Appendix B: A two-stage optimization approach for the model OM (TS-OM)

(1) *The first stage*

The first stage aims to optimize the train skip-stop patterns. To formulate this problem, we additionally introduce the following variables and then build the first-stage optimization model on the basis of the model OM.

y_i^s : equal to 1 if train *i* stops at station *s* and 0 otherwise;

$z_{i,k}^s$: equal to 1 if train *i* departs from station *s* within period *k* and 0 otherwise.

The first-stage optimization model

$$\min\ 60\cdot\sum_{s\in S}\sum_{s'>s}\sum_{k\in K} r_k^{s,s'} + \sum_{i\in I}\sum_{s\in S_i\setminus\{o(i),d(i)\}} t_{i,s}^{\text{wait}} + \sum_{i\in I}\sum_{s\in S_i\setminus\{d(i)\}} p_{i,s}^{\text{in-train}}\cdot r_{i,s}^{\text{run}}$$

$s.t.$

$$t_{i,s}^{\text{wait}} \ge w_{i,s}^{\min}\cdot p_{i,s}^{\text{in-train}} + (y_i^s - 1)\cdot M \qquad s\in S_i\setminus\{o(i),d(i)\}, i\in I \tag{B1}$$

$$p_{i,s}^{\text{in-train}} = \sum_{k\in K}\sum_{s'\le s, s'\in S_i}\sum_{s''>s, s''\in S_i} b_{i,k}^{s',s''} \qquad s\in S_i\setminus\{d(i)\}, i\in I \tag{B2}$$

$$\sum_{s'>s, s'\in S_i} b_{i,k}^{s,s'} \le z_{i,k}^s\cdot M \qquad s\in S_i\setminus\{d(i)\}, i\in I, k\in K \tag{B3}$$

$$\sum_{k\in K}\Big(\sum_{s'>s, s'\in S_i} b_{i,k}^{s,s'} + \sum_{s'<s, s'\in S_i} b_{i,k}^{s',s}\Big) \le y_i^s\cdot M \qquad s\in S_i\setminus\{o(i),d(i)\}, i\in I \tag{B4}$$

$$r_k^{s,s'} = r_{k-1}^{s,s'} + p_k^{s,s'} - \sum_{i\in I: o(i)\le s<s'\le d(i)} b_{i,k}^{s,s'} \qquad s'>s, s\in S, k\in K \tag{B5}$$

$$r_{|K|}^{s,s'} = 0 \qquad s'>s, s\in S \tag{B6}$$

$$\sum_{s'\le s, s'\in S_i}\sum_{s''>s, s''\in S_i}\sum_{k\in K} b_{i,k}^{s',s''} \le q_i \qquad s\in S_i, i\in I \tag{B7}$$

$$\sum_{k\in K} z_{i,k}^s = 1 \qquad s\in S_i\setminus\{d(i)\}, i\in I \tag{B8}$$

Note that, except for the variables y_i^s and $z_{i,k}^s$, the other symbols have the same meaning as those of the model OM. In the first-stage model, since we do not consider the specific train timetable, the train dwell time at stations cannot be determined. Therefore, we use the minimum dwell time of trains to estimate the waiting time of in-train passengers in inequalities (B1). Considering that the above model is a mixed integer linear programming model, we thus employ GUROBI to solve it.

(2) *The second stage*

The second stage is only to determine train timetables. Specifically, we first fix the skip-stop pattern of each train using the output of the first-stage model, and then search the space-time of each train one by one in the space-time network. The solution process is similar to Algorithm 1, except that the trains are not sorted and the departure period constraints are not required.