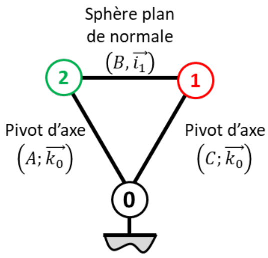
Barrière Sympact ★

C2-06

Question 1 Tracer le graphe des liaisons.



Question 2 Exprimer $\varphi(t)$ en fonction de $\theta(t)$. On a $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{0}$ soit $\lambda(t)\overrightarrow{i_2} - R\overrightarrow{i_1} - h\overrightarrow{j_0} = \overrightarrow{0}$.

En exprimant l'équation vectorielle dans le repère $\mathcal{R}_0$, on a $\lambda(t)\left(\cos\varphi(t)\overrightarrow{i_0} + \sin\varphi(t)\overrightarrow{j_0}\right) - R\left(\cos\theta(t)\overrightarrow{i_0} + \sin\theta(t)\overrightarrow{j_0}\right) - h\overrightarrow{j_0} = \overrightarrow{0}$.

On a alors $\left\{\begin{array}{l}\lambda(t)\cos\varphi(t) - R\cos\theta(t) = 0\\ \lambda(t)\sin\varphi(t) - R\sin\theta(t) - h = 0\end{array}\right.$

soit $\left\{\begin{array}{l}\lambda(t)\cos\varphi(t) = R\cos\theta(t)\\ \lambda(t)\sin\varphi(t) = R\sin\theta(t) + h\end{array}\right.$.

En faisant le rapport des équations, on a donc : $\tan\varphi(t) = \dfrac{R\sin\theta(t) + h}{R\cos\theta(t)}$ (pour $\theta(t) \neq \dfrac{\pi}{2} mod \pi$).

Question 3 Exprimer $\dot{\varphi}(t)$ en fonction de $\dot{\theta}(t)$. On a : $\varphi(t) = \arctan\left(\dfrac{R\sin\theta(t) + h}{R\cos\theta(t)}\right)$.

Pour commencer, $(R\sin\theta(t) + h)' = R\dot{\theta}(t)\cos\theta(t)$ et $(R\cos\theta(t))' = -R\dot{\theta}(t)\sin\theta(t)$.

De plus, $\left(\dfrac{R\sin\theta(t) + h}{R\cos\theta(t)}\right)'$

$= \dfrac{R\dot{\theta}(t)\cos\theta(t) R\cos\theta(t) + R\dot{\theta}(t)\sin\theta(t)\left(R\sin\theta(t) + h\right)}{R^2\cos^2\theta(t)}$

$= \dfrac{R^2\dot{\theta}(t)\cos^2\theta(t) + R\dot{\theta}(t)\sin\theta(t)\left(R\sin\theta(t) + h\right)}{R^2\cos^2\theta(t)}$

$= \dfrac{R\dot{\theta}(t)\cos^2\theta(t) + R\sin^2\theta(t)\dot{\theta}(t) + h\dot{\theta}(t)\sin\theta(t)}{R\cos^2\theta(t)}$

$= \dot{\theta}(t)\dfrac{R + h\sin\theta(t)}{R\cos^2\theta(t)}$.

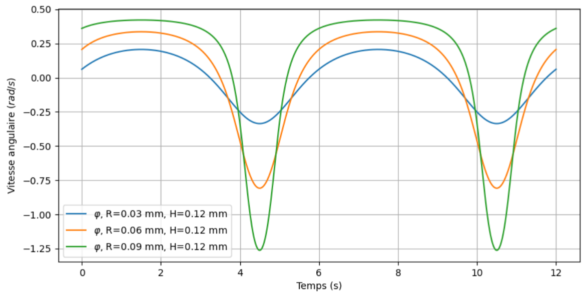
Au final,

$$\dot{\varphi}(t) = \frac{\dot{\theta}(t)\dfrac{R + h\sin\theta(t)}{R\cos^2\theta(t)}}{1 + \left(\dfrac{R\sin\theta(t) + h}{R\cos\theta(t)}\right)^2} \; \frac{\dot{\theta}(t)\dfrac{R + h\sin\theta(t)}{R\cos^2\theta(t)}}{1 + \dfrac{\left(R\sin\theta(t) + h\right)^2}{R^2\cos^2\theta(t)}}.$$

$$\dot{\varphi}(t) = R^2\cos^2\theta(t)\frac{\dot{\theta}(t)\dfrac{R+h\sin\theta(t)}{R\cos^2\theta(t)}}{R^2\cos^2\theta(t)+\dfrac{(R\sin\theta(t)+h)^2}{R^2\cos^2\theta(t)}} = \frac{R\dot{\theta}(t)\,(R+h\sin\theta(t))}{R^2\cos^2\theta(t)+(R\sin\theta(t)+h)^2}.$$

$$\dot{\varphi}(t) = \frac{R\dot{\theta}(t)\,(R+h\sin\theta(t))}{R^2\cos^2\theta(t)+R^2\sin^2\theta(t)+h^2+2Rh\sin\theta(t)} = \frac{R\dot{\theta}(t)\,(R+h\sin\theta(t))}{R^2+h^2+2Rh\sin\theta(t)}.$$

Question 4 En utilisant Python, tracer $\dot{\varphi}(t)$ en fonction de $\dot{\theta}(t)$. On considérera que la fréquence de rotation de la pièce **1** est de 10 tours par minute.



```python
#!/usr/bin/env python
# -*- coding: utf-8 -*-

"""14_Sympact.py"""

__author__ = "Xavier Pessoles"
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import numpy as np
import matplotlib.pyplot as plt
import math as m
from scipy.optimize import newton
from scipy.optimize import fsolve

R = 0.03 # m
H = 0.12     # m
w = 10 # tours /min
w = 10*2*m.pi/60  # rad/s

def calc_phi(theta):
    num = R*np.sin(theta)+H
    den = R*np.cos(theta)
    return np.arctan2(num,den)

def calc_phip(theta):
    num = R*w*(R+H*np.sin(theta))
    den = R*R+H*H+2*R*H*np.sin(theta)
    return np.arctan2(num,den)

def plot_phi():
    les_t = np.linspace(0,12,1000)
    les_theta = w*les_t
    les_phi = calc_phi(les_theta)
    plt.grid()
```

```python
    plt.xlabel("Temps (s)")
    plt.ylabel("Position angulaire ($rad$)")
    #plt.plot(les_t,les_theta,label=str("$\\theta$, R=")+str(R)+" mm,"+str("H=")+str(H)+" mm")
    plt.plot(les_t,les_phi,label=str("$\\varphi$, R=")+str(R)+" mm, "+str("H=")+str(H)+" mm")
    plt.legend()
    plt.show()


def plot_phip():
    les_t = np.linspace(0,12,1000)
    les_theta = w*les_t
    les_phip = calc_phip(les_theta)

    plt.grid()
    plt.xlabel("Temps (s)")
    plt.ylabel("Vitesse angulaire ($rad/s$)")
    #plt.plot(les_t,les_theta,label=str("$\\theta$, R=")+str(R)+" mm,"+str("H=")+str(H)+" mm")
    plt.plot(les_t,les_phip,label=str("$\\varphi$, R=")+str(R)+" mm, "+str("H=")+str(H)+" mm")
    plt.legend()
    plt.show()

for R in [0.03,0.06,0.09]:
    plot_phip()
```