Application 1
Kart – Corrigé

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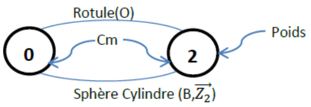
C1-05

C2-09

Eléments de corrigé:

Question 1:



$$\begin{Bmatrix} X_o.\overrightarrow{X_2}+Y_0.\overrightarrow{Y_2}+Z_0.\overrightarrow{Z_2} \\ \vec{0} \end{Bmatrix}_O ; \begin{Bmatrix} X_B.\overrightarrow{X_2}+Y_B.\overrightarrow{Y_2} \\ \vec{0} \end{Bmatrix}_B = \begin{Bmatrix} X_B.\overrightarrow{X_2}+Y_B.\overrightarrow{Y_2} \\ a\left(X_B.\overrightarrow{Y_2}-Y_B.\overrightarrow{X_2}\right) \end{Bmatrix}_O$$

Question 2 :

On isole 2: BAME: Torseurs de 0 → 2 et Cm: $\begin{Bmatrix} \vec{0} \\ C_m.\vec{Z} \end{Bmatrix}_0$; poids: $\begin{Bmatrix} m_2.g\,\vec{Y} \\ \vec{0} \end{Bmatrix}_G = \begin{Bmatrix} m_2.g\,\vec{Y} \\ m_2.g.l_2.\vec{X} \end{Bmatrix}_0$

Calcul du torseur cinétique:

$$\{C(2/R)\} = \begin{Bmatrix} m_2\overrightarrow{VG\in 2/0} \\ \sigma(\overrightarrow{G_2,2/0}) \end{Bmatrix}_G ;\overrightarrow{VG\in 2/0}=\vec{0};\overrightarrow{\sigma(G_2;2/0)}=I(G_2,2)\overrightarrow{\Omega(2/0)}=-D.\dot{\theta}_2.\overrightarrow{Y_2}+C\dot{\theta}_2.\vec{Z}$$

Calcul du torseur dynamique:

$$\{D(2/R)\} = \begin{Bmatrix} m_2\Gamma\overrightarrow{G\in 2/0} \\ \delta(\overrightarrow{G_2,2/0}) \end{Bmatrix}_G ;\overrightarrow{\Gamma G\in 2/0}=\vec{0};\overrightarrow{\delta(G_2;2/0)}=-D.\ddot{\theta}_2.\overrightarrow{Y_2}+D\dot{\theta}_2^2.\overrightarrow{X_2}+C\ddot{\theta}_2.\vec{Z}$$

PFS en O dans la base 2:

$$Sur\ \overrightarrow{X_2} : X_O + X_B = 0$$

$$Sur\ \overrightarrow{Y_2} : Y_O + Y_B = 0$$

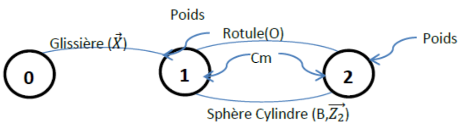
$$Sur\ \vec{Z} : Z_0 = 0$$

$$Sur\ \overrightarrow{X_2} : -a.Y_B + m_2.g.l_2.\cos(\theta_2) = D\dot{\theta}_2^2$$

$$Sur\ \overrightarrow{Y_2} : a.X_B - m_2.g.l_2.\sin(\theta_2) = -D.\ddot{\theta}_2$$

$$Sur\ \vec{Z} : C_m = C\ddot{\theta}_2$$

Question 3:



Question 4:

On isole {1+2} BAME: $\begin{Bmatrix} Y_{01}.\vec{Y}+Z_{01}.\vec{Z} \\ L_{01}.\vec{X}+M_{01}\vec{Y}+N_{01}.\vec{Z} \end{Bmatrix}_O$

$$\{C(1+2/R)\} = \{C(1/R)\} + \{C(2/R)\}$$

$$\{C(1/R)\} = \begin{Bmatrix} m_1.\overrightarrow{V(G \in 1/0)} \\ \vec{0} \end{Bmatrix}_O = \begin{Bmatrix} m_1.\dot{\lambda}.\vec{X} \\ \vec{0} \end{Bmatrix}_0$$

$$\{C(2/R)\} = \begin{Bmatrix} m_2\overrightarrow{VG \in 2/0} \\ \overrightarrow{\sigma(G_2, 2/0)} \end{Bmatrix}_G ; \overrightarrow{VG \in 2/0} = \dot{\lambda}.\vec{X};$$

$$\overrightarrow{\sigma(G_2; 2/0)} = I(0,2)\overrightarrow{\Omega(2/0)} = -D.\dot{\theta}_2.\overrightarrow{Y_2} + C\dot{\theta}_2.\vec{Z}$$

Torseur dynamique:

$$\{D(1+2/R)\} = \{D(1/R)\} + \{D(2/R)\}$$

$$\{D(2/R)\} = \begin{Bmatrix} m_2\overrightarrow{\Gamma G \in 2/0} \\ \overrightarrow{\delta(G_2, 2/0)} \end{Bmatrix}_G ; \overrightarrow{\Gamma G \in 2/0} = \ddot{\lambda}.\vec{X};$$

$$\overrightarrow{\delta(G_2; 2/0)} = -D.\ddot{\theta}_2.\overrightarrow{Y_2} + D\dot{\theta}_2^2.\overrightarrow{X_2} + C\ddot{\theta}_2.\vec{Z}$$

$$\overrightarrow{\delta(O; 2/0)} = \overrightarrow{\delta(G_2; 2/0)} + \overrightarrow{OG_2}\wedge m_2.\overrightarrow{\Gamma(G_2 \in 2/0)}$$

$$\overrightarrow{\delta(O; 2/0)} == -D.\ddot{\theta}_2.\overrightarrow{Y_2} + D\dot{\theta}_2^2.\overrightarrow{X_2} + C\ddot{\theta}_2.\vec{Z} + l_2.\vec{Z}\wedge m_2.\ddot{\lambda}.\vec{X}$$

$$\overrightarrow{\delta(O; 2/0)} == -D.\ddot{\theta}_2.\overrightarrow{Y_2} + D\dot{\theta}_2^2.\overrightarrow{X_2} + C\ddot{\theta}_2.\vec{Z} + l_2.m_2.\ddot{\lambda}.\vec{Y}$$

PFS à {1+2}:

$$Sur\ \overrightarrow{X_0} : 0 = (m_1 + m_2).\ddot{\lambda}$$

$$Sur\ \overrightarrow{Y_0} : Y_{01} = m_1 g + m_2 g$$

$$Sur\ \vec{Z} : Z_0 = 0$$

$$Sur\ \overrightarrow{X_0} : L_{01} = D\dot{\theta}_2^2 \cos(\theta_2) + D.\ddot{\theta}_2 \sin(\theta_2)$$

$$Sur\ \overrightarrow{Y_0} : M_{01} = D\dot{\theta}_2^2 \sin(\theta_2) - D.\ddot{\theta}_2 \cos(\theta_2) + m_2.l_2.\ddot{\lambda}$$

$$Sur\ \vec{Z} : N_{01} = 0$$

Question 5:

Idem question 4 mais ajouter tous les termes complémentaires.