Petits déplacements

(1) $\{\mathcal{U}_{1/R}\} = \left\{\begin{matrix} \delta\theta_x & \delta U_{Ax} \\ \delta\theta_y & 0 \\ \delta\theta_z & \delta U_{Az} \end{matrix}\right\}_{A,R}$

$= \left\{\begin{matrix} \delta\theta_x & \delta U_{Bx} \\ \delta\theta_y & e \\ \delta\theta_z & \delta U_{Bz} \end{matrix}\right\}_{B,R} = \left\{\begin{matrix} \delta\theta_x & \delta U_{Cx} \\ \delta\theta_y & 0 \\ \delta\theta_z & \delta U_{Cz} \end{matrix}\right\}_{C,R}$

$= \left\{\begin{matrix} \delta\theta_x & \delta U_{Dx} \\ \delta\theta_y & \delta U_{Dy} \\ \delta\theta_z & 0 \end{matrix}\right\}_{D,R} = \left\{\begin{matrix} \delta\theta_x & \delta U_{Ex} \\ \delta\theta_y & \delta U_{Ey} \\ \delta\theta_z & 0 \end{matrix}\right\}_{R}$

$= \left\{\begin{matrix} \delta\theta_x & 0 \\ \delta\theta_y & \delta U_{Fy} \\ \delta\theta_z & \delta U_{Fz} \end{matrix}\right\}_{F,R} = \left\{\begin{matrix} \delta\theta_x & \delta U_{Mx} \\ \delta\theta_y & \delta U_{My} \\ \delta\theta_z & \delta U_{Mz} \end{matrix}\right\}_{M,R}$

(2) Aux différents points de contact (A, B, C, D, E, F) sur les 6 coordonnées du torseur, il y a 5 inconnues.
A l'aide de la relation de Varignon il est possible d'établir 5 relations. [A, B] [A, C] [A, D] [A, E], [A, F] donc potentiellement 5 équations.

- $\overrightarrow{\delta U_{A\in 1/R}} = \overrightarrow{\delta U_{B\in 1/R}} + \overrightarrow{AB} \wedge \overrightarrow{\delta\theta_{1/R}}$

$\begin{vmatrix} \delta U_{Ax} \\ 0 \\ \delta U_{Az} \end{vmatrix}_R = \begin{vmatrix} \delta U_{Bx} \\ e \\ \delta U_{Bz} \end{vmatrix}_R + \begin{vmatrix} a \\ 0 \\ 0 \end{vmatrix}_R \wedge \begin{vmatrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \end{vmatrix}_R$

Utilisation de la projection sur $\vec{y}$

$0 = e - a\,\delta\theta_z$ (1)

- $\overrightarrow{\delta U_{A\in 1/R}} = \overrightarrow{\delta U_{C\in 1/R}} + \overrightarrow{AC} \wedge \overrightarrow{\delta \Theta_{1/R}}$

$$\left|\begin{matrix} \delta U_{Ax} \\ 0 \\ \delta U_{Az} \end{matrix}\right._{R} = \left|\begin{matrix} \delta U_{Cx} \\ 0 \\ \delta U_{Cz} \end{matrix}\right._{R} + \left|\begin{matrix} c \\ 0 \\ d \end{matrix}\right._{R} \wedge \left|\begin{matrix} \delta\Theta_x \\ \delta\Theta_y \\ \delta\Theta_z \end{matrix}\right._{R}$$

Utilisation de la projection sur $\vec{y}$:

$$0 = 0 - c\,\delta\Theta_z + d\,\delta\Theta_x \qquad (2)$$

- $\overrightarrow{\delta U_{A\in 1/R}} = \overrightarrow{\delta U_{D\in 1/R}} + \overrightarrow{AD} \wedge \overrightarrow{\delta \Theta_{1/R}}$

$$\left|\begin{matrix} \delta U_{Ax} \\ 0 \\ \delta U_{Az} \end{matrix}\right._{R} = \left|\begin{matrix} \delta U_{Dx} \\ \delta U_{Dy} \\ 0 \end{matrix}\right._{R} + \left|\begin{matrix} 0 \\ f \\ -b \end{matrix}\right._{R} \wedge \left|\begin{matrix} \delta\Theta_x \\ \delta\Theta_y \\ \delta\Theta_z \end{matrix}\right._{R}$$

Utilisation de la projection sur $\vec{z}$

$$\delta U_{Az} = 0 - f\,\delta\Theta_x \qquad (3)$$

- $\overrightarrow{\delta U_{A\in 1/R}} = \overrightarrow{\delta U_{E\in 1/R}} + \overrightarrow{AE} \wedge \overrightarrow{\delta \Theta_{1/R}}$

$$\left|\begin{matrix} \delta U_{Ax} \\ 0 \\ \delta U_{Az} \end{matrix}\right._{R} = \left|\begin{matrix} \delta U_{Ex} \\ \delta U_{Ey} \\ 0 \end{matrix}\right._{R} + \left|\begin{matrix} a \\ f \\ -b \end{matrix}\right._{R} \wedge \left|\begin{matrix} \delta\Theta_x \\ \delta\Theta_y \\ \delta\Theta_z \end{matrix}\right._{R}$$

Utilisation de la projection sur $\vec{z}$

$$\delta U_{Az} = 0 + a\,\delta\Theta_y - f\,\delta\Theta_x \qquad (4)$$

- $\overrightarrow{\delta U_{A\in 1/R}} = \overrightarrow{\delta U_{F\in 1/R}} + \overrightarrow{AF} \wedge \overrightarrow{\delta \Theta_{1/R}}$

$$\left|\begin{matrix} \delta U_{Ax} \\ 0 \\ \delta U_{Az} \end{matrix}\right._{R} = \left|\begin{matrix} 0 \\ \delta U_{Fy} \\ \delta U_{Fz} \end{matrix}\right._{R} + \left|\begin{matrix} e' \\ f \\ 0 \end{matrix}\right._{R} \wedge \left|\begin{matrix} \delta\Theta_x \\ \delta\Theta_y \\ \delta\Theta_z \end{matrix}\right._{R}$$

Utilisation de la projection sur $\vec{x}$

$$\delta U_{Ax} = 0 + f\,\Theta_z \qquad (5)$$

③ ① $\longrightarrow$ $\boxed{\delta\theta_z = \frac{e}{a}}$

② $\longrightarrow$ $\delta\theta_x = \frac{c}{d}\delta\theta_z = \boxed{\frac{ce}{ad} = \delta\theta_x}$

③ $\longrightarrow$ $\delta U_{Az} = -f\,\delta\theta_x = \boxed{-\frac{fce}{ad} = \delta U_{Az}}$

④ $\longrightarrow$ $\delta\theta_y = \dfrac{\delta U_{Az} + \frac{fec}{ad}}{a} = \dfrac{-\frac{fce}{ad} + \frac{fec}{ad}}{a} = \boxed{0 = \delta\theta_y}$

⑤ $\longrightarrow$ $\delta U_{Ax} = \boxed{f\frac{e}{a} = \delta U_{Ax}}$

$$\boxed{\left\{\mathcal{U}_{1/R}\right\} = {}_A\left\{\begin{array}{c|c} \frac{ce}{ad} & \frac{fe}{a} \\ 0 & 0 \\ \frac{e}{a} & -\frac{fce}{ad} \end{array}\right\}_R.}$$

④ La cale, si on raisonne en petits déplacements, provoque une rotation autour de la droite AC. On a donc de façon intuitive $\delta\theta_x \neq 0$, $\delta\theta_z \neq 0$, $\boxed{\delta\theta_y = 0}$

⑤ $\overrightarrow{\delta U_{M\in 1/R}} = \overrightarrow{\delta U_{A\in 1/R}} + \overrightarrow{MA} \wedge \overrightarrow{\delta\theta_{1/R}}$

$$_R\begin{vmatrix} \delta U_{Mx} \\ \delta U_{My} \\ \delta U_{Mz} \end{vmatrix} = {}_R\begin{vmatrix} \frac{fe}{a} \\ 0 \\ -\frac{fce}{ad} \end{vmatrix} + {}_R\begin{vmatrix} -x \\ -y \\ -z \end{vmatrix} \wedge {}_R\begin{vmatrix} \frac{ce}{ad} \\ 0 \\ \frac{e}{a} \end{vmatrix}$$

$$
\begin{cases}
\delta U_{Mx} = \dfrac{fe}{a} - y\dfrac{e}{a} = \dfrac{e}{a}(f-y) \\[2ex]
\delta U_{My} = \dfrac{xe}{a} + \dfrac{ce}{ad}z = \dfrac{e}{a}\left(x - z\dfrac{c}{d}\right) \\[2ex]
\delta U_{Mz} = -\dfrac{fce}{ad} + y\dfrac{ce}{ad} = \dfrac{ce}{ad}(y-f)
\end{cases}
$$

6

$$\delta\theta_x = \frac{ce}{ad} = \frac{40 \times 1}{77 \times 46} \rightarrow 0{,}647^\circ$$

$$\delta\theta_z = \frac{e}{a} = \frac{1}{77} \longrightarrow 0{,}744^\circ$$

$$\delta U_{Mx} = \frac{e}{a}(f-y) = -2{,}18 \text{ mm}.$$

$$\delta U_{My} = \frac{e}{a}\left(x - z\frac{c}{d}\right) = \frac{1}{77}\left(-3 - \frac{48 \times 40}{46}\right) = -0{,}58 \text{ mm}$$

$$\delta U_{Mz} = \frac{ce}{ad}(y-f) = \frac{40 \times 1}{77 \times 46}(180 - 12{,}5) = 1{,}89 \text{ mm}$$

$$
\left|
\begin{array}{l}
\delta U_{Mx} = -2{,}18 \text{ mm}. \\
\delta U_{My} = -0{,}58 \text{ mm} \\
\delta U_{Mz} = 1{,}89 \text{ mm}
\end{array}
\right.
$$

7 Mesures sur
- x, $-2{,}14$ pour $-2{,}18$, erreur $= 1{,}86\%$
- y, $-0{,}5$ pour $-0{,}58$, erreur $= 16\%$
- z, $1{,}75$ pour $1{,}89$, erreur $= 8\%$

Liaison de Boys

$$\{\mathcal{U}_{1/a}\} = {}_{A}\left\{ \overrightarrow{\delta\theta_{1/a}},\ a\,\overrightarrow{x_0} \right\}$$

$$= {}_{B}\left\{ \overrightarrow{\delta\theta_{1/a}},\ b\,\overrightarrow{x_1} \right\}$$

$$= {}_{C}\left\{ \overrightarrow{\delta\theta_{1/a}},\ c\,\overrightarrow{x_2} \right\}$$

- $\overrightarrow{\delta \mathcal{U}_A} = \overrightarrow{\delta \mathcal{U}_B} + \overrightarrow{AB} \wedge \overrightarrow{\delta\theta_{1/a}}$

$${}_{a}\left|\begin{matrix} a \\ 0 \\ 0 \end{matrix}\right. = {}_{a}\left|\begin{matrix} b\cos 120° \\ b\sin 120 \\ 0 \end{matrix}\right. + {}_{a}\left|\begin{matrix} -r + r\cos 120° \\ r\sin 120° \\ 0 \end{matrix}\right. \wedge {}_{a}\left|\begin{matrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \end{matrix}\right.$$

$${}_{a}\left|\begin{matrix} a \\ 0 \\ 0 \end{matrix}\right. = {}_{a}\left|\begin{matrix} -\frac{b}{2} \\ b\sqrt{3}/2 \\ 0 \end{matrix}\right. + {}_{a}\left|\begin{matrix} -\frac{3r}{2} \\ r\frac{\sqrt{3}}{2} \\ 0 \end{matrix}\right. \wedge {}_{a}\left|\begin{matrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \end{matrix}\right. .$$

On obtient donc 3 équations

$$\begin{cases} a = -\frac{b}{2} + r\frac{\sqrt{3}}{2}\,\delta\theta_z & (1) \\ 0 = b\frac{\sqrt{3}}{2} + \frac{3r}{2}\,\delta\theta_z & (2) \\ 0 = -\frac{3r}{2}\,\delta\theta_y - r\frac{\sqrt{3}}{2}\,\delta\theta_x & (3) \end{cases}$$

- $\overrightarrow{\delta \mathcal{U}_A} = \overrightarrow{\delta \mathcal{U}_C} + \overrightarrow{AC} \wedge \overrightarrow{\delta\theta_{1/a}}$

$${}_{a}\left|\begin{matrix} a \\ 0 \\ 0 \end{matrix}\right. = {}_{B}\left|\begin{matrix} -\frac{c}{2} \\ -c\frac{\sqrt{3}}{2} \\ 0 \end{matrix}\right. + {}_{a}\left|\begin{matrix} -\frac{3r}{2} \\ -r\frac{\sqrt{3}}{2} \\ 0 \end{matrix}\right. \wedge {}_{a}\left|\begin{matrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \end{matrix}\right.$$

On obtient alors 3 équations.

$$\begin{cases} a = -\dfrac{c}{2} - r\dfrac{\sqrt{3}}{2}\,\delta\theta_z & (4) \\ 0 = -c\dfrac{\sqrt{3}}{2} + \dfrac{3}{2}r\,\delta\theta_z & (5) \\ 0 = -\dfrac{3}{2}r\,\delta\theta_y + r\dfrac{\sqrt{3}}{2}\,\delta\theta_x & (6) \end{cases}$$

(3) et (6) $\longrightarrow$ $\boxed{\delta\theta_x = \delta\theta_y = 0}$

(2) $\longrightarrow$ $b = -\dfrac{3r\,\delta\theta_z}{\cancel{3}\sqrt{3}}$

(5) $\longrightarrow$ $c = \dfrac{3r\,\delta\theta_z}{\cancel{3}\sqrt{3}}$

(1) $\longrightarrow$ $a = -\dfrac{1}{2}\left(-\dfrac{3r\,\delta\theta_z}{\cancel{3}\sqrt{3}}\right) + \dfrac{r\sqrt{3}}{2}\,\delta\theta_z$

(4) $\longrightarrow$ $a = -\dfrac{1}{2}\left(\dfrac{3r\,\delta\theta_z}{\cancel{3}\sqrt{3}}\right) - \dfrac{r\sqrt{3}}{2}\,\delta\theta_z$

$\longrightarrow$ $\boxed{\delta\theta_z = 0}$

(2) $\longrightarrow$ $b = 0$
(5) $\longrightarrow$ $c = 0$
(1) $\longrightarrow$ $a = 0$

$\boxed{\{\mathcal{U}_{1/0}\} = \{0\}}$ $\longrightarrow$ aucun mouvement possible
$\longrightarrow$ liaison encastrement

Liaison de Kelvin

en A $\rightarrow \{\mathcal{V}_{1/0}\} = {}_A\begin{Bmatrix} \delta\theta_x & x_A \\ \delta\theta_y & y_A \\ \delta\theta_z & 0 \end{Bmatrix}_{B_0}$ liaison ponctuelle

en B $\rightarrow \{\mathcal{V}_{1/0}\} = {}_B\begin{Bmatrix} \delta\theta_x & 0 \\ \delta\theta_y & y_B \\ \delta\theta_z & 0 \end{Bmatrix}_{B_0}$ liaison cylindre plan

en C $\rightarrow \{\mathcal{V}_{1/0}\} = {}_C\begin{Bmatrix} \delta\theta_x & 0 \\ \delta\theta_y & 0 \\ \delta\theta_z & 0 \end{Bmatrix}_{B_0}$ liaison sphérique

$$\overrightarrow{\delta U_{A\in 1/0}} = \overrightarrow{\delta U_{B\in 1/0}} + \overrightarrow{AB} \wedge \overrightarrow{\delta\Theta_{1/0}}$$

$$= {}_B\begin{vmatrix} 0 \\ y_B \\ 0 \end{vmatrix} + {}_B\begin{vmatrix} -a \\ b \\ 0 \end{vmatrix} \wedge {}_B\begin{vmatrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \end{vmatrix} = {}_B\begin{vmatrix} b\,\delta\theta_z \\ y_B + a\,\delta\theta_z \\ -a\,\delta\theta_y - b\,\delta\theta_x \end{vmatrix}$$

$$\overrightarrow{\delta U_{A\in 1/0}} = \overrightarrow{\delta U_{C\in 1/0}} + \overrightarrow{AC} \wedge \overrightarrow{\delta\Theta_{1/0}}$$

$$= {}_B\begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} + \begin{vmatrix} -a \\ -c \\ 0 \end{vmatrix} \wedge \begin{vmatrix} \delta\theta_x \\ \delta\theta_y \\ \delta\theta_z \end{vmatrix} = \begin{vmatrix} -c\,\delta\theta_z \\ +a\,\delta\theta_z \\ -a\,\delta\theta_y + c\,\delta\theta_x \end{vmatrix}$$

$$\begin{cases} (1) & x_A = b\,\delta\theta_z = -c\,\delta\theta_z \\ (2) & y_A = y_B + a\,\delta\theta_z = a\,\delta\theta_z \\ (3) & 0 = -a\,\delta\theta_y - b\,\delta\theta_x = -a\,\delta\theta_y + c\,\delta\theta_x \end{cases}$$

(1) une seule possibilité $\delta\theta_z = 0$ car $(b + c\,\delta\theta_z) = 0$

d'où $x_A = 0$

(2) $y_A = y_B + 0 = 0 \longrightarrow y_B = y_A = 0$

(3) $\left.\begin{array}{l} 0 = -a\,\delta\theta_y - b\,\delta\theta_x \\ 0 = -a\,\delta\theta_y + c\,\delta\theta_x \end{array}\right\} \longrightarrow (c+b)\,\delta\theta_x = 0$

donc $\delta\theta_x = 0$

d'où $\delta\theta_y = 0$

$\{\mathcal{U}_{1/0}\} = \{0\} \longrightarrow$ aucune liberté $\longrightarrow$

liaison encastrement.