Mouvement RT – RSG ★★

C2-08

C2-09

Question 1 Déterminer $\overrightarrow{R_d(2/0)} \cdot \overrightarrow{i_1}$

Par définition, $\overrightarrow{R_d(2/0)} = m_2 \overrightarrow{\Gamma(G_2, 2/0)} = m_2 \overrightarrow{\Gamma(B, 2/0)}$.

Calcul de $\overrightarrow{V(B, 2/0)}$:

$\overrightarrow{V(B, 2/0)} = \overrightarrow{V(B, 2/1)} + \overrightarrow{V(B, 1/0)}$.

D'une part, $\overrightarrow{V(B, 2/1)} = \dot{\lambda}\overrightarrow{i_1}$.

D'autre part, en utilisant le roulement sans glissement en I, $\overrightarrow{V(B, 1/0)} = \overrightarrow{V(I, 1/0)} + \overrightarrow{BI} \wedge \overrightarrow{\Omega(1/0)} = \overrightarrow{0} + \left(-\lambda(t)\overrightarrow{i_1} - R\overrightarrow{j_0}\right) \wedge \dot{\theta}\overrightarrow{k_0} = -\dot{\theta}\left(\lambda(t)\overrightarrow{i_1} \wedge \overrightarrow{k_0} + R\overrightarrow{j_0} \wedge \overrightarrow{k_0}\right) = \dot{\theta}\left(\lambda(t)\overrightarrow{j_1} - R\overrightarrow{i_0}\right)$.

Au final, $\overrightarrow{V(B, 2/0)} = \dot{\lambda}\overrightarrow{i_1} + \dot{\theta}\left(\lambda(t)\overrightarrow{j_1} - R\overrightarrow{i_0}\right)$.

Calcul de $\overrightarrow{\Gamma(B, 2/0)}$:

$\overrightarrow{\Gamma(B, 2/0)} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left[\overrightarrow{V(B, 2/0)}\right]_{\mathcal{R}_0} = \ddot{\lambda}(t)\overrightarrow{i_1} + \dot{\lambda}(t)\dot{\theta}\overrightarrow{j_1} + \ddot{\theta}(t)\left(\lambda(t)\overrightarrow{j_1} - R\overrightarrow{i_0}\right) + \dot{\theta}(t)\left(\dot{\lambda}(t)\overrightarrow{j_1} - \lambda(t)\dot{\theta}\overrightarrow{i_1}\right)$.

Au final, $\overrightarrow{R_d(2/0)} = m_2\left(\ddot{\lambda}(t)\overrightarrow{i_1} + \dot{\lambda}(t)\dot{\theta}\overrightarrow{j_1} + \ddot{\theta}(t)\left(\lambda(t)\overrightarrow{j_1} - R\overrightarrow{i_0}\right) + \dot{\theta}(t)\left(\dot{\lambda}(t)\overrightarrow{j_1} - \lambda(t)\dot{\theta}\overrightarrow{i_1}\right)\right)$

Question 2 Déterminer $\overrightarrow{\delta(I, 1+2/0)} \cdot \overrightarrow{k_0}$ On a $\overrightarrow{\delta(I, 1+2/0)} = \overrightarrow{\delta(I, 1/0)} + \overrightarrow{\delta(I, 2/0)}$.

Calcul $\overrightarrow{\delta(I, 1/0)}$

Par déplacement du moment dynamique, on a $\overrightarrow{\delta(I, 1/0)} = \overrightarrow{\delta(G_1, 1/0)} + \overrightarrow{IG_1} \wedge \overrightarrow{R_d(1/0)}$

- $\overrightarrow{\delta(G_1, 1/0)} \cdot \overrightarrow{k_0} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left[\overrightarrow{\sigma(G_1, 1/0)}\right]_{\mathcal{R}_0} \cdot \overrightarrow{k_0} = C_1\ddot{\theta}$.
- $\overrightarrow{R_d(1/0)} = m_1 \dfrac{\mathrm{d}}{\mathrm{d}t}\left[\overrightarrow{V(G_1, 1/0)}\right]_{\mathcal{R}_0}$ et $\overrightarrow{V(G_1, 1/0)} = \overrightarrow{V(I, 1/0)} + \overrightarrow{G_1 I} \wedge \overrightarrow{\Omega(1/0)} = \overrightarrow{0} + \left(\ell\overrightarrow{i_1} - R\overrightarrow{j_0}\right) \wedge \dot{\theta}\overrightarrow{k_0} = \dot{\theta}\left(-\ell\overrightarrow{j_1} + R\overrightarrow{i_0}\right)$. On a donc $\overrightarrow{R_d(1/0)} = m_1\ddot{\theta}\left(-\ell\overrightarrow{j_1} + R\overrightarrow{i_0}\right) + m_1\ell\dot{\theta}^2\overrightarrow{i_1}$.
- $\left(\overrightarrow{IG_1} \wedge \overrightarrow{R_d(1/0)}\right) \cdot \overrightarrow{k_0} = \left(\left(R\overrightarrow{j_0} - \ell\overrightarrow{i_1}\right) \wedge \left(m_1\ddot{\theta}\left(-\ell\overrightarrow{j_1} + R\overrightarrow{i_0}\right) + m_1\ell\dot{\theta}^2\overrightarrow{i_1}\right)\right) \cdot \overrightarrow{k_0}$
$= \left(R\overrightarrow{j_0} \wedge \left(m_1\ddot{\theta}\left(-\ell\overrightarrow{j_1} + R\overrightarrow{i_0}\right) + m_1\ell\dot{\theta}^2\overrightarrow{i_1}\right) - \ell\overrightarrow{i_1} \wedge \left(m_1\ddot{\theta}\left(-\ell\overrightarrow{j_1} + R\overrightarrow{i_0}\right) + m_1\ell\dot{\theta}^2\overrightarrow{i_1}\right)\right) \cdot \overrightarrow{k_0}$
$= m_1\left(R\overrightarrow{j_0} \wedge \left(\ddot{\theta}\left(-\ell\overrightarrow{j_1} + R\overrightarrow{i_0}\right) + \ell\dot{\theta}^2\overrightarrow{i_1}\right) - \ell\ddot{\theta}\overrightarrow{i_1} \wedge \left(-\ell\overrightarrow{j_1} + R\overrightarrow{i_0}\right)\right) \cdot \overrightarrow{k_0}$
$= m_1\left(R\left(\ddot{\theta}\left(-\ell\overrightarrow{j_0} \wedge \overrightarrow{j_1} + R\overrightarrow{j_0} \wedge \overrightarrow{i_0}\right) + \ell\dot{\theta}^2\overrightarrow{j_0} \wedge \overrightarrow{i_1}\right) - \ell\ddot{\theta}\left(-\ell\overrightarrow{i_1} \wedge \overrightarrow{j_1} + R\overrightarrow{i_1} \wedge \overrightarrow{i_0}\right)\right) \cdot \overrightarrow{k_0}$
$= m_1\left(R\left(\ddot{\theta}\left(-\ell\sin\theta - R\right) - \ell\dot{\theta}^2\cos\theta\right) - \ell\ddot{\theta}\left(-\ell - R\sin\theta\right)\right)$
$= m_1\left(-R\left(\ddot{\theta}\left(\ell\sin\theta + R\right) + \ell\dot{\theta}^2\cos\theta\right) + \ell\ddot{\theta}\left(\ell + R\sin\theta\right)\right)$
$= m_1\left(-R\ddot{\theta}\ell\sin\theta - R^2\ddot{\theta} - R\ell\dot{\theta}^2\cos\theta + \ell^2\ddot{\theta} + R\ell\ddot{\theta}\sin\theta\right)$
$= m_1\left(-R^2\ddot{\theta} - R\ell\dot{\theta}^2\cos\theta + \ell^2\ddot{\theta}\right)$
- Au final, $\overrightarrow{\delta(I, 1/0)} = C_1\ddot{\theta} + m_1\left(-R^2\ddot{\theta} - R\ell\dot{\theta}^2\cos\theta + \ell^2\ddot{\theta}\right)$.

Calcul $\overrightarrow{\delta\left(I,2/0\right)}$

Par déplacement du moment dynamique, on a $\overrightarrow{\delta\left(I,2/0\right)} = \overrightarrow{\delta\left(G_2,2/0\right)} + \overrightarrow{IG_2} \wedge \overrightarrow{R_d\left(2/0\right)}$

- $\overrightarrow{\delta\left(G_2,2/0\right)} \cdot \overrightarrow{k_0} = \dfrac{\mathrm{d}}{\mathrm{d}t}\left[\overrightarrow{\sigma\left(G_2,2/0\right)}\right]_{\mathcal{R}_0} \cdot \overrightarrow{k_0} = C_2\ddot{\theta}$.
- $\left(\overrightarrow{IG_2} \wedge \overrightarrow{R_d\left(2/0\right)}\right)\cdot\overrightarrow{k_0} = \left(R\overrightarrow{j_0} \wedge \left(m_2\left(\ddot{\lambda}(t)\overrightarrow{i_1} + \dot{\lambda}(t)\dot{\theta}\overrightarrow{j_1} + \ddot{\theta}(t)\left(\lambda(t)\overrightarrow{j_1} - R\overrightarrow{i_0}\right) + \dot{\theta}(t)\left(\dot{\lambda}(t)\overrightarrow{j_1}\right.\right.\right.\right.$
  $\overrightarrow{k_0} = m_2R\left(\overrightarrow{j_0} \wedge \left(\ddot{\lambda}(t)\overrightarrow{i_1} + \dot{\lambda}(t)\dot{\theta}\overrightarrow{j_1} + \ddot{\theta}(t)\left(\lambda(t)\overrightarrow{j_1} - R\overrightarrow{i_0}\right) + \dot{\theta}(t)\left(\dot{\lambda}(t)\overrightarrow{j_1} - \lambda(t)\dot{\theta}\overrightarrow{i_1}\right)\right)\right)\cdot$
  $\overrightarrow{k_0}$
  $\lambda(t)\overrightarrow{i_1}\wedge\left(m_2\left(\ddot{\lambda}(t)\overrightarrow{i_1} + \dot{\lambda}(t)\dot{\theta}\overrightarrow{j_1} + \ddot{\theta}(t)\left(\lambda(t)\overrightarrow{j_1} - R\overrightarrow{i_0}\right) + \dot{\theta}(t)\left(\dot{\lambda}(t)\overrightarrow{j_1} - \lambda(t)\dot{\theta}\overrightarrow{i_1}\right)\right)\right)\cdot$
  $\overrightarrow{k_0}$
  $= m_2R\left(-\ddot{\lambda}(t)\cos\theta + \dot{\lambda}(t)\dot{\theta}\sin\theta + \ddot{\theta}(t)\left(-\lambda(t)\sin\theta + R\right) + \dot{\theta}(t)\left(\dot{\lambda}(t)\sin\theta + \lambda(t)\dot{\theta}\cos\theta\right)\right.$
  $+\lambda(t)m_2\left(2\dot{\lambda}(t)\dot{\theta} + \ddot{\theta}(t)\left(\lambda(t) + R\sin\theta\right)\right)$
  $= -m_2R\ddot{\lambda}(t)\cos\theta + m_2R\dot{\lambda}(t)\dot{\theta}\sin\theta - m_2R\ddot{\theta}(t)\lambda(t)\sin\theta + m_2R^2\ddot{\theta}(t) + m_2R\dot{\theta}(t)\dot{\lambda}(t)\sin\theta +$
  $m_2R\dot{\theta}^2(t)\lambda(t)\cos\theta + 2\lambda(t)m_2\dot{\lambda}(t)\dot{\theta} + \ddot{\theta}(t)m_2\lambda(t)^2 + \ddot{\theta}(t)\lambda(t)m_2R\sin\theta$
  $= -m_2R\ddot{\lambda}(t)\cos\theta + m_2R\dot{\lambda}(t)\dot{\theta}\sin\theta + m_2R^2\ddot{\theta}(t) + m_2R\dot{\theta}(t)\dot{\lambda}(t)\sin\theta + m_2R\dot{\theta}^2(t)\lambda(t)\cos\theta +$
  $2m_2\lambda(t)\dot{\lambda}(t)\dot{\theta} + m_2\ddot{\theta}(t)\lambda(t)^2$
  Au final, $\overrightarrow{\delta\left(I,2/0\right)} = -m_2R\ddot{\lambda}(t)\cos\theta + m_2R\dot{\lambda}(t)\dot{\theta}\sin\theta + m_2R^2\ddot{\theta}(t) + m_2R\dot{\theta}(t)\dot{\lambda}(t)\sin\theta +$
  $m_2R\dot{\theta}^2(t)\lambda(t)\cos\theta + 2m_2\lambda(t)\dot{\lambda}(t)\dot{\theta} + m_2\ddot{\theta}(t)\lambda(t)^2 + C_2\ddot{\theta}$