Appendix A-Conventional Lagrangian relaxation

The model **OM** is shown as follows.

$$\min \ e_1 \cdot \sum_{s\in S}\sum_{s'>s}\sum_{k\in K} r_k^{s,s'} + e_2 \cdot \sum_{i\in I}\sum_{a\in A} c^a \cdot x_i^a \tag{1}$$

$$\sum_{a\in A_{o(i)}^{\text{travel}}} x_i^a = 1 \qquad i\in I \tag{2}$$

$$\sum_{a\in A: a^- = v} x_i^a - \sum_{a\in A: a^+ = v} x_i^a = 0 \qquad i\in I, v\in\{V_s \mid o(i)<s<d(i)\} \tag{3}$$

$$\sum_{a\in A_{d(i)}^{\text{travel}}} x_i^a = 1 \qquad i\in I \tag{4}$$

$$\sum_{i\in I}\sum_{a\in A_v^{\text{con}}} x_i^a \le 1 \qquad v\in V \tag{5}$$

$$r_k^{s,s'} = r_{k-1}^{s,s'} + p_k^{s,s'} - \sum_{i\in I} b_{k,i}^{s,s'} \qquad s'>s, s\in S, k\in K \tag{7}$$

$$r_{|K|}^{s,s'} = 0 \qquad s'>s, s\in S \tag{8}$$

$$\sum_{s'\le s, s'\in S_i}\ \sum_{s''>s, s''\in S_i}\ \sum_{k\in K} b_{k,i}^{s',s''} \le q_i \qquad s\in S_i, i\in I \tag{9}$$

$$\sum_{s'>s, s'\in S_i} b_{k,i}^{s,s'} / q_i \le \sum_{a\in A_{s,k}^{\text{travel}}} x_i^a \qquad s\in S_i \setminus \{d(i)\}, i\in I, k\in K \tag{10}$$

$$\sum_{k\in K}\Big(\sum_{s'>s, s'\in S_i} b_{k,i}^{s,s'} + \sum_{s'<s, s'\in S_i} b_{k,i}^{s',s}\Big) / 2q_i \le \sum_{a\in A_s^{\text{wait}}} x_i^a \qquad s\in S_i \setminus \{o(i), d(i)\}, i\in I \tag{11}$$

Under Lagrangian relaxation framework, we can transform constraints (5), (10) and (11) to the objective function (1) by introducing multipliers α_v, $\bar{\beta}_{i,k}^s$, and $\bar{\gamma}_i^s$, and then obtain the Lagrangian dual problem with the following function and the remaining constraints.

$$\max_{\boldsymbol{\alpha},\bar{\boldsymbol{\beta}},\bar{\boldsymbol{\gamma}}} LR = \min_{\boldsymbol{x}} \sum_{i\in I}\sum_{a\in A} \bar{c}_i^a \cdot x_i^a + \min_{\boldsymbol{r},\boldsymbol{b}} \left[e_1 \cdot \sum_{s\in S}\sum_{s'>s}\sum_{k\in K} r_k^{s,s'} + \sum_{i\in I}\sum_{s\in S_i}\sum_{k\in K} b_{k,i}^{s',s} \cdot \left[\bar{\beta}_{i,k}^s + (\bar{\gamma}_i^s + \bar{\gamma}_i^{s'})/2 \right] / q_i \right] - \sum_{v\in V} \alpha_v$$

where $\bar{c}_i^a = \begin{cases} e_2 \cdot c^a + \sum_{v\in V: a\in A_v^{\text{con}}} \alpha_v - \bar{\beta}_{i,\bar{k}(a)}^s & a\in A_s^{\text{travel}} \\ 0 & a\in A_s^{\text{pass}} \\ e_2 \cdot c^a - \bar{\gamma}_i^s & a\in A_s^{\text{wait}} \end{cases} \quad s\in S_i, i\in I$.

The above Lagrangian dual problem can be decomposed into the following two subproblems.

Subproblem 1 (ASP1): Train space-time path subproblem

$$\min \ \sum_{i\in I}\sum_{a\in A} \bar{c}_i^a \cdot x_i^a$$

Constraints (2)-(4).

Subproblem 2 (ASP2): Demand loading subproblem

$$\min \ e_1 \cdot \sum_{s\in S}\sum_{s'>s}\sum_{k\in K} r_k^{s,s'} + \sum_{i\in I}\sum_{s\in S_i}\sum_{k\in K} b_{k,i}^{s',s} \cdot \left[\bar{\beta}_{i,k}^s + (\bar{\gamma}_i^s + \bar{\gamma}_i^{s'})/2 \right] / q_i$$

Constraints (7)-(9).