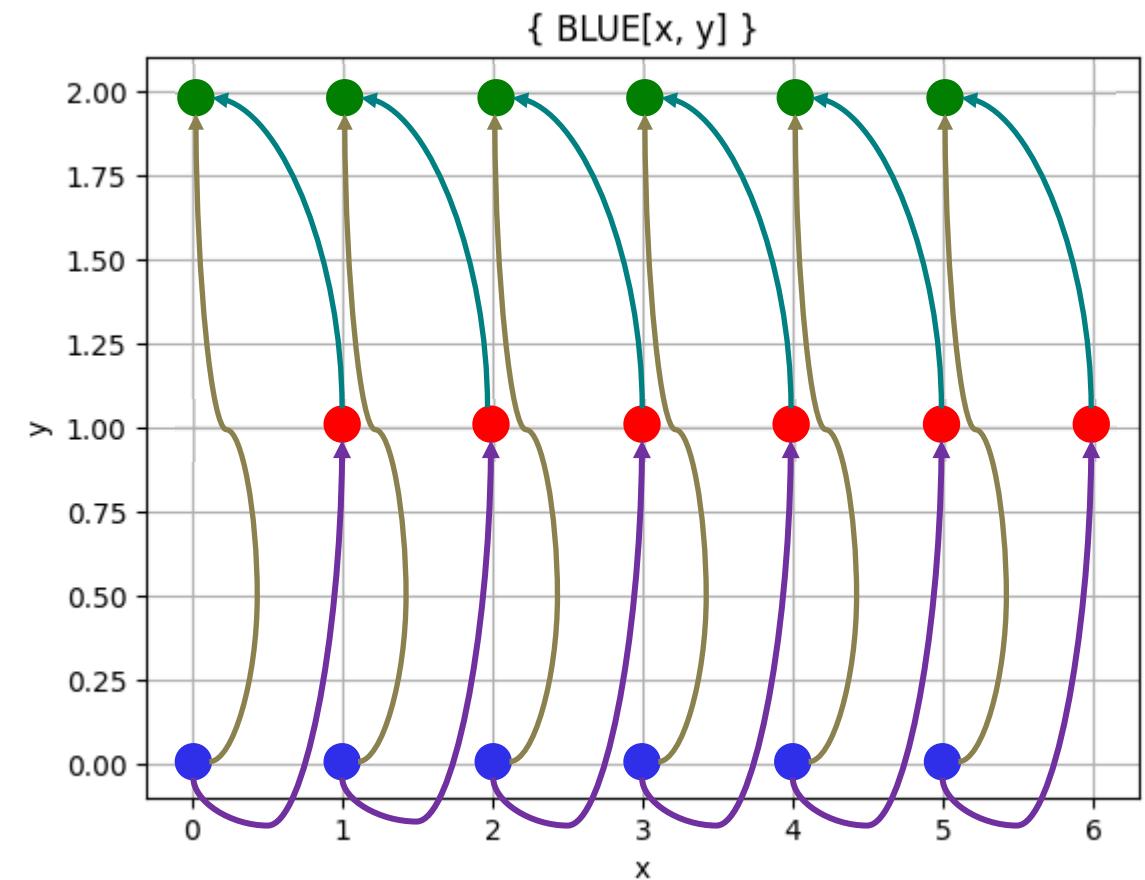
```
blue2red = isl.Map("{ BLUE[x,y] -> RED[x1,y1] :  
x1=x+1 and y1=y+1 }")
```

```
red2green = isl.Map("{ RED[x,y] -> GREEN[x1,y1] :  
x1=x-1 and y1=y+1 }")
```

```
blue2green = blue2red.apply_range(red2green)
```

Output:

```
blue2green = { BLUE[x, y = 0] -> GREEN[x1 = x, y1 = 2] : 0 <= x <= 5 }
```



Timestamp to Weight

Timestamp -> Weight

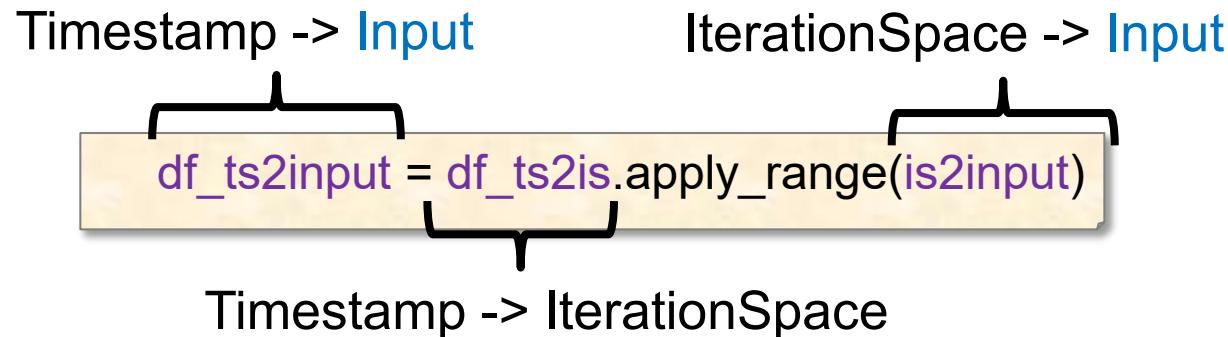
```
df_ts2weight_current = df_ts2is.apply_range(is2weight)
```

Timestamp -> IterationSpace

IterationSpace -> Weight

```
{ [Timestamp[t1 = 0, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Weight[2]] }
```

Timestamp to Input



```
{ [Timestamp[t1 = 0, t0 = 0] -> Input[w = 0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> Input[w = 1]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Input[w = 2]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Input[w = 1]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Input[w = 2]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Input[w = 3]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Input[w = 2]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Input[w = 3]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Input[w = 4]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Input[w = 3]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Input[w = 4]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Input[w = 5]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Input[w = 4]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Input[w = 5]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Input[w = 6]] }
```

Timestamp to Output

Timestamp -> Output

IterationSpace -> Output

```
df_ts2output = df_ts2is.apply_range(is2output)
```

Timestamp -> IterationSpace

```
{
  [Timestamp[t1 = 0, t0 = 0] -> Output[0]]
}
{
  [Timestamp[t1 = 0, t0 = 1] -> Output[0]]
}
{
  [Timestamp[t1 = 0, t0 = 2] -> Output[0]]
}
{
  [Timestamp[t1 = 1, t0 = 0] -> Output[1]]
}
{
  [Timestamp[t1 = 1, t0 = 1] -> Output[1]]
}
{
  [Timestamp[t1 = 1, t0 = 2] -> Output[1]]
}
{
  [Timestamp[t1 = 2, t0 = 0] -> Output[2]]
}
{
  [Timestamp[t1 = 2, t0 = 1] -> Output[2]]
}
{
  [Timestamp[t1 = 2, t0 = 2] -> Output[2]]
}
{
  [Timestamp[t1 = 3, t0 = 0] -> Output[3]]
}
{
  [Timestamp[t1 = 3, t0 = 1] -> Output[3]]
}
{
  [Timestamp[t1 = 3, t0 = 2] -> Output[3]]
}
{
  [Timestamp[t1 = 4, t0 = 0] -> Output[4]]
}
{
  [Timestamp[t1 = 4, t0 = 1] -> Output[4]]
}
{
  [Timestamp[t1 = 4, t0 = 2] -> Output[4]]
}
```

Probing the Schedule

```
print('last timestamp =', df_is2ts.range().lexmax())
```

```
last timestamp = { Timestamp[t1 = 4, t0 = 2] }
```

```
print(isl.Set('{ Timestamp[1,2] }').apply(df_ts2weight_current))
```

```
{ Weight[2] }
```

```
print(df_ts2weight_current.intersect_domain(isl.Set('{ Timestamp[1,2] }')))
```

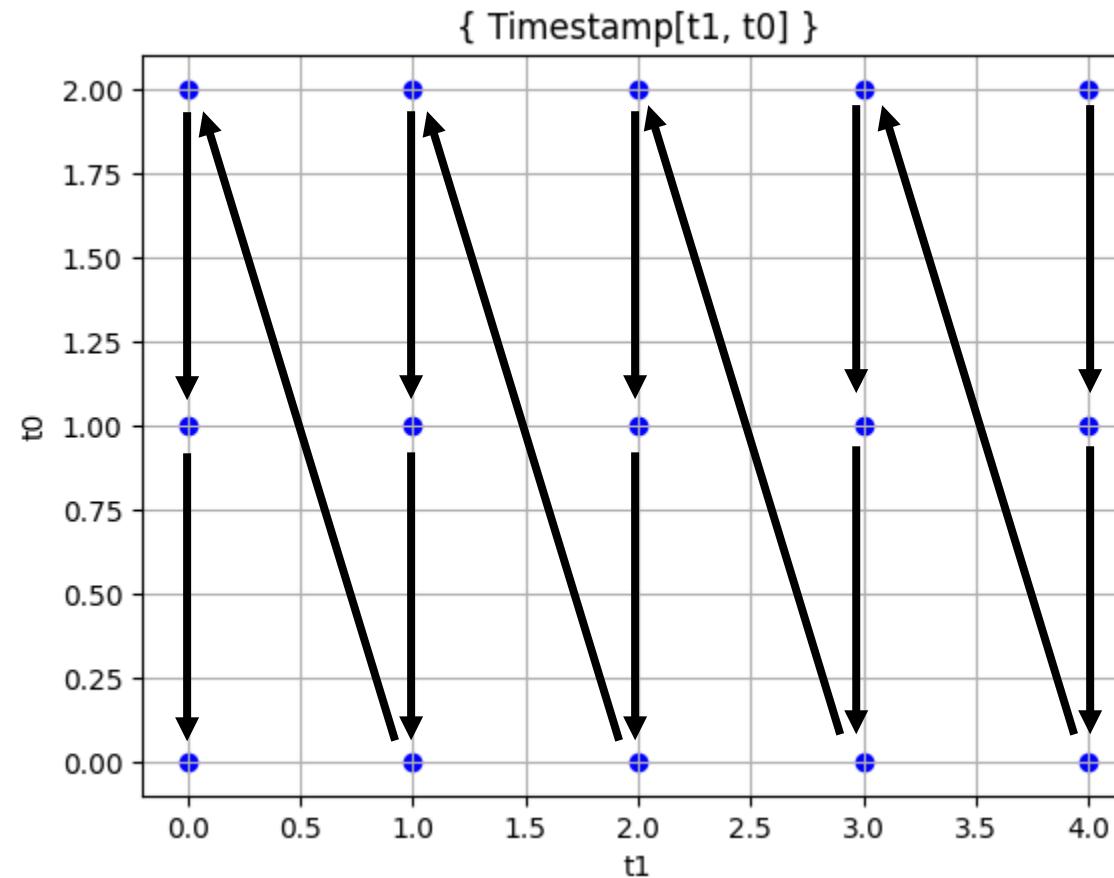
```
{ Timestamp[t1 = 1, t0 = 2] -> Weight[2] }
```

Calculating Data Movement

- Create a map from a timestamp to the previous timestamp
- Determine weight access in each timestep
- Determine weight access in previous timestep
- Find changes
- Repeat for inputs and outputs

Previous Timestamp

```
T0_MAX = S # Ideally should get this from lexmax  
timestamp_previous = isl.Map(f"{{ Timestamp[t1',t0'] -> Timestamp[t1,t0] :  
t1'=t1 and t0'=t0+1 or t1'=t1+1 and t0'=0 and t0={T0_MAX}-1 }}")
```



Previous Timestamp Map

```
T0_MAX = S # Ideally should get this from lexmax  
timestamp_previous = isl.Map(f"{{ Timestamp[t1',t0'] -> Timestamp[t1,t0] :  
t1'=t1 and t0'=t0+1 or t1'=t1+1 and t0'=0 and t0={T0_MAX}-1 }}")
```

```
{ [Timestamp[t1 = 0, t0 = 1] -> Timestamp[t1' = 0, t0' = 0]] }  
{ [Timestamp[t1 = 0, t0 = 2] -> Timestamp[t1' = 0, t0' = 1]] }  
{ [Timestamp[t1 = 1, t0 = 0] -> Timestamp[t1' = 0, t0' = 2]] }  
{ [Timestamp[t1 = 1, t0 = 1] -> Timestamp[t1' = 1, t0' = 0]] }  
{ [Timestamp[t1 = 1, t0 = 2] -> Timestamp[t1' = 1, t0' = 1]] }  
{ [Timestamp[t1 = 2, t0 = 0] -> Timestamp[t1' = 1, t0' = 2]] }  
{ [Timestamp[t1 = 2, t0 = 1] -> Timestamp[t1' = 2, t0' = 0]] }  
{ [Timestamp[t1 = 2, t0 = 2] -> Timestamp[t1' = 2, t0' = 1]] }  
{ [Timestamp[t1 = 3, t0 = 0] -> Timestamp[t1' = 2, t0' = 2]] }  
{ [Timestamp[t1 = 3, t0 = 1] -> Timestamp[t1' = 3, t0' = 0]] }  
{ [Timestamp[t1 = 3, t0 = 2] -> Timestamp[t1' = 3, t0' = 1]] }  
{ [Timestamp[t1 = 4, t0 = 0] -> Timestamp[t1' = 3, t0' = 2]] }  
{ [Timestamp[t1 = 4, t0 = 1] -> Timestamp[t1' = 4, t0' = 0]] }  
{ [Timestamp[t1 = 4, t0 = 2] -> Timestamp[t1' = 4, t0' = 1]] }
```

Weight Use in Previous Timestamp

```
df_ts2weight_previous = timestamp_previous.apply_range(df_ts2weight_current)
```

```
{ [Timestamp[t1 = 0, t0 = 1] -> Weight[0]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Weight[1]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Weight[2]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Weight[0]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Weight[1]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Weight[2]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Weight[0]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Weight[1]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Weight[2]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Weight[0]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Weight[1]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Weight[2]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Weight[0]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Weight[1]] }
```

Previous

```
{ [Timestamp[t1 = 0, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Weight[2]] }
```

Current

Timesteps Using a New Weight

```
df_ts2weight_delta = df_ts2weight_current.subtract(df_weight_previous)
```

```
{ [Timestamp[t1 = 0, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Weight[2]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Weight[0]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Weight[1]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Weight[2]] }
```

Input Use in Previous Timestamp

```
df_ts2input_previous = timestamp_previous.apply_range(df_ts2input_current)
```

```
{ [Timestamp[t1 = 0, t0 = 1] -> Input[w = 0]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Input[w = 1]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Input[w = 2]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Input[w = 1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Input[w = 2]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Input[w = 3]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Input[w = 2]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Input[w = 3]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Input[w = 4]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Input[w = 3]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Input[w = 4]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Input[w = 5]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Input[w = 4]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Input[w = 5]] }
{ [Timestamp[t1 = 5, t0 = 0] -> Input[w = 6]] }
```

Previous

```
{ [Timestamp[t1 = 0, t0 = 0] -> Input[w = 0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> Input[w = 1]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Input[w = 2]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Input[w = 1]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Input[w = 2]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Input[w = 3]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Input[w = 2]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Input[w = 3]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Input[w = 4]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Input[w = 3]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Input[w = 4]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Input[w = 5]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Input[w = 4]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Input[w = 5]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Input[w = 6]] }
```

Current

Timesteps Using a New Input

```
df_ts2input_delta = df_ts2input_current.subtract(df_ts2input_previous)
```

```
{ [Timestamp[t1 = 0, t0 = 0] -> Input[w = 0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> Input[w = 1]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Input[w = 2]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Input[w = 1]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Input[w = 2]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Input[w = 3]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Input[w = 2]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Input[w = 3]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Input[w = 4]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Input[w = 3]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Input[w = 4]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Input[w = 5]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Input[w = 4]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Input[w = 5]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Input[w = 6]] }
```

Output Use in Previous Timestamp

```
df_ts2output_previous = timestamp_previous.apply_range(df_ts2output_current)
```

```
{ [Timestamp[t1 = 0, t0 = 1] -> Output[0]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Output[0]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Output[0]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Output[1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Output[1]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Output[1]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Output[2]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Output[2]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Output[2]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Output[3]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Output[3]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Output[3]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Output[4]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Output[4]] }
```

Previous



```
{ [Timestamp[t1 = 0, t0 = 0] -> Output[0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> Output[0]] }
{ [Timestamp[t1 = 0, t0 = 2] -> Output[0]] }
{ [Timestamp[t1 = 1, t0 = 0] -> Output[1]] }
{ [Timestamp[t1 = 1, t0 = 1] -> Output[1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> Output[1]] }
{ [Timestamp[t1 = 2, t0 = 0] -> Output[2]] }
{ [Timestamp[t1 = 2, t0 = 1] -> Output[2]] }
{ [Timestamp[t1 = 2, t0 = 2] -> Output[2]] }
{ [Timestamp[t1 = 3, t0 = 0] -> Output[2]] }
{ [Timestamp[t1 = 3, t0 = 1] -> Output[3]] }
{ [Timestamp[t1 = 3, t0 = 2] -> Output[3]] }
{ [Timestamp[t1 = 4, t0 = 0] -> Output[4]] }
{ [Timestamp[t1 = 4, t0 = 1] -> Output[4]] }
{ [Timestamp[t1 = 4, t0 = 2] -> Output[4]] }
```

Current

Timesteps Using a New Output

```
df_ts2output_delta = df_ts2output_current.subtract(df_ts2output_previous)
```

```
{ [Timestamp[t1 = 0, t0 = 0] -> Output[0]] }  
{ [Timestamp[t1 = 1, t0 = 0] -> Output[1]] }  
{ [Timestamp[t1 = 2, t0 = 0] -> Output[2]] }  
{ [Timestamp[t1 = 3, t0 = 0] -> Output[3]] }  
{ [Timestamp[t1 = 4, t0 = 0] -> Output[4]] }
```

L1 Buffering Recipe

- **Group sets of timestamp points into tiles**
- **Create map of tiles to corresponding set of timestamp points**
- **Create map of timesteps of previous tiles**
- **Calculate deltas**

Tiling Timestamps

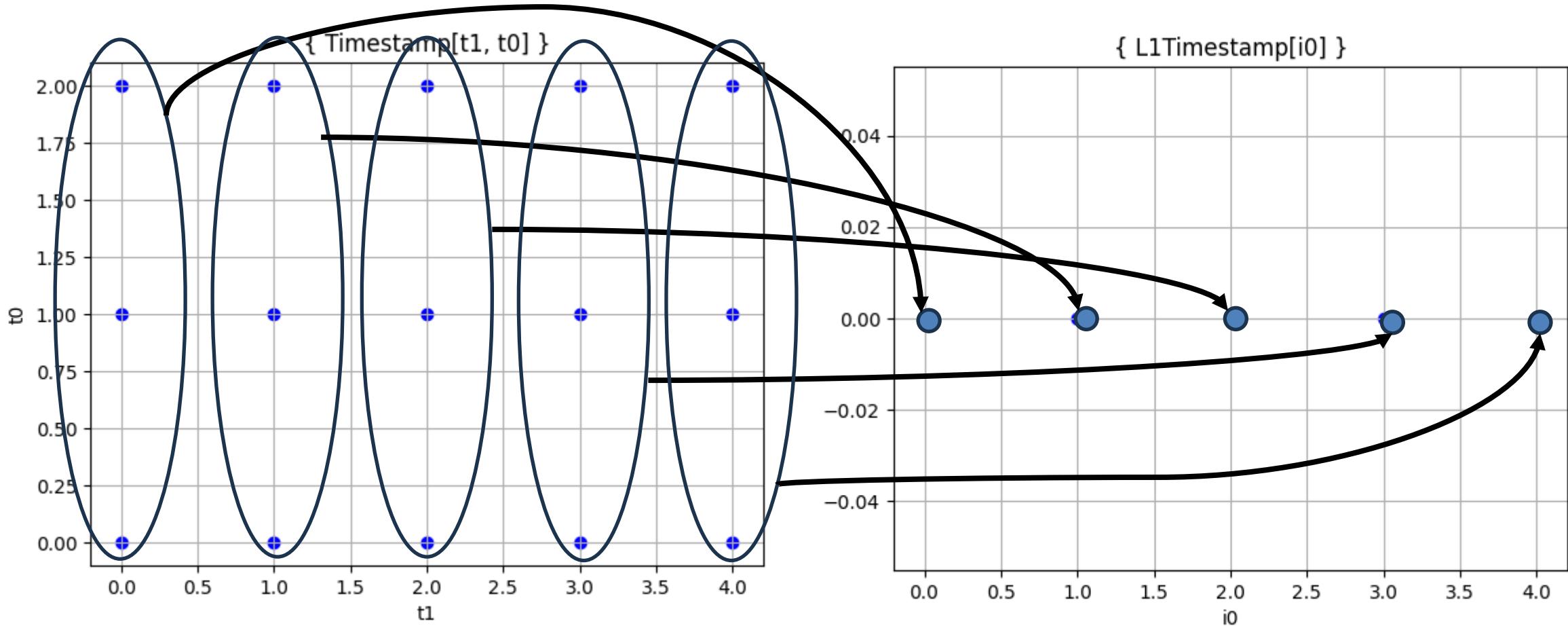
```
timestamp2L1timestamp = isl.Map('{ Timestamp[t1,t0] -> L1Timestamp[t1] }')
```

```
{ [Timestamp[t1 = 0, t0 = 0] -> L1Timestamp[0]] }
{ [Timestamp[t1 = 0, t0 = 1] -> L1Timestamp[0]] }
{ [Timestamp[t1 = 0, t0 = 2] -> L1Timestamp[0]] }
{ [Timestamp[t1 = 1, t0 = 0] -> L1Timestamp[1]] }
{ [Timestamp[t1 = 1, t0 = 1] -> L1Timestamp[1]] }
{ [Timestamp[t1 = 1, t0 = 2] -> L1Timestamp[1]] }
{ [Timestamp[t1 = 2, t0 = 0] -> L1Timestamp[2]] }
{ [Timestamp[t1 = 2, t0 = 1] -> L1Timestamp[2]] }
{ [Timestamp[t1 = 2, t0 = 2] -> L1Timestamp[2]] }
{ [Timestamp[t1 = 3, t0 = 0] -> L1Timestamp[3]] }
{ [Timestamp[t1 = 3, t0 = 1] -> L1Timestamp[3]] }
{ [Timestamp[t1 = 3, t0 = 2] -> L1Timestamp[3]] }
{ [Timestamp[t1 = 4, t0 = 0] -> L1Timestamp[4]] }
{ [Timestamp[t1 = 4, t0 = 1] -> L1Timestamp[4]] }
{ [Timestamp[t1 = 4, t0 = 2] -> L1Timestamp[4]] }
```

} L1Timestamp = 0
} L1Timestamp = 1
} L1Timestamp = 2
} L1Timestamp = 3
} L1Timestamp = 4

Tiling Timestamps

```
timestamp2L1timestamp = isl.Map('{ Timestamp[t1,t0] -> L1Timestamp[t1] }')
```



Tiling Timestamps

```
L1timestamp2timestamp = timestamp2L1timestamp.reverse()
```

```
{ [L1Timestamp[0] -> Timestamp[t1 = 0, t0 = 0]] }
{ [L1Timestamp[0] -> Timestamp[t1 = 0, t0 = 1]] }
{ [L1Timestamp[0] -> Timestamp[t1 = 0, t0 = 2]] }

{ [L1Timestamp[1] -> Timestamp[t1 = 1, t0 = 0]] }
{ [L1Timestamp[1] -> Timestamp[t1 = 1, t0 = 1]] }
{ [L1Timestamp[1] -> Timestamp[t1 = 1, t0 = 2]] }

{ [L1Timestamp[2] -> Timestamp[t1 = 2, t0 = 0]] }
{ [L1Timestamp[2] -> Timestamp[t1 = 2, t0 = 1]] }
{ [L1Timestamp[2] -> Timestamp[t1 = 2, t0 = 2]] }

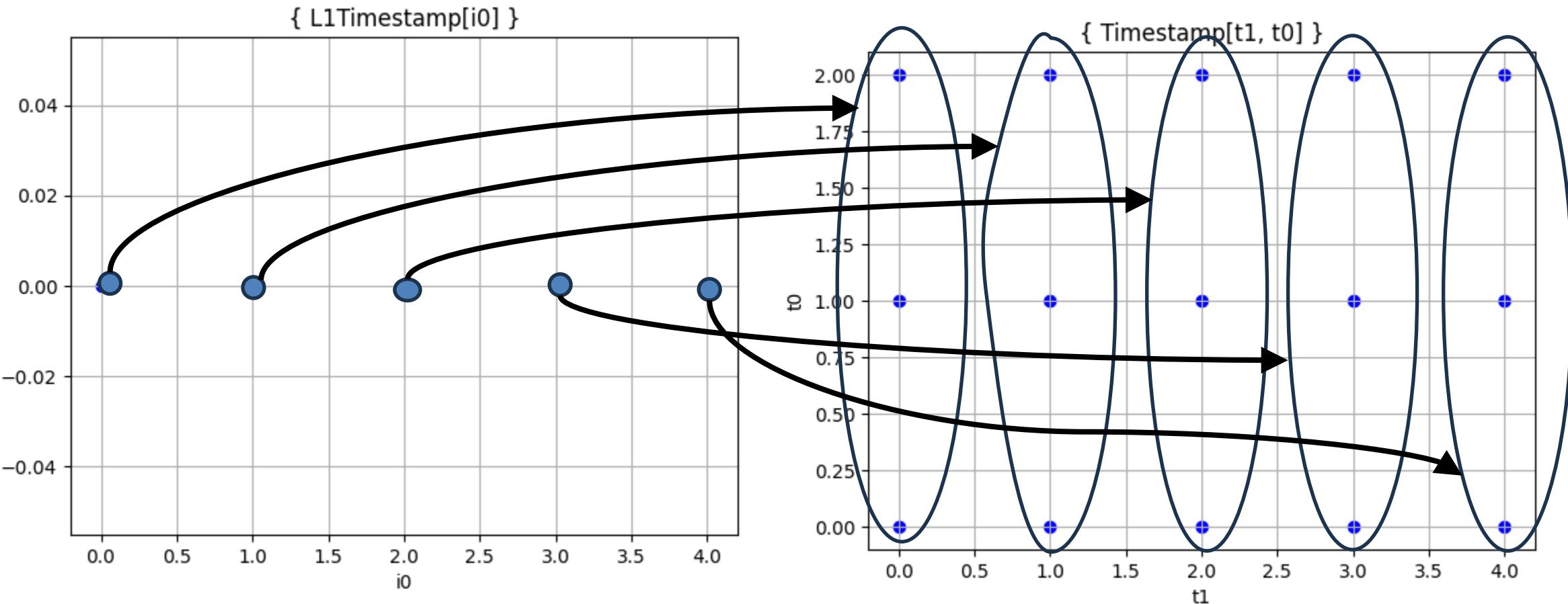
{ [L1Timestamp[3] -> Timestamp[t1 = 3, t0 = 0]] }
{ [L1Timestamp[3] -> Timestamp[t1 = 3, t0 = 1]] }
{ [L1Timestamp[3] -> Timestamp[t1 = 3, t0 = 2]] }

{ [L1Timestamp[4] -> Timestamp[t1 = 4, t0 = 0]] }
{ [L1Timestamp[4] -> Timestamp[t1 = 4, t0 = 1]] }
{ [L1Timestamp[4] -> Timestamp[t1 = 4, t0 = 2]] }
```

} L1Timestamp = 0
} L1Timestamp = 1
} L1Timestamp = 2
} L1Timestamp = 3
} L1Timestamp = 4

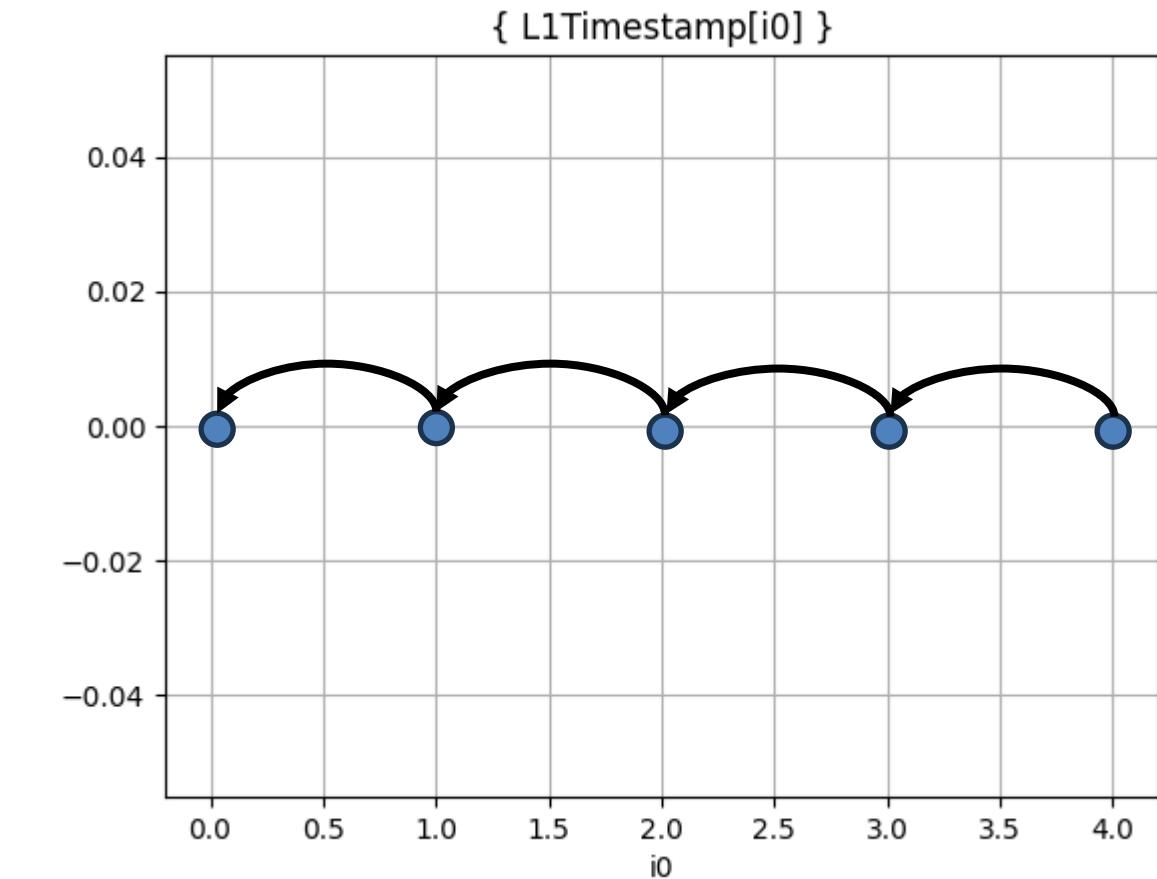
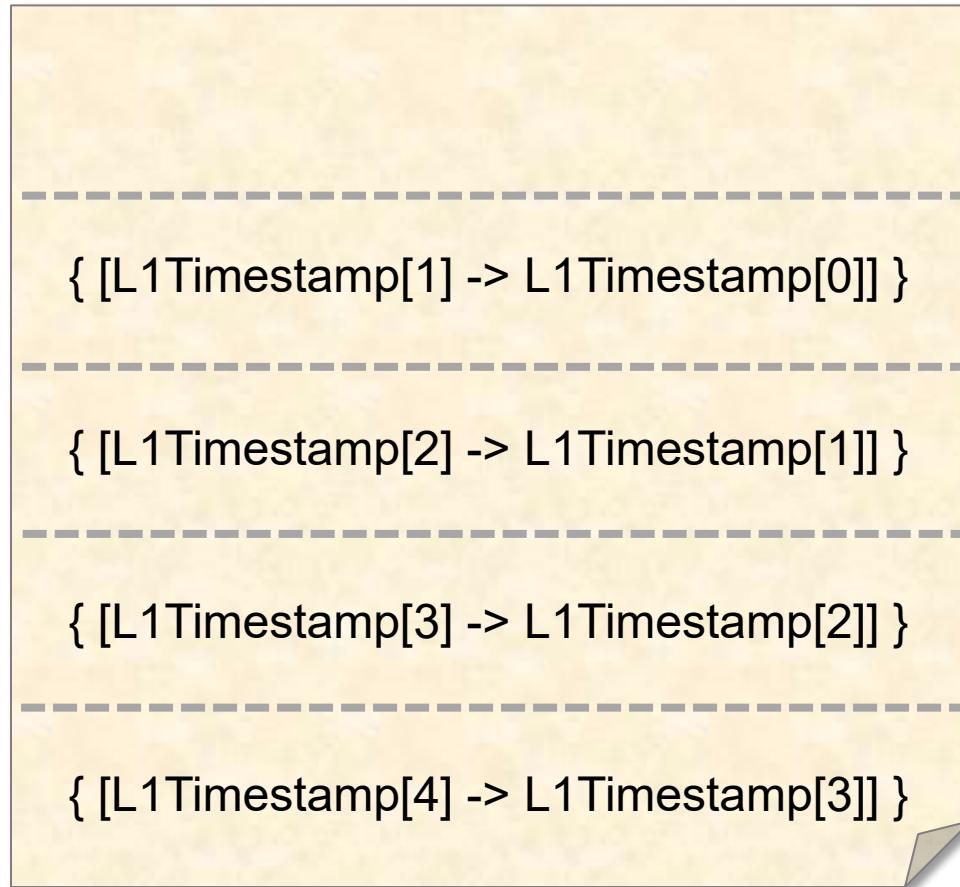
Tiling Timestamps

`L1timestamp2timestamp = timestamp2L1timestamp.reverse()`



Calculating Tile Data Movement

```
L1timestamp_previous = isl.Map(f"{{ L1Timestamp[t'] -> L1Timestamp[t] : t'=t+1 }}")
```



L1 Weight Accesses

```
df_L1ts2weight_current = L1timestamp2timestamp.apply_range(df_ts2weight_current)
```

```
{ [L1Timestamp[0] -> Weight[0]] }
{ [L1Timestamp[0] -> Weight[1]] }
{ [L1Timestamp[0] -> Weight[2]] }
{ [L1Timestamp[1] -> Weight[0]] }
{ [L1Timestamp[1] -> Weight[1]] }
{ [L1Timestamp[1] -> Weight[2]] }
{ [L1Timestamp[2] -> Weight[0]] }
{ [L1Timestamp[2] -> Weight[1]] }
{ [L1Timestamp[2] -> Weight[2]] }
{ [L1Timestamp[3] -> Weight[0]] }
{ [L1Timestamp[3] -> Weight[1]] }
{ [L1Timestamp[3] -> Weight[2]] }
{ [L1Timestamp[4] -> Weight[0]] }
{ [L1Timestamp[4] -> Weight[1]] }
{ [L1Timestamp[4] -> Weight[2]] }
```

L1 Weight Accesses

```
df_L1ts2weight_current = L1timestamp2timestamp.apply_range(df_ts2weight_current)
df_L1ts2weight_previous = L1timestamp_previous.apply_range(df_L1ts2weight_current)
```

{ [L1Timestamp[1] -> Weight[0]] }
{ [L1Timestamp[1] -> Weight[1]] }
{ [L1Timestamp[1] -> Weight[2]] }
{ [L1Timestamp[2] -> Weight[0]] }
{ [L1Timestamp[2] -> Weight[1]] }
{ [L1Timestamp[2] -> Weight[2]] }
{ [L1Timestamp[3] -> Weight[0]] }
{ [L1Timestamp[3] -> Weight[1]] }
{ [L1Timestamp[3] -> Weight[2]] }
{ [L1Timestamp[4] -> Weight[0]] }
{ [L1Timestamp[4] -> Weight[1]] }
{ [L1Timestamp[4] -> Weight[2]] }

{ [L1Timestamp[0] -> Weight[0]] }
{ [L1Timestamp[0] -> Weight[1]] }
{ [L1Timestamp[0] -> Weight[2]] }
{ [L1Timestamp[1] -> Weight[0]] }
{ [L1Timestamp[1] -> Weight[1]] }
{ [L1Timestamp[1] -> Weight[2]] }
{ [L1Timestamp[2] -> Weight[0]] }
{ [L1Timestamp[2] -> Weight[1]] }
{ [L1Timestamp[2] -> Weight[2]] }
{ [L1Timestamp[3] -> Weight[0]] }
{ [L1Timestamp[3] -> Weight[1]] }
{ [L1Timestamp[3] -> Weight[2]] }
{ [L1Timestamp[4] -> Weight[0]] }
{ [L1Timestamp[4] -> Weight[1]] }
{ [L1Timestamp[4] -> Weight[2]] }

Previous

Current

L1 Weight Deltas

```
df_L1ts2weight_delta = df_L1ts2weight_current.subtract(df_L1ts2weight_previous)
```

{ [L1Timestamp[0] -> Weight[0]] }
{ [L1Timestamp[0] -> Weight[1]] }
{ [L1Timestamp[0] -> Weight[2]] }

L1 Input Accesses

```
df_L1ts2input_current = L1timestamp2timestamp.apply_range(df_ts2input_current)
df_L1ts2input_previous = L1timestamp_previous.apply_range(df_L1ts2input_current)
```

{ [L1Timestamp[1] -> Input[0]] }
{ [L1Timestamp[1] -> Input[1]] }
{ [L1Timestamp[1] -> Input[2]] }
{ [L1Timestamp[2] -> Input[1]] }
{ [L1Timestamp[2] -> Input[2]] }
{ [L1Timestamp[2] -> Input[3]] }
{ [L1Timestamp[3] -> Input[2]] }
{ [L1Timestamp[3] -> Input[3]] }
{ [L1Timestamp[3] -> Input[4]] }
{ [L1Timestamp[4] -> Input[3]] }
{ [L1Timestamp[4] -> Input[4]] }
{ [L1Timestamp[4] -> Input[5]] }

{ [L1Timestamp[0] -> Input[0]] }
{ [L1Timestamp[0] -> Input[1]] }
{ [L1Timestamp[0] -> Input[2]] }
{ [L1Timestamp[1] -> Input[1]] }
{ [L1Timestamp[1] -> Input[2]] }
{ [L1Timestamp[1] -> Input[3]] }
{ [L1Timestamp[2] -> Input[2]] }
{ [L1Timestamp[2] -> Input[3]] }
{ [L1Timestamp[2] -> Input[4]] }
{ [L1Timestamp[3] -> Input[3]] }
{ [L1Timestamp[3] -> Input[4]] }
{ [L1Timestamp[3] -> Input[5]] }
{ [L1Timestamp[4] -> Input[4]] }
{ [L1Timestamp[4] -> Input[5]] }
{ [L1Timestamp[4] -> Input[6]] }

L1 Input Deltas

```
df_L1ts2input_delta = df_L1ts2input_current.subtract(df_L1ts2input_previous)
```

{ [L1Timestamp[0] -> Input[0]] }

{ [L1Timestamp[0] -> Input[1]] }

{ [L1Timestamp[0] -> Input[2]] }

{ [L1Timestamp[1] -> Input[3]] }

{ [L1Timestamp[2] -> Input[4]] }

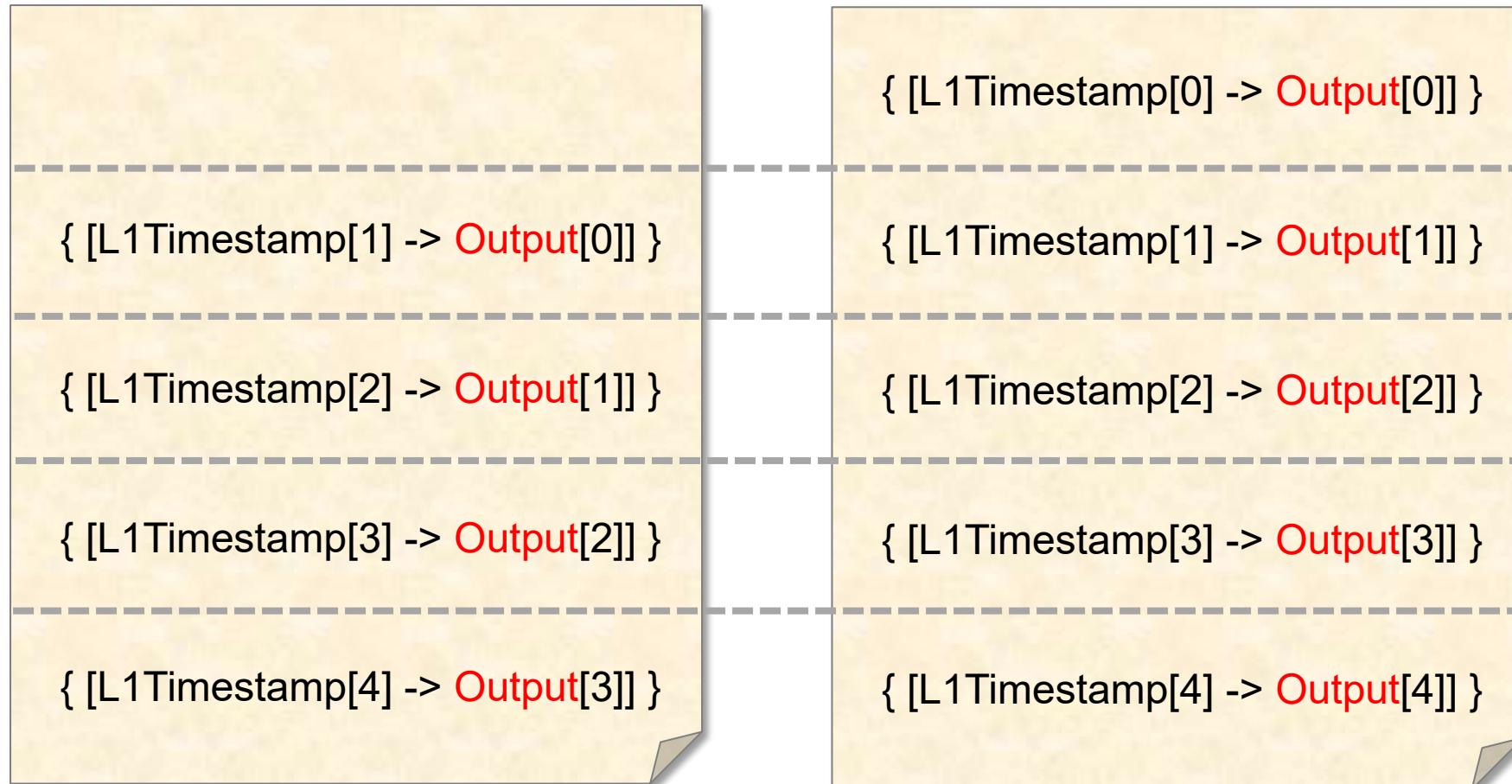
{ [L1Timestamp[3] -> Input[5]] }

{ [L1Timestamp[4] -> Input[6]] }

It's a sliding
window!

L1 Output Accesses

```
df_L1ts2output_current = L1timestamp2timestamp.apply_range(df_ts2weight_current)  
df_L1ts2output_previous = L1timestamp_previous.apply_range(df_L1ts2output_current)
```



L1 Weight Deltas

```
df_L1ts2output_delta = df_L1ts2output_current.subtract(df_L1ts2output_previous)
```

{ [L1Timestamp[0] -> Output[0]] }

{ [L1Timestamp[1] -> Output[1]] }

{ [L1Timestamp[2] -> Output[2]] }

{ [L1Timestamp[3] -> Output[3]] }

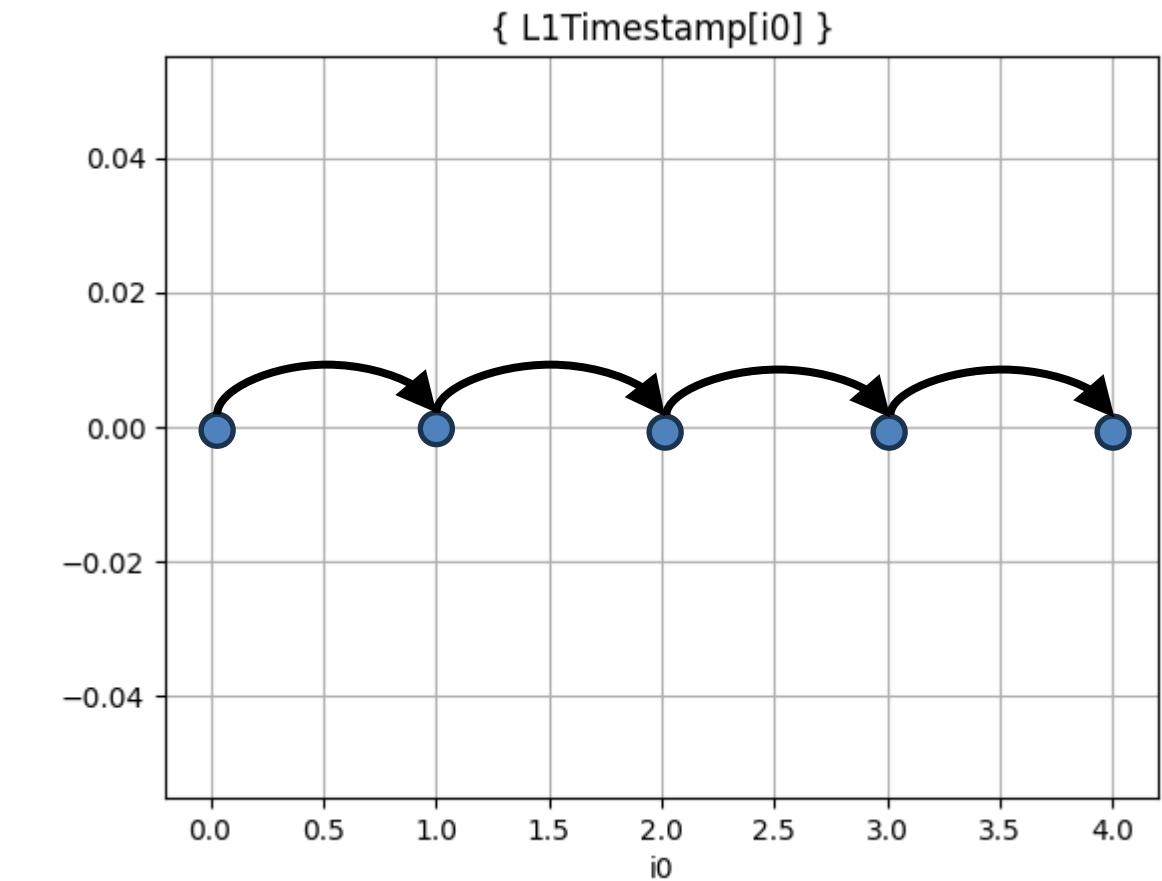
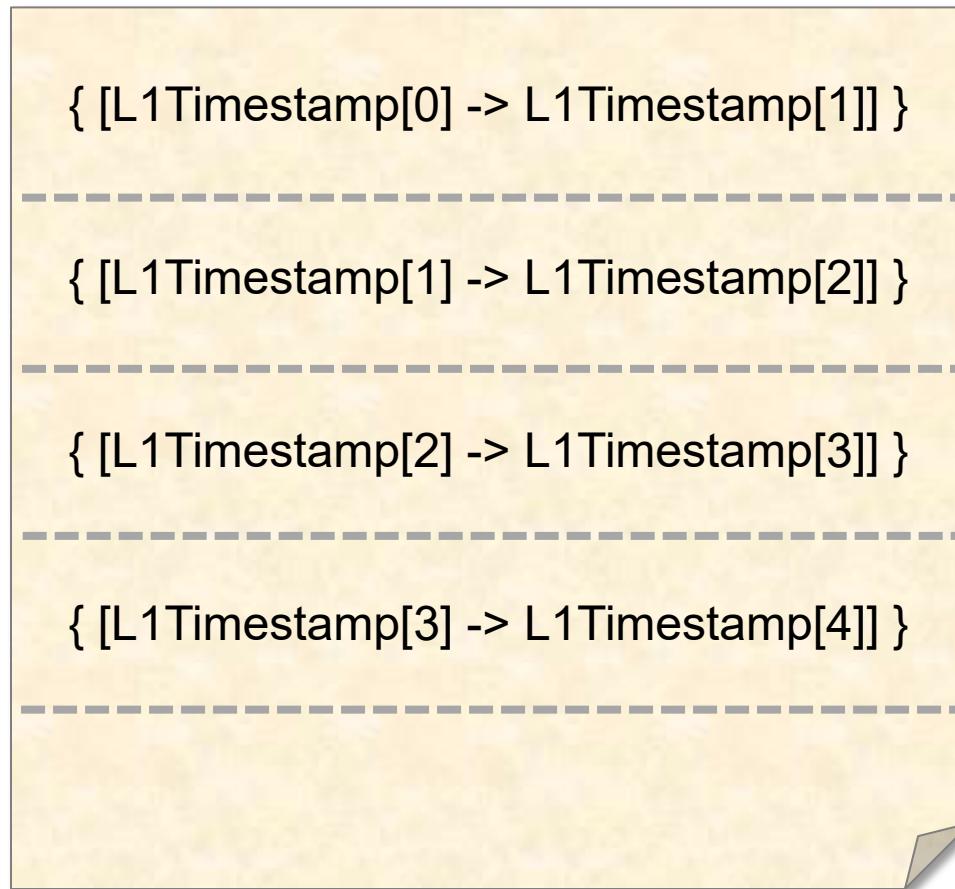
{ [L1Timestamp[4] -> Output[4]] }

Shrink Recipe

- Calculate data needed in next cycle
- Take difference from data needed in previous cycle

Tiling Timestamps

```
L1timestamp_next = L1timestamp_previous.reverse()
```



L1 Weight Accesses (next)

```
df_L1ts2weight_current = L1timestamp2timestamp.apply_range(df_ts2weight_current)
df_L1ts2weight_next = L1timestamp_next.apply_range(df_L1ts2weight_current)
```

{ [L1Timestamp[0] -> Weight[0]] }
{ [L1Timestamp[0] -> Weight[1]] }
{ [L1Timestamp[0] -> Weight[2]] }
{ [L1Timestamp[1] -> Weight[0]] }
{ [L1Timestamp[1] -> Weight[1]] }
{ [L1Timestamp[1] -> Weight[2]] }
{ [L1Timestamp[2] -> Weight[0]] }
{ [L1Timestamp[2] -> Weight[1]] }
{ [L1Timestamp[2] -> Weight[2]] }
{ [L1Timestamp[3] -> Weight[0]] }
{ [L1Timestamp[3] -> Weight[1]] }
{ [L1Timestamp[3] -> Weight[2]] }
{ [L1Timestamp[4] -> Weight[0]] }
{ [L1Timestamp[4] -> Weight[1]] }
{ [L1Timestamp[4] -> Weight[2]] }

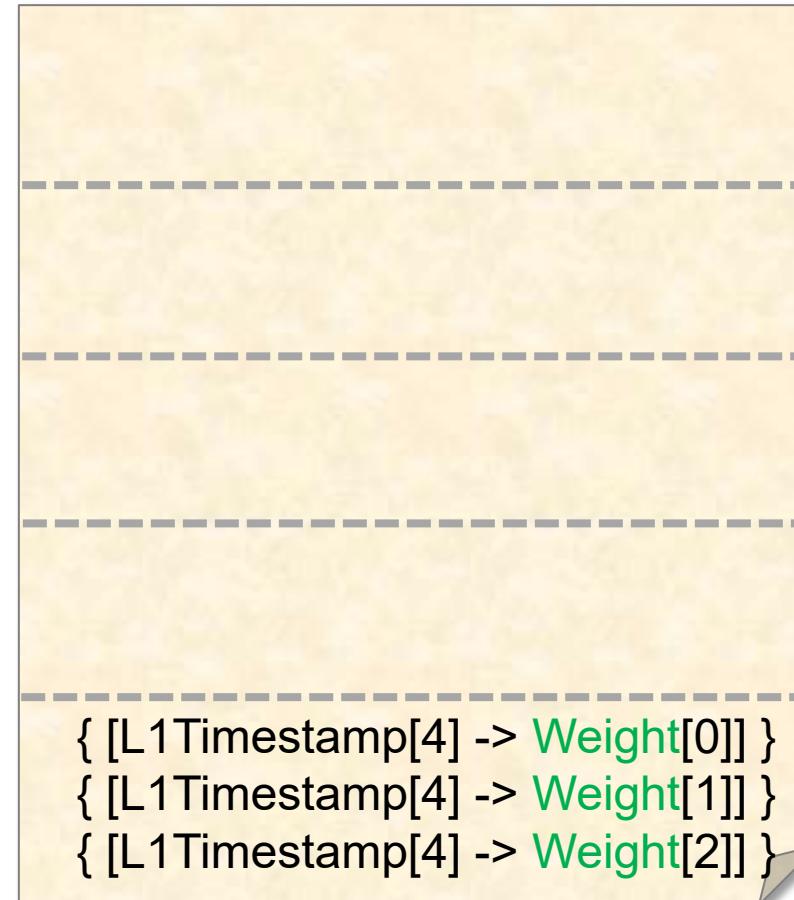
Current

{ [L1Timestamp[0] -> Weight[0]] }
{ [L1Timestamp[0] -> Weight[1]] }
{ [L1Timestamp[0] -> Weight[2]] }
{ [L1Timestamp[1] -> Weight[0]] }
{ [L1Timestamp[1] -> Weight[1]] }
{ [L1Timestamp[1] -> Weight[2]] }
{ [L1Timestamp[2] -> Weight[0]] }
{ [L1Timestamp[2] -> Weight[1]] }
{ [L1Timestamp[2] -> Weight[2]] }
{ [L1Timestamp[3] -> Weight[0]] }
{ [L1Timestamp[3] -> Weight[1]] }
{ [L1Timestamp[3] -> Weight[2]] }

Next

L1 Weight Shrinks

```
df_L1ts2weight_shrink = df_L1ts2weight_current.subtract(df_L1ts2weight_next)
```



L1 Input Accesses (Next)

```
df_L1ts2input_current = L1timestamp2timestamp.apply_range(df_ts2input_current)
df_L1ts2input_next = L1timestamp_next.apply_range(df_L1ts2input_current)
```

```
{ [L1Timestamp[0] -> Input[0]] }
{ [L1Timestamp[0] -> Input[1]] }
{ [L1Timestamp[0] -> Input[2]] }
{ [L1Timestamp[1] -> Input[1]] }
{ [L1Timestamp[1] -> Input[2]] }
{ [L1Timestamp[1] -> Input[3]] }
{ [L1Timestamp[2] -> Input[2]] }
{ [L1Timestamp[2] -> Input[3]] }
{ [L1Timestamp[2] -> Input[4]] }
{ [L1Timestamp[3] -> Input[3]] }
{ [L1Timestamp[3] -> Input[4]] }
{ [L1Timestamp[3] -> Input[5]] }
{ [L1Timestamp[4] -> Input[4]] }
{ [L1Timestamp[4] -> Input[5]] }
{ [L1Timestamp[4] -> Input[6]] }
```

Current

```
{ [L1Timestamp[0] -> Input[1]] }
{ [L1Timestamp[0] -> Input[2]] }
{ [L1Timestamp[0] -> Input[3]] }
{ [L1Timestamp[1] -> Input[2]] }
{ [L1Timestamp[1] -> Input[3]] }
{ [L1Timestamp[1] -> Input[4]] }
{ [L1Timestamp[2] -> Input[3]] }
{ [L1Timestamp[2] -> Input[4]] }
{ [L1Timestamp[2] -> Input[5]] }
{ [L1Timestamp[3] -> Input[4]] }
{ [L1Timestamp[3] -> Input[5]] }
{ [L1Timestamp[3] -> Input[6]] }
```

Next

L1 Input Shrink

```
df_L1ts2input_shrink = df_L1ts2input_current.subtract(df_L1ts2input_next)
```

{ [L1Timestamp[0] -> Input[0]] }

{ [L1Timestamp[1] -> Input[1]] }

{ [L1Timestamp[2] -> Input[2]] }

{ [L1Timestamp[3] -> Input[3]] }

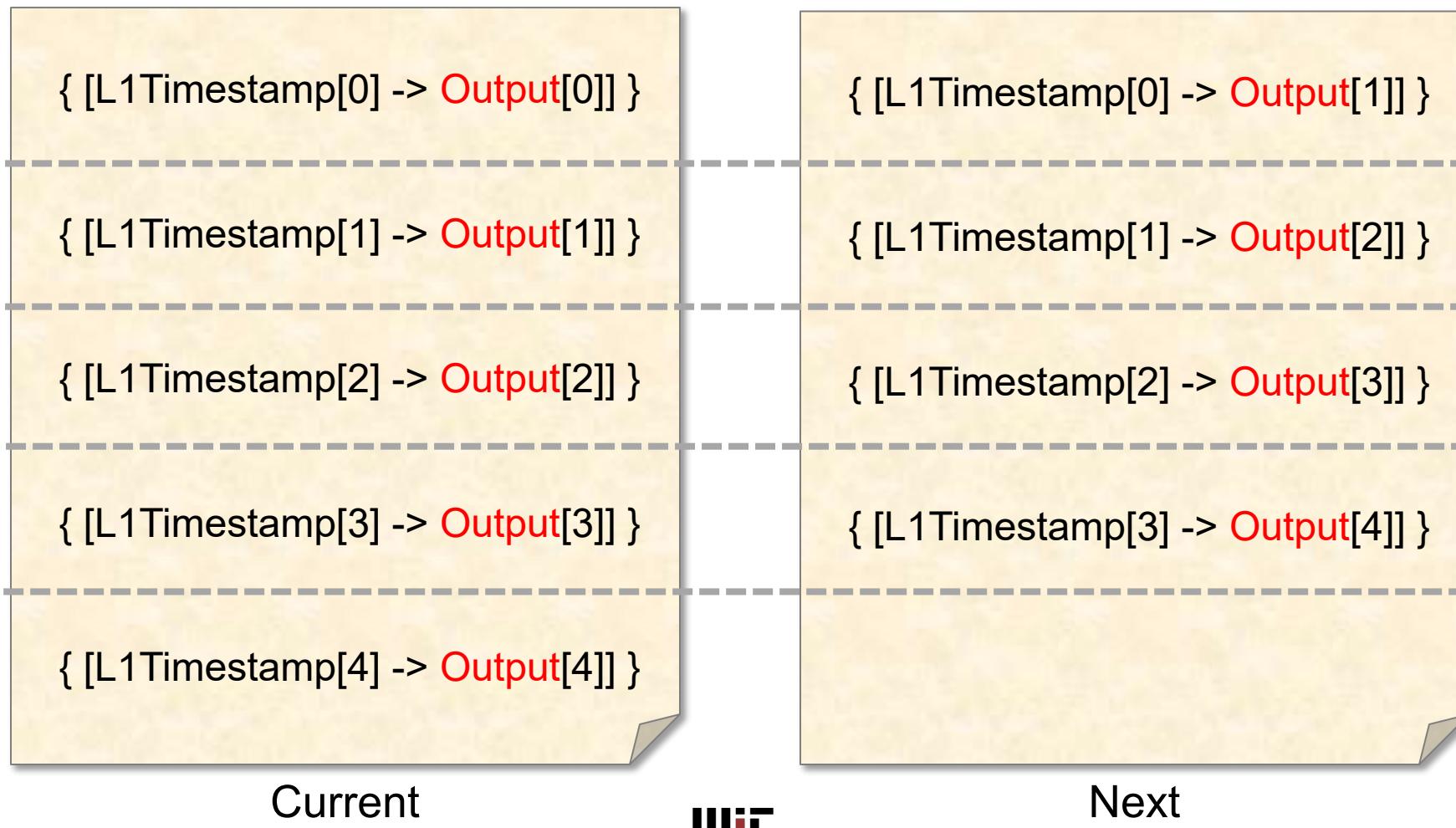
{ [L1Timestamp[4] -> Input[4]] }

{ [L1Timestamp[4] -> Input[5]] }

{ [L1Timestamp[4] -> Input[6]] }

L1 Output Accesses (Next)

```
df_L1ts2output_current = L1timestamp2timestamp.apply_range(df_ts2weight_current)  
df_L1ts2output_next = L1timestamp_next.apply_range(df_L1ts2output_current)
```



L1 Output Shrink

```
df_L1ts2output_shrink = df_L1ts2output_current.subtract(df_L1ts2output_next)
```

{ [L1Timestamp[0] -> Output[0]] }

{ [L1Timestamp[1] -> Output[1]] }

{ [L1Timestamp[2] -> Output[2]] }

{ [L1Timestamp[3] -> Output[3]] }

{ [L1Timestamp[4] -> Output[4]] }

OS -> WS

```
os_schedule = isl.Map(f'{{ IterationSpace[q,s] -> Timestamp[t1,t0] : t1=q and t0=s }}')
```



```
ws_schedule = isl.Map(f'{{ IterationSpace[q,s] -> Timestamp[t1,t0] : t1=s and t0=q }}')
```

Next Lecture: Sparsity

Thank you!