In [1]:
```
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
%matplotlib inline
```

1. Markov Chain

Define a Markov chain according to the network structure, such that from each node a random walker will jump to its neighbors with equal probability,
i.e. $P = D^{-1}A$ where $D = diag(d_i)$
and $d_i = \sum_j A_{ij}$

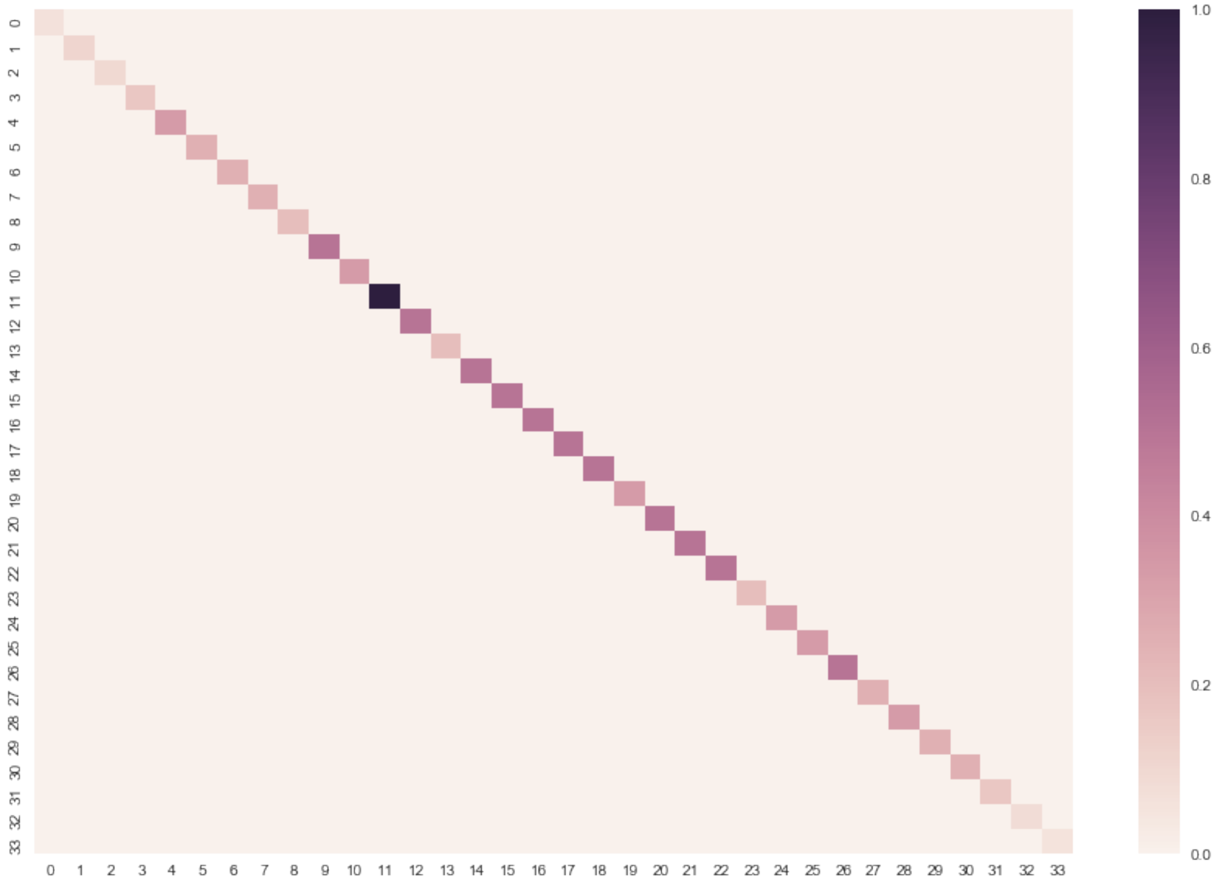
In [2]:
```
import pickle
data = pickle.load(open('data/karate_cleaned.p','rb'))
matrix = data['matrix']
```

In [3]:
```
D = np.diag(np.sum(matrix,axis=1))
print D
# plt.figure(figsize=(15,10))
# sns.heatmap(D)
```

```
[[16  0  0 ...,  0  0  0]
 [ 0  9  0 ...,  0  0  0]
 [ 0  0 10 ...,  0  0  0]
 ...,
 [ 0  0  0 ...,  6  0  0]
 [ 0  0  0 ...,  0 12  0]
 [ 0  0  0 ...,  0  0 17]]
```

In [4]:
```
#Inverse of D
inv_D = np.diag(1./np.sum(matrix,axis=1))
inv_D
plt.figure(figsize=(15,10))
sns.heatmap(inv_D)
```

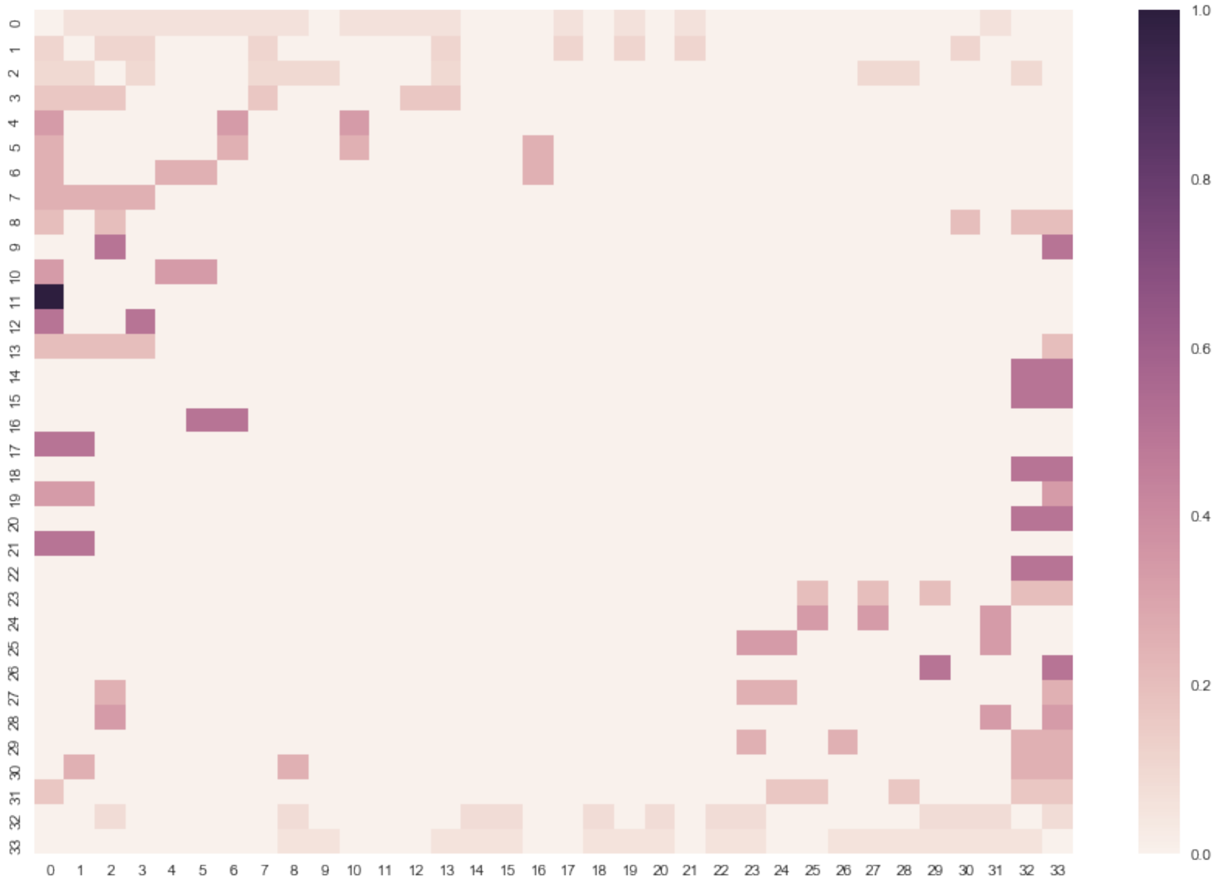
Out[4]: <matplotlib.axes._subplots.AxesSubplot at 0x1131c8410>



In [5]:
```
#P-1 * A
markov = np.dot(inv_D,matrix.T.astype(int))
print markov
plt.figure(figsize=(15,10))
sns.heatmap(markov)
np.sum(markov,axis=1)
```

```
[[ 0.          0.0625      0.0625     ...,  0.0625      0.
0.        ]
 [ 0.11111111  0.          0.11111111 ...,  0.          0.
0.        ]
 [ 0.1         0.1         0.         ...,  0.          0.1
0.        ]
 ...,
 [ 0.16666667  0.          0.         ...,  0.          0.16666667
   0.16666667]
 [ 0.          0.          0.08333333 ...,  0.08333333  0.
0.08333333]
 [ 0.          0.          0.         ...,  0.05882353  0.05882353
0.        ]]
```

Out[5]:
```
array([ 1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.,
1.,
        1.,  1.,  1.,  1.,  1.,  1.,  1.,  1.])
```



2. Stationnary Distribution

In [6]:
```
from scipy.linalg import eig

S, U = eig(markov, right=False, left=True)
# print S
# print U
```

In [ ]:

In [7]:
```
# Eigenvector corresponding to the eigenvalue 1
print np.abs(S - 1.)
np.argsort(np.abs(S - 1.))
# Position 0!
```

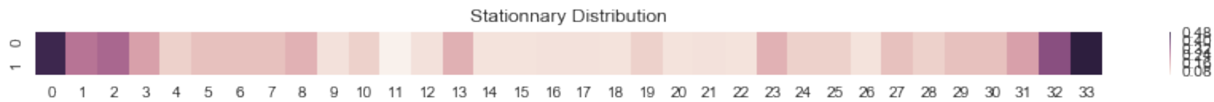
```
[  2.22044605e-15   1.32272329e-01   2.87048985e-01   3.87313233e-01
   1.71461135e+00   6.12230540e-01   6.48992947e-01   7.07208202e-01
   7.39957989e-01   7.70910617e-01   8.22942852e-01   8.64832945e-01
   9.06816002e-01   1.10538084e+00   1.15929996e+00   1.26802355e+00
   1.61190959e+00   1.56950660e+00   1.35177826e+00   1.49703011e+00
   1.39310454e+00   1.41691585e+00   1.44857938e+00   1.58333333e+00
   1.00000000e+00   1.00000000e+00   1.00000000e+00   1.00000000e+00
   1.00000000e+00   1.00000000e+00   1.00000000e+00   1.00000000e+00
   1.00000000e+00   1.00000000e+00]
```

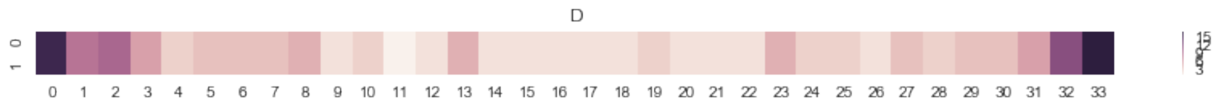
Out[7]:
```
array([ 0,  1,  2,  3,  5,  6,  7,  8,  9, 10, 11, 12, 25, 26, 30, 2
9, 28,
        27, 31, 33, 32, 24, 13, 14, 15, 18, 20, 21, 22, 19, 17, 23, 1
6,  4])
```

In [8]:
```
stationary = np.array(U.T[0])
# stationary /= np.sum(stationary)
print stationary
print D

plt.figure(figsize=(15,0.5))
sns.heatmap([np.abs(stationary),np.abs(stationary)]).set_title('Statio

plt.figure(figsize=(15,0.5))
sns.heatmap([np.sum(matrix,axis=0),np.sum(matrix,axis=1)]).set_title('I
```

```
[-0.45958799+0.j -0.25851825+0.j -0.28724249+0.j -0.17234550+0.j
 -0.08617275+0.j -0.11489700+0.j -0.11489700+0.j -0.11489700+0.j
 -0.14362125+0.j -0.05744850+0.j -0.08617275+0.j -0.02872425+0.j
 -0.05744850+0.j -0.14362125+0.j -0.05744850+0.j -0.05744850+0.j
 -0.05744850+0.j -0.05744850+0.j -0.05744850+0.j -0.08617275+0.j
 -0.05744850+0.j -0.05744850+0.j -0.05744850+0.j -0.14362125+0.j
 -0.08617275+0.j -0.08617275+0.j -0.05744850+0.j -0.11489700+0.j
 -0.08617275+0.j -0.11489700+0.j -0.11489700+0.j -0.17234550+0.j
 -0.34469099+0.j -0.48831224+0.j]
[[16  0  0 ...,  0  0  0]
 [ 0  9  0 ...,  0  0  0]
 [ 0  0 10 ...,  0  0  0]
 ...,
 [ 0  0  0 ...,  6  0  0]
 [ 0  0  0 ...,  0 12  0]
 [ 0  0  0 ...,  0  0 17]]
```

Out[8]: <matplotlib.text.Text at 0x1176155d0>





$\pi(i) \sim d_i$!!

Finally worked this out!

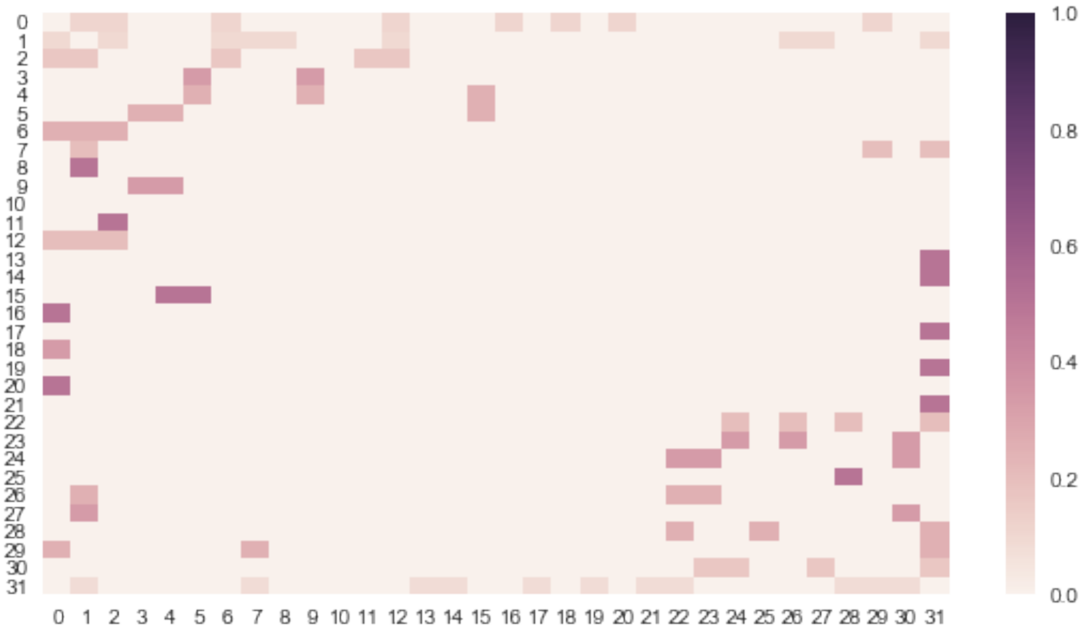
3. Cut using probability of reaching V1 before V0

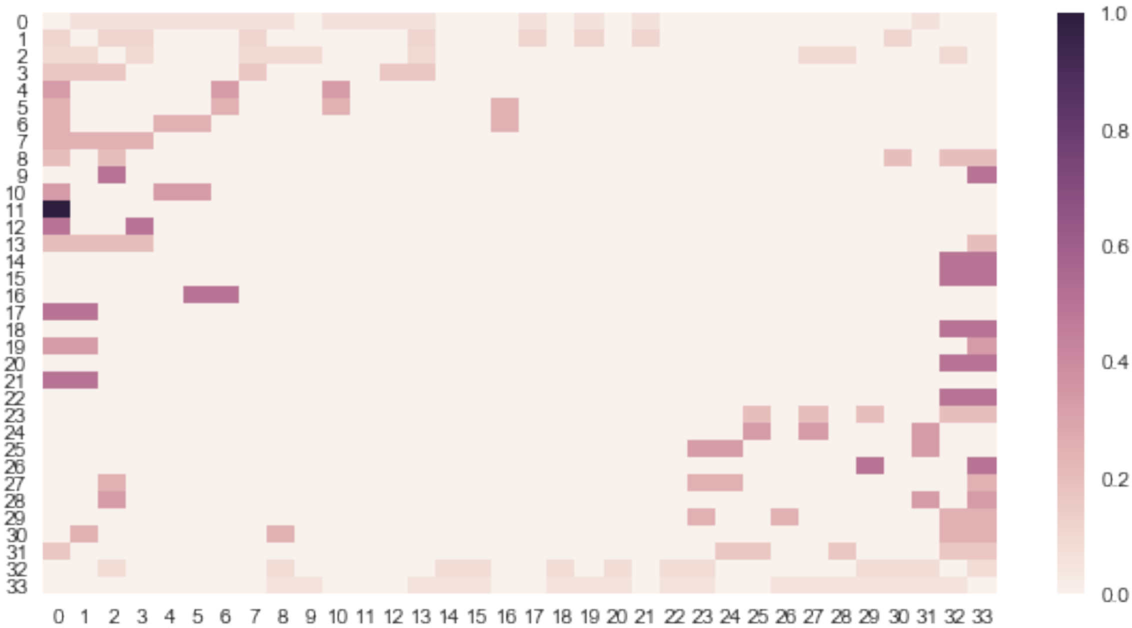
Here I'm using a different method than the presented in the PDF.
I'm using the Q Matrix,matrix transition probabilities from {2,..32,33} to itself,

In [9]:
```
Q = markov[1:33,1:33]
print Q.shape, markov.shape
Q
plt.figure(figsize=(9.5,5))
sns.heatmap(Q,vmax=1)
plt.figure(figsize=(10,5))
sns.heatmap(markov)
```

```
(32, 32) (34, 34)
```

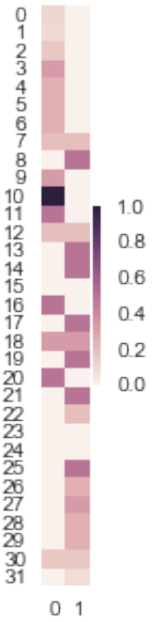
Out[9]: <matplotlib.axes._subplots.AxesSubplot at 0x117900510>





In [10]:
```
S1 = markov.T[0][1:33]
S2 = markov.T[33][1:33]
S = np.column_stack((S1,S2))
plt.figure(figsize=(0.5,5))
sns.heatmap(S,vmax=1)
```

Out[10]: <matplotlib.axes._subplots.AxesSubplot at 0x117aeac90>



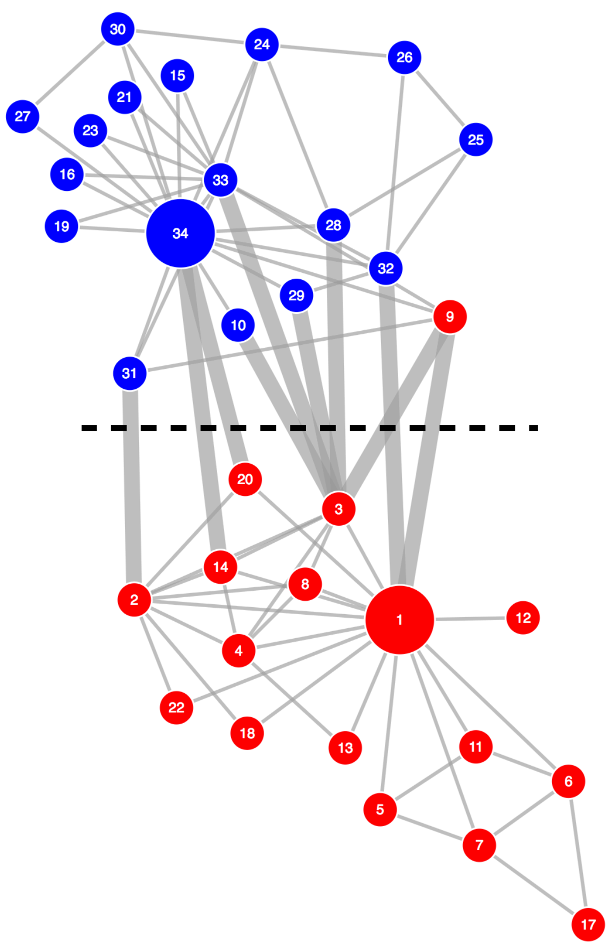
In [11]:
```
prob = np.dot(np.linalg.inv(np.identity(n-2)-Q),S)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-11-1e1cdd960aad> in <module>()
----> 1 prob = np.dot(np.linalg.inv(np.identity(n-2)-Q),S)

NameError: name 'n' is not defined
```

In [ ]:
```
ground_truth = [0 if x in data['blue_list'] else 1 for x in range(1,35
results = [0 if x >= 0.5 else 1 for x in prob.T[1]]
results = [1] + results + [0]

print ground_truth
plt.figure(figsize=(15,0.5))
sns.heatmap([ground_truth,ground_truth],cmap='coolwarm').set_title('Gr

plt.figure(figsize=(15,0.5))
sns.heatmap([results,results],cmap='coolwarm').set_title('Predicted wi
```

In [ ]:
```
# Cut in the graph

def value(node):
    if node == 1:
        return 0
    if node == 34:
        return 1
    return prob.T[1][node-2]

for link in data['graph']['links']:
    if (value(link['source'])>=0.5 and value(link['target'])<0.5) or (v
        link['value'] = 20
    else:
        link['value'] = 1
```

In [ ]:
```
import json
with open('cluster_karate_B.json', 'w') as outfile:
    json.dump(data['graph'], outfile)
```



Effective Flux

In [ ]:
```
def J(x,y):
    if x == y:
        return 0
    else:
        return stationary[x-1]*(1-value(x))*markov[x-1][y-1]*value(y)

def J_p(x,y):
    return max([0,J(x,y)-J(y,x)])
```

In [ ]:
```
SET = range(1,35)
```

In [ ]:
```
def T(x):
    if x == 1:
        return sum([J_p(x,y) for y in SET])
    if x == 34:
        return sum([J_p(y,x) for y in SET])
    return sum([J_p(x,y) for y in SET])
```

In [ ]:
```
[abs(T(x)) for x in SET]
```

In [ ]:

In [ ]:
```
# Matlab Code

% Transition Path Analysis for Karate Club network
%
%   Reference:
%       Weinan E, Jianfeng Lu, and Yuan Yao (2013)
%       The Landscape of Complex Networks: Critical Nodes and A Hierar
%       Methods and Applications of Analysis, special issue in honor o

% load the Adjacency matrix of Karate Club network
%   replace it by your own data
load karate_rand1.mat A

D = sum(A, 2);
N = length(D);
Label = [0:N-1];
TransProb = diag(1./D) * A;
LMat = TransProb - diag(ones(N, 1));

% source set A contains the coach
% target set B contains the president
SetA = 1; % [44:54];%[find(ind==19)];%[44:54];%18 + 1;
SetB = 34; %[find(ind==11)];%10 + 1; % seems to be 11 instead of 10

[EigV, EigD] = eig(LMat');
EquiMeasure = EigV(:, 1)./sign(EigV(1,1));

for i = 1:N
  localmin = true;
```

```
  for j = setdiff(1:N, i)
    if ((LMat(i,j)>0)&(EquiMeasure(j)>EquiMeasure(i)))
      localmin = false;
      break
    end
  end
  if (localmin)
    i
  end
end

mfpt = zeros(N, 1);
SourceSet = 11;
RemainSet = setdiff(1:N, SourceSet);
mfpt(RemainSet) = - LMat(RemainSet, RemainSet) \ ones(N-1, 1);

TransLMat = diag(EquiMeasure) * LMat * diag(1./EquiMeasure);

SourceSet = SetA;
TargetSet = SetB;
RemainSet = setdiff(1:N, union(SourceSet, TargetSet));

% Initialization of Committor function: transition probability of reac
% the target set before returning to the source set.
CommitAB = zeros(N, 1);
CommitAB(SourceSet) = zeros(size(SourceSet));
CommitAB(TargetSet) = ones(size(TargetSet));

LMatRestrict = LMat(RemainSet, RemainSet);
RightHandSide = - LMat(RemainSet, TargetSet) * CommitAB(TargetSet);

% Solve the Dirchelet Boundary problem
CommitAB(RemainSet) = LMatRestrict \ RightHandSide;

% Clustering into two basins according to the transition probability
ClusterA = find(CommitAB <= 0.5);
ClusterB = find(CommitAB > 0.5);

% The inverse transition probability (committor function)
CommitBA = zeros(N, 1);
CommitBA(SourceSet) = ones(size(SourceSet));
CommitBA(TargetSet) = zeros(size(TargetSet));

LMatRestrict = LMat(RemainSet, RemainSet);
RightHandSide = - LMat(RemainSet, SourceSet) * CommitBA(SourceSet);

% Dirichelet Boundary Problem with inverse transition probability
CommitBA(RemainSet) = LMatRestrict \ RightHandSide;

RhoAB = EquiMeasure .* CommitAB .* CommitBA;

% Current or Flux on edges
CurrentAB = diag(EquiMeasure .* CommitBA) * LMat * diag(CommitAB);
CurrentAB = CurrentAB - diag(diag(CurrentAB));
```

```
CurrentAB = CurrentAB - diag(diag(CurrentAB));

% Effective Current Flux
EffCurrentAB = max(CurrentAB - CurrentAB', 0);

% Transition Current or Flux on each node
TransCurrent = zeros(N, 1);
TransCurrent(ClusterA) = sum(EffCurrentAB(ClusterA, ClusterB), 2);
TransCurrent(ClusterB) = sum(EffCurrentAB(ClusterA, ClusterB), 1);
```

In [ ]:

```
```

In [ ]:

```
```