In [1]:
```
import pickle
data = pickle.load(open('data/karate_cleaned.p','rb'))
# data
```

Spectral clustering via the Cheeger vector

In [2]:
```
import numpy as np
from scipy.sparse import csgraph
```

In [3]:
```
matrix = data['matrix']
for a in matrix:
    print a
```

```
[0 1 1 1 1 1 1 1 1 0 1 1 1 1 0 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 0
]
[1 0 1 1 0 0 0 1 0 0 0 0 0 1 0 0 0 1 0 1 0 1 0 0 0 0 0 0 0 0 1 0 0 0
]
[1 1 0 1 0 0 0 1 1 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 0
]
[1 1 1 0 0 0 0 1 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 0 0 0 0 0 1 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 1
]
[0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
]
[1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
]
[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
]
[0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
[1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
]
```

```
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
 ]
 [1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
 ]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
 ]
 [1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
 ]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
 ]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 0 1 1
 ]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 1 0 0
 ]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 0 0
 ]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
 ]
 [0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1
 ]
 [0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
 ]
 [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 1 1
 ]
 [0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
 ]
 [1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 0 0 0 1 1
 ]
 [0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 1 0 1 0 1 1 0 0 0 0 0 1 1 1 0 1
 ]
 [0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1 0 0 1 1 1 0 1 1 0 0 1 1 1 1 1 1 1 0
 ]]
```

Normalized Graph Laplacian

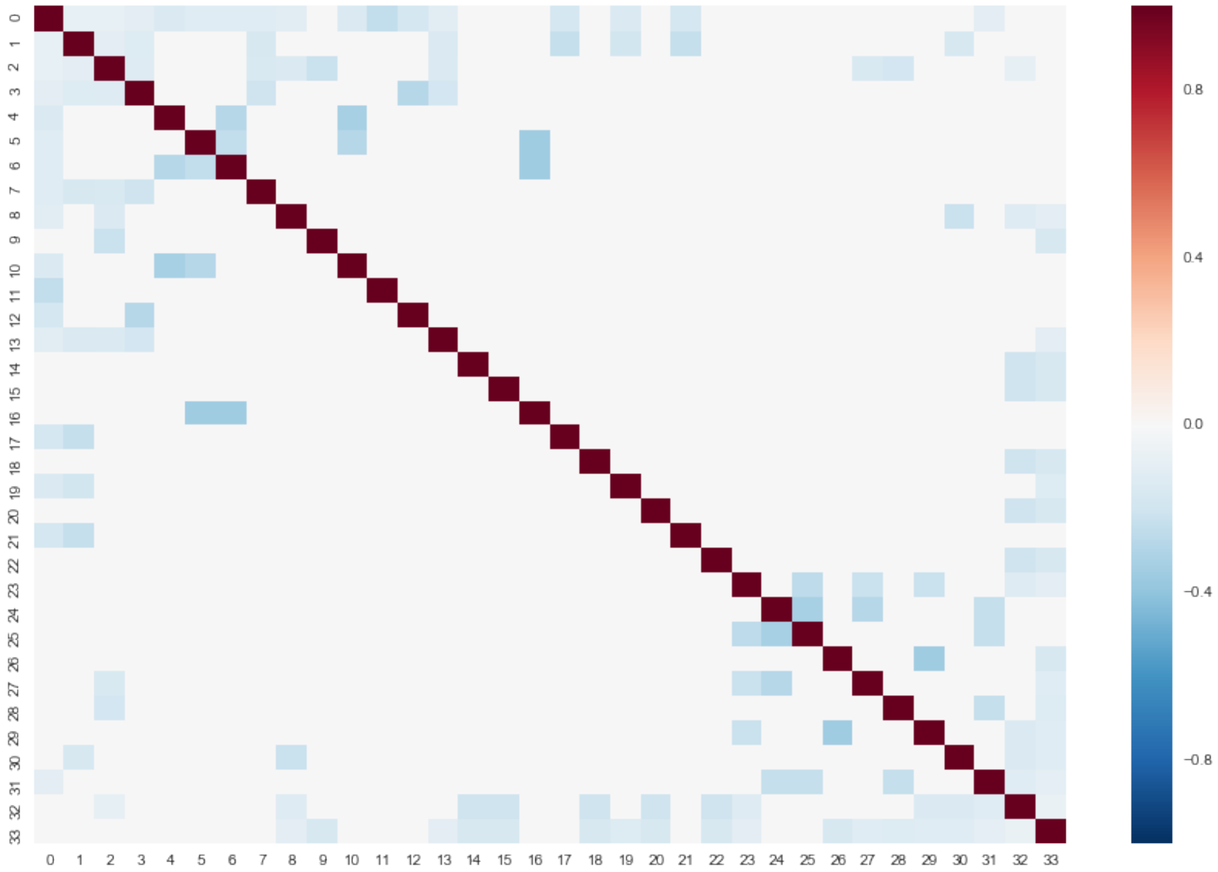
In [4]:
```
ngl = csgraph.laplacian(matrix.astype(float), normed=True)
ngl
```

Out[4]:
```
array([[ 1.        , -0.08333333, -0.07905694, ..., -0.10206207,
        -0.        , -0.        ],
       [-0.08333333,  1.        , -0.10540926, ..., -0.        ,
        -0.        , -0.        ],
       [-0.07905694, -0.10540926,  1.        , ..., -0.        ,
        -0.09128709, -0.        ],
       ...,
       [-0.10206207, -0.        , -0.        , ...,  1.        ,
        -0.11785113, -0.09901475],
       [-0.        , -0.        , -0.09128709, ..., -0.11785113,
         1.        , -0.070014  ],
       [-0.        , -0.        , -0.        , ..., -0.09901475,
        -0.070014  ,  1.        ]])
```

In [5]:
```
import seaborn as sns
import matplotlib.pyplot as plt
%matplotlib inline
```

In [6]:
```
plt.figure(figsize=(15,10))
sns.heatmap(ngl)
```

Out[6]: <matplotlib.axes._subplots.AxesSubplot at 0x114204fd0>



Eigenvectors and Eigenvalues

In [7]:
```
from scipy.linalg import eig
```

In [8]:
```
(eigenvalues, eigenvectors) = eig(ngl, right=False, left=True)
print eigenvectors.shape
print eigenvalues
eigenvectors
```

```
(34, 34)
[ -2.49800181e-16+0.j   1.32272329e-01+0.j   2.87048985e-01+0.j
   3.87313233e-01+0.j   1.71461135e+00+0.j   6.12230540e-01+0.j
   6.48992947e-01+0.j   7.07208202e-01+0.j   7.39957989e-01+0.j
   7.70910617e-01+0.j   8.22942852e-01+0.j   8.64832945e-01+0.j
   9.06816002e-01+0.j   1.10538084e+00+0.j   1.15929996e+00+0.j
   1.26802355e+00+0.j   1.61190959e+00+0.j   1.56950660e+00+0.j
   1.35177826e+00+0.j   1.39310454e+00+0.j   1.41691585e+00+0.j
   1.44857938e+00+0.j   1.49703011e+00+0.j   1.58333333e+00+0.j
   1.00000000e+00+0.j   1.00000000e+00+0.j   1.00000000e+00+0.j
   1.00000000e+00+0.j   1.00000000e+00+0.j   1.00000000e+00+0.j
   1.00000000e+00+0.j   1.00000000e+00+0.j   1.00000000e+00+0.j
   1.00000000e+00+0.j]
```

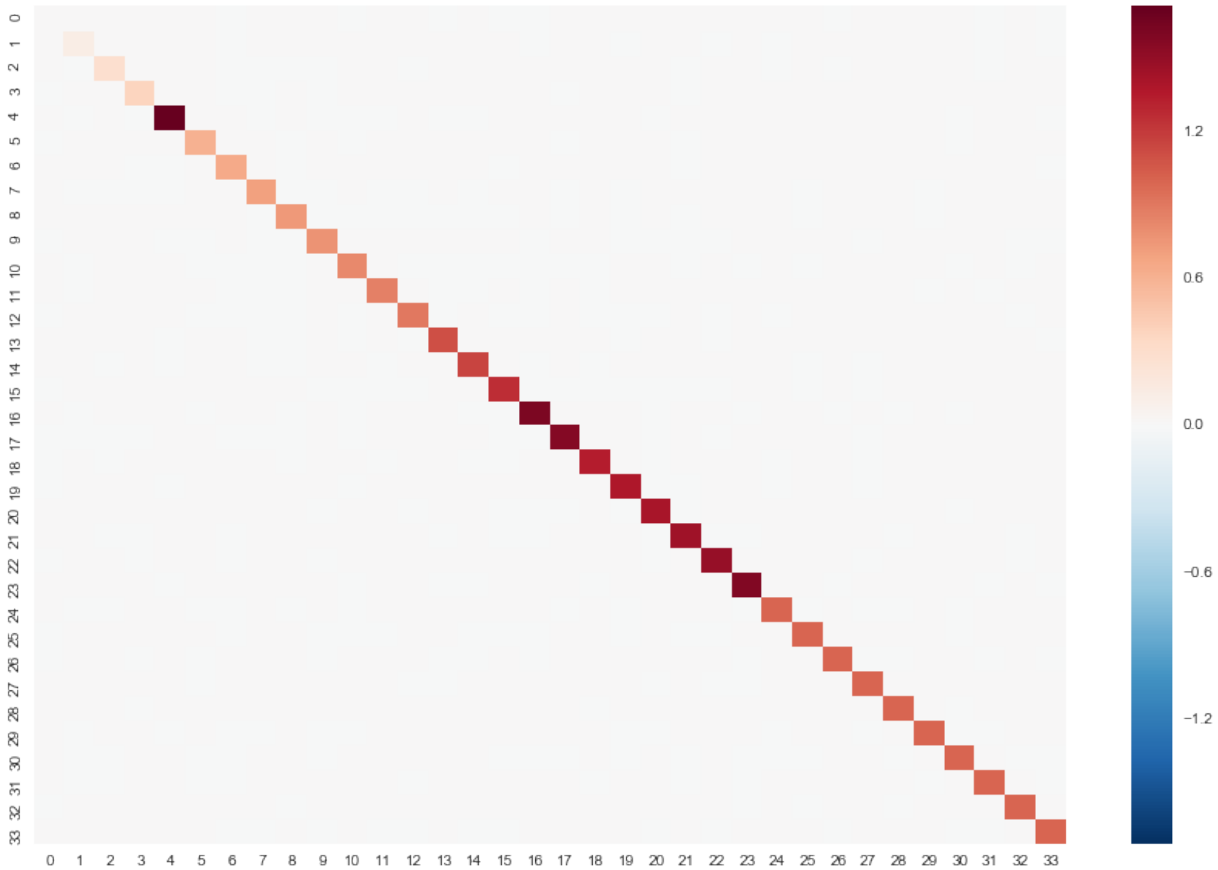
Out[8]:
```
array([[ -3.20256308e-01,  -2.96399797e-01,   1.44586983e-01, ...,
          0.00000000e+00,   0.00000000e+00,   0.00000000e+00],
       [ -2.40192231e-01,  -1.13413889e-01,   3.50466912e-01, ...,
          1.02381971e-16,   1.06364095e-17,   1.59074765e-16],
       [ -2.53184842e-01,   8.97112612e-03,   2.11481660e-01, ...,
         -2.19799805e-17,  -1.03550800e-16,   4.96462301e-16],
       ...,
       [ -1.96116135e-01,   1.28108134e-01,  -1.10871456e-01, ...,
          1.24049165e-16,  -1.59003548e-16,   4.68327043e-17],
       [ -2.77350098e-01,   2.51627460e-01,  -1.12649647e-01, ...,
         -4.80002082e-16,  -4.22277765e-16,   8.22532117e-16],
       [ -3.30112646e-01,   2.69793542e-01,  -9.23627895e-02, ...,
          5.37478772e-16,   2.97317487e-16,  -7.58639980e-16]])
```

In [9]:
```
print 'Eigenvalues\n', eigenvalues
print '\nEigenvectors\n', eigenvectors
from numpy.linalg import inv
(eigenvalues, eigenvectors) = eig(ngl, left=True,right=False)
a = np.dot(np.dot(inv(eigenvectors),ngl),eigenvectors)
plt.figure(figsize=(15,10))
print '\nInvert in this base (sanity check)\n'
sns.heatmap(a)
```

```
Eigenvalues
[ -2.49800181e-16+0.j   1.32272329e-01+0.j   2.87048985e-01+0.j
   3.87313233e-01+0.j   1.71461135e+00+0.j   6.12230540e-01+0.j
   6.48992947e-01+0.j   7.07208202e-01+0.j   7.39957989e-01+0.j
   7.70910617e-01+0.j   8.22942852e-01+0.j   8.64832945e-01+0.j
   9.06816002e-01+0.j   1.10538084e+00+0.j   1.15929996e+00+0.j
   1.26802355e+00+0.j   1.61190959e+00+0.j   1.56950660e+00+0.j
   1.35177826e+00+0.j   1.39310454e+00+0.j   1.41691585e+00+0.j
   1.44857938e+00+0.j   1.49703011e+00+0.j   1.58333333e+00+0.j
   1.00000000e+00+0.j   1.00000000e+00+0.j   1.00000000e+00+0.j
   1.00000000e+00+0.j   1.00000000e+00+0.j   1.00000000e+00+0.j
```

```
   1.00000000e+00+0.j   1.00000000e+00+0.j   1.00000000e+00+0.j
   1.00000000e+00+0.j]

Eigenvectors
[[ -3.20256308e-01  -2.96399797e-01   1.44586983e-01 ...,   0.000000
00e+00
    0.00000000e+00   0.00000000e+00]
 [ -2.40192231e-01  -1.13413889e-01   3.50466912e-01 ...,   1.023819
71e-16
    1.06364095e-17   1.59074765e-16]
 [ -2.53184842e-01   8.97112612e-03   2.11481660e-01 ...,  -2.197998
05e-17
   -1.03550800e-16   4.96462301e-16]
 ...,
 [ -1.96116135e-01   1.28108134e-01  -1.10871456e-01 ...,   1.240491
65e-16
   -1.59003548e-16   4.68327043e-17]
 [ -2.77350098e-01   2.51627460e-01  -1.12649647e-01 ...,  -4.800020
82e-16
   -4.22277765e-16   8.22532117e-16]
 [ -3.30112646e-01   2.69793542e-01  -9.23627895e-02 ...,   5.374787
72e-16
    2.97317487e-16  -7.58639980e-16]]

Invert in this base (sanity check)
```

Out[9]: <matplotlib.axes._subplots.AxesSubplot at 0x1177e3290>



Sorting the eigenvectors by norm (but that's useless)

In [10]:
```
# Sort by norm
indexlist = np.argsort(np.linalg.norm(eigenvectors,axis=1))

# Just for test purposes
# indexlist = np.arange(34)
# indexlist=reversed(indexlist)
print indexlist
sorted_eigenvectors = np.array([eigenvectors[:,k] for k in indexlist])
```

```
[20 22  9 18 19  7 28 15 27 25 12 11  3  8  2 30 16  0 33 29  1 31 3
2 26 24
 10  4  6  5 23 14 21 17 13]
```

In [11]:
```
# a = np.dot(np.dot(inv(sorted_eigenvectors),ngl),sorted_eigenvectors)
# plt.figure(figsize=(15,10))
# sns.heatmap(a)
```

Sort by eigenvalues

In [12]:
```
(eigenvalues, eigenvectors) = eig(ngl, left=True,right=False)
print eigenvalues.shape
eigenvalues
```

```
(34,)
```

Out[12]:
```
array([ -2.49800181e-16+0.j,   1.32272329e-01+0.j,   2.87048985e-01+
0.j,
         3.87313233e-01+0.j,   1.71461135e+00+0.j,   6.12230540e-01+
0.j,
         6.48992947e-01+0.j,   7.07208202e-01+0.j,   7.39957989e-01+
0.j,
         7.70910617e-01+0.j,   8.22942852e-01+0.j,   8.64832945e-01+
0.j,
         9.06816002e-01+0.j,   1.10538084e+00+0.j,   1.15929996e+00+
0.j,
         1.26802355e+00+0.j,   1.61190959e+00+0.j,   1.56950660e+00+
0.j,
         1.35177826e+00+0.j,   1.39310454e+00+0.j,   1.41691585e+00+
0.j,
         1.44857938e+00+0.j,   1.49703011e+00+0.j,   1.58333333e+00+
0.j,
         1.00000000e+00+0.j,   1.00000000e+00+0.j,   1.00000000e+00+
0.j,
         1.00000000e+00+0.j,   1.00000000e+00+0.j,   1.00000000e+00+
0.j,
         1.00000000e+00+0.j,   1.00000000e+00+0.j,   1.00000000e+00+
0.j,
         1.00000000e+00+0.j])
```

In [13]:
```
print np.argsort(eigenvalues)
[eigenvalues[k] for k in np.argsort(eigenvalues)]
```

```
[ 0  1  2  3  5  6  7  8  9 10 11 12 24 30 32 31 27 26 33 25 29 28 1
3 14 15
 18 19 20 21 22 17 23 16  4]
```

Out[13]:
```
[(-2.4980018054066022e-16+0j),
 (0.13227232922951659+0j),
 (0.28704898538503493+0j),
 (0.38731323261013068+0j),
 (0.61223054020030909+0j),
 (0.64899294666920038+0j),
 (0.70720820249415162+0j),
 (0.73995798930084333+0j),
 (0.77091061685113016+0j),
 (0.82294285233819053+0j),
 (0.86483294458061954+0j),
 (0.90681600158647535+0j),
 (0.999999999999998+0j),
 (0.99999999999999822+0j),
 (0.99999999999999933+0j),
 (0.99999999999999978+0j),
 (0.99999999999999989+0j),
 (0.99999999999999989+0j),
 (0.99999999999999989+0j),
 (1+0j),
 (1.0000000000000004+0j),
 (1.0000000000000007+0j),
 (1.1053808390082949+0j),
 (1.1592999555430796+0j),
 (1.2680235467032606+0j),
 (1.3517782590320488+0j),
 (1.39310454092137+0j),
 (1.4169158506381598+0j),
 (1.4485793824675033+0j),
 (1.4970301128853551+0j),
 (1.5695066032433369+0j),
 (1.5833333333333335+0j),
 (1.6119095875050404+0j),
 (1.7146113474736213+0j)]
```

so the second smallest eigenvalues is the second in the list eigenvalues (0.132272329229)

In [14]:
```
second_smallest_eigenvalues = eigenvalues.T[1]
print second_smallest_eigenvalues
associated_eigenvectors = eigenvectors.T[1]
v = associated_eigenvectors
```

```
(0.13227232923+0j)
```

v is our **Fiedler Vector**

In [15]:
```
print v
```

```
[-0.2963998  -0.11341389  0.00897113 -0.11512758 -0.2671717  -0.3463
8736
 -0.34638736 -0.08992931  0.05282964  0.05563406 -0.2671717  -0.0853
954
 -0.09868424 -0.04671125  0.11251508  0.11251508 -0.28226926 -0.0911
9046
  0.11251508 -0.03091925  0.11251508 -0.09119046  0.11251508  0.1960
2288
  0.13544115  0.14515535  0.12748466  0.1349113   0.08022324  0.1820
1707
  0.07139028  0.12810813  0.25162746  0.26979354]
```

According to the sign, it give us the grouping:

In [16]:
```
# Every negative and corresponding indexes
REDS = filter(lambda a: a!='', [index+1 if v[index] <= 0 else '' for i
print REDS
print data['red_list']
```

```
[1, 2, 4, 5, 6, 7, 8, 11, 12, 13, 14, 17, 18, 20, 22]
[1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 17, 18, 20, 22]
```

In [17]:
```
correct = filter(lambda a:a in REDS,data['red_list'])
print 'Correct',correct ,len(correct)
wrong = filter(lambda a:a not in REDS,data['red_list'])
print 'Wrong',wrong ,len(wrong)
```

```
Correct [1, 2, 4, 5, 6, 7, 8, 11, 12, 13, 14, 17, 18, 20, 22] 15
Wrong [3, 9] 2
```

In [18]:
```
# Every positive
BLUES = filter(lambda a: a!='', [index+1 if v[index] > 0 else '' for i
print BLUES
print data['blue_list']
```

```
[3, 9, 10, 15, 16, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 3
3, 34]
[10, 15, 16, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34]
```

In [19]:
```
correct = filter(lambda a:a in BLUES,data['blue_list'])
print 'Correct',correct,len(correct)
wrong = filter(lambda a:a not in BLUES,data['blue_list'])
print 'Wrong',wrong ,len(wrong)
```

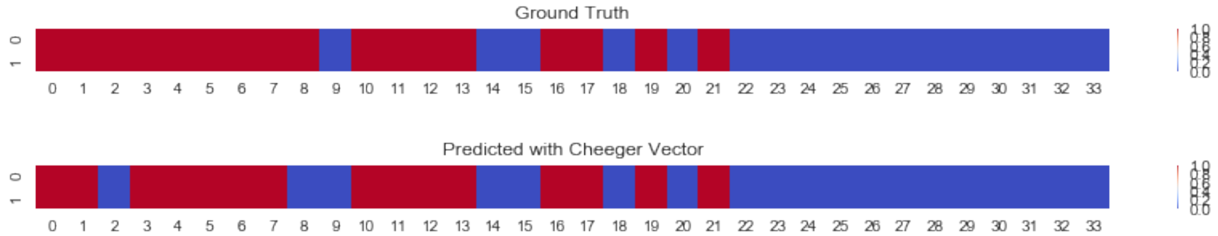
```
Correct [10, 15, 16, 19, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32,
33, 34] 17
Wrong [] 0
```

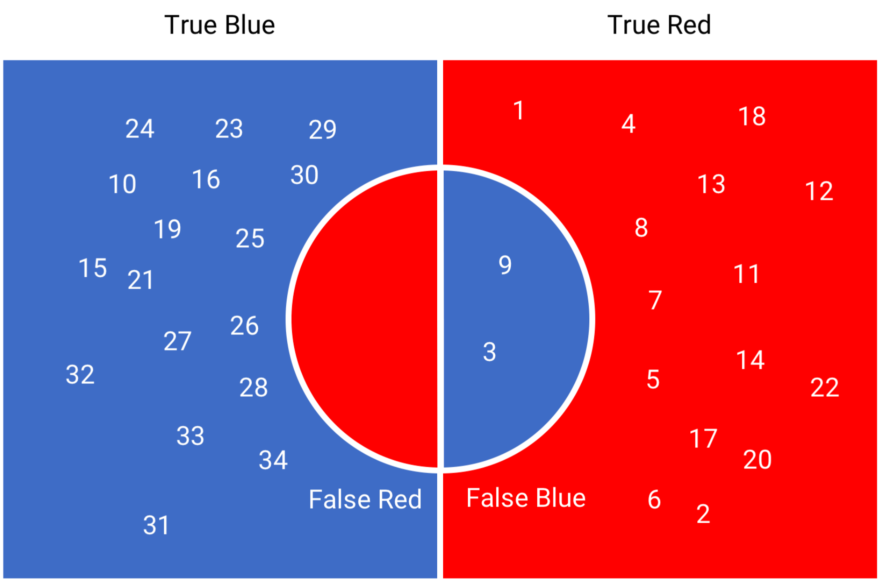
Just visualisations

In [20]:
```
ground_truth = [0 if x in data['blue_list'] else 1 for x in range(1,35

print ground_truth
plt.figure(figsize=(15,0.5))
sns.heatmap([ground_truth,ground_truth],cmap='coolwarm').set_title('Gr

results = [0 if x in BLUES else 1 for x in range(1,35)]
print results
plt.figure(figsize=(15,0.5))
sns.heatmap([results,results],cmap='coolwarm').set_title('Predicted wi
```

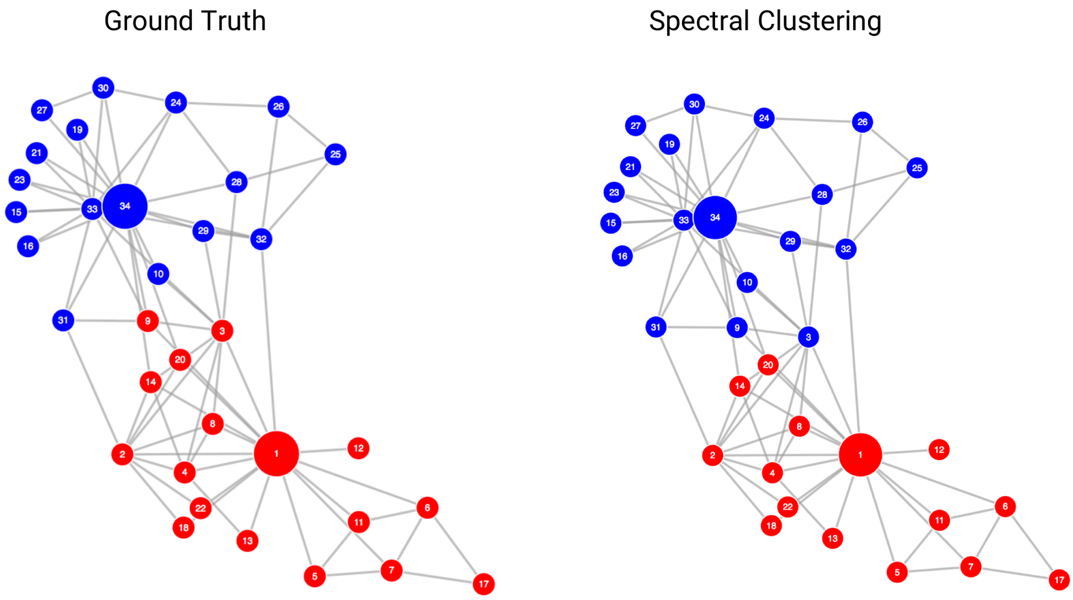
```
[1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0
, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
[1, 1, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0
, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
```

Out[20]: `<matplotlib.text.Text at 0x118f919d0>`





In [21]:
```
for a in data['graph']['nodes']:
    a['group'] = 'red' if a['id'] in REDS else 'blue'

import json
with open('cluster_karate_B.json', 'w') as outfile:
    json.dump(data['graph'], outfile)
```



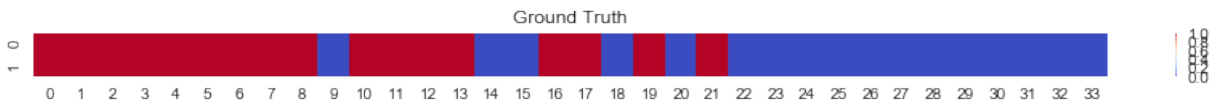
...not that efficient

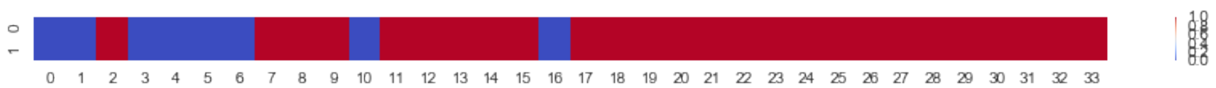
there might be something wrong with my code

Using Scikit Learn

In [22]:
```python
from sklearn.cluster import SpectralClustering
from sklearn import metrics

# Cluster
sc = SpectralClustering(2, affinity='precomputed', n_init=100)
sc.fit(matrix)


print 'Ground Truth'
ground_truth = [0 if x in data['blue_list'] else 1 for x in range(1,35
print ground_truth
plt.figure(figsize=(15,0.5))
sns.heatmap([ground_truth,ground_truth],cmap='coolwarm').set_title('Gr


print('Spectral Clustering')
print(sc.labels_)
plt.figure(figsize=(15,0.5))
sns.heatmap([sc.labels_,sc.labels_],cmap='coolwarm')

# Calculate some clustering metrics
print(metrics.adjusted_rand_score(ground_truth, sc.labels_))
print(metrics.adjusted_mutual_info_score(ground_truth, sc.labels_))
```

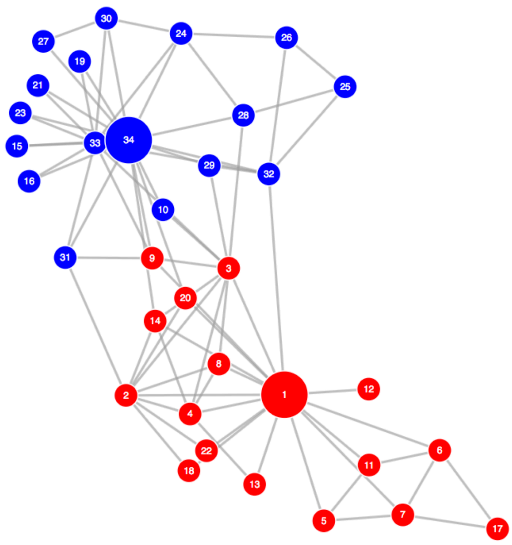
```
Ground Truth
[1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0
, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
Spectral Clustering
[0 0 1 0 0 0 0 1 1 1 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
]
0.204094758281
0.271689477828
```



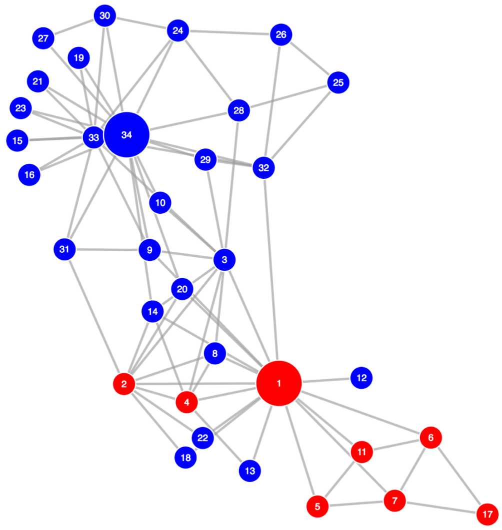


those are pretty good results!

Ground Truth



Spectral Clustering - Scikit



Tuning parameters and adding KNN

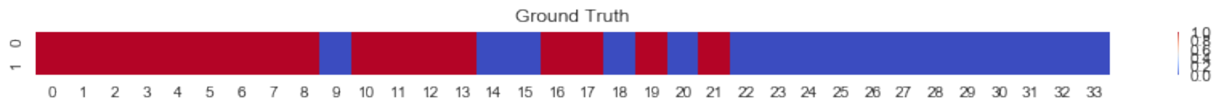
In [23]:
```
sc = SpectralClustering(2, affinity='precomputed', n_init=100, assign_l
sc.fit(matrix)
```

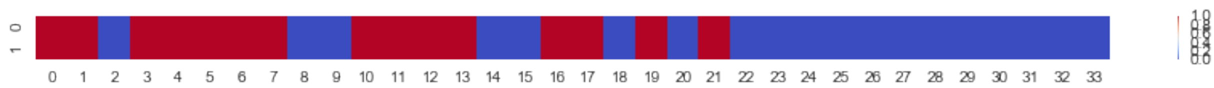
Out[23]:
```
SpectralClustering(affinity='precomputed', assign_labels='discretize
',
          coef0=1, degree=3, eigen_solver=None, eigen_tol=0.0, gamma
=1.0,
          kernel_params=None, n_clusters=2, n_init=100, n_jobs=1,
          n_neighbors=10, random_state=None)
```

In [24]:

```
print 'Ground Truth'
ground_truth = [0 if x in data['blue_list'] else 1 for x in range(1,35
print ground_truth
plt.figure(figsize=(15,0.5))
sns.heatmap([ground_truth,ground_truth],cmap='coolwarm').set_title('Gr


print('Spectral Clustering')
print(sc.labels_)
plt.figure(figsize=(15,0.5))
sns.heatmap([sc.labels_,sc.labels_],cmap='coolwarm')

# Calculate some clustering metrics
print(metrics.adjusted_rand_score(ground_truth, sc.labels_))
print(metrics.adjusted_mutual_info_score(ground_truth, sc.labels_))
```

```
Ground Truth
[1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0
, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
Spectral Clustering
[1 1 0 1 1 1 1 1 0 0 1 1 1 1 0 0 1 1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0
]
0.771725032425
0.722546051351
```

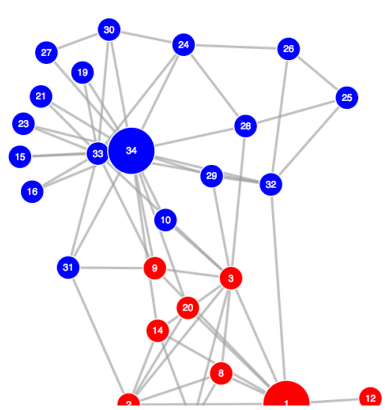




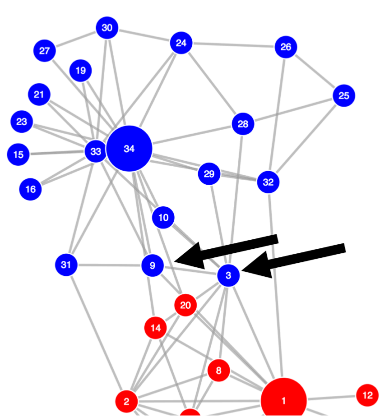
In [25]:

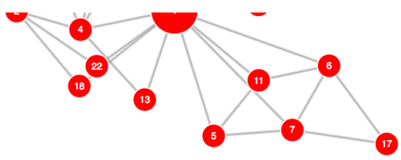
```
for a in data['graph']['nodes']:
    a['group'] = 'red' if sc.labels_[a['id']-1] == 0 else 'blue'

import json
with open('cluster_karate_B.json', 'w') as outfile:
    json.dump(data['graph'], outfile)
```

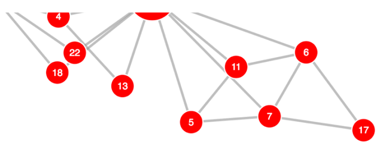
Ground Truth



Fine Tuned







In [ ]: