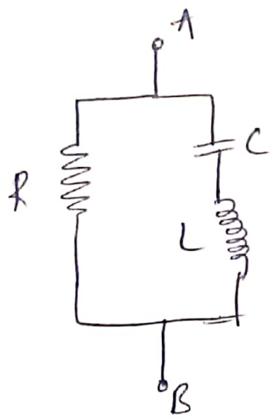
Tuned Amplifiers

→ Tuned amplifiers
- ↳ Small signal tuned amplifiers
- ↳ Large signal tuned amplifiers

→ To get narrow range of frequencies we use tuned amplifiers.

→ In tuned amplifiers, a tuned circuit is attached to the collector of the transistor.

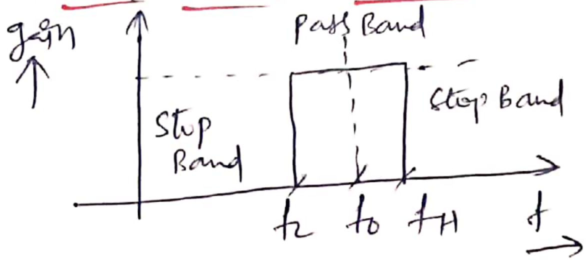


Tuned Circuit

≡

Frequency Response:

Ideal Tuned Circuit:

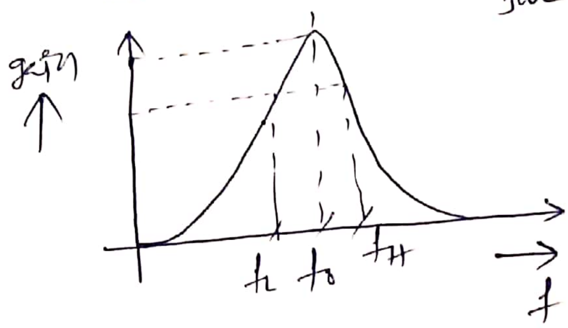


At 'f_0' circuit becomes purely resistive
$(X_L = X_C)$

Bandwidth $= (f_H - f_L)$

Practical tuned circuit:

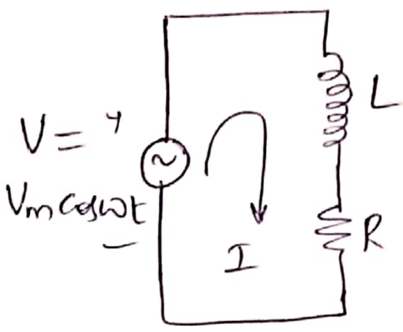
$Z_L = R + j\omega L$ & $Z_C = R + \frac{1}{j\omega C}$



→ Quality factor : (Q) :– It is defined as

$$\boxed{Q = 2\pi \times \frac{\text{Maximum Energy stored per cycle}}{\text{Energy dissipated per cycle}}}$$

→ For inductor:



Let I_m → Peak value of current 'I', then

→ Max. Energy stored in inductor per cycle is $\boxed{E_{max} = \frac{1}{2} L I_m^2}$

↳ $P = V.I$ & $V = L\frac{dI}{dt}$

$\Rightarrow P = L.I.\frac{dI}{dt}$

$\rightarrow E = \int P.dt = \int\left[L.I.\frac{dI}{dt}\right]dt$

$\Rightarrow E_{max} = \frac{1}{2} L I_m^2$

→ Average power dissipated per cycle

$$P = I_{rms}^2 . R = \left(\frac{I_m}{\sqrt{2}}\right)^2 . R = \frac{I_m^2 R}{2}$$

⇒ Energy dissipated per cycle $(E) = P \times T$

$$= \frac{I_m^2 . R}{2} . \frac{1}{f}$$

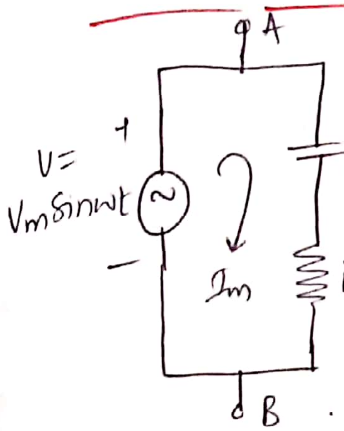
$$\rightarrow Q = 2\pi \times \frac{\frac{1}{2} L . I_m^2}{\frac{I_m^2 R}{2} . \frac{1}{f}} = \frac{2\pi f . L}{R}$$

$$\Rightarrow \boxed{Q = \frac{\omega L}{R}}$$

→ If inductor is pure $\Rightarrow R = 0$

$$\Rightarrow \boxed{Q = \infty}$$

→ For Capacitor:-



→ $E_{max} = \frac{1}{2} C V_{max}^2$

$(V_{max} \neq V_m)$

→ $V_{max} = I_m\left[\frac{1}{\omega C} + R\right]$

$P = V \cdot I$

$= C \cdot V \cdot \frac{dV}{dt}$

$E = \int P \cdot dt$

If $\frac{1}{\omega C} >> R \Rightarrow V = V_{max}$

$\Rightarrow \boxed{V_{max} = \frac{I_m}{\omega C}}$

$\Rightarrow E_{max} = \frac{1}{2} C \cdot \frac{I_m^2}{\omega^2 C^2} = \frac{1}{2} \frac{I_m^2}{\omega^2 C}$

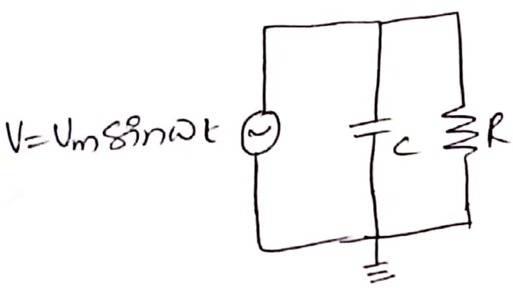
→ $E_{dissipated} = \frac{1}{2} I_m^2 \cdot \frac{R}{f}$

→ $Q = 2\pi \cdot \frac{\frac{1}{2} \frac{I_m^2}{\omega^2 C}}{\frac{1}{2} I_m^2 \cdot \frac{R}{f}} = \frac{2\pi f}{\omega^2 C R} = \frac{\omega}{\omega^2 C R}$

$\Rightarrow \boxed{Q = \frac{1}{\omega R C}}$

→ If $R = 0 \Rightarrow \boxed{Q = \infty}$ ← For pure Capacitor.

→ For lossy Capacitors



$E_{max} = \frac{1}{2} C V_m^2 \quad (\because V_{max} = V_m)$

$E_{dissipated} = \frac{V_m^2}{2} \cdot \frac{1}{R \cdot f}$

$Q = \frac{2\pi \times \frac{1}{2} C V_m^2}{\frac{V_m^2}{2} \cdot \frac{1}{Rf}}$

$\Rightarrow \boxed{Q = \omega R C}$

→ Loaded and Unloaded (Q):-

- Unloaded (Q): No load is connected to the tank circuit

$$Q_{unload} = 2\pi \times \frac{\text{Max. Energy stored per cycle}}{\text{Energy dissipated per cycle}}$$

- Loaded (Q):-

$$Q_{loaded} = 2\pi \times \frac{\text{Max. Energy stored per cycle}}{\left[\text{Energy dissipated per cycle}\right] + \left[\text{Energy dissipated due to the presence of External load}\right]}$$

→ Bandwidth decreases as Q-factor increases

↳ selectivity ~~decreases~~ increases

$$\boxed{BW = \frac{f_r}{Q}}$$

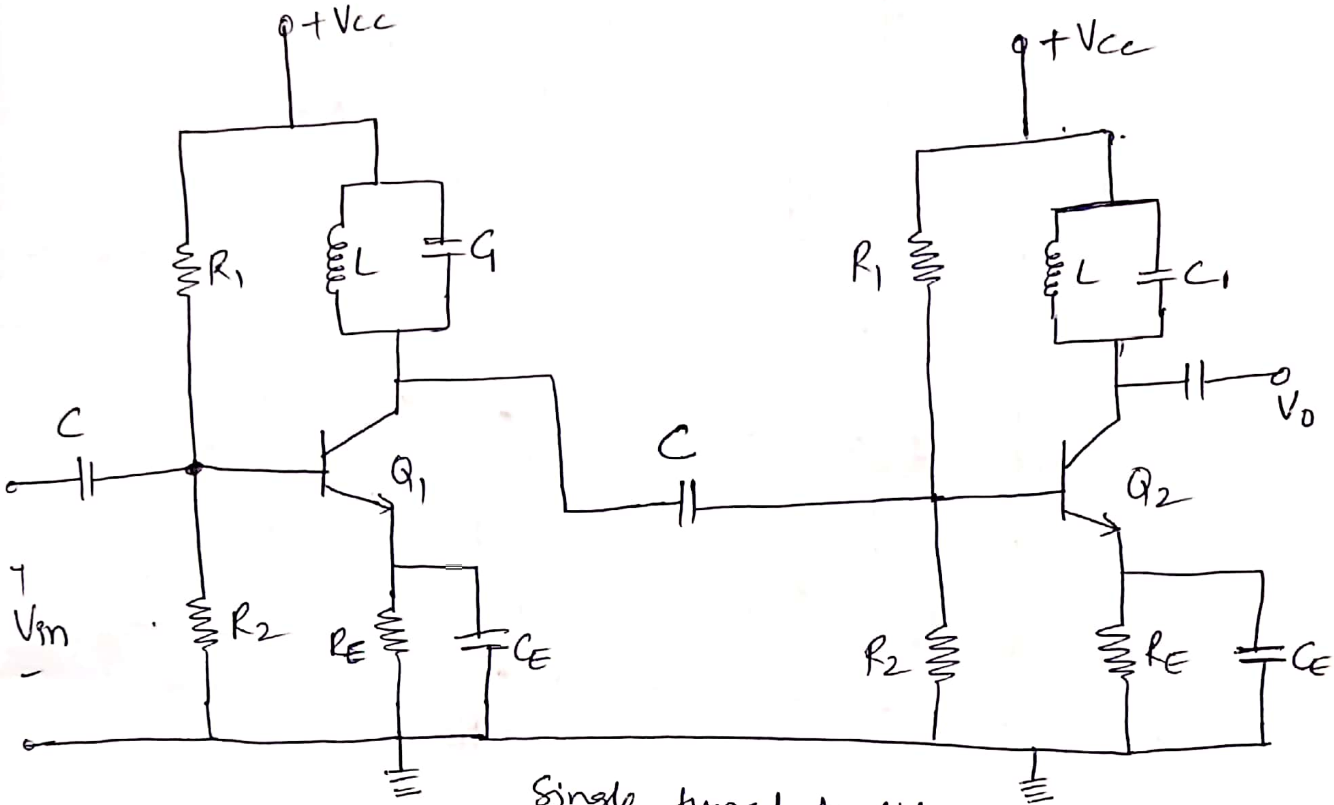
→ BW = Bandwidth

f_r = Resonant frequency of the tuned circuit

→ Small Signal Tuned amplifiers:-

- Single tuned amplifiers
  - Capacitor Coupled
  - Inductor Coupled
- Double tuned amplifiers
- Stagger tuned amplifiers.

→ Capacitor Coupled single tuned amplifier:-



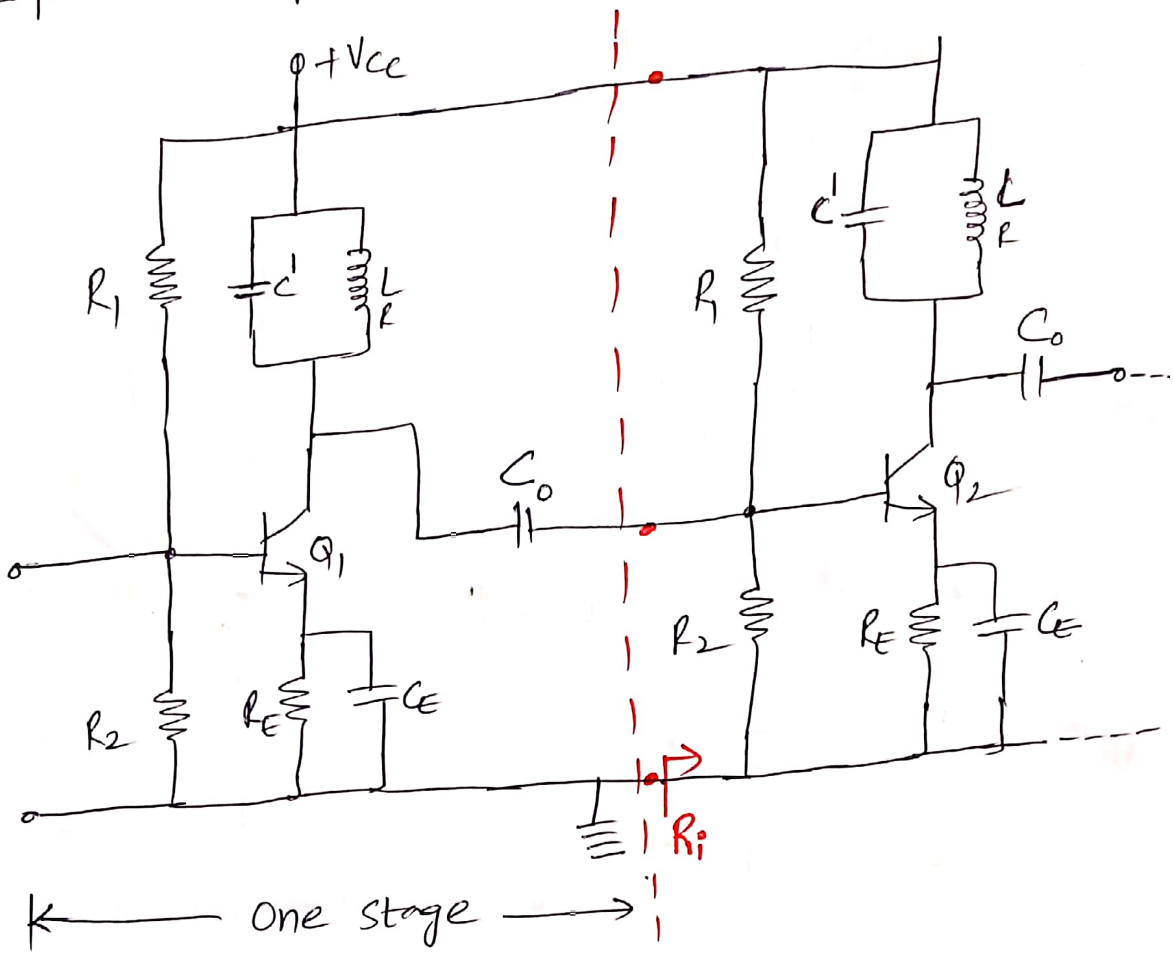
Single tuned Amplifier (STA)

→ Single tuned amplifiers use one parallel resonant circuit as load impedance in each stage and all the tuned circuits are tuned to the same frequency.

→ Double tuned amplifiers use two inductively coupled tuned circuits per stage, both the tuned circuits being tuned to the same frequency.

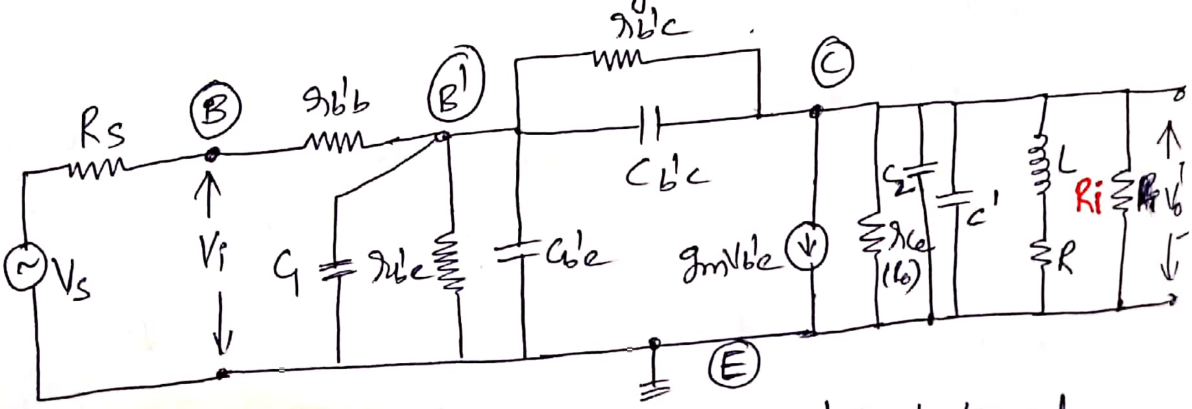
→ Stagger tuned amplifiers use a number of single tuned stages in cascade, the successive tuned circuits being tuned to slightly different frequencies.

→ Capacitor coupled single tuned Amplifier :-



AC Analysis :- (Using high-frequency π model) :-

↳ Consider one stage.

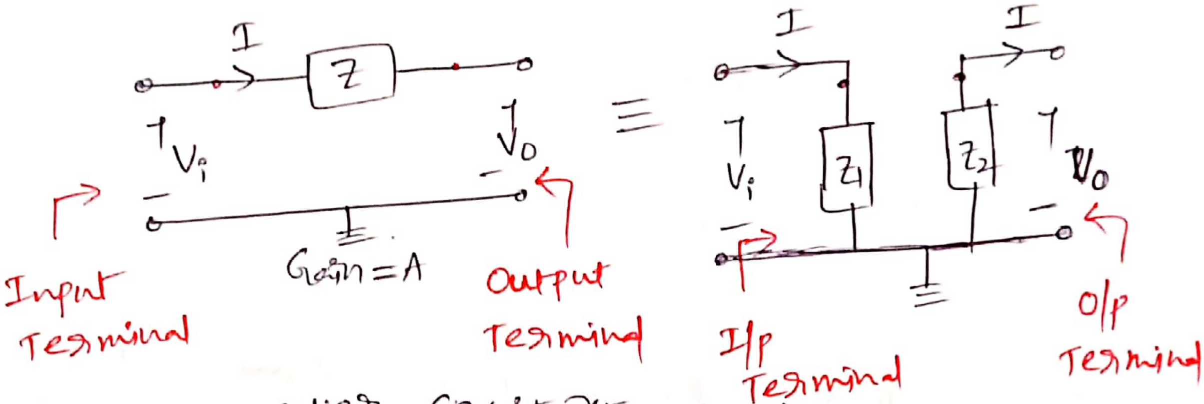


Equivalent circuit of single tuned amplifier

→ Where

- R_i : is the input resistance of the next stage.
- C_1 & C_2 are stray wiring capacitances at i/p & o/p ports respectively.

→ Miller's Theorem:-

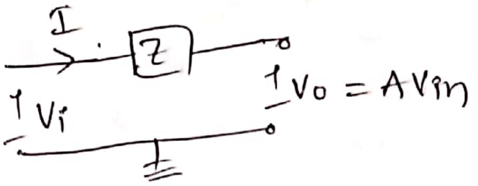


→ In an amplifier circuit with gain 'A', An impedance 'Z' connected between input and output terminals can be distributed to input and output terminals separately as Z_1 and Z_2

where $\boxed{Z_1 = \frac{Z}{(1-A)}}$ and $\boxed{Z_2 = Z\left(\frac{A}{A-1}\right)}$.

Proof:-

→

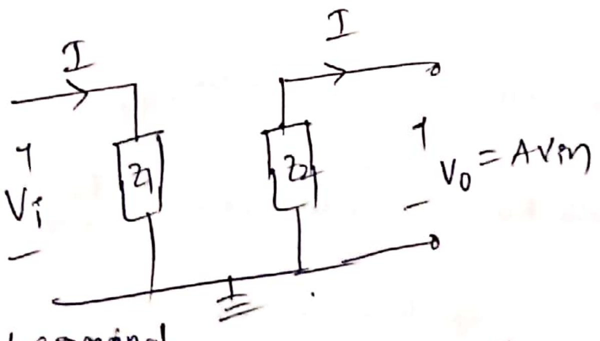


& $I = \frac{V_i - V_o}{Z}$

$= \frac{V_i - AV_{in}}{Z}$

$\Rightarrow \boxed{I = \frac{V_i(1-A)}{Z}}$.

→



→ At input terminal

$I = \frac{V_i - 0}{Z_1}$. $\Rightarrow$ ~~V_i~~ $Z_1 = \frac{V_i}{I} = \frac{V_i Z}{V_i(1-A)}$

$\Rightarrow \boxed{Z_1 = \frac{Z}{(1-A)}}$

→ At output terminal

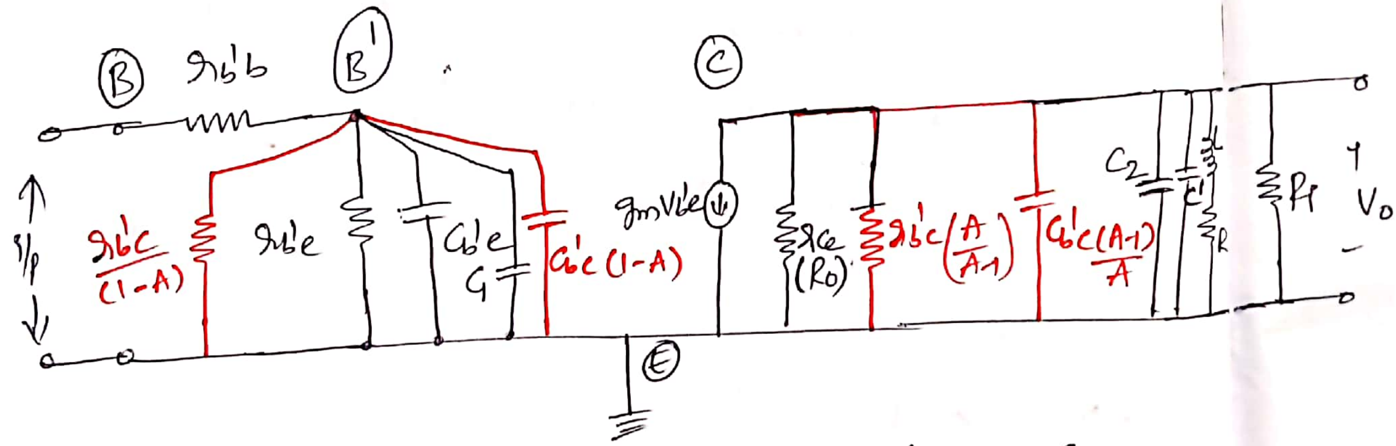
$$\rightarrow I = \frac{0 - V_o}{Z_2} \Rightarrow Z_2 = \frac{-V_o}{I}$$

$$\Rightarrow Z_2 = \frac{-AV_i}{I} \quad (\because V_o = AV_i)$$

$$= -\frac{AV_i}{\frac{V_i(1-A)}{Z}}$$

$$\Rightarrow \boxed{Z_2 = \frac{ZA}{(A-1)}}$$

→ Applying Miller's theorem to the equivalent circuit of single tuned amplifier.



where A: is the amplifier gain & $A >> 1$

and $r_{ce} = R_o$

At output side,

$$\rightarrow \because A >> 1 \Rightarrow r_{b'c}\frac{(A)}{(A-1)} = r_{b'c}$$

and $r_{b'c} >> R_o \Rightarrow \boxed{r_{b'c} \| R_o \simeq R_o} \Rightarrow \boxed{\text{Neglect } r_{b'c}}$

($r_{b'c}$ is the reverse biased resistance of the junction 'J_c')

Let

→ $C_{in} = C_{b'c}(1-A) + C_1 + C_{b'e}$

→ $C = C' + C_{b'c}\left(\frac{A-1}{A}\right) + C_2$.

→ At the input side

$\frac{r_{b'c}}{(1-A)} >> r_{b'e}$

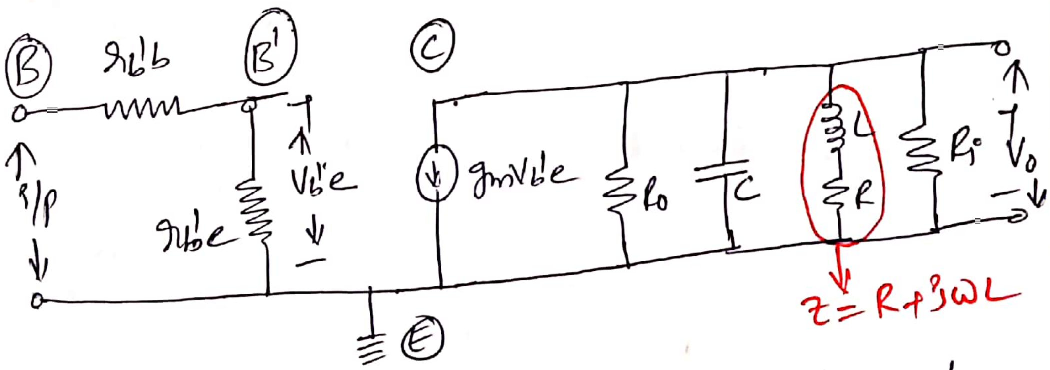
↳ Forward resistance of junction 'J_E'

$\Rightarrow \left[\frac{r_{b'c}}{(1-A)} \parallel r_{b'e}\right] \simeq r_{b'e}$.

$\Rightarrow \boxed{\text{Neglect } \frac{r_{b'c}}{(1-A)}}$

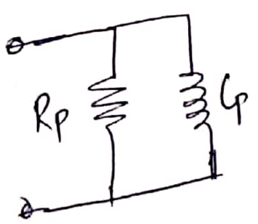
→ Input time constant is much less than output time constant } → (Time constant ~~R'XC'~~) = RC)

$\therefore$ $\boxed{C_{in} \text{ is neglected.}}$



Simplified equivalent circuit of single tuned amplifier.

→ Consider



$$\frac{1}{Z} = \frac{1}{R_p} + \frac{1}{L_p} = Y.$$

$$\Rightarrow \boxed{Y = \frac{1}{R_p} + \frac{1}{L_p}}.$$

→ From the simplified equivalent circuit

$Z = R + j\omega L$

$\Rightarrow Y = \frac{1}{R + j\omega L} = \frac{R - j\omega L}{R^2 + \omega^2 L^2}$

$\Rightarrow Y = \frac{R}{R^2 + \omega^2 L^2} - \frac{j\omega L}{R^2 + \omega^2 L^2}$

Comparing with $Y = \frac{1}{R_p} + \frac{1}{j\omega L_p}$

$R_p = \frac{R^2 + \omega^2 L^2}{R}$, $L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L}$

→ ~~Redrawing~~

Let Q_0 be the Q-factor at resonance given by

$Q_0 = \frac{\omega_0 L}{R}$ $\quad \because \omega L >> R$

$\Rightarrow \frac{\omega L}{R} >> 1 \;\Big|\; \frac{R}{\omega L} << 1$

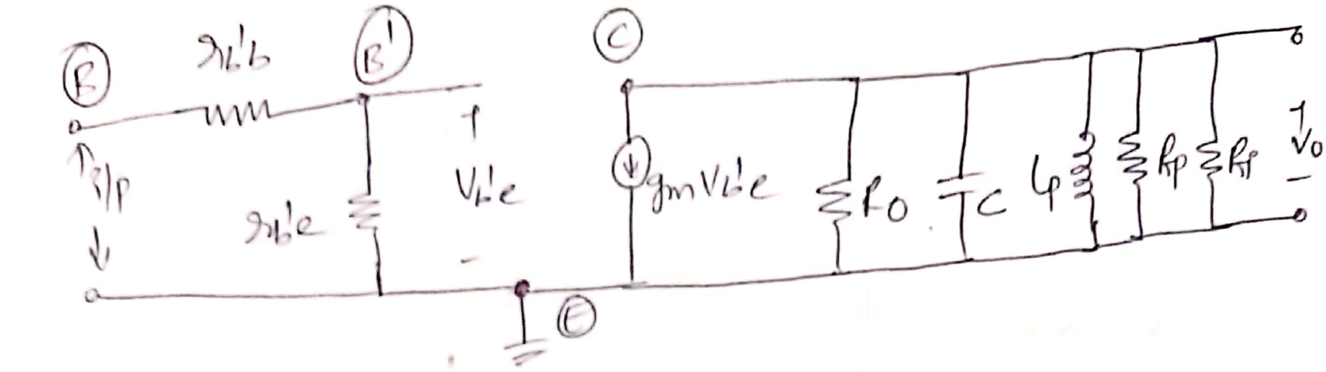
$\Rightarrow R_p = \frac{R^2 + \omega^2 L^2}{R} = R + \frac{\omega^2 L^2}{R}$.

$\Rightarrow \boxed{R_p \simeq \frac{\omega^2 L^2}{R}}$ $\quad (\because \frac{\omega L}{R} >> 1)$

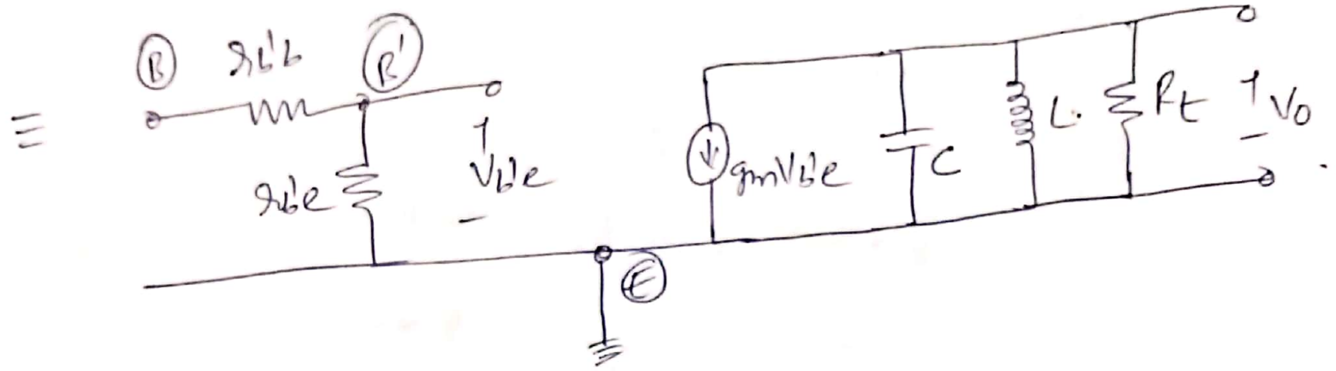
& $\Rightarrow L_p = \frac{R^2 + \omega^2 L^2}{\omega^2 L} = \frac{R^2}{\omega^2 L} + L$

$\Rightarrow \boxed{L_p \simeq L}$ $\quad (\because \frac{R}{\omega L} << 1)$

→ Re-drawing simplified equivalent circuit



Let $R_o \| R_p \| R_i = R_t$



Q-factor:

The effective Q-factor of the entire output circuit at resonant frequency 'ω_0' is given by

$$Q_e = \frac{\text{Susceptance of the inductor (L) (or) Capacitor (C)}}{\text{Conductance of resistance (R)}}$$

Note: Conductance $= \frac{1}{\text{Resistance}}$, Susceptance $= \frac{1}{\text{Reactance}}$

Admittance $= \frac{1}{\text{impedance}}$

& Impedance = Resistance + j (Reactance)

Admittance = Conductance + j (Susceptance)

$$\therefore Q_{e(L)} = \frac{1/\omega_0 L}{1/R_t} = \frac{R_t}{\omega_0 L}$$

$$Q_{e(C)} = \frac{\omega_0 C}{1/R_t} = \omega_0 C R_t$$

$$\Rightarrow \boxed{Q_e = \frac{R_t}{\omega_0 L} = \omega_0 C R_t}$$

$\rightarrow$ Gain $= \frac{V_o}{V_{in}}$

& $\boxed{V_o = -g_m V_{b'e} \cdot Z}$

where $Z = R_t \parallel j\omega L \parallel \frac{1}{j\omega C}$

$$\Rightarrow Y = \frac{1}{Z} = \frac{1}{R_t} + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{R_t}\left[1 + \frac{R_t}{j\omega L} + j\omega C R_t\right]$$

$$= \frac{1}{R_t}\left[1 + \frac{R_t}{j\omega_0 L}\cdot\left(\frac{\omega_0}{\omega}\right) + j\omega_0 C R_t \cdot \left(\frac{\omega}{\omega_0}\right)\right]$$

$$\Rightarrow Y = \frac{1}{R_t}\left[1 + \frac{Q_e}{j}\left(\frac{\omega_0}{\omega}\right) + jQ_e\left(\frac{\omega}{\omega_0}\right)\right]$$

$$\Rightarrow Y = \frac{\left[1 + jQ_e\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]}{R_t}$$

$$\Rightarrow \boxed{Z = \frac{1}{Y} = \frac{R_t}{\left[1 + jQ_e\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]}}$$

→ Let 'δ' indicates the fractional frequency variation i.e, variation in frequency expressed as a fraction of the resonant frequency

$$\Rightarrow \delta = \frac{\omega - \omega_0}{\omega_0} = \frac{\omega}{\omega_0} - 1$$

$$\Rightarrow \frac{\omega}{\omega_0} = \delta + 1$$

$$\rightarrow Z = \frac{R_t}{\left[1 + jQ_e\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]} = \frac{R_t}{\left[1 + jQ_e\left(\delta + 1 - \frac{1}{\delta + 1}\right)\right]}$$

$$\Rightarrow Z = \frac{R_t}{1 + jQ_e\left(\frac{\delta^2 + 2\delta}{\delta + 1}\right)}$$

$$\Rightarrow \boxed{Z = \frac{R_t}{1 + j2\delta Q_e\left[\frac{\delta/2 + 1}{\delta + 1}\right]}}.$$

→ If ω is close to 'ω_0'

$$\delta << 1 \Rightarrow \boxed{Z = \frac{R_t}{1 + j2\delta Q_e}}$$

→ If $\omega = \omega_0 \Rightarrow \delta = 0$.

$$\Rightarrow \boxed{Z = R_t}$$

→ We know $R_p = \frac{\omega^2 L^2}{R}$

and at resonance $X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C}$.

$$\Rightarrow \omega^2 = \frac{1}{LC}.$$

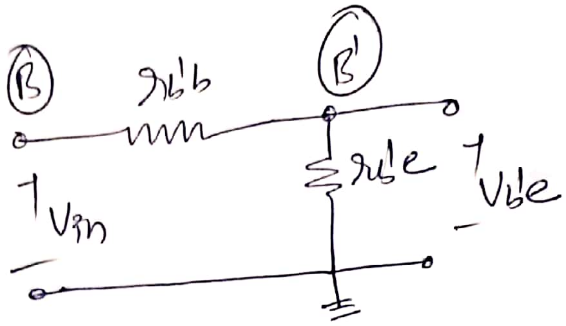
$$\Rightarrow \boxed{R_p = \frac{L^2}{LCR} = \frac{L}{RC}}$$

$$\longrightarrow Q_0 = \frac{\omega_0 L}{R}$$

$$\Rightarrow \boxed{R_p = \frac{\omega_0^2 L^2}{R} = Q_0^2 \cdot R = \omega_0 L Q_0}$$

$\rightarrow \because R_p = Q_0^2 \cdot R \Rightarrow$ Q-factor is also known as circuit magnification factor.

$\rightarrow$ V_{in} :



$$\Rightarrow V_{b'e} = V_{in} \cdot \frac{r_{b'e}}{r_{b'b} + r_{b'e}}$$

$$\Rightarrow \boxed{V_{in} = \frac{V_{b'e} \cdot (r_{b'b} + r_{b'e})}{r_{b'e}}}$$

$$\rightarrow A_V = \frac{V_o}{V_{in}} = \frac{-g_m V_{b'e} \cdot Z}{V_{b'e} \cdot \frac{(r_{b'b} + r_{b'e})}{r_{b'e}}}$$

$$\Rightarrow A_V = -g_m \left[\frac{r_{b'e}}{r_{b'e} + r_{b'b}}\right] \cdot Z$$

$$\Rightarrow \boxed{A_V = -g_m \left[\frac{r_{b'e}}{r_{b'e} + r_{b'b}}\right] \left[\frac{R_t}{1 + j2\delta Q_e}\right]}$$

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→ At resonance $\delta = 0$

$$\Rightarrow (A_V)_{Resonance} = -\frac{g_m R'_{be} \cdot R_t}{(r'_{be} + r'_{bb})} \leftarrow \text{Max. gain}$$

$$\Rightarrow \boxed{\frac{A_V}{(A_V)_{Resonance}} = \frac{1}{1 + j2\delta Q_e}}$$

$$\Rightarrow \left| \frac{(A_V)}{(A_V)_{Resonance}} \right| = \frac{1}{\sqrt{1 + (2\delta Q_e)^2}}$$

$$\& \quad \phi = -\tan^{-1}(2\delta Q_e)$$

→ At frequency 'ω_1' below the resonant frequency

let $\delta = -\frac{1}{2Q_e}$

$$\Rightarrow \left| \frac{A_V}{(A_V)_{Resonance}} \right| = \frac{1}{\sqrt{2}} = 0.707$$

$$\& \quad (A_V)_{dB} = (A_{V resonance})_{dB} - 3dB$$

∴ The frequency 'ω_1' is the lower '3dB' frequency.

→ Similarly at frequency 'ω_2' above 'ω_0' let

$$\delta = \frac{1}{2Q_e}$$

$$\Rightarrow \left| \frac{A_V}{A_{V resonance}} \right| = \frac{1}{\sqrt{2}} = 0.707$$

Hence frequency 'ω_2' is the upper '3dB' frequency.

$$[A_{V dB} = (A_{V resonance})_{dB} - 3dB]$$

→ Bandwidth

$$BW = \omega_2 - \omega_1$$

$$= \left[\frac{(\omega_2 - \omega_0) + (\omega_0 - \omega_1)}{\omega_0}\right] \times \omega_0 .$$

$$\Rightarrow BW = \left[\left(\frac{\omega_2 - \omega_0}{\omega_0}\right) + \left(\frac{\omega_0 - \omega_1}{\omega_0}\right)\right] \omega_0 .$$

$$= (\delta + \delta)\,\omega_0$$

$$\Rightarrow \boxed{BW = 2\delta\omega_0}$$

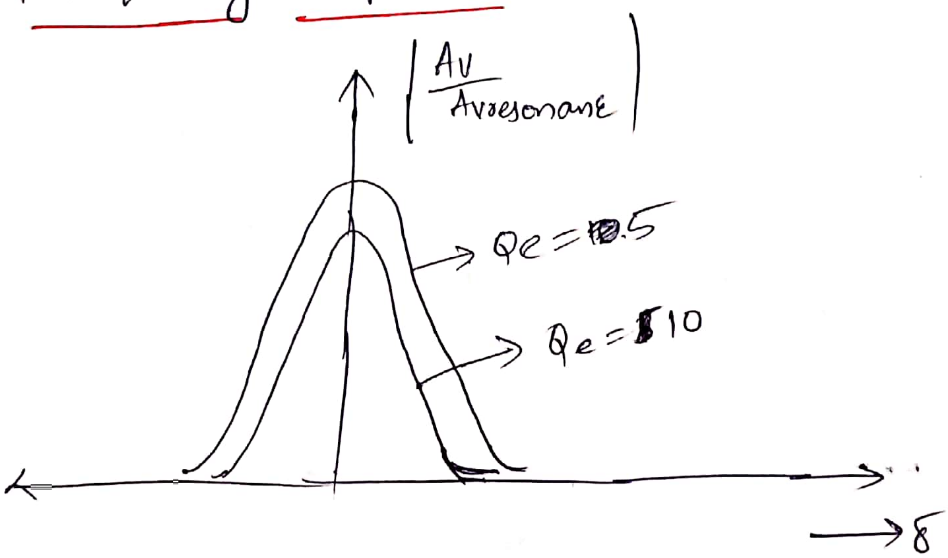
→ But $\delta = \frac{1}{2Q_e}$ $\Rightarrow \boxed{BW = \frac{\omega_0}{Q_e}} = \frac{\omega_0}{\omega_0 R_t . C} = \frac{1}{CR_t}$

(or)

$$= \frac{\omega_0 \, \omega_0 L}{R_t} = \frac{\omega_0^2 L}{R_t}$$

$$\Rightarrow \boxed{BW = 2\delta\omega_0 = \frac{\omega_0}{Q_e} = \frac{1}{CR_t} = \frac{\omega_0^2 L}{R_t}}$$

→ Frequency Response :-



Frequency Response .

→ Effect of Cascading single tuned Amplifiers on Bandwidth :-

→ Relative gain $\left[\frac{A}{A_{resonance}}\right] = \frac{1}{1+j2\delta Q_e}$ for single stage

→ For 'n' stages

$$\left[\frac{A}{A_{resonance}}\right]^n = \left[\frac{1}{1+j2\delta Q_e}\right]^n$$

$$\Rightarrow \left|\frac{A}{A_{res.}}\right|^n = \left[\frac{1}{\sqrt{1+(2\delta Q_e)^2}}\right]^n = \frac{1}{\sqrt{2}}$$

$$\Rightarrow 1+(2\delta Q_e)^2 = 2^{1/n}.$$

$$\Rightarrow 2\delta Q_e = \pm\sqrt{(2^{1/n}-1)}$$

$$\Rightarrow \delta = \pm\frac{1}{2Q_e}\sqrt{(2^{1/n}-1)}$$

& $\delta = \frac{\omega-\omega_0}{\omega_0} = \frac{f-f_0}{f_0}$

$$\Rightarrow (f-f_0) = \pm\frac{f_0}{2Q_e}\sqrt{2^{1/n}-1} \quad : \quad + \to f_H, \; - \to f_L$$

$$\Rightarrow (f_H-f_0) = \frac{f_0}{2Q_e}\sqrt{2^{1/n}-1}$$

$$\& \; (f_L-f_0) = \frac{-f_0}{2Q_e}\sqrt{2^{1/n}-1}$$

$$\Rightarrow f_0-f_L = \frac{f_0}{2Q_e}\sqrt{2^{1/n}-1}.$$

$\rightarrow (BW)_{overall} = f_H - f_L$

$$= (f_H - f_0) + (f_0 - f_L)$$

$$= \frac{f_0}{Q_e}\sqrt{(2^{1/n}-1)}$$

$$\Rightarrow \boxed{(BW)_{overall} = BW_1\sqrt{2^{1/n}-1}}$$ ← Bandwidth for n-stages

where $BW_1 = \frac{f_0}{Q_e}$ → Bandwidth for single stage.

→ Let $n=2 \Rightarrow (BW)_{overall} = BW_1\sqrt{2^{1/2}-1}$

$$= BW_1 \times 0.643.$$

$n=3 \Rightarrow (BW)_{overall} = BW_1\sqrt{2^{1/3}-1}$

$$= BW_1 \times 0.509.$$

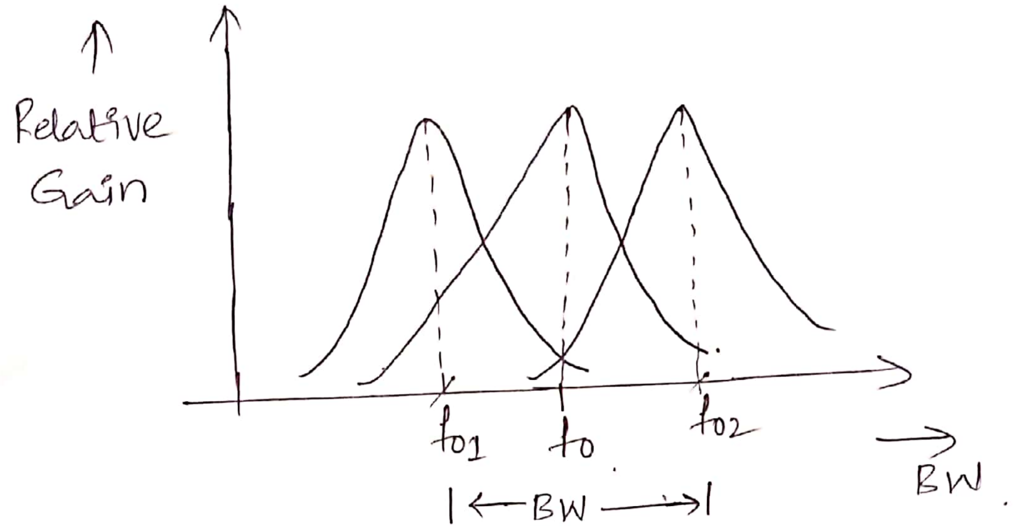
→ Stagger Tuned Amplifier :-

- Stagger tuned amplifiers use a no.of single tuned stages in cascade, the successive tuned circuits being tuned to slightly different frequencies.

→ Let in a stagger tuned circuit, 2 single tuned cascade amplifiers having a certain bandwidth are taken.

→ The resonant frequencies of the two tuned circuits are adjusted such that they are separated by an amount equal to bandwidth of each stage.

→



Conditions

1. $f_{02} - f_{01} = BW$.

2. $BW_{stagger} = \sqrt{2}\,BW$.

→ $$\frac{A}{A_{res}} = \frac{1}{1 + j2\delta Q_e}$$

let $x = 2\delta Q_e$.

then $\frac{A}{A_{res}} = \frac{1}{1+jx}$, At 3dB $\delta = \frac{1}{2Q_e}$

$$\boxed{BW = \frac{f_0}{Q_e} = 2\delta f_0}$$

→ $$\left[\frac{A}{A_{res}}\right]_1 = \frac{1}{1+j(x-1)}, \quad \left[\frac{A}{A_{res}}\right]_2 = \frac{1}{1+j(x+1)}$$

$$\Rightarrow \left[\frac{A}{A_{res}}\right]_{stagger} = \left[\frac{A}{A_{res}}\right]_1 \left[\frac{A}{A_{res}}\right]_2$$

$$\Rightarrow \left[\frac{A}{A_{res}}\right]_{stagger} = \frac{1}{2 - x^2 + 2jx}$$

$$\Rightarrow \left|\frac{A}{A_{res}}\right|_{stagger} = \frac{1}{2\sqrt{1+4\delta^4 Q_e^4}}$$