# Calculation and Analysis of Flux Leakage Coefficient of Interior Permanent Magnet Synchronous Motors With Fractional Slot Concentrated Windings

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***Abstract*—Permanent magnet (PM) motors adopt interior V-type PMs and fractional-slot concentrated winding for improving the torque density and efficiency. However, the no-load magnetic coefficient will be increased, which would reduce the utilization rate of the PM. The structure parameters of the rotor should be designed reasonably, especially the bridge. This paper calculates the no-load leakage coefficient by establishing the analytical model of the no-load magnetic coefficient. It shows that, the bridge size has an important effect on the no-load magnetic coefficient. The analysis results are verified by the two-dimensional finite element analysis and tests.**

***Index Terms*—FSCW, IPMSM, iron bridge, V-type magnets.**

## I. Introduction

INTERIOR permanent magnet (PM) synchronous motors (IPMSM) with fractional slot concentrated windings (FSCW) are widely used because of their high torque density and high efficiency. Unlike other motors with surface-mounted PM, due to the asymmetry of the reluctance between d-axis and q-axis, the additional torque are utilized to improve their overload capacity and running efficiency [1]–[3]. In the meanwhile, the structure of the rotor can be designed flexibly, such as interior flat type, V-type, U-type and multilayer PMs. However, IPMSM produces a large leakage flux which jumps from one magnet to the next without passing into the stator especially FSCW is designed. The stator structure with FSCW is used to reduce the motor size. It is also beneficial to improve the coil space factor of motors, the end windings and the cogging torque [4].

The leakage flux coefficient reflects the effective utilization of total flux provided by PMs to the external magnetic circuit. It also indicates the diversion effort of the armature reaction and the resistance to the demagnetization of the PM. Therefore, an accurate calculation and reasonable design of the leakage flux coefficient should be taken into consideration in the process of motor design. Due to the complicated distribution of magnetic field in IPMSM, an accurate calculation of the leakage flux coefficient is needed with numerical method such as the finite element method (FEM). But it consumes a lot of time and computational resources, and it is adverse to the quick calculation and optimization design of the motor [5], [6].

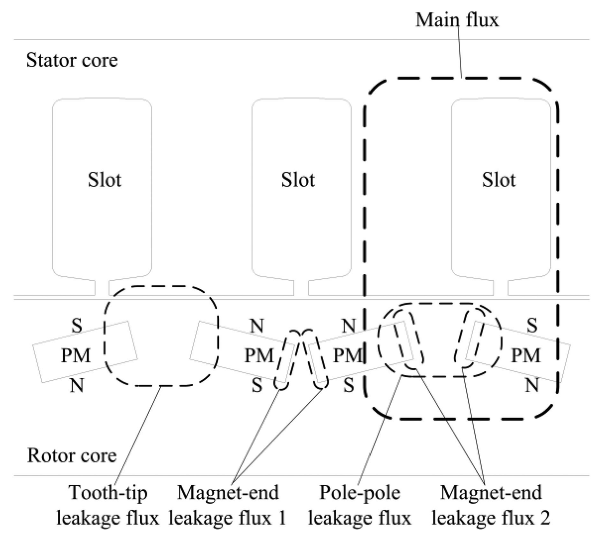


Fig. 1. V-type magnets IPMSM.

In this paper, a magnetic circuit model of IPMSM with V-type magnets and FSCW is proposed. The analytical method (AM) based on the magnetic circuit model is used to estimate the magnetic field and calculate the leakage flux coefficient. It is more practical and efficient to realize the initial design and schemes assessment comparing with FEM. To verify the proposed magnetic circuit model, a 35 kW IPMSM with V-type magnets and FSCW is analyzed by AM and FEM. The experimental tests have also been carried out. The comparison results show that the proposed model is suitable for magnetic field analysis and motor design.

## II. Analysis

### A. Magnetic Circuit Model

Fig. 1 shows the magnetic circuit diagram of an IPMSM with V-type magnets and FSCW. The PM as the magnetic sources,

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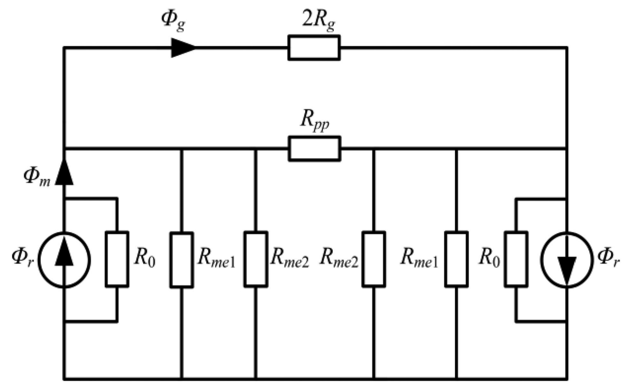


Fig. 2. Simple magnetic equivalent network for V-Type magnets PMSM.

the magnet flux leaving North poles crosses through rotor core, air-gap, tooth and yoke of the stator to South poles. This magnetic flux is linked by coils and the magnetic path is defined as the primary flux path. In addition to the primary flux path, some flux jump from North poles to South poles without passing into the stator is defined as the leakage flux paths. These leakage flux of the tooth-tip, pole-pole and magnet-ends are shown in Fig. 1.

### B. Magnetic Equivalent Network

To simplify the calculation, the permeability of the rotor and stator core except for the iron bridges is assumed infinite. The magnetic equivalent network is shown in Fig. 2.

In Fig. 2, $\Phi_r$ and $R_0$ are the flux source and the reluctance of each PM, respectively. $\Phi_g$ is the air-gap flux. $\Phi_m$ is the total flux provided by one PM to external magnetic circuit. $R_{me1}$ and $R_{me2}$ are the reluctances corresponding to the magnet-end leakage flux 1 and the magnet-end leakage flux 2, respectively. $R_g$ is the air-gap reluctance. $R_{pp}$ is the pole-pole reluctance between the adjacent PMs of different poles.

Applied Kirchhoff's law to magnetic field analysis, the air-gap flux, and air-gap flux density $B_g$ can be described respectively as

$$\Phi_g = \frac{\Phi_r}{1 + \frac{2R_g}{R_{pp}} + \frac{R_g(R_0 R_{me1} + R_0 R_{me2} + R_{me1} R_{me2})}{R_0 R_{me1} R_{me2}}}, \tag{1}$$

$$B_g = \frac{B_r C_\Phi}{1 + \frac{2R_g}{R_{pp}} + \frac{R_g(R_0 R_{me1} + R_0 R_{me2} + R_{me1} R_{me2})}{R_0 R_{me1} R_{me2}}}, \tag{2}$$

where $B_r$ is the remanent flux density of PM, $C_\Phi$ is the flux concentration factor [7], described as

$$C_\Phi = \frac{A_m}{A_g}. \tag{3}$$

Based on the magnetic equivalent network, $\Phi_m$ can be expressed as

$$\Phi_m = \frac{\Phi_r R_0 + \Phi_g \left(1 + \frac{2R_g}{R_{pp}}\right) \frac{R_{me1} R_{me2}}{R_{me1} + R_{me2}}}{R_0 + \frac{R_{me1} R_{me2}}{R_{me1} + R_{me2}}}. \tag{4}$$

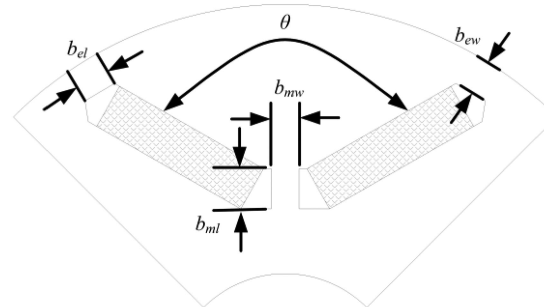


Fig. 3. Rotor structure.

## III. Calculation of the Tooth-Tip Leakage Flux

The tooth-tip leakage flux is the flux which leaving the PMs cross through rotor core, air-gap and stator core without linking with coils. The tooth-tip leakage flux is calculated simply by equalizing all the tooth-tip leakage flux to all the poles. The tooth-tip leakage flux equal to the average tooth-tip leakage flux [8], which can be calculated by

$$\Phi_{ave} = \frac{\sum_{i=1}^{z/\mathrm{GCD}(z,2p)} \Phi_{Lti}(x_i)}{2p/\mathrm{GCD}(z,2p)}, \tag{5}$$

where GCD($z$,$2p$) is the greatest common divisor (GCD) of the number of the slots $z$ and the number of the poles $2p$. $x_i$ is the shortest distance between the tooth and the corresponding q-axis. $\Phi_{Lti}$ is the leakage of one tooth, which can be described as

$$\Phi_{Lti} = \begin{cases} -\frac{2p\Phi_g}{\alpha_p \pi D_g} x + \frac{\Phi_g}{\alpha_p}\left(\frac{p}{z} + \frac{\alpha_p}{2} - \frac{1}{2}\right) & [0, |\xi|] \\ 0 & \text{others} \end{cases}, \tag{6}$$

$$\xi = \pm \frac{\pi D_g}{2}\left(\frac{1}{z} + \frac{\alpha_p}{2p} - \frac{1}{2p}\right), \tag{7}$$

where $D_g$ is the diameter of the air-gap. $\alpha_\mathrm{p}$ is the pole-arc coefficient.

## IV. Calculation of Magnetic Reluctance

The rotor structure is shown in Fig. 3. The reluctance of PM is described as

$$R_0 = \frac{h_{Mp}}{\mu_0 \mu_r l_w L}, \tag{8}$$

where $h_{Mp}$ is the magnet length in the direction of magnetization; $\mu_0$ and $\mu_r$ are the permeability of vacuum and the relative permeability of PM, respectively. $l_w$ is the thickness of a magnet pole. $L$ is the stack length of the laminated steel.

The reluctance of air-gap is described as

$$R_g = \frac{\delta}{\mu_0 A_g}, \tag{9}$$

$$A_g = L\left(\frac{\pi R_r}{180^\circ} \arctan \frac{l_w \tan\frac{\theta}{2} + \frac{b_{mw}}{2}}{R_r} + b_{el}\right), \tag{10}$$

where $\delta$ is the length of air-gap. $R_r$ is the outer diameter of the rotor.

TABLE I
MACHINE PARAMETER

| Parameters | Value |
|---|---|
| Rated output(kW) | 35 |
| Rated speed(r/min) | 3600 |
| Poles/Slots | 12/18 |
| Stator outer diameter(mm) | 300 |
| Stator inner diameter(mm) | 200 |
| Air-gap length(mm) | 0.9 |
| Rotor diameter(mm) | 198.2 |
| Stack length(mm) | 60 |
| Number of magnets per pole | 2 |
| Magnet width(mm) | 20 |
| Magnet thickness(mm) | 5 |

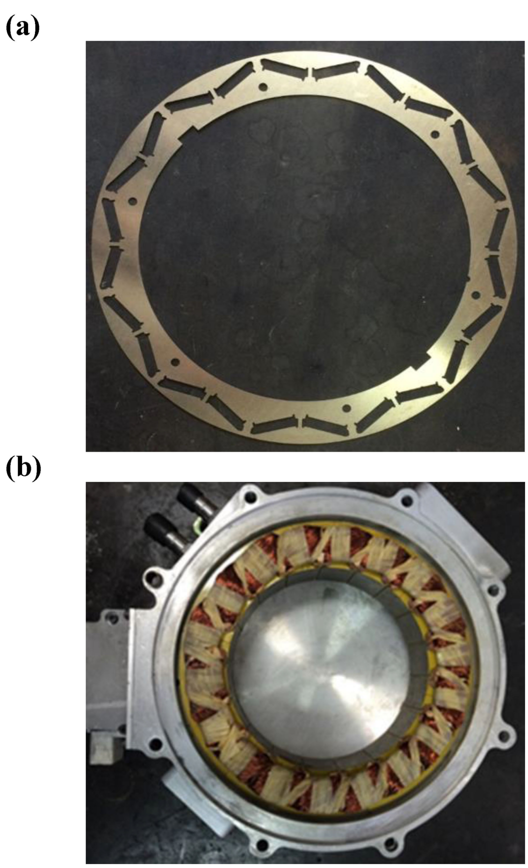


Fig. 4. (a) Rotor of prototype motor. (b) Stator of prototype motor.

The magnet-end leakage reluctance $R_{me1}$ and the pole-pole leakage reluctance $R_{pp}$ can be described as

$$R_{me1} = \frac{2b_{ml}}{\mu_{r(2.2\mathrm{T})} b_{mw} L}, \tag{11}$$

$$R_{pp} = \frac{b_{el}}{\mu_{r(2.2\mathrm{T})} b_{ew} L}, \tag{12}$$

where $\mu_{\mathrm{r}(2.2\mathrm{T})}$ is the relative permeability of rotor core and stator core material at the flux density 2.2 T, which is considered as the high saturation point. In fact, the flux density in the magnetic bridge is greater than or equal to 2.2 T.

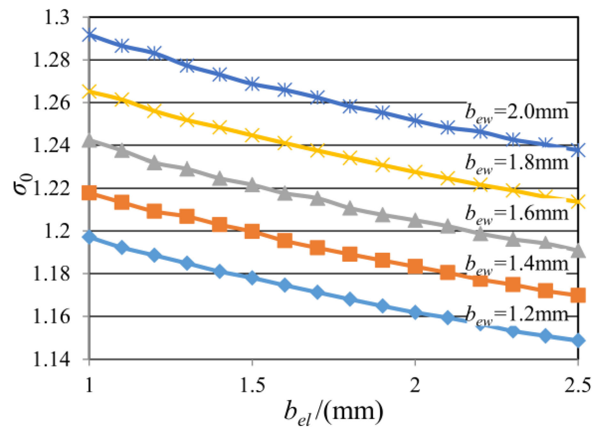


Fig. 5. Plot of $\sigma_0$ versus $b_{el}$.

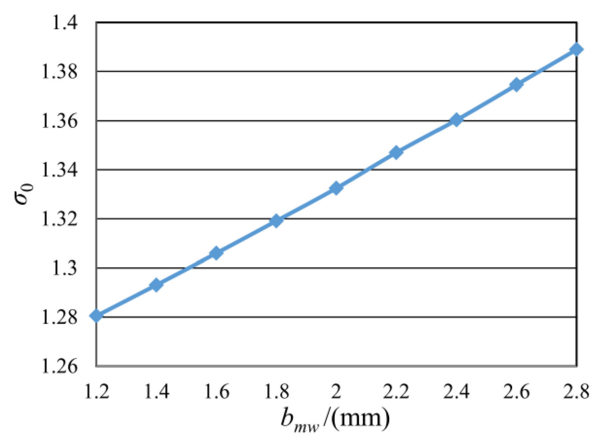


Fig. 6. Plot of $\sigma_0$ versus $b_{mw}$.

## V. Calculation of Flux Leakage Coefficient

Flux leakage coefficient $\sigma_0$ is acquired by the ratio of total magnetic flux to main magnetic flux. Substituting equation (9), (12) into (1) and combining with (5), the flux leakage coefficient can be expressed as

$$\sigma_0 = \frac{\Phi_m}{\Phi_g - \Phi_{ave}}. \tag{13}$$

The AM is used to calculate the leakage flux coefficient of a 35 kW IPMSM with V-type magnets and FSCW. The specifications are indicated in Table I and the motor prototype is shown in the Fig. 4.

There are five important parameters ($b_{el}$, $b_{ew}$, $b_{ml}$, $b_{mw}$, and $\theta$) to describe the rotor structure as shown in Fig. 4. In general, $b_{ml}$ is fixed to confine the PMs. The simplified relations between $b_{el}$, $b_{ew}$, $b_{mw}$, $\theta$ and $\sigma_0$ are shown in Figs. 5–7. It is concluded that $\sigma_0$ decreases when $b_{el}$ and $\theta$ increase. On the contrary, $\sigma_0$ increases when $b_{ew}$ and $b_{mw}$ increase. As expected, the smaller magnetic bridges width are, the lower the $\sigma_0$. A crucial problem of the rotor structure design for IPMSM is that, the bridges have sufficient mechanical strength in order to resist the mechanical stresses at the highest speed.

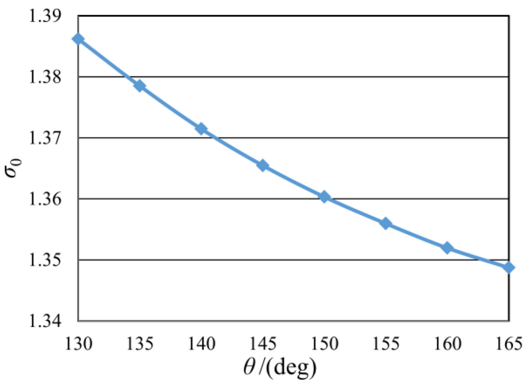


Fig. 7. Plot of $\sigma_0$ versus $\theta$.

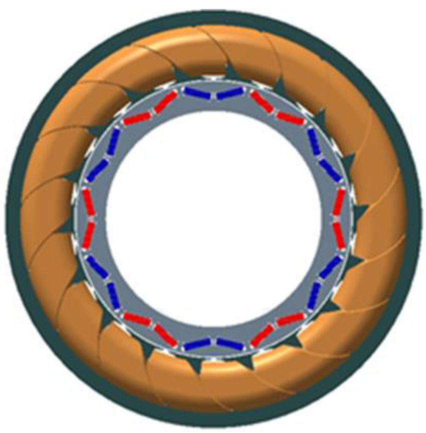

Fig. 8. FEM model.

Fig. 9. Experimental setup.

## VI. Experimental Test

In order to verify the analytical model, the AM and FEM are used to calculate $\sigma_0$ of a 35 kW IPMSM with V-type magnets and FSCW. The electromotive force (EMF) is also tested to reflect indirectly the $\sigma_0$ of the motor. The FEM model is shown in Fig. 8. The experimental setup is shown in Fig. 9. The comparison results are shown in Table II. The results show that the flux leakage coefficient calculated by AM is approximately equal to the finite element analysis result. The values of EMF gained by three methods are very close to each other.

TABLE II
Units for Magnetic Properties

| | AM | FEA | Test |
|---|---|---|---|
| σ0 | 1.23 | 1.29 | — |
| EMF(V/krpm) | 38.2 | 36.9 | 36.5 |

## VII. Conclusion

An analytical model taking into account, the flux leakage coefficient calculation of PMSM with interior V-type magnets and FSCW has been presented in this paper. The calculation results show that the structural parameters of the bridge have an effect of on the magnetic flux leakage coefficient, especially its breadth of it and the spacing distance between the bridges of the same poles. The validity of this analytical model has been verified by FEA and experimental tests. It is fast and efficient to compute the motor flux-leakage coefficient by the analytical method.

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