Chapter 8

ROTATION

This lecture will help you understand:

- Circular Motion
- Rotational Inertia
- Torque
- Center of Mass and Center of Gravity
- Centripetal Force
- Centrifugal Force
- Angular Momentum
- Conservation of Angular Momentum

Circular Motion

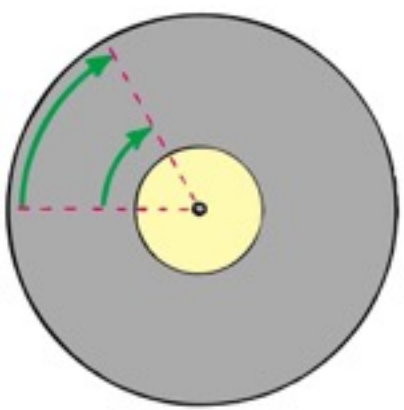
- When an object turns about an internal axis, it is undergoing circular motion or rotation.

- Circular Motion is characterized by two kinds of speeds:
  - tangential (or linear) speed.
  - rotational (or circular) speed.

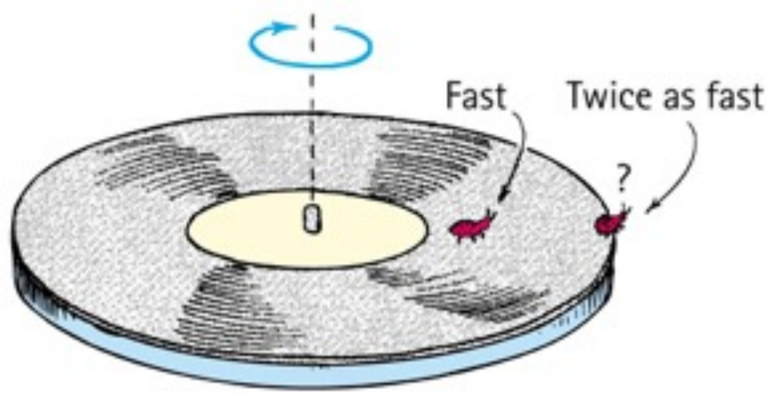
Circular Motion—Tangential Speed

The distance traveled by a point on the rotating object divided by the time taken to travel that distance is called its *tangential* speed (symbol *v*).

- Points closer to the circumference have a higher tangential speed that points closer to the center.



(a)



(b)

Circular Motion – Rotational Speed

- Rotational (angular) speed is the *number of rotations or revolutions per unit of time* (symbol ω).
- All parts of a rigid merry-go-round or turntable turn about the axis of rotation in the same amount of time.
- So, all parts have the same rotational speed.

$$\begin{aligned} &\text{Tangential speed} \\ &\quad = \text{Radial Distance} \times \text{Rotational Speed} \\ &\nu \;= r\omega \end{aligned}$$

A ladybug sits halfway between the rotational axis and the outer edge of the turntable . When the turntable has a rotational speed of 20 RPM and the bug has a tangential speed of 2 cm/s, what will be the rotational and tangential speeds of her friend who sits at the outer edge?

A. 1 cm/s
B. 2 cm/s
C. 4 cm/s
D. 8 cm/s

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D. 8 cm/s

Explanation:
Tangential speed
= $r\omega$
Rotational speed of both bugs is the same, so if radial distance doubles, tangential speed also doubles.

So, tangential speed is 2 cm/s $\times$ 2 = 4 cm/s.

Rotational Inertia

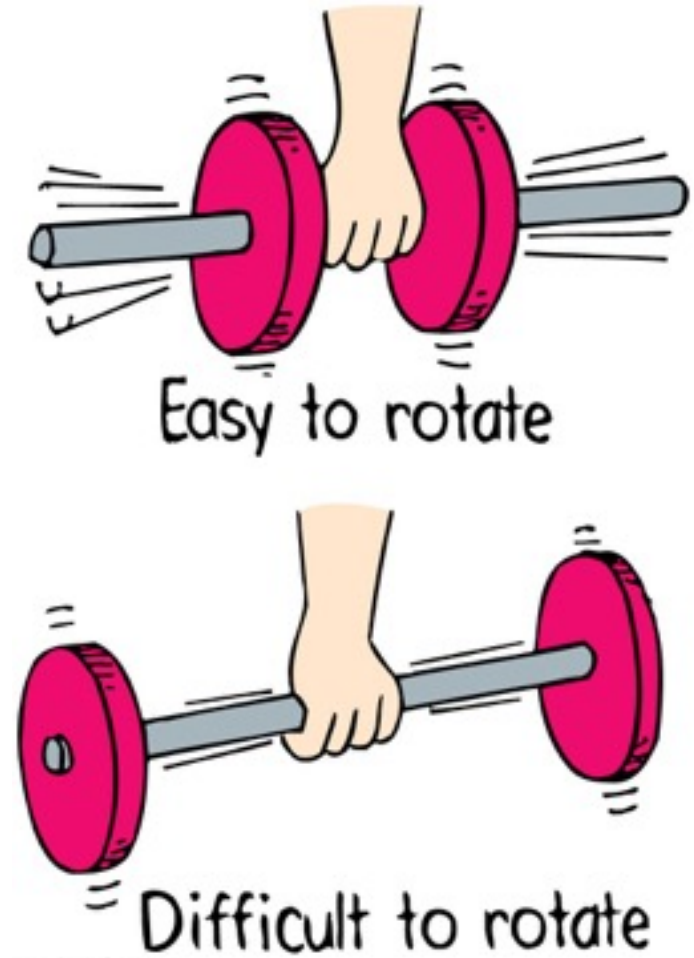
- An object rotating about an axis tends to remain rotating about the same axis at the same rotational speed unless interfered with by some external influence.

- The property of an object to resist changes in its rotational state of motion is called **rotational inertia (symbol *I*).**

Rotational Inertia

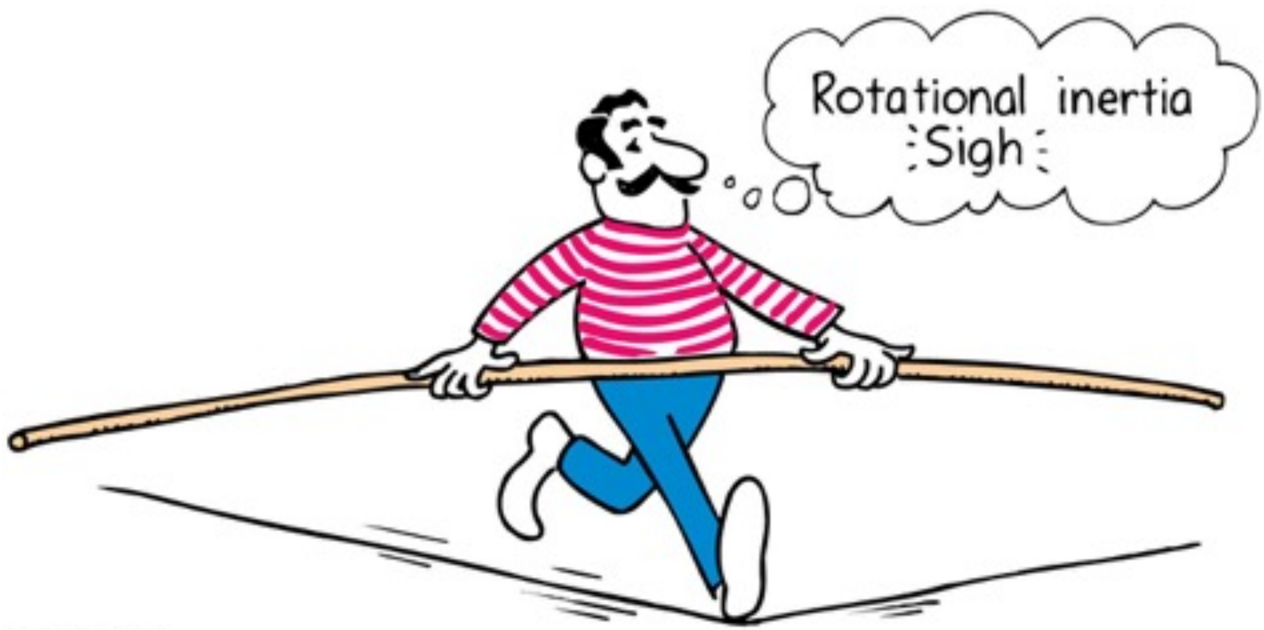
Depends upon

- mass of object.
- distribution of mass around axis of rotation.
  - The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia.



Rotational Inertia

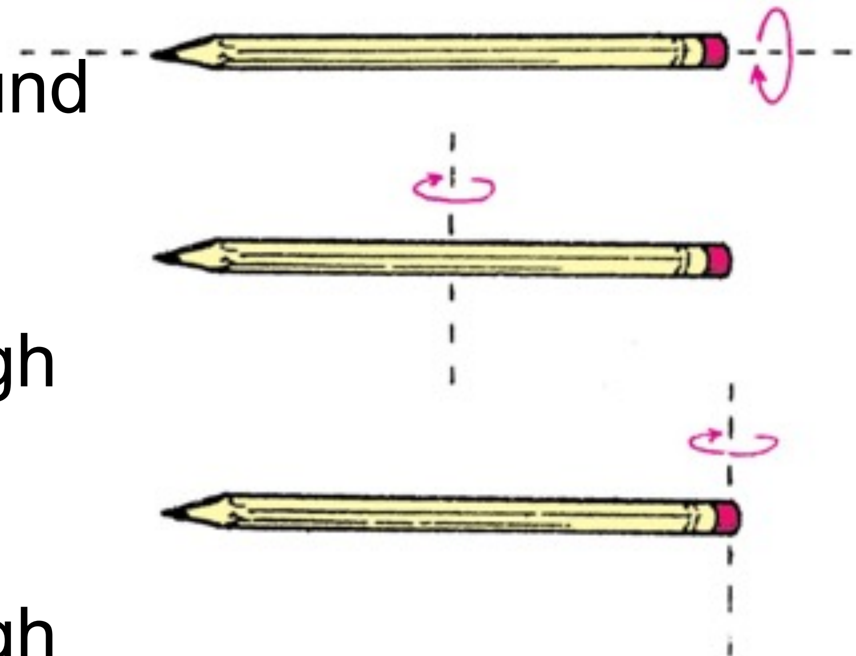
- The greater the rotational inertia, the harder it is to change its rotational state.
  - A tightrope walker carries a long pole that has a high rotational inertia, so it does not easily rotate.
  - Keeps the tightrope walker stable.



Rotational Inertia

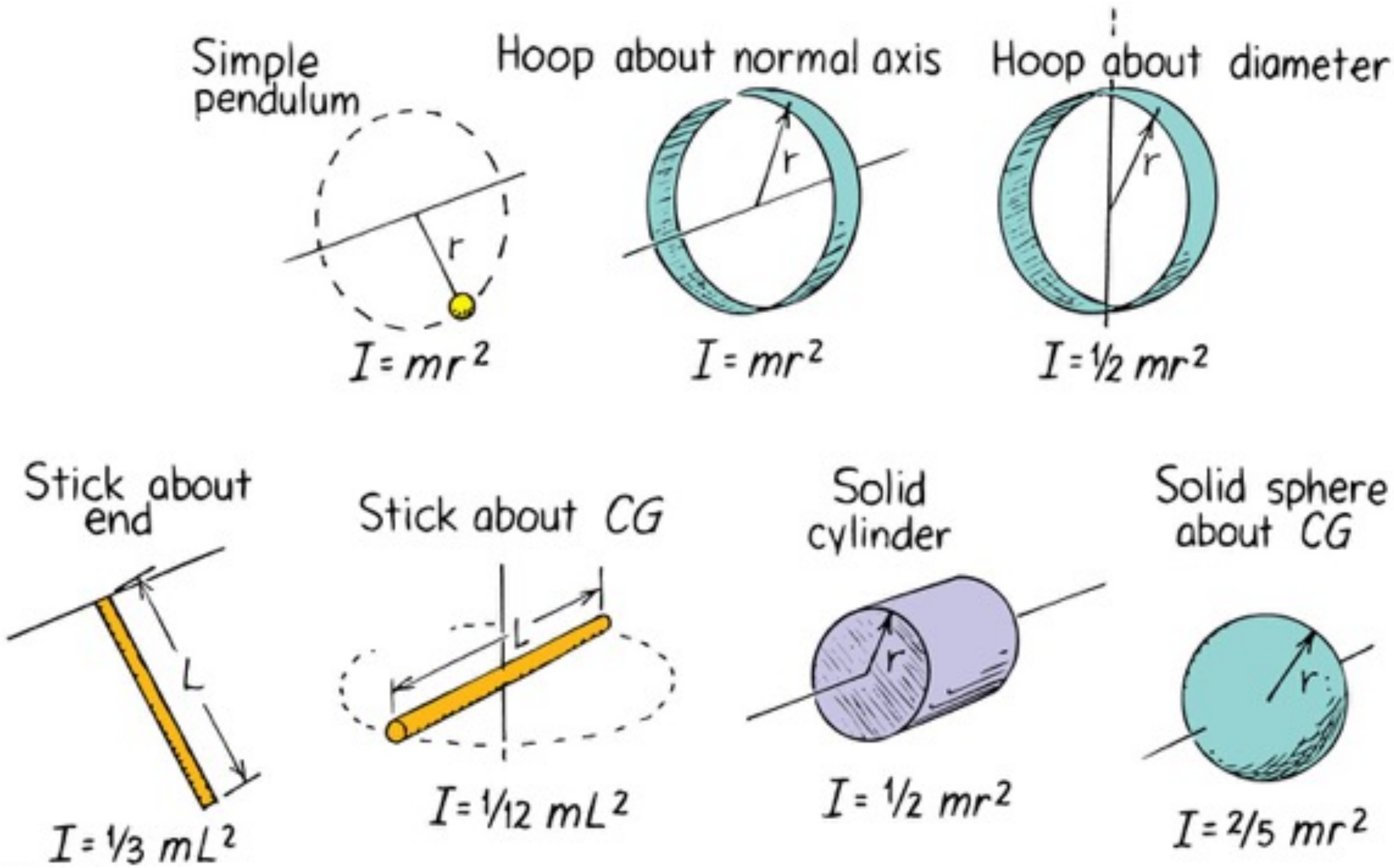
Depends upon the axis around which it rotates

- Easier to rotate pencil around an axis passing through it.
- Harder to rotate it around vertical axis passing through center.
- Hardest to rotate it around vertical axis passing through the end.



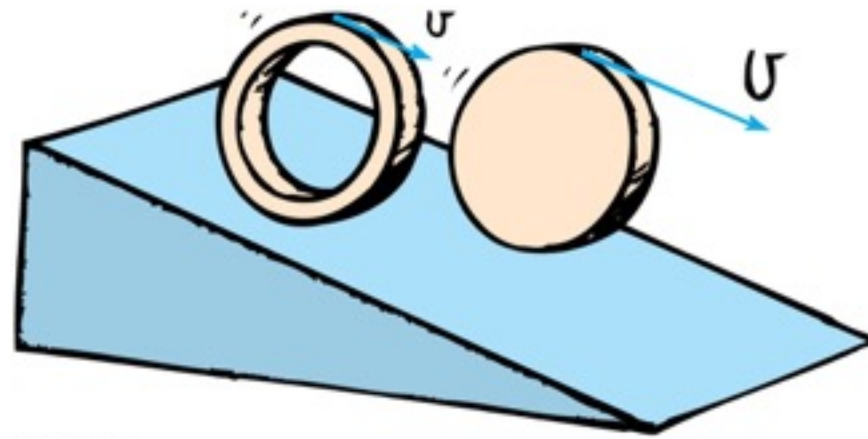
Rotational Inertia

The rotational inertia depends upon the shape of the object and its rotational axis.



A hoop and a disk are released from the top of an incline at the same time. Which one will reach the bottom first?

A. Hoop
B. Disk
C. Both together
D. Not enough information



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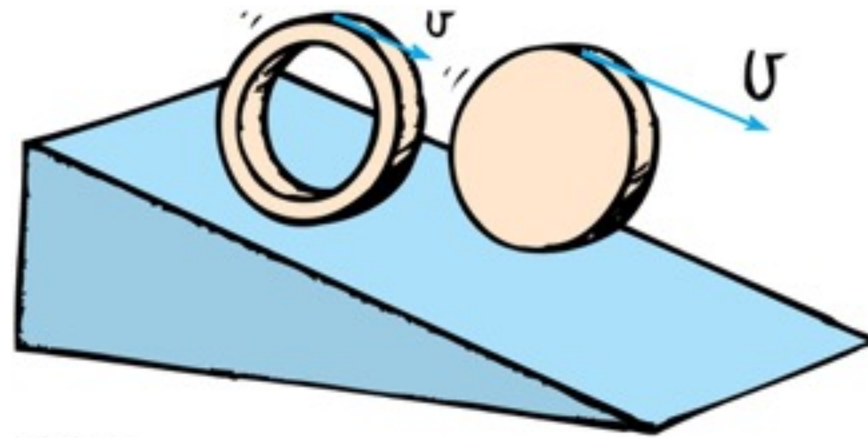


Explanation:
Hoop has larger rotational inertia, so it will be slower in gaining speed.

The mass of the hoop is more toward the circumference, compared to the disk in which it is uniformly spread out.

So, the rotational inertia of the hoop is larger than the disk

So, the hoop will be harder to rotate, and will reach the bottom later.

Torque

- The tendency of a force to cause rotation is called **torque.**

- Torque depends upon three factors:
  – Magnitude of the force
  – The direction in which it acts
  – The point at which it is applied on the object

Torque

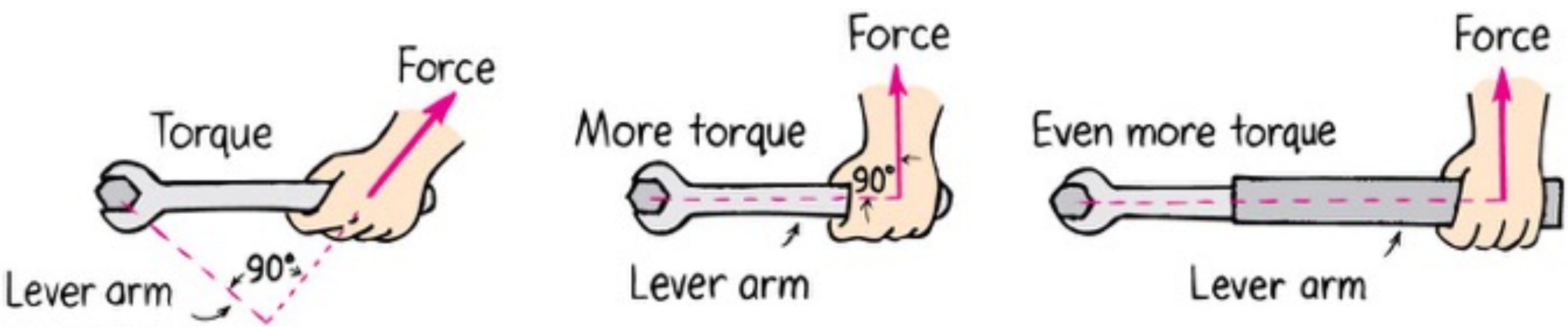
- The equation for Torque is

$$\text{Torque} = \text{lever arm} \times \text{force}$$

- The lever arm depends upon
  - where the force is applied.
  - the direction in which it acts.

Torque—Example

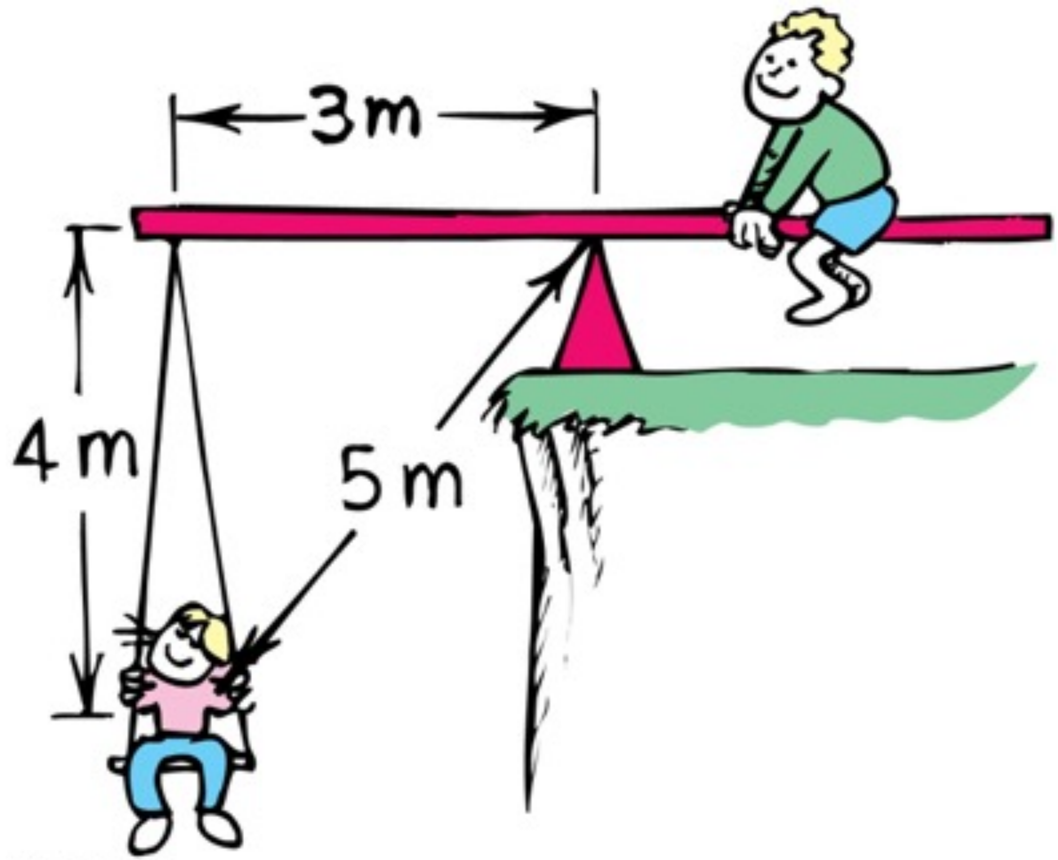
- 1st picture: Lever arm is *less than* length of handle because of direction of force.
- 2nd picture: Lever arm is equal to length of handle.
- 3rd picture: Lever arm is longer than length of handle.



Girl’s weight = 250N, boy’s weight = 500N. The boy is 1.5 m from the fulcrum.

Suppose the girl on the left suddenly is handed a bag of apples weighing 50 N. Where should she sit order to balance, assuming the boy does not move?

A. 1 m from pivot
B. 1.5 m from pivot
C. 2 m from pivot
D. 2.5 m from pivot



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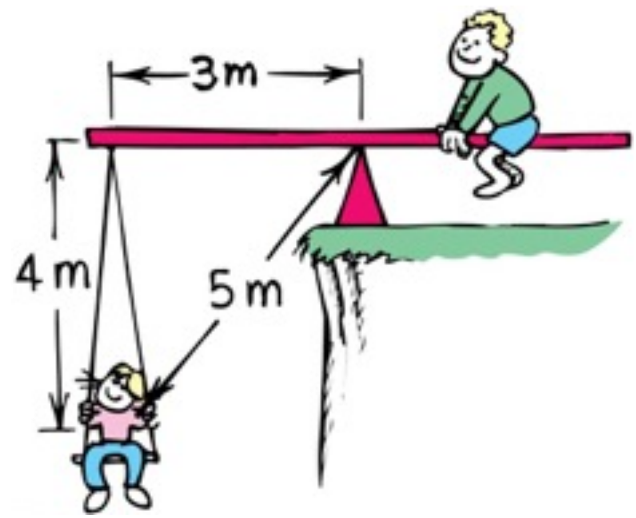


Explanation:
She should exert same torque as before.
Torque = lever arm × force
= 3 m × 250 N   = 1.5m x 500N
= 750 Nm
Torque = new lever arm × force
750 Nm = new lever arm × 300N
⇒ New lever arm = 750 Nm / 300 N = 2.5 m

Center of Mass and Center of Gravity

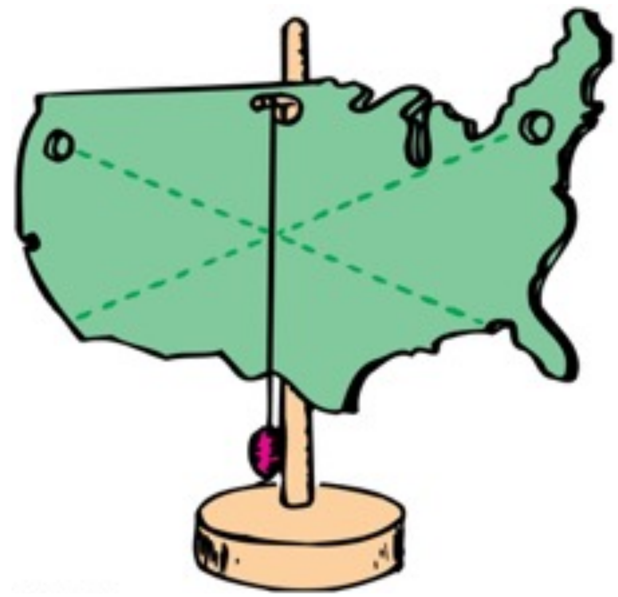
- **Center of mass** is the average position of all the mass that makes up the object.
- **Center of gravity (CG)** is the average position of weight distribution.
  - Since weight and mass are proportional, center of gravity and center of mass usually refer to the same point of an object.

Center of Mass and Center of Gravity

To determine the center of gravity,

- suspend the object from a point and draw a vertical line from suspension point.
- repeat after suspending from another point.

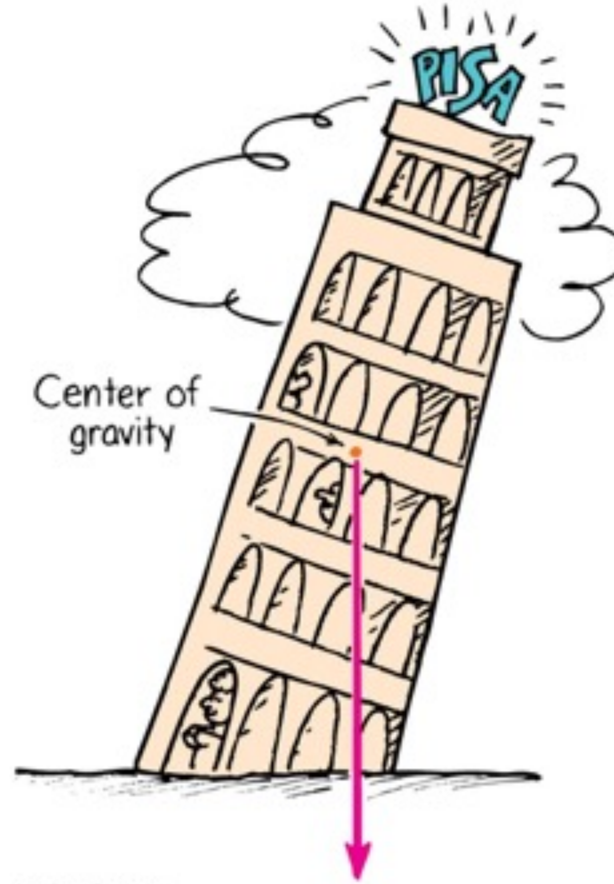
- The center of gravity lies where the two lines intersect.



Center of Gravity—Stability

The location of the center of gravity is important for stability.

- If we draw a line straight down from the center of gravity and it falls inside the base of the object, it is in stable **equilibrium;** it will balance.
- If it falls outside the base, it is unstable.



Centripetal Force

- Any force directed toward a fixed center is called a **centripetal force.**
- *Centripetal* means “center-seeking” or “toward the center.”

Example: To whirl a tin can at the end of a string, you pull the string toward the center and exert a centripetal force to keep the can moving in a circle.

Centripetal Force

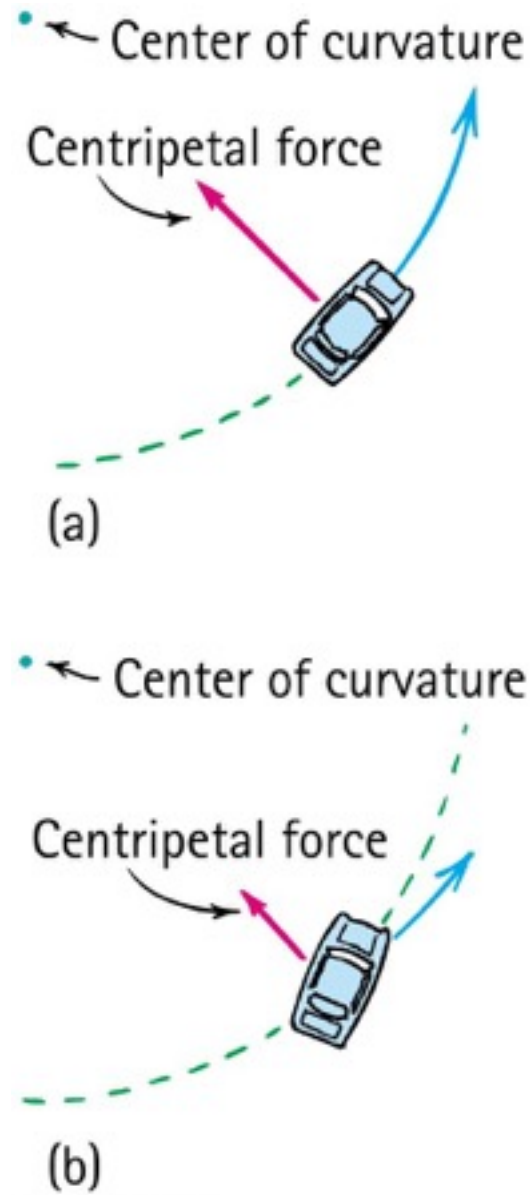
- Depends upon
  - mass of object.
  - tangential speed of the object.
  - radius of the circle.

- In equation form:

$$\text{Centripetal force} = \frac{\text{mass} \times \text{tangential speed}^2}{\text{radius}}$$

Centripetal Force—Example

- When a car rounds a curve, the centripetal force prevents it from skidding off the road.
- If the road is wet, or if the car is going too fast, the centripetal force is insufficient to prevent skidding off the road.



Suppose you double the speed at which you round a bend in the curve, by what factor must the centripetal force change to prevent you from skidding?

A. Double
B. Four times
C. Half
D. One-quarter

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B. **Four times**
C. Half
D. One-quarter

Explanation:

$$\text{Centripetal force} = \frac{\text{mass} \times \text{tangential speed}^2}{\text{radius}}$$

Because the term for "tangential speed" is squared, if you *double* the tangential speed, the centripetal force will be *double* squared, which is **four times**.

Suppose you take a sharper turn than before and *halve* the radius, by what factor will the centripetal force need to change to prevent skidding?

A. Double
B. Four times
C. Half
D. One-quarter

Suppose you take a sharper turn than before and *halve* the radius; by what factor will the centripetal force need to change to prevent skidding?

A. **Double**
B. Four times
C. Half
D. One-quarter

Explanation:

$$\text{Centripetal force} = \frac{\text{mass} \times \text{tangential speed}^2}{\text{radius}}$$

Because the term for “radius” is in the denominator, if you *halve* the radius, the centripetal force will double.

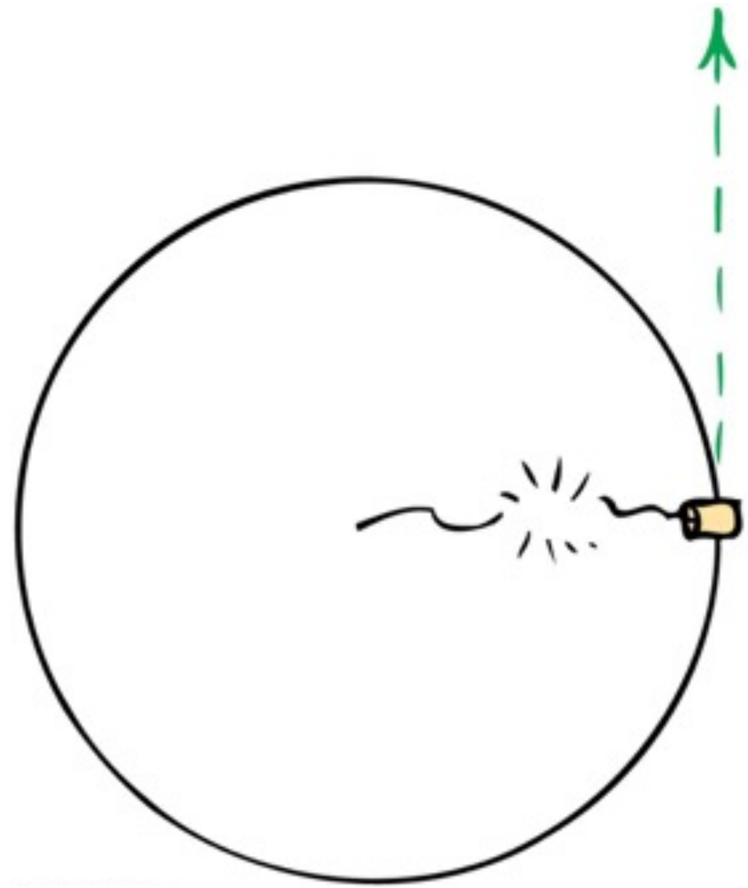
Centrifugal Force

- Although centripetal force is center directed, an occupant inside a rotating system seems to experience an outward force. This apparent outward force is called **centrifugal force.**

- *Centrifugal* means “center-fleeing” or “away from the center.”

Centrifugal Force – *A Common Misconception*

- It is a *common misconception* that a *centrifugal force pulls **outward*** on an object.
- Example:
  - If the string breaks, the object *doesn't move radially outward*.
  - It continues along its tangent straight-line path—because *no* force acts on it. (Newton's first law)



Angular Momentum

- The "inertia of rotation" of rotating objects is called **angular momentum.**
  - This is analogous to "inertia of motion", which was momentum.

- Angular momentum
  = rotational inertia × angular velocity

  - This is analogous to
    Linear momentum = mass × velocity

Angular Momentum

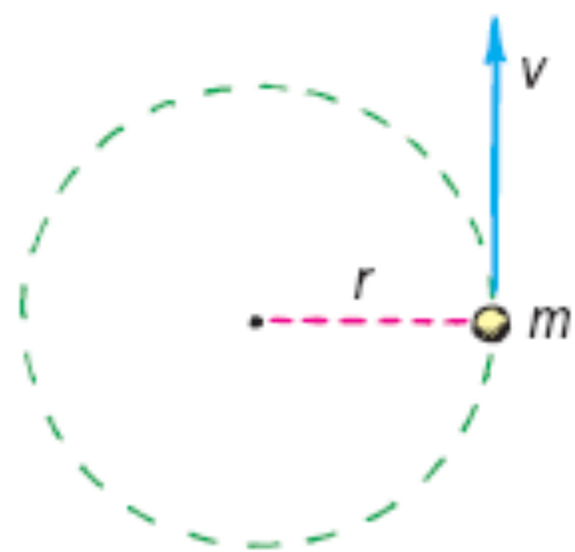
- For an object that is small compared with the radial distance to its axis, magnitude of

Angular momentum = mass tangential speed × radius

  – This is analogous to magnitude of

Linear momentum = mass × speed

- Examples:
  – Whirling ball at the end of a long string
  – Planet going around the Sun



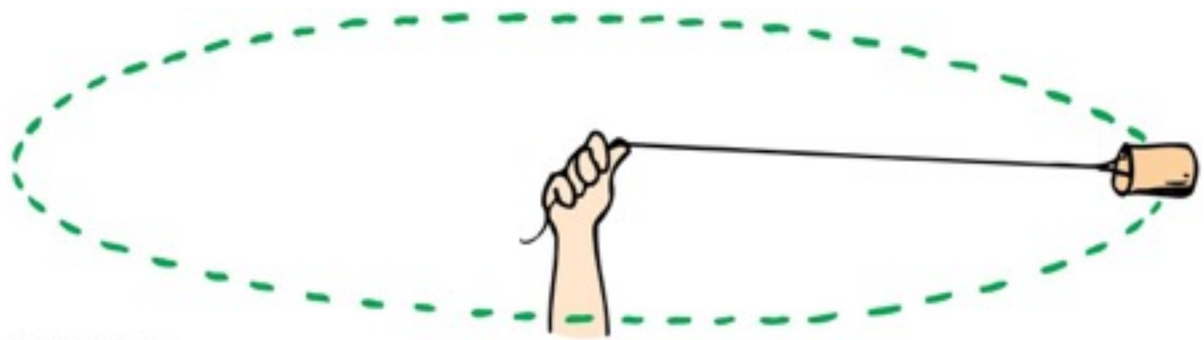
Angular Momentum

- An external net torque is required to change the angular momentum of an object.

- Rotational version of Newton's first law:
  - **An object or system of objects will maintain its angular momentum unless acted upon by an external net torque.**

Suppose you are swirling a can around and suddenly decide to pull the rope in *half*way; by what factor would the speed of the can change?

A. Double
B. Four times
C. Half
D. One-quarter



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A. **Double**
B. Four times
C. Half
D. One-quarter

Explanation:

Angular momentum
= mass tangential speed × radius

Angular Momentum is proportional to radius of the turn.

No external torque acts with inward pull, so angular momentum is conserved. Half radius means speed **doubles**.

Conservation of Angular Momentum

The **law of conservation of angular momentum** states:

- If **no external net torque** acts on a rotating system, the **angular momentum of that system remains constant**.

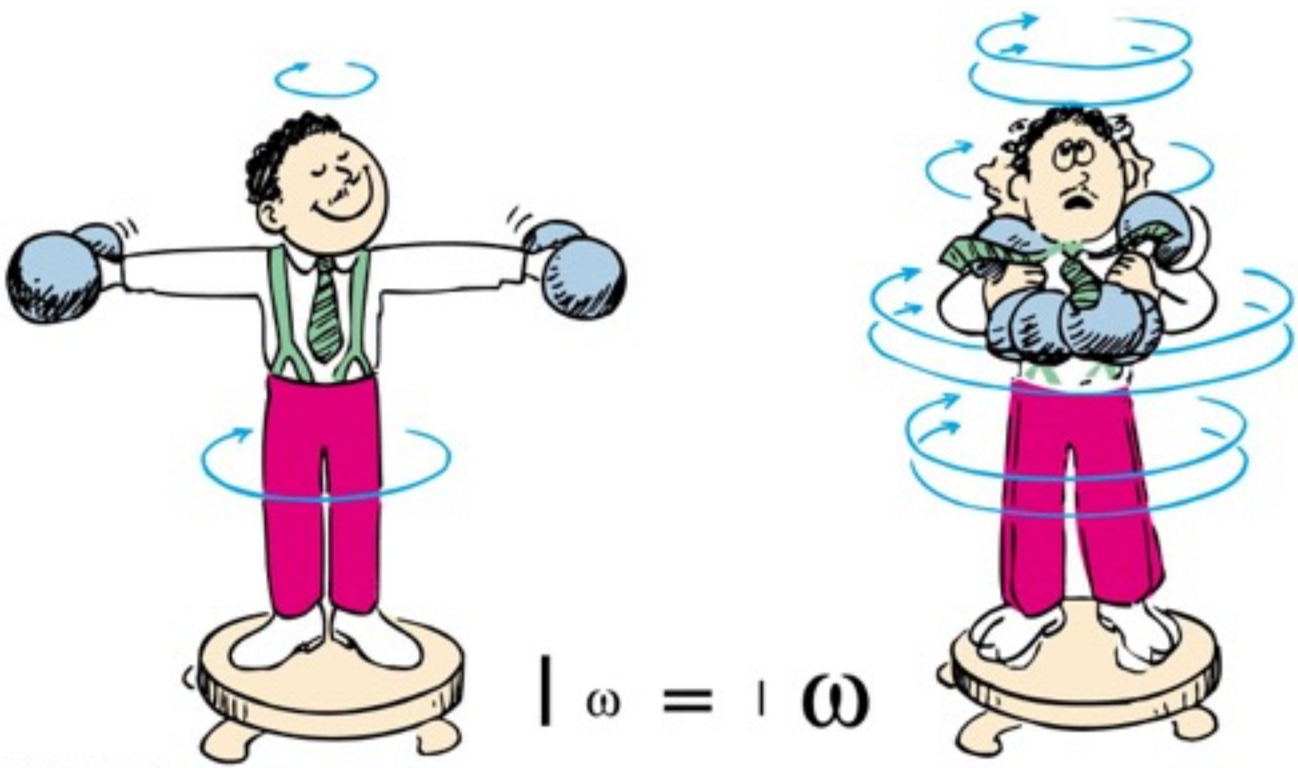
Analogous to the law of conservation of linear momentum:

- If **no external force** acts on a system, the total **linear momentum** of that system remains constant.

Conservation of Angular Momentum

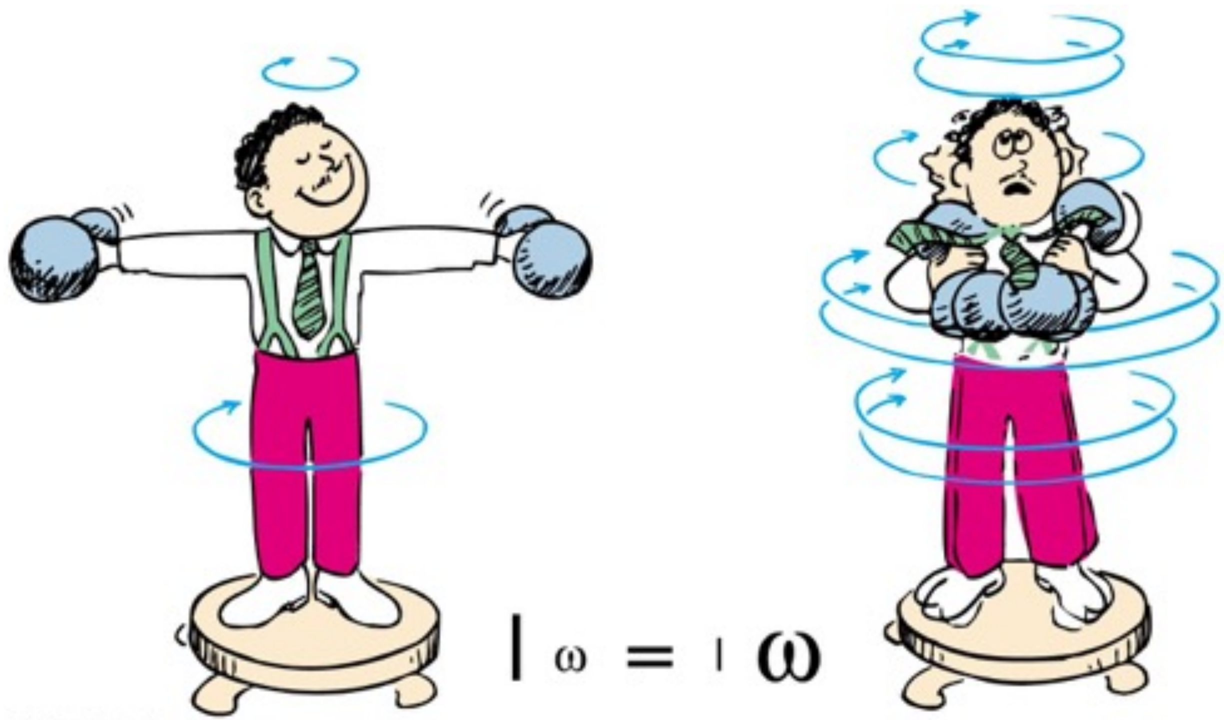
Example:

- When the man pulls the weights inward, his rotational speed increases!



Suppose by pulling the weights inward, the rotational inertia of the man reduces to half its value. By what factor would his angular velocity change?

A. Double
B. Three times
C. Half
D. One-quarter



Suppose by pulling the weights in, if the rotational inertia of the man decreases to half of his initial rotational inertia, by what factor would his angular velocity change?

A. **Double**
B. Three times
C. Half
D. One-quarter

Explanation:

Angular momentum
= rotational inertia × angular velocity

Angular momentum is proportional to "rotational inertia".

If you *halve* the rotational inertia, to keep the angular momentum constant, the angular velocity would **double**.

Chapter 8 Problems

4. Mary Beth uses a torque feeler that consists of a meter-stick held at the 0-cm end with a weight dangling from various positions along the stick (see Figure 8.17). When the stick is held horizontally, what torque is produced when a 1-kg mass hangs from the 50cm mark?

How much more torque is exerted when it is hung from the 75-cm mark?

τ = 10Nx0.75m = **7.5 Nm**

τ he 100-cm mark?

τ = 10Nx1m = **10 Nm**

Chapter 8 Problems

4. Mary Beth uses a torque feeler that consists of a meter-stick held at the 0-cm end with a weight dangling from various positions along the stick (see Figure 8.17). When the stick is held horizontally, what torque is produced when a 1-kg mass hangs from the 50cm mark?

$\tau = F\,d = mgd$

$\tau = 1\text{kg} \times 10\text{m/s}^2 \times 0.5\text{m}$

$\boldsymbol{\tau = 5\ \text{Nm}}$

How much more torque is exerted when it is hung from the 75-cm mark?

Chapter 8 Problems

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τ = F d = mgd

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τ **= 5 Nm**

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Chapter 8 Problems

8. If a trapeze artist rotates once each second while sailing through the air and contracts to reduce her rotational inertia to one-third of what it was, how many rotations
per second will result?

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   Since no external forces are applied then conservation of angular momentum states that:

   (Rotational Inertia, I) x (angular velocity, ω) must not change

   $I_{before}\ \omega_{before} = I_{after} \times \omega_{after}$ or $I_{before}\ \omega_{before} = (1/3)I_{before}$

   So $\omega_{after} = I_{before}\ \omega_{before} / (1/3)I_{before}$

   $$\boldsymbol{\omega_{after} = 3\ \omega_{before} = 3 \times 1 \text{ rps} = 3 \text{ rps}}$$

Homework

- Read Chapter 8 in Detail.
- Do Ranking 3, 4
- Do Exercises 1, 33, 36, 39
- Do problems 2, 3.

(You must show your calculations !!!)

- **Homework due: June 18**